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What is Bounded Rationality?

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by REINHARD SELTEN

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Abstract

The paper deals with bounded rationality understood in the tradition of H.A. Simon. Fundamental problems and theoretical issues are discussed. Special emphasis is put on aspiration adaptation theory. Further remarks concern basic models of decision behavior (like learning and expectation formation), reasoning, and the connection between bounded rationality and motivation.

Keywords

Bounded rationality, learning, aspiration adaptation theory

JEL Classification Codes

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Address

Laboratorium für experimentelle Wirtschaftsforschung
Uni Bonn
Adenauerallee 24-42
53113 Bonn, Germany
Phone 49-228-73-9192
Fax 49-228-73-9193

1. Introduction

Modern mainstream economic theory is largely based on an unrealistic picture of human decision making. Economic agents are portrayed as fully rational Bayesian maximizers of subjective utility. This view of economics is not based on empirical evidence, but rather on the simultaneous axiomization of utility and subjective probability. In the fundamental book of Savage the axioms are consistency requirements on actions with actions defined as mappings from states of the world to consequences (Savage 1954). One can only admire the imposing structure built by Savage. It has a strong intellectual appeal as a concept of ideal rationality. However, it is wrong to assume that human beings conform to this ideal.

1.1 Origins

At about the same time when Savage published his book, H.A. Simon created the beginnings of a theory of bounded rationality (Simon 1957). He described decision making as a search process guided by aspiration levels. An aspiration level is a value of a goal variable which must be reached or surpassed by a satisfactory decision alternative. In the context of the theory of the firm one may think of goal variables like profit and market share.

Decision alternatives are not given but found one after the other in a search process. In the simplest case the search process goes on until a satisfactory alternative is found which reaches or surpasses the aspiration levels on the goal variables and then this alternative is taken. Simon coined the word Asatisficing@ for this process.

Often satisficing is seen as the essence of Simon=s approach. However, there is more to it than just satisficing. Aspiration levels are not fixed once and for all, but dynamically adjusted to the situation. They are raised, if it is easy to find satisfactory alternatives and lowered if satisfactory alternatives are hard to come by. This adaptation of aspiration levels is a central idea in Simon=s early writings on bounded rationality.

Three features characterize Simon=s original view of bounded rationality: Search for alternatives, satisficing, and aspiration adaptation.

1.2 Aim of this essay

It is difficult to gain an overview of the literature on bounded rationality accumulated since Simon=s seminal work. No attempts in this direction will be made here. Instead of this, only a few selected topics will be discussed with the aim of conveying insights into the essential features of bounded rationality.

The author looks at the subject matter from the point of view of economic theory. He is convinced of the necessity of reconstructing microeconomics on the basis of a more realistic picture of economic decision making. Moreover he thinks that there are strong reasons for modelling boundedly rational economic behavior as non-optimizing. The material presented here reflects this conviction. More about the non-optimizing character of boundedly rational decision making will be said in the remaining sections of the introduction.

A comprehensive coherent theory of bounded rationality is not available. This is a task for the future. At the moment we must be content with models of limited scope.

1.3 Bounds of rationality

Full rationality requires unlimited cognitive capabilities. Fully rational man is a mythical hero who knows the solutions of all mathematical problems and can immediately perform all computations, regardless of how difficult they are. Human beings are very different. Their cognitive capabilities are quite limited. For this reason alone the decision behavior of human beings cannot conform to the ideal of full rationality.

It could be the case that in spite of obvious cognitive limitations the behavior of human beings is approximately correctly described by the theory of full rationality. Confidence in this conjecture of approximate validity explains the tenacity with which many economists stick to the assumption of Bayesian maximization of subjectively expected utility. However, there is overwhelming experimental evidence for substantial deviations from Bayesian rationality (Kahneman, D. P. Slovic and A. Tversky, 1982). People do not obey Bayes' rule, their probability judgements fail to satisfy basic requirements like monotonicity with respect to set inclusion, and they do not have consistent preferences, even in situations involving no risk and uncertainty. A more detailed discussion of these matters will not be given here but can be found elsewhere (e.g. Selten 1991).

The cognitive bounds of rationality are not the only ones. A decision maker may think that a choice is the only rational one, e.g. to stop smoking, but nevertheless not take it. Conclusions reached by rational deliberations may be overridden by strong emotional impulses. The lack of complete control over behavior is not due to motivational bounds of behavior rather than to cognitive ones.

1.4 Concept

In this paper the use of the term bounded rationality follows the tradition of H. A. Simon. It refers to rational principles underlying non-optimizing adaptive behavior of real people. Bounded rationality cannot be precisely defined. It is a problem which needs to be explored. However, to some extent it is possible to say what it is not.

Bounded rationality is not irrationality. A sharp distinction should be made here. The theory of bounded rationality does not try to explain trust in lucky numbers or abnormal behavior of mentally ill people. In such cases one may speak of irrationality. However, behavior should not be called irrational simply because it fails to conform to norms of full rationality. A decision maker who is guided by aspiration adaptation rather than utility maximization may be perfectly rational in the sense of everyday language use.

Sometimes the word bounded rationality is used in connection with theories about optimization under some cognitive bounds. An example for this is the game theoretic analysis of supergames under constraints on the operating memory (Aumann and Sorin, 1989). The task the players have to solve is much more complicated with these constraints than without them. The paper by Aumann and Sorin is a remarkable piece of work but it is not a contribution to the theory of bounded rationality. The same must be said about the recent book on Abounded rationality macroeconomics@ (Sargent 1993). There, the assumption of rational expectations is replaced by least square learning but otherwise an optimization approach is taken without any regards to cognitive bounds of rationality. Here, too, we see a highly interesting theoretical exercise which, however, is far from adequate as a theory of boundedly rational behavior.

Subjective expected utility maximization modified by some isolated cognitive constraints does not lead to a realistic description of boundedly rational decision making in a complex environment. Moreover, there are reasons to believe that an optimization approach fails to be feasible in many situations in which not only an optimal solution must be found but also a method of how to find it. More will be said about this in the next section.

Boundedly rational decision making necessarily involves non-optimizing procedures. This is a central feature of the concept of bounded rationality proposed. Other features will become clear in later parts of this paper.

Much of human behavior is automatized in the sense that it is not connected to any conscious deliberation. In the process of walking one does not decide after each step which leg to move next and by how much. Such automatized routines can be interrupted and modified by decisions but while they are executed they do not require any decision making. They may be genetically preprogrammed like involuntary body activities or they may be the result of learning. Somebody who begins to learn driving a car has to pay conscious attention to much detail which later becomes automatized.

One might want to distinguish between bounded rationality and automatized routine. However, it is difficult to do this. Conscious attention is not a good criterion. Even thinking is based on automatized routine. We may decide what to think about but not what to think. The results of thinking become conscious, but most of the procedure of thinking remains unconscious and not even accessible to introspection. Obviously the structure of these hidden processes is important for a theory of bounded rationality.

