THE COLOR VISION OF DICHROMATS*

I. Wavelength Discrimination, Brightness Distribution, and Color Mixture

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I

Purpose of Investigation As a Whole

Our interest in colorblindness springs from the desire to determine the unknown spectral sensibilities of the retinal cones, and to correlate with them the quantitative properties of *normal* color vision. This is a complex task (*cf.* Hecht, 1930; 1931; 1932), because it involves the manipulation of nine variables. Of these only five or six have actually been measured; the others are hypothetical. Therefore the value of the theoretical treatment is always uncertain.

Colorblindness is a simpler form of color vision because the colorblind recognizes fewer colors than the normal, and his spectrum may be matched with two primaries instead of the three required for the normal (Young, 1807). A complete description should therefore be possible in terms of four variables, obtained by measuring four independent aspects of colorblindness. Moreover, since Thomas Young it has been generally recognized that the various types of colorblindness[1] are most probably derivatives of normal color vision. Hence,

* The measurements for this group of papers were begun in 1931 and finished in 1933. The main results were reported to the Optical Society in February, 1934 (Hecht and Shlaer, 1934) and in October, 1935 (Hecht and Shlaer, 1936), and to the XV International Physiological Congress in Leningrad, August, 1935.

[1] In order that our references to different types of colorblindness be easily understood, we give here a diagnostic classification of all kinds of color vision, based on the accumulated knowledge of a hundred years, and not on any theory.

I. MONOCHROMATS are persons who confuse any part of the spectrum with any other part, and who can match any part with white. There are (*a*) *Scotopic*

information obtained from such a study should serve as a critical supplement to the theoretical treatment of the data from color-normals.

With certain exceptions, the measurements necessary for our purpose have not been available. We therefore began the study of colorblindness to supply them. Our aim has been to investigate as many properties as possible with a few selected individuals. The results so far obtained are presented in the present group of papers.

II

Apparatus

All our measurements were made either with a Helmholtz Color Mixer (Koenig and Dieterici, 1892) built by Schmidt and Haensch, or with an apparatus composed of two monochrometers. A diagrammatic representation of the optical essentials of both arrangements is shown in Fig. 1 where *A* describes the Helmholtz Color Mixer and *B* the two-monochrometer apparatus.

The Helmholtz Color Mixer is essentially a spectrometer having one telescope and two collimators each with a light source. A 100 watt concentrated filament lamp *L* illuminates a finely ground glass *G*, placed 5 cm. from it, which is then

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Monochromats, who have a brightness distribution in the spectrum corresponding to rod vision; and (*b*) *Photopic Monochromats*, who have a brightness distribution in the spectrum corresponding to cone vision.

II. DICHROMATS are persons who confuse large sections of the spectrum, who can match a particular part of it (either in the blue-green or in the yellow) with white, and who can match any part of it with a mixture of two primaries. Of these the (*a*) *Protanopes* confuse green, yellow, and red; they match a point in the blue-green with white; in particular, their brightness distribution in the spectrum even at high intensities is depressed in the red, and they are therefore frequently called red-blind. They are to be distinguished from the (*b*) *Deuteranopes*, who also confuse green, yellow, and red, and similarly have a point in the blue-green which they match with white, by the fact that the deuteranopes have a brightness distribution in the spectrum much like the normal. These are often but incorrectly called green-blind. To be distinguished from both of these green-yellow-red confusers are the (*c*) *Tritanopes*, who confuse blue and green, and match a point in the yellow with white. These are often called violet-blind or blue-blind, but such names involve a theory of colorblindness which is probably incorrect.

III. ANOMALOUS TRICHROMATS are persons who confuse parts of the spectrum, but who still require three primaries to match the spectrum. They form intermediates of all grades between dichromats and color-normals, and may be (*a*) *Protanomalous*, or (*b*) *Deuteranomalous*, or (*c*) *Tritanomalous*, depending on which type of dichromat they resemble.

