The computational complexity of satisfiability of temporal Horn formulas in propositional linear-time temporal logic

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Communicated by D.A. Plaisted
Received 21 April 1992
Revised 23 November 1992

Keywords: Computational complexity; temporal logic; temporal Horn formula; temporal logic programming; satisfiability

1. Introduction

Since the invention of Prolog, a programming language based on classical first-order logic, many people have tried to extend it using similar ideas and redefine the semantics of the extended Prolog in terms of nonclassical logics [3,5,8]. The success of a programming language based on nonclassical logics usually lies in the new definition of Horn formulas and SLD-resolution-like inference rule. For modal logic and temporal logic, the corresponding definitions of Horn formulas have been available [1,4,5]. It is thus theoretically interesting to know the inherent complexity of the satisfiability problem for propositional temporal Horn formulas. In this paper we shall investigate the complexity of the satisfiability problem for two Horn fragments in propositional linear-time temporal logic. The first one contains two temporal connectives $\Diamond$ (eventually) and $\Box$ (always) only and the second one contains an additional next-time connective $\circ$.

It has been shown that the satisfiability problem for linear-time temporal logic whose temporal connectives include $\Diamond$ and $\Box$ only is NP-complete and is PSPACE-complete if additional next-time connective $\circ$ is permitted [9]. In this paper we shall show that the complexity of the satisfiability problem remains unchanged even if the input is restricted to temporal Horn formulas. In other words, if the next-time connective is not allowed to occur in temporal Horn formulas, the problem is NP-complete; otherwise, it is PSPACE-complete.

Linear-time temporal logic without the next-time connective and the modal logic $S5$ are two of a few nonclassical logics whose satisfiability problems are known to be NP-complete. The NP algorithms to decide satisfiability of formulas for both logics are usually derived by the same small-model property (cf. [7,9]). It is thus natural to expect that both logics have also the same complexity when considering their Horn fragments. However, it has been proved by Farinas del Cerro and Penttonen [4] that the satisfiability of modal Horn formulas for $S5$ is solvable in polynomial time. Compared with the result for $S5$, our first complexity result for linear-time temporal logic seems surprising.

The rest of this paper is structured as follows. In Section 2 we review linear-time temporal logic and introduce temporal Horn formula. In the