Reinforcement learning models have a long tradition in psychology (Bush and Mosteller 1955) and have recently become popular in research on experimental games (Roth and Erev 1995, Erev and Roth 1998). These models describe automatized routine behavior. Reinforcement learning occurs in men as well as animals of relatively low complexity and one may therefore hesitate to call it even boundedly rational. However, a theory of bounded rationality cannot avoid this basic mode of behavior (see section 3.3)

The concept of bounded rationality has its roots in H. A. Simon's attempt to construct a more realistic theory of human economic decision making. Such a theory cannot cover the whole area of cognitive psychology. The emphasis must be on decision making. Learning in decision situations and reasoning supporting decisions belong to the subject matter, but visual perception and recognition, a marvelously powerful and complex cognitive process, seems to be far from it.

Undoubledly biological and cultural evolution as well as the accquistion of motivational dispositions in ontogenetic development are important influences on structure and content of decision behavior. However, boundedly rational decision making happens on much smaller time scale. For the purpose of examining decision processes the results of biological and cultural evolution and ontogenetic development can be taken as given. The emphasis on decision making within the bounds of human rationality is maybe more important for the concept of bounded rationality than the boundaries of its applicability.

1.5. Impossibility of unfamiliar optimization when decision time is scarce

Imagine a decision maker, who has to solve an optimization problem in order to maximize his utility over a set of decision alternatives. Assume that decision time is scarce in the sense that there is a deadline for choosing one of the alternatives. The decision maker has to do his best within the available time.

It is useful to distinguish between familiar and unfamiliar problems of this kind. A problem is familiar if the decision maker knows the optimal way to attack it. This means that he knows what to do by prior training or mathematical investigation or that the problem is so simple that a suitable method immediately suggests itself.

In the case of an unfamiliar problem the decision maker must devise a method for finding the alternative to be chosen before it can be applied. This leads to two levels of decision making activities which both take time.

<u>level 1:</u> Finding the alternative to be chosen

level 2: Finding a method for level 1

What is the optimal approach to the problem of level 2 ? One can hardly imagine that this problem is familiar. Presumably a decision maker who does not immediately know what to do on level 1 will also not be familiar with the task of level 2. Therefore he has to spend some time finding an optimal method for solving the task of level 2. We arrive at level 3.

It is clear that in this way we obtain an infinite sequence of levels k = 2, 3, ... provided that finding an optimal method for level k continues to be unfamiliar for every k.

level k: Finding a method for level $k-1$

It is reasonable to assume that there is a positive minimum time which is required for the decision making activities at each level k. Obviously this has the consequence that an optimization approach is not feasible when decision time is scarce.

Admittedly the reasoning which has led to the impossibility conclusion is not based on a precise mathematical framework and therefore cannot claim the rigor of a formal proof. Nevertheless it strongly suggests that a truly optimizing approach to unfamiliar decision problems with constrained decision time is not feasible.

Trying to optimize in such situations is like trying to build a computer which must be used in order to determine its own design, an attempt which is doomed to fail. The activity of optimizing cannot optimize its own procedure.

The impossibility of unfamiliar optimization, when decision time is scarce, would not be of great importance if most optimization problems faced by real people were familiar to them in the strict sense explained above. It is clear that the opposite is true.

2. Aspiration adaptation theory

As has been argued in the preceding section, there are reasons to believe that unfamiliar optimization is impossible within the cognitive bounds of rationality, when decision time is scarce. This raises the following question: How can we model the non-optimizing behavior of boundedly rational economic agents? The author was involved in an early attempt to answer this question (Sauermann and Selten 1962). Only recently this aspiration adaptation theory has been made available in English (Selten 1998). This sign of a continued interest in the theory has encouraged the author to present a condensed exposition here. Experiences with a course on bounded rationality suggest that aspiration adaptation theory is a good starting point for conveying insights into the problem area and the modelling possibilities.

Aspiration adaptation theory cannot claim to be an empirically validated description of decision making behavior. Experimental evidence points to a need of extensions and modifications. However, the basic ideas have been fruitful in experimentation, e.g. in the development of descriptive theories of two-person bargaining (Tietz and Weber 1972, Tietz 1976) and may continue to be valuable in the future.

2.1 The aspiration scheme

It is one of the features of aspiration adaptation theory that it models decision making as a multigoal problem without aggregation of the goals into a complete preference order over all decision alternatives. The decision maker has a number of real valued goal variables. For each goal variable more is better. This is a convenient modelling convention. If, e.g., it is one of the goals to keep costs low, this can be modelled by negative costs as a goal variable. Consider two different vectors of values for the goal variables. If one of them has a higher or equal value for each goal variable then this vector is prefered to the other, but if this is not the case then there is no comparability. We refer to this feature as <u>goal incomparability</u>.

In aspiration adaptation theory an <u>aspiration level</u> is a vector of values for the goal variables. These values, the <u>partial aspiration levels</u> vary in discrete steps. The possible aspiration levels form a grid in the space of the goal variables. We refer to it as the <u>aspiration grid</u>. Aspiration adaptation takes the form of a sequence of <u>adjustment steps</u>. An adjustment step shifts the current aspiration level to a neighboring point on the aspiration grid by change of only one goal variable. An <u>upward</u> adjustment step is an increase and a <u>downward</u> adjustment step is a decrease of a goal variable.

Aspiration adaptation takes the form of a sequence of adjustment steps. Upward adjustment steps are governed by an <u>urgency order</u>, a ranking of the goal variables without ties. Downward adaptations decrease a <u>retreat variable</u>. The retreat variable is not necessarily the least urgent one. Urgency order and retreat variable may depend on the grid point. The aspiration grid together with the assignment of an urgency order and a retreat variable to every grid point forms an <u>aspiration scheme</u>.

In aspiration adaptation theory the aspiration scheme takes the place of the preference order in the usual decision theory. Goal incomparability is not removed. Urgency order and retreat variable do not express global preferences over decision alternatives but <u>local procedural preferences</u> over adjustment steps. There is an asymmetry between upward adjustment steps and downward adjustment steps. One needs an urgency order to select among different feasible upward adjustment steps, but one can always choose the least painful downward adjustment and continue the adaptation process from there.

Aspiration adaptation theory offers a coherent modelling approach to bounded rationality. It provides a first orientation in the problem area. Therefore a condensed exposition will be presented in this and the following sections with some minor extensions of previously published material.

As we have seen aspiration adaptation models two features which seem to be important ingredients of bounded rationality: goal incomparability and local procedural preferences.

2.2 Selection of one of many given alternatives

The exposition of aspiration adaptation theory given here restricts itself to the case of a borderless aspiration grid without maximal or minimal values for partial aspiration levels. This avoids tedious detail without loss of important aspects of the theory.