IV. TRICHROMATS possess normal color vision.

focussed by a condenser through a Nicol prism N of the Glan-Thompson variety on to the slit S of the collimator. Between the slit and the collimator lens is a Rochon prism R whose position may be set anywhere in the collimator tube by a

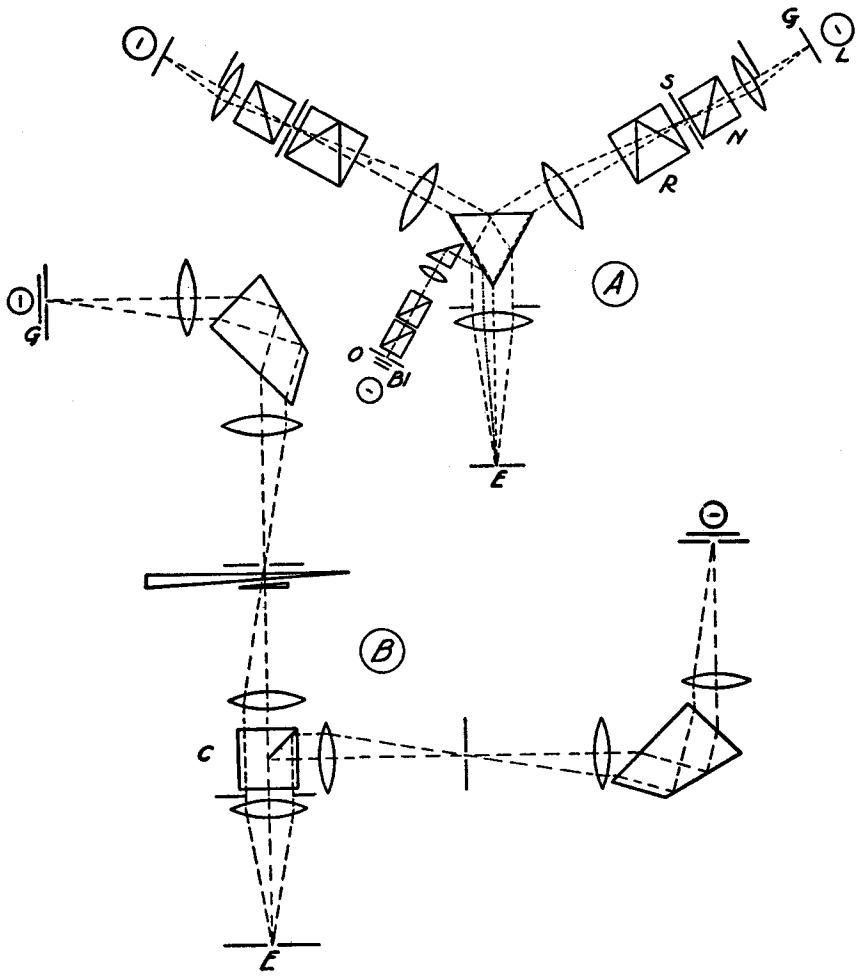

FIG. 1. Diagrammatic representation of the optical arrangements. A shows the main features of the Helmholtz Color Mixer, while B shows the apparatus composed of two separate monochrometers and a photometric cube.

rack and pinion sliding over a scale. The eye at the exit pupil E of the telescope sees the two prism faces as contiguous semicircular fields, one from each collimator. The field as a whole has a diameter of 1.2°, and thus falls entirely in the rod-free area of the fovea.

The Rochon prism splits the beam of light into two components polarized at right angles. The ordinary beam passes through undeviated; the extraordinary beam is deviated at a constant angle. At the exit pupil of the telescope, the λ of the ordinary beam depends solely on the angular position of the collimator, while the λ of the extraordinary beam depends in addition on the position of the Rochon prism. The angular position of each collimator is set by a Brown and Sharpe micrometer screw which we added to the instrument, but which is not shown in the diagram. The light from the collimator slit is plane polarized by the Nicol prism; the angle of the prism then determines the fraction of each beam which passes to the exit pupil. In this way different amounts of light of two chosen wavelengths may be made to appear superimposed in each half of the visual field. Their combined intensity may then be controlled by the collimator slit which is symmetrical and whose width may be accurately set by an appropriate micrometer screw.

When only one band of homogeneous light is needed from the collimator at the exit pupil, the Rochon prism is placed close to the collimating lens, thus completely excluding the extraordinary beam at all positions of the collimator. The Rochon now acts like a Nicol prism and enables the ordinary beam to be varied in intensity by the Nicol prism; and the color mixer becomes an ordinary double spectrometer.

We have added an arrangement for reflecting white light of color temperature 5000°K from the left prism face; its intensity is controlled by a pair of Nicol prisms. The color temperature was achieved with a selected blue glass *Bl*, a piece of ground glass *O*, and a three-volt battery lamp whose amperage was adjusted by color-matching the combination against a lamp of standard color temperature plus a Davis-Gibson standard filter (Davis and Gibson, 1931).