Imagine a decision maker who has to select one of finitely many alternatives with specified values for all goal variables. The vector of these values for an alternative is called its goal vector. The aspiration levels, satisfied by the goal vector of at least one alternative, are called feasible. The decision maker starts with a previous aspiration level taken over from the past. The selection is made by an adaptation process which generates a sequence of intermediate aspiration levels with the initial aspiration level as the first one and a new aspiration level as the last one. At the end a choice is made which satisfies the new aspiration level.

An upward adjustment step is feasible if it leads to a feasible aspiration level. The most urgent feasible upward adjustment step among all feasible ones is the one which raises the most urgent goal variable. The adaptation process is governed by three adaptation rules:

1) Downward rule: If an intermediate aspiration level is not feasible, the downward adjustment step is taken which lowers the partial aspiration level of the retreat variable.

2) Upward rule: If an intermediate aspiration level is feasible and an upward adjustment step is feasible, then the most urgent feasible upward adjustment step is taken.

3) End rule: If an intermediate aspiration level is feasible and no upward adjustment step is feasible, then this aspiration level is the new one.

The process may involve a first phase of downward adjustment steps until a feasible aspiration level is reached followed by a second phase of upward adjustments leading to the new aspiration level.

2.3 Search for alternatives with integrated decisions on decision resources

Imagine a decision maker engaged in a sequential search for decision alternatives. The search generates a sequence of alternatives with specified goal vectors. Some of the goal variables may be stocks of decision resources like decision cost resources out of which decision costs must be paid or decision time reserves, i.e. time saved compared to maximum decision time. These decision resource stocks diminuish with search.

The decision maker knows the first alternative, the default alternative, before she has the opportunity to start searching and she can end searching any time. Search may also be stopped exogenously by the end of decision time. The decision maker starts with an initial aspiration level. Consider the situation after k alternatives have been found, including the default alternative. For k = 1 the previous aspiration level is the initial one. An adaptation process running through intermediate aspiration levels, like the one in section 2.2, leads to a new aspiration level which becomes the next previous aspiration level.

Unlike in section 2.2 the term Afeasible@ will now be used in a dynamic sense: An aspiration level is feasible if it is satisfied by the goal vector of at least one alternative found already. An aspiration level which is not feasible at the moment is potentially feasible, if it is possible that it becomes feasible in further search and recognizably infeasible otherwise. In order to clarify this classification into three non-overlapping categories, it must be explained what is meant by Apossible@ in this context. It is assumed that everything is possible as far as goal variables other than decision resource stocks are concerned. However, decision resource stocks are diminuished by search. As soon as some of them become so low that they exclude further search at the current aspiration level, this aspiration level becomes recognizably unfeasible.

An upward adjustment step is permissible if it leads from a feasible aspiration level to another feasible one or to a potentially feasible one. The definition of the most urgent permissible upward adjustment step is analogous to that of the most urgent feasible upward adjustment step in the preceding section. We now define permissible aspiration levels by the following three conditions:

1) A feasible aspiration level is permissible.

2) A potentially feasible aspiration level is permissible if it is the previous aspiration level or if it can be reached by a permissible upward adjustment step.

3) Aspiration levels other than those permissible by 1) or 2) are not permissible.

In this section the permissible aspiration levels have the same role as the feasible ones in the preceding section. Aspiration adaptation is analogous with the only difference that everywhere in the three adaptation rules Afeasible@ has to be replaced by Apermissible@. The following continuation rule determines whether search is continued or not.

Continuation rule: If the new aspiration level is potentially feasible then search is continued; otherwise search ends with the new aspiration level as the final aspiration level and an alternative is chosen whose goal vector satifies this final aspiration level.

Since permissible aspiration levels are either potentially feasible or feasible the final aspiration level is feasible.

The definition of permissibility embodies a principle of cautious optimism. Only recognizable infeasibility is a reason to lower the previous aspiration level, not just lack of feasibility. As long as an aspiration level is potentially feasible one can hope to satisfiy it by further search. However, upward adjustment should not lead more than one step away from what is feasible now. Therefore upward adjustment steps from potentially feasible aspiration levels are not permissible. Thereby some caution is imposed on the optimism necessary for search.

The model of this section shows how aspiration adaptation theory simultaneously deals with the choice of a decision alternative and decisions on decision resources. We refer to this feature as integrated decisions on decision resources. This feature seems to be a necessary ingredient of rational behavior within the cognitive bounds of human beings.

2.4 Aspiration adaptation on the basis of qualitative expectations

Boundedly rational decision makers do not necessarily form quantitative expectations. Instead of this they may rely on qualitative expectations connected to decision alternatives. This means that the decision maker has expectations about the direction of change compared with the present state of affairs. A decision alternative may be expected to raise a goal variable, or to lower it, or to have a negligible influence on it. We use the symbol A+@, A!@, and A0" in order to represent these three possibilities formally. In this way aspiration adaptation theory describes the qualitative expectations connected to a decision alternative by a qualitative goal vector with +, !, or 0 as possible entries.

In a simple model called the Aroutine model@ a firm makes period by period decisions on a number of instrument variables like price, production, advertising etc.. The decision alternatives are finitely many possibilities of change like Aincrease price by 5% and advertising by 10%@. For each alternative the firm has a qualitative expectation in the form of a qualitative goal vector.

We refer to the goal vector realized in the last period as the realization. One of the alternatives is doing nothing i.e., leaving the instrument variables at their last period:s values. For this alternative a quantitative expectation is formed, namely the realization. The decision maker expects that nothing is changed if nothing is done. The realized aspiration level is the highest aspiration level (highest in each component) which is satisfied by the realization.

The decision maker constructs an expected feasible aspiration level for each alternative. The realized aspiration level is the expected feasible aspiration level for doing nothing. For other alternatives four expectation rules determine the expected feasible aspiration level by relating its partial aspiration level for a goal variable to the corresponding entry of the qualitative goal vector:

1) If the entry is A+@ then the partial aspiration level is one step above that of the realized aspiration level.

2) If the entry is A!@ and the value of the goal variable in the realization is above that in the realized aspiration level, then the partial aspiration level is that of the realized aspiration level.

3) If the entry is A!@ and the value of the goal variable in the realization is equal to that in the realized aspiration level, then the partial aspiration level is one step below that of the realized aspiration level.

4) If the entry is A0" then the partial aspiration level is that of the realized aspiration level.

Aspiration adaptation follows after the construction of the expected feasible aspiration levels. Starting with the previous aspiration level taken over from the last period the procedure for selecting one alternative out of many is applied for this purpose (see section 2.2) with the only difference that now the expected feasible aspiration levels take the place of the feasible ones. This yields a new aspiration level for the current period. An alternative with this expected feasible aspiration level is chosen. If there are several such alternatives the choice is narrowed by the successive application of two criteria:

1) Minimization of the number of entries A!@ in the qualitative goal vector of the alternative.