The arrangement shown in *B* of Fig. 1 furnishes to the eye at the exit pupil *E* a bipartite, circular field also 1.2° in diameter, each part being illuminated by homogeneous light from a separate, constant deviation spectrometer. The circular field is produced by the photometer cube *C*. Each spectrometer is illuminated by a 100 watt lamp and ground glass *G*. The intensity of one field is varied by a neutral gelatine wedge and balancer.

Both pieces of apparatus were calibrated with sources of known wavelength. In particular, the Helmholtz Color Mixer was calibrated frequently, because it showed a tendency to vary over long periods.

III

Wavelength Discrimination

(A) Previous Work

The color-normal can divide the visible spectrum into about 180 short stretches which differ in appearance even when their brightness differences have been eliminated. The size ($\Delta\lambda$) of these spectral patches is not uniform, but shows two distinct minima found by all

observers,—in the blue-green between 490 and 500 mμ, and in the yellow between 570 and 580 mμ, where Δλ is about 1 mμ. Nearly all observers show also one or two secondary minima: in the violet at 440 mμ, and in the orange near 620 mμ, where Δλ is between 2 and 3 mμ. At the two ends of the spectrum Δλ quickly rises to about 7 mμ (for a summary of the literature see Judd, 1932; Ladekarl, 1934; and Wright and Pitt, 1934).

Colorblinds possess only one minimum of Δλ in the spectrum. Brodhun (see Koenig, 1903 *a*) found this minimum near 500 mμ for the deuteranope. Here Δλ is about the same as for the color-normal, but to either side Δλ rapidly becomes large. This has been confirmed by Steindler (1906), by us (Hecht and Shlaer, 1934), by Ladekarl (1934), and very recently by Pitt (1935). Steindler reported the same minimum near 500 mμ for deuteranopes and protanopes, and this has also been corroborated by later work (Laurens and Hamilton, 1923; Rosencrantz, 1926; Sachs, 1928; Hecht and Shlaer, 1934; Ladekarl, 1934; and Pitt, 1935). In addition, Steindler found a second minimum for protanopes at about 575 mμ; this, however, was undoubtedly due to lack of brightness control, because it has not been found by those investigators who controlled this source of error (Laurens and Hamilton, Sachs, Hecht and Shlaer, and Pitt). Measurements with an extreme case of tritanomaly by Engelking (1925) show a minimum between 575 and 600 mμ, and indicate that a tritanope would probably have a minimum in that region of the spectrum.

With the exception of Steindler's measurements, which suffer because brightness differences were not eliminated, all of the published measurements are restricted to a small region of the spectrum covering about 30 mμ to either side of the neutral point near 500 mμ. For a complete description, it is obviously necessary to have data which cover the whole spectrum. We have measured two deuteranopes and one protanope[2] for this purpose.

[2] The first deuteranope is Dr. Alan W. Greenwood (A.W.G.) of the Department of Genetics of the University of Edinburgh, a mature and skilled investigator, who at the time (1931) was in New York, and to whom we shall always be grateful for the time and patience he devoted to our work. The second deuteranope (S.R.F.) was a senior (1933) at Columbia College. The protanope (H.J.) was a high school senior (1932–33). In spite of our efforts of the last three years to find a tritanope so as to make this study complete, we have been unable to secure one for measurement.

(B) *Method*

In making the measurements we set the wavelength of both halves of the field, while the subject determined whether by varying the brightness of one side only he could match the two sides perfectly. Light of λ_1 was first put on one side, then lights of other wavelengths were successively put on the other side until the wavelength λ_2 was found beyond which the observer could not match with λ_1. Each final observation was checked at least once before being recorded. The difference between λ_1 and λ_2 is $\Delta\lambda$ and represents the just discriminable interval.

The subject was light-adapted throughout, and given a few minutes rest between each determination of $\Delta\lambda$. All judgments were made by looking freshly into the exit pupil, since differences which are apparent at once tend to disappear on prolonged examination. Measurements never lasted more than 2 hours and were interrupted by two or three 15 minute periods of relaxation.

All the measurements were made with the Helmholtz Color Mixer, except the November series for the protanope which was made with the two-monochrometer system. The exit slit in both arrangements was kept at 0.4 mm. This is a compromise involving the desire to have a high purity of spectrum which demands a narrow slit, the elimination of diffraction which requires a wide slit, and a good brightness which also requires a wide slit. Since the homogeneity of the spectrum at the exit slit is maximal when collimator slit and exit slit have the same width, it is useless to have a fine collimator slit with a wide exit slit as used by Pitt (1935). We kept the collimator slits at 0.5 mm. With the Helmholtz Color Mixer, the exit pupil contained a band 4 mμ wide at 500 mμ; with the monochrometer system it was somewhat less. The brightness at the eye under these conditions is equivalent to between 200 and 500 millilamberts viewed through a 2 mm. pupil, or between 2000 and 5000 photons.