2) Maximization of the number of entries A+@ in the qualitative goal vector of the alternative.

In the preceding section upward adjustment steps from feasible aspiration levels to potentially feasible ones were modelled as permissible. The expected feasible aspiration levels are similar to the potentially feasible aspiration levels reached in this way. One can hope that the chosen alternative will result in a new realization which satisfies its expected feasible aspiration level but one cannot count on this. Choosing an alternative on this basis of qualitative expectations requires the optimism expressed by expectation rules 1 and 2. The partial aspiration levels of expected feasible aspiration levels are at most one step away from the realized ones. This adds an element of caution.

Cautious optimism seems to be a necessary feature of boundedly rational behavior where there is ignorance about the consequences of decisions. This includes costly search for new alternatives as well as situations in which alternatives have to be tried out on the basis of qualitative expectations.

Decision making largely based on only qualitative information seems to be an important mode of boundedly rational behavior. Aspiration adaptation theory offers a modelling approach for this.

2.5 Risk related goal variables

Everyday experience suggests that risky decisions are rarely based on explicit probability judgements. In some special contexts probability estimates are found on the basis of frequency information. Life insurance companies use mortuary tables for this purpose. However, their customers usually do not even think of the relevant probabilities. Thus a buyer of life insurance may decide that he wants to be insured for an amount which enables his family to survive for three years if this can be done for a premium not higher than 5% of his income. This involves a two-dimensional aspiration level with premium and amount as goal variables, but no probability estimates.

In some cases probability judgements are also formed on the basis of a priori considerations. However, this seems to be restricted to very special contexts like that of the gambling house. In practical business and daily life there is usually no opportunity for such a priori judgements.

The risk of a catastrophe like bankruptcy can be limited by paying attention to goal variables with the property that the risk is the smaller the greater the value of this variable is. We call such goal variables <u>risk related</u>. Thus a firm may use a liquidity ratio (liquid assets as a percentage of obligations) as a risk related goal variable.

An interesting example of a risk related goal variable is the safety margin applied in engineering computations. On static computations for buildings or bridges one requires that the structure withstands k times the maximal forces expected in use, where k is an integer like 2 or 3. The computations are based on a deterministic model. Nevertheless the risk of breakdown can be limited by a partial aspiration level on k.

Safety margins may be imposed by law. In this case they are the result of collective rather than individual aspiration adaptation.

2.6 Features of bounded rationality modelled by aspiration adaptation theory

The decision behavior modelled by aspiration adaptation theory has some features which seem to be of significance for the description of bounded rationality independently of modelling details. These features are listed below:

1. Goal incomparability
2. Local procedural preferences
3. Integrated decisions on decision resources
4. Decisions based on qualitative expectations
5. Cautious optimism in the search for alternatives and the use of qualitative expectations
6. Risk related goal variables

Aspiration adaptation theory models decision making in a multi-goal framework with goal incomparability and local procedural preferences. These properties are embodied in the aspiration scheme. Integrated decisions on decision resources are modelled by aspiration adaptation involving decision resource stocks as goal variables. Aspiration adaptation theory also describes the use of qualitative expectations on the directions of change as the basis of a cautiously optimistic construction of expected feasible goal vectors for alternatives and aspiration adaptation among them. Search for alternatives is modelled as cautiously optimistic, too. Risk related goal variables explain how risks can be limited without probability judgements.

2.7. Questions not answered by aspiration adaptation theory

Aspiration adaptation theory is a coherent approach to rational decision making. However, it has its weaknesses. Extensions and modifications are necessary. Moreover, the way in which aspiration adaptation is modelled could be brought into closer agreement with the behavioral theory of the firm (Cyert and March 1963, 1992). This mainly concerns aspiration adaptation as an organisational process, a problem area intentionally avoided here in order to concentrate attention on features of bounded rationality already found on the individual level.

It is possible that experimental research will lead to a fundamentally different theory of non-optimizing decision making. However, it seems to be more likely that aspiration adaptation in a multigoal framework will have to be a part of a comprehensive theory of boundedly rational decision making, even if the modelling details are different. Aspiration adaptation theory suffers from its neglect of aspects of boundedly rational behavior which often are indispensible for the explanation of experimental results. These aspects are relevant for a number of questions not answered by aspiration adaptation theory, at least not in its present form.

Decision makers do not always know what they want. In new situations goals must be formed. Where does the aspiration scheme come from? Often only a finite number of decision alternatives is considered, even if in principle infinitely many are available. How is this selection made? If quantitative or qualitative expectations about goal variables need to be formed, how is this done? Aspiration adaptation theory leaves processes of goal formation, construction of alternatives and expectation formation largely unmodelled.

3. Some basic modes of choice behavior

Human decision making seems to involve a multitude of basic modes of choice behavior. One can think of them as forming an Aadaptive toolbox@ with many special instruments used for different purposes, alone or in combination with others. Probably we do not yet know more than a small fraction of the content of this toolbox. Without any claim of completeness the following sections will loosely describe examples of what is known. These examples are basic modes of choice behavior used for the performance of tasks of judgement, expectation formation and learning.

3.1 Prominence in the decimal system

The theory of prominence in the decimal system is due to Albers and Albers (1983) Roughly speaking, it is a theory of the perception of degrees of roundness of numbers. Recently Wulf Albers has developed a new version of the theory which is more powerful but also more complex. In this essay not even the old version can be described in detail. Instead of this we sketch a process of selecting a spontaneous response, which makes it understandable why such responses tend to be rounder numbers than randomly expected.

Consider a person who has to guess the number of inhabitants of Islamabad. The first step is the perception of a broad range in which the answer must be, say between 0 and 20 million. Then attention is focused on the midpoint, 10 million, and the person ponders the question whether the number of inhabitants is higher or lower than 10 million. If the person feels that she cannot answer this question then the process stops here and her response is 10 million.

Suppose that this person decides that the number is lower: This narrows the range to the numbers between 0 and 10 million. Again attention is focused on the midpoint, 5 million, and the person asks herself whether the number is lower or higher. If she cannot decide 5 million is the spontaneous response. Suppose that she thinks that the number is greater. In this situation some people will focus on 7.5 million but others on 7 or 8 million, since they perceive these as rounder. Suppose that she focuses on 7 million, then she decides that the number is smaller, considers the new midpoint and ends up with the response 6 million, since she feels that she is unable to judge whether the number of inhabitants is smaller or greater.

In this way it becomes understandable that direct numerical responses tend to be round, the rounder the less is known about the subject matter. The question arises how the judgements required by this process are made. Maybe these judgements are based on a procedure like Atake the best@, in which one criterion after the other is checked, e.g. whether the person knows the name, whether the town is a capital etc. (Gigerenzer 1997). The first criterion which points in one direction decides the issue unless the criteria are exhausted and the process stops. Presumably different criteria are used at different midpoints.