(C) *Measurements*

Table I gives the data for the two deuteranopes and the protanope. The April data for A. W. G. were secured at the beginning of his work. After measuring a variety of visual properties, he made the May determinations. S. R. F. made only one set of measurements. H. J. made two sets; the first at the very beginning, and the second several months later after having acquired skill, but after an absence of about 2 months from the laboratory. Each item in the table is the average of at least two separate measurements; in the region between 540 and 580 mμ the daily variation was such that we made four or five, and occasionally more determinations for one point.

The table shows that for an individually variable stretch of the spectrum between 480 and 550 mμ, $\Delta\lambda$ for the colorblind is of the

same magnitude as for the normal. To either side of this stretch $\Delta\lambda$ rises very rapidly, reaching nearly 50 mμ at the two ends. The nature of the data is best illustrated in Fig. 2, where $\Delta\lambda$ is plotted against both λ_1 and λ_2. The upper box of Fig. 2 also shows for comparison $\Delta\lambda$ for the normal eye of Laurens (Laurens and Hamilton, 1923).

TABLE I

Wavelength Discrimination of Dichromats

Deuteranope A. W. G.						Deuteranope S.R.F.			Protanope H. J.					
April, 1931			May, 1931			June, 1931			May, 1933			November, 1933		
λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$	λ_1	λ_2	$\Delta\lambda$
421.3	470.4	49.1	423.8	471.8	48.0	425.0	468.0	43.0	438.9	469.6	31.7	449.9	464.6	14.7
428.7	470.5	41.8	434.1	475.6	41.5	440.0	471.8	31.8	450.0	473.6	23.6	458.1	469.5	11.4
442.6	476.4	33.8	443.8	476.6	32.8	450.0	471.8	21.8	458.8	477.3	18.5	469.1	473.0	3.9
455.2	480.3	25.1	453.6	477.8	24.2	460.0	474.5	14.5	470.3	479.9	9.6	481.2	482.5	1.3
462.2	482.4	20.2	462.4	479.4	17.0	470.0	478.9	8.9	480.5	485.4	4.9	485.8	486.5	0.7
468.1	481.6	13.5	472.8	482.9	10.1	474.5	480.0	6.4	485.6	487.5	1.9	491.8	492.0	0.2
482.6	486.4	3.8	483.9	486.3	2.4	480.0	482.4	2.4	489.6	491.2	1.6	495.3	496.0	0.7
492.3	494.2	1.9	491.7	492.5	0.8	489.6	490.2	0.6	496.2	497.2	1.0	500.2	501.0	0.8
500.3	502.4	2.1	505.0	505.5	0.5	498.9	500.0	1.1	499.6	501.1	1.5	504.8	506.3	1.5
508.9	511.0	2.1	517.8	518.8	1.0	509.0	510.0	1.0	504.9	506.0	1.1	513.3	515.2	1.9
523.3	525.8	2.5	524.4	525.5	1.1	519.4	520.0	0.6	508.7	511.1	2.4	523.7	525.2	1.5
531.6	537.3	5.7	530.3	533.4	3.1	529.2	530.0	0.8	516.8	521.5	4.7	532.0	534.8	2.8
537.4	563.0	25.6	535.0	541.7	6.7	538.7	540.0	1.3	526.9	532.0	5.1	540.8	544.0	3.2
540.5	572.8	32.3	541.7	555.2	13.5	547.1	550.0	2.3	538.6	553.4	14.8	550.6	553.3	2.7
543.4	593.5	50.1	547.1	580.0	32.9	553.2	559.0	5.8	549.7	576.0	26.3	560.0	561.8	1.8
			550.2	608.3	58.1	563.3	569.0	5.7	554.1	602.8	48.7	568.0	571.7	3.7
						570.2	580.0	9.8				570.1	582.6	12.5
						571.0	595.0	24.0				573.0	593.0	20.0
						572.5	610.0	37.5				573.5	604.2	30.7
						575.1	620.0	44.9						

The neutral points (A. W. G. at 495.0 mμ; S. R. F. at 500.1 mμ; and H. J. at 491.5 mμ) are marked with a vertical line in Fig. 2. The part of the spectrum which the observer matches with white of 5000° K is usually a band less than 1 mμ wide; its midpoint is given by the line.

Note that $\Delta\lambda$ is minimal near but not exactly at the neutral point. The colorblind thus distinguishes wavelength best near the region of the spectrum which to him resembles white. To either side of this, $\Delta\lambda$ increases with the distance from the neutral point, on one side more

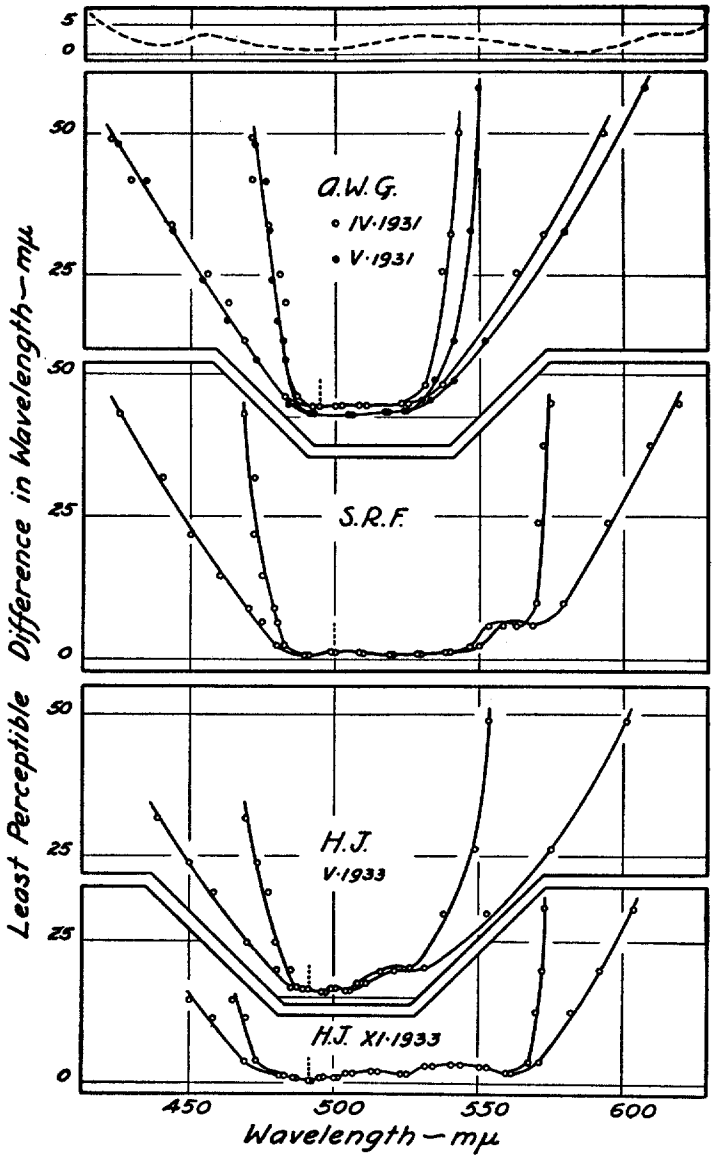

FIG. 2. The data of wavelength discrimination for one normal (upper box) and for the two deuteranopes (A. W. G. and S. R. F.) and one protanope (H. J.). For the dichromats, $\Delta\lambda$ is plotted against both λ_1 and λ_2.

rapidly than on the other. The distribution of wavelength discrimination around the neutral point is not symmetrical; there is a more extensive stretch of moderately good discrimination toward the long-wave end of the spectrum than toward the short end. The data of Laurens and Hamilton's protanope and of Sach's protanope, though quite meager, also show an asymmetrical distribution of $\Delta\lambda$ around the neutral point; the asymmetry, however, is reversed in that the more extensive portion is on the blue side. Pitt's recent data, which cover only about a third the spectrum, indicate a more restricted region of good discrimination, distributed nearly but not quite symmetrically about the neutral point. The symmetry is probably due to Pitt's method which uses an average of $\Delta\lambda$ to each side of a given λ. This is not a correct procedure, since $\Delta\lambda$ is not the same in the two directions, as is obvious from Sach's work and from our data in Fig. 2. Ladekarl's data resemble ours in the small range which he measured. They are less symmetrical than Pitt's, but more than ours and Laurens and Hamilton's and Sachs'. Here again the method obscures the measurements because Ladekarl used the average-error-of-setting method which automatically averages the two directions.

The sharply rising inner lines in the data of Fig. 2 represent two regions of striking sensory change. Starting with any wavelength below 460 mμ, our colorblinds see no differences in the spectrum until they hit this