A much better elaborated theory about a way of making rough estimates is the procedure QuickEst (Hertwig, Hoffrage, and Martiguon 1999). Here, the estimates are restricted to a predetermined scale of prominent numbers. Each number on the scale is connected with a criterion. Beginning with the smallest number, one criterion after the other is examined until one of them leads to a negative answer. Then the process stops with the associated number on the scale as the estimate.

3.2. Expectation formation

O. Becker and U. Leopold (1996) have developed an interesting experimentally based theory of expectation formation in an environment in which a subject predicts the next value of a univariate time series on the basis of past observations. In the experiments the time series was generated by a stochastic second order difference equation. The average forecasts of the subjects are well described by a surprisingly simple rule, which they call the "bounds and likelihood procedure". In order to explain this rule we need some definitions and notations.

Let x_t be the value of the time series at period t and let f_{t+1} be the forecast for period t + 1. The average variation b_t is the average of all absolute changes $|x_j - x_{j-1}|$ for j = 2, ..., t. The average variation is an upper bound for the absolute value of the predicted change. This bound is modified by the likelihood h_t of the event that x_t will be a turning point. h_t will be defined below. Let M_t be the number of local maxima of the time series observed up to period t and let m_t be the number of those local maxima among them which are smaller or equal to x_t. Similarily let N_t be the number of local minima up to period t and n_t the number of those among them which are greater or equal to x_t. The likelihood h_t is defined as follows:

$$h_t = (1 + m_t) / (2 + M_t) \text{ for } x_t > x_{t-1}$$

$$h_t = (1 + n_t) / (2 + N_t) \text{ for } x_t < x_{t-1}$$

The bounds and likelihood procedure specifies the following forecast:

$$f_{t+1} = x_t + b_t\,(1 - 2h_t)\,\text{sign}\,(x_t - x_{t-1})$$

where sign $(x_t - x_{t-1})$ is +1, -1 and 0 for $x_{t+1} > x_t$, $x_{t+1} < x_t$, $x_{t+1} = x_t$ resp.

The more previous local maxima are surpassed by x_t the less likely is a continuation of an increase. An analogous statement applies to the continuation of a decrease. This is the rationale of the procedure. It is very interesting that the variance of the best prediction based on an exact knowledge of the stochstic difference equation is 87% of the variance of the bounds and likelihood procedure. This shows that this procedure is surprizingly efficient, in spite of the fact that it is very different from the usual forecasting techniques and much simpler. However, it must be kept in mind that it describes average forecasts rather than individual behavior. Nevertheless it suggests that the spontaneous response of individuals is also guided by the recent direction of the time series, by past average variation and by comparisons of the present value with past local extrema.

3.3 Reinforcement learning

In this essay there is no room for a thorough discussion of reinforcement learning models. The payoff sum model has been very successful in experimental game research (Roth and Erev ??). In the simplest version of this model the probability of choosing an alternative is proportional to its payoff sum, defined as follows: Before period 1 the payoff sum is equal to a positive initial value.The payoff sum of an alternative is increased by the period payoff just obtained after a period in which the alternative has been chosen; otherwise it remains unchanged.

This model is applicable to situations in which payoffs are always non-negative, but modifications also cover the case that payoffs may be negative. The behavior described by the payoff sum model is characterized by information parsimony in the sense that it uses no other information than the feedback about payoffs. This is a remarkable property which makes this model applicable to situations in which the decision maker is ignorant about his environment.

3.4 Learning direction theory

Learning direction theory (Selten and Stoecker 1986, Selten and Buchta 1998) is another surprisingly sucessful approach to learning which is quite different from reinforcement theory. The basic idea can be illustrated by a simple example: Consider an archer who repeatedly tries to hit the trunk of a tree by bow and arrow. After a miss he will have the tendency to aim more to the left if the arrow passed the trunk at the right hand side and more to the right in the opposite case.

The example is not as trivial as it may seem to be. The archer is guided by a qualitative causal model of his environment. This model relates changes of the angle at which the bow is held to changes of the direction in which the arrow flies. After a miss he sees on which side of the trunk the arrow has passed. This feedback and the qualitative causal model enable the archer to draw a qualitative conclusion about what would have been better in the past period. He can determine in which direction of what he did a better alternative could have been found.

The term ex post rationality refers to the analysis of what could have been done better, in contrast to ex ante rationality which reasons about future consequences of possible actions. Learning direction theory is based on ex post rationality. It requires reasoning, but only about the past, not about the future.

Learning direction theory can be applied to repeated decision tasks in which a parameter p_t has to be chosen in a sequence of periods t = 1, ..., T, provided that the decision maker has a qualitative causal model and receives feedback which enables him to infer in which direction from what he did a better choice could have been found.

The theory predicts that the parameter tends to be changed in the direction indicated by the inference, if it is changed at all. This is a weak prediction which, however, has proved to be successful in about a dozen experimental studies (see Selten 1998).

Learning direction theory differs from reinforcement learning by a property referred to as improvement orientation. It does not matter whether the choice of last period resulted in a high or low payoff. What matters is the direction in which a better choice could have been found.

4. Reasoning

Following Johnson-Laird (1983) the author thinks of reasoning as based on the inspection of mental models rather than something more akin to the use of a predicate calculus of formal logic. As we have seen in part 3, reasoning is a part of some basic modes of choice behavior, e.g. learning direction theory. Obviously it has a great influence on human decision making, even at a very basic level. In the following sections some selected topics connected to reasoning will be looked at but admittedly not very closely.

4.1 Intuitíve and analytical approaches to decision tasks

It is useful to distinguish between two kinds of approaches to a decision task. An analytical approach tries to base the decision on the structure of the problem, on the relationship between choice and outcome, and as far as possible on the use of numerical information for the calculation of a solution. In contrast to this an intuitive approach is not based on an understanding of the task, but on its perceived similarity to other situations, for which an appropriate behavior in known, which can be transferred to the problem. The term Aappropriate@ means that this is what one does in such situations.

Analytical approaches are not necessarily superior to intuitve ones. They may be based on a wrong understanding of the situation or on faulty calculation. Intuitive approaches run the danger that the similarities which seem to justify the transfer of behavior, are only superficial ones which hide crucial differences. However, in the face of a lack of understanding, an intuitive approach may be the only available one.

In some cases the same experiment has been run with groups of students and with groups of professionals as subjects. The professionals had practical experience with similar tasks but their performance was worse than that of the students. This happened to professional wool buyers in sequential auctions (Burns 1985) and to dealers on financial markets (Abbink, Kuon 1996). In both cases there are reasons to suspect that the experimental situation did not really have the same structure as the practical one and that the professionals wrongly transferred their practical behavior to the experiment. Since the students had no practical experience they had to look more closely at the situation.

Undoubtedly analytical approaches involve reasoning. Calculations must be based on some kind of mental model. This seems to be different for intuitive approaches. The perception of similarities is maybe a more basic process than reasoning.

4.2 Superficial analysis

Somebody who wants to take an analytical approach to a new decision task must begin the analysis somewhere. At the start of the analysis it cannot be clear yet, which features of the problem are important and which are inessential. A judgement about this is not an input but at best an output of the analysis. Reasoning must begin at the surface, even though it may go deeper later. In this sense the decision maker starts with a superficial analysis.

A study on Face-to-Face duopoly experiments provides an example (Selten and Berg 1970). Payoffs depended on final assets, composed of initial assets and total profits. Initial assets and profits were varied systematically in a way which does not change the relationship between behavior and final payoffs. Only the presentation of the game was varied, not its game theoretically essential features. However, this presentation influenced behavior.

One of two observed modes of cooperation was an agreement at a nearly Pareto-optimal combination with equal profits. This equal profit solution depends on the presentation. How does this presentation effect arise?

Each subject had access to profit tables of both firms. The borders between the regions of positive and negative profits are a conspicuous feature of the situation, even if they are not essential from the point of view of game theory. The superficial analysis starts there. Both want positive profits. From there it is only a small step to the equal profit solution.

In the case of an agreement at the equal profit solution the analysis stops before it is discovered that it is superficial. Boundedly rational analysis is a dynamic process. Initial conditions matter.

Superficial analysis explains the great importance of presentation or framing effects. The author thinks that such effects should not be dismissed as due to misleading instructions. It is important to understand in which way superficial analysis can be misled.

4.3. Human problem solving

The heading of this section refers to the famous book by Newell and Simon (1972) with this title. They have built up an experimentally well supported theory about how people deal with sharply defined problems. An example is the puzzle about the three missionaries and the three cannibals who have to be brought from the left bank of a river to the right one by repeated trips of a boat carrying at most two people. This problem has to be solved under the constraints that at no time more cannibals than missionaries are on a river bank including the persons just landing there.

The problem solving process takes place in a <u>problem space</u>, a set of possible <u>problem states</u>. In the case of the example of the missionaries and the cannibals a problem state is a vector (m,c, b) where m and c are the numbers of missionaries and cannibals, resp. on the left bank and b is the position of the boat at one of the two banks. b may have the values L (left bank) or R (right bank). A problem state is permissible, if it satisfies the constraints imposed by the task.

The description of the problem also specifies a set of operations, by which a transition from one problem state to another is possible, in our case the transport of 1 or 2 persons from the bank where the boat is to the other one. A solution of the problem is a sequence of permissible states starting with the initial state to the end state in the example from (3, 3, L) to (3, 3, R). In this sequence each state before the final one leads to the next one by one of the operations.

At each point of the search for the solution, the problem solver knows states which he can reach from the initial one (including the initial one) and others from which he can reach the end state (including the end state). He tries to narrow the gap between one of the former and one of the latter; i.e. he forms a subgoal of reducing a difference between them. The subgoal suggests the operation to be applied. In the search he sometimes may have to backtrack because he has met a dead end.

The problem space has to be constructed by the problem solver. Sometimes there are several possibilities. In our example the problem space could also represent the situations with the boat on the middle of the river (b = M). However, this is a relatively minor variation. In other cases it can make a great difference for the case of finding a solution which problem space is constructed by the problem solver. The presentation of the problem is important here.

Newell and Simon point out that tic-tac-toe is equivalent to a game in which the two players alternate in picking one of the numbers 1, ..., 9 not yet taken by the other; the first player with three numbers summing to 15 wins. The equivalence is due to the fact that the numbers 1, ..., 9 can be arranged in a magical square.

The problem solving theory of Newell and Simon is a great contribution to the understanding of boundedly rational reasoning. However, the problems to which it applies are very special. Extensions are necessary in order to make the theory fruitful for modelling economic decision making. Consider the case of a tram company who has to work out a new time schedule. Various considerations like maximum waiting times, avoidance of overcrowding and underuse, etc. have to be met. One can think of such requirements as partial aspiration levels on goal variables.

Clearly aspiration adaptation must enter the picture as an integrated part of the search for a solution. It must be decided whether an attempt to satisfy an aspiration should be continued or given up as hopeless. This may involve decision resource stocks as goal variables.

4.4. Qualitative causal reasoning

Qualitative statements about the casual influence of one variable on another concern the direction of the influence. The influence of x on y is positive if y is increasing in x and negative if y is decreasing in x. Qualitative reasoning is the derivation of qualitative conclusions from qualitative assumptions. The notion of a causal diagram, introduced for the purpose of explaining behavior in oligopoly experiments (Selten 1972) describes a mental model for the representation of qualitative causal relationships in a system.

A causal diagram is a directed signed graph whose nodes stand for the variables of a system and whose edges represent influences; the direction goes from the influencing to the influenced variable. A positive influence is indicated by a A+@ at the edge and a negative one by a A!@. The influences represented by edges are <u>direct</u> in contrast to <u>indirect</u> influences exerted along a <u>chain</u>, i.e. a sequence of variables such that each of them except the last one has a direct influence on the next one. An indirect influence along a chain is positive, if the number of negative influences on the chain is even. Otherwise it is negative. In figure 1 we find three indirect influences of price on total profits:

1. price $\overset{+}{ÿ}$ unit profits $\overset{+}{ÿ}$ total profits

2. price $\overset{!}{ÿ}$ sales $\overset{!}{ÿ}$ unit costs $\overset{!}{ÿ}$ unit profits $\overset{+}{ÿ}$ total profits

3. price $\overset{!}{ÿ}$ sales $\overset{+}{ÿ}$ total profits

The indirect influence of price on total profits is positive along the first chain and negative along the other two chains. The diagram is <u>unbalanced</u> in the sense that it does not yield unambiguous qualitative causal conclusions. A <u>balanced</u> diagram is defined by the requirement that for any two variables x and y with indirect influences of x on y either all of them are positive or all of them are negative.

An unbalanced diagram can be changed to a balanced one by the removal of influences or variables. Experiments suggest that subjects tend to balance their qualitative beliefs in this way. Thus, in the example of figure 1 the direct influence of price on unit profits may be removed on the basis of a judgement that it is relatively unimportant. Thereby a balanced diagram is obtained, in which all direct influences of price on total profits are negative. Suppose that the management of the monopoly forms its beliefs in this way. Then it will come to the conclusion that price must be lowered in order to increase total profits.

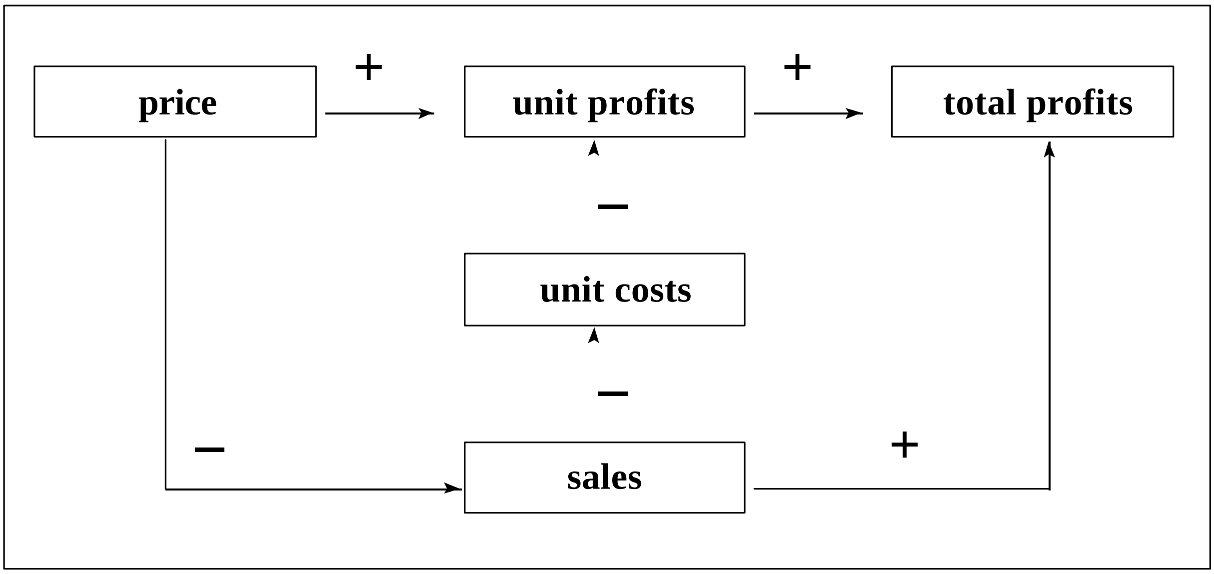

Figure 1: Causal diagram for a monopoly with decreasing unit costs

The decision to decrease the price needs to be quantified, in the sense that the amount of the decrease has to be fixed. Since qualitative beliefs are insecure and maybe only locally correct, management may decide to decrease price by a small percentage, say 5%, which seems to be great enough to have non-negligible effect but not greater. One can think of the quantification decision as reached by the process described in 3.1, the section on prominence in the decimal system.

Opinions about questions of economic policy expressed by journal articles seem to be largely based on qualitative reasoning. Quantitative information is used to argue that some influences are important and others unimportant, but only rarely are any arithmetic calculations made. The author admits that this is an impression, not yet substantiated by systematic empirical research.

5. Motivation

The human motivational system determines the goal pursued by boundedly rational decision making. Unfortunately we have no clear understanding of the interaction of different motivational forces. This is a serious difficulty for the development of a comprehensive theory of bounded rationality. Some decision problems are easy and others cause serious inner conflicts. What is an inner conflict? One approach to this question going back to Freudian psychoanalytic theory is the idea that the self is composed of several parts with different interests. Conflicts may arise among these components of the self. This view of inner conflicts suggests modelling them as games. What are the rules of these games? How should we model them? The following two sections try to throw light on these questions.

5.1 Hyperbolic discounting

In economic theory the usual assumption about discounting streams of payoffs is that of a constant discount rate q with $0 < q < 1$. The payoffs are utilities u_t obtained in period t = 1, 2, ... This means that u_t enters the discounted sum of the payoff stream with the weight q^t. Experimental evidence shows that behavior is much better described by hyperbolic discounting with weights $1 / (A + t)$ for u_t, where A is a positive constant (Ainslie 1992). Thus a subject may prefer 95 money units now to 100 tomorrow, but 100 in a year and a day to 95 in a year. This involves a time inconsistency since after a year the second choice will look like the first one today.

Ainslie models decision making as a game among multiple selves, one for each time period. The self of time t decides what is done at time t with the aim of maximizing its own hyperbolically discounted utility, taking into account what later selves are expected to do. This means that a subgame perfect equilibrium is played. However, the game can have more than one such equilibrium, e.g. one in which the present self and all future selves continue to smoke and another one in which the present self stops to smoke and all future selves follow this example. The second equilibrium may be better for all of them. Assume that this is the case.

The two equilibria have not yet been fully described. In the first one, the <u>smoking equilibrium</u>, all selves smoke independently of prior history. In the second one, the <u>non-smoking equilibrium</u>, the present self does not smoke and the later ones do not either, as long as none of them has deviated. If one of them smokes all the later ones will smoke under all circumstances.

In order to make these equilibria work, one has to assume that the sequence of future selves is infinite. Even if this is wrong one may argue that an analysis based on this idea nevertheless is correct, since it is known that boundedly rational game players can show behavior akin to equilibria of infinite supergames in finite supergames. (Selten and Stoecker 1986)

Suppose that the person is in the smoking equilibrium. It would be better to switch to the non-smoking equilibrium. However, there may be many other subgame perfect equilibria, among them a <u>delayed non-smoking equilibrium</u> in which the present self smokes, but all future selves don't. Under plausible assumptions on payoffs this is the case and the delayed non-smoking equilibrium is better for the present self than the smoking equilibrium.

In this way the inner conflict between stopping or continuing to smoke can be modelled as a problem of equilibrium selection. This is a very interesting modelling approach to the phenomenon of inner conflict, even if the game theoretic reasoning is not fully worked out by Ainslie. However, it is based on strong rationality assumptions. The multiple selves are modelled as fully rational game players. A more plausible picture of inner conflicts faced by boundedly rational players requires another kind of decision behavior . Maybe one should try to modify Ainslie`s theory in this direction.

The split of the person into muliple selves with conflicting goals in itself is a bound of rationality for the person as a whole, even if it is not cognitive but motivational. Not only cognitive, but also motivational bounds of rationality must be taken into account by a comprehensive theory of bounded ratonality.

5.2 Want generator and administrator

Otwin Becker (1967) has proposed a theory of household behavior which extends aspiration adaptation theory to this context. The household divides its monthly income into a number of funds for different kinds of expenditures like a fund for food, a fund for clothing, a fund for entertainment, etc.. The goal variables are the fund sizes and upper price limits for wants, like the desire for a pair of shoes seen in the window of a shop, or an excursion advertised by a travel agency. Such wants are produced by a want generator, modelled as a random mechanism.

When a want is generated by the want generator another instance, the administrator, checks whether there is still enough money in the appropriate fund and whether the want remains under the price limit for such desires. If the price limit is violated, the want is rejected. If the want remains under the price limit but there is not enough money in the fund then the want will still be granted if transfer rules permit the transfer of the missing amount from another fund. The structure of these transfer rules will not be explained here. If such a transfer is not permissible, then the want is rejected.

At the end of the spending period a new aspiration level for the next one is formed by aspiration adaptation in the light of recent experience. The details will not be explained here. If the household theory of Otwin Becker is applied to the spending behavior of a single person, then want generator and administrator are different personalty components. Conflicts between them are not modelled by the theory but it may be possible to extend it in this direction. Everyday experience suggests that sometimes wants are realized against the will of the administrator.

The split of a person into a mechanistically responding want generator and a boundedly rational administrator seems to be a promising modelling approach not only to household theory but also for other areas of decision making.

6. Concluding remarks

The author hopes that he succeeded in conveying the essential features of bounded rationality as he understands it. In the introductory part it was argued that rational decision making within the cognitive bounds of human beings must be non-optimizing. The exposition of aspiration adaptation theory served the purpose of demonstrating the possibility of a coherent modelling approach to non-optimizing but nevertheless systematic and reasonable boundedly rational behavior.

Some of the questions left open by aspiration adaptation theory are related to the topics which have been discussed under the heading Abasic modes of choice behavior@, the role of decimal prominence in decision making, expectation formation in the case of a univariate time series, reinforcement learning, and learning direction theory. Of course, this is only a small sample of what can be found in the relevant literature. The remarks about reasoning in this essay do even less justice to important theoretical and empirical developments like Johnson Laird=s work on mental models (1983), the impressive book by Holland, Holyoak, Nisbett and Thagard (1989) and the illuminating analysis of analogy in Amental leaps@ by Holyoak and Thagard (1995). Unfortunately these and other very interesting advances do not lend themselves to very short condensed descriptions. the discussion of motivation was restricted to only one aspect of it, the idea that a person is subdidvided into several components which may be in conflict with each other. A subject matter which was left out altogether is the strategic interaction of boundedly rational players in game situations, an area of research discussed in a recent paper by the author (1998 b).

What is bounded rationality? A complete answer to this question cannot be given at the present state of the art. However, empirical findings put limits to the concept and they indicate in which direction further inquiry should go.

Bibliography

Abbink, K. and B. Kuon, 1998, An option pricing experiment with professional traders, SFB discussion paper B 426, University of Bonn.

Ainslie, G., 1992, Picoeconomics, Cambridge University Press.

Albers, W. and G. Albers, 1983, On the prominence structure in the decimal system, in: R.W. Scholz (ed.), Decision making under uncertainty, Elsevier (North Holland), 271-287.

Aumann, R. and S. Sorin, 1989, Cooperation and bounded recall, Games and Economic Behavior 1, 5-39

Becker, O., 1967, Die wirtschaftliche Entscheidungen des Haushalt, Duncker and Humblot.

Becker, O. and U. Leopold, 1996, The bounds & likelihood procedure, A simulation study concerning the efficiency of visual forecasting techniques, Central European Journal of Operations Research and Economics, 4, 2-3.

Burns, P., 1985, Experience and decision making: A comparison of students and businessmen in a simulated progressive auction, in: V. Smith (ed.) Research in Experimental Economics, vol. 3, Greenwich, CT, JAI Press, 139-157.

Bush, R.R. and F. Mosteller, 1955, Stochastic Models of Learning, New York, John Wiley & Sons.

Cyert, R. and J.G. March, 1963, 1992 (second ed.), A behavioral theory of the firm, New Jersey, Prentice Hall.

Gigerenzer, G., 1997, Bounded rationality: Models of fast and frugal inference, Swiss Journal of Economics and Statistics, 133, 201-218.

Hertwig R., U. Hoffrage, and L. Martiguon, 1999, Quick estimation: Letting the environment do some of the work, in: G. Gigerenzer, P.M. Todd, and the ABC Research Group, Simple Heuristic that make us smart, New York, Oxford University Press.

Holland, J., K. Holyoak, R. Nisbett and P. Thagard, 1986, Induction, MIT Press.

Holyoak, K., and P. Thagard, 1995, Mental leaps, MIT Press.

Johnson-Laird, Ph., 1983 Mental models, Harvard University Press.

Kahnemann, D., P. Slovic and A. Tversky (eds.), 1982, Judgement under Uncertainty, Heuristics and Biases, Cambridge University Press.

Newell, A. and H. Simon, 1972, Human Problem Solving, Prentice Hall

Roth, A. and I. Erev, 1995, Learning in extensive form games, Games and Economic Behavior, 8, 164-212.

Sargent, Th., 1993, Bounded rationality in macroeconomics, Oxford, Clarendon Press.

Sauermann, H. and R. Selten, 1962, Anspruchsanpassungstheorie der Unternehmung, Zeitschrift für die gesamte Staatswissenschaft, 118, 577-597.
Savage, L., 1954, The foundation of statistics, New York, John Wiley & Sons.
Selten, R., 1967, Investitionsverhalten im Oligoexperiment, in: H. Sauermann (ed.), Beiträge zur experimentellen Wirtschaftsforschung, Vol. 1, Tübingen, J.C.B. Mohr, 60-102.
Selten, R. and C.C. Berg, 1970, Drei experimentelle Oligospielserien mit kontinuierlichem Zeitverlauf, in: H. Sauermann (ed.), Beiträge zur experimentellen Wirtschaftsforschung, vol. 2, Tübingen, J.C.B. Mohr, 162-221.
Selten, R. and R. Stoecker, 1986, End behavior in sequences of finite prisoner`s dilemma supergames, Journal of Economic Behavior and Organization, 7, 47-70.
Selten R., 1991, Evolution, Learning and Economic Behavior, Games and Economic Behavior, 3, 3-24.
Selten, R., 1998 a, Aspiration adaption theory, Journal of Mathematical Psychology, 42, 191-214.
Selten, R., 1998 b, Features of experimentally observed bounded rationality, European Economic Review, 42, 413-436.
Selten, R. and J. Buchta, 1999, Experimental scaled bid first price auctions with directly observed bid functions, in: D. Budescu, I. Erev and R. Zwick (eds.), Games and Human Behavior, Lawrence Erlbaum, 79-102.
Simon, H.A., 1957, Models of Man, New York, Wiley & Sons.
Tietz, R. and H.J. Weber, 1972, On the nature of the bargaining process in the KRESKO-game, in: H. Sauermann (ed.), Contributions to Experimental Economics, vol. 3, Tübingen, J.C.B. Mohr, 305-334.
Tietz, R., 1976, Anspruchsausgleich in experimentellen Zwei-Personenverhandlungen mit verbaler Kommunikation, Zeitschrift für Sozialpsychologie, Beiheft 2, Entscheidungsprozesse in Gruppen, 123-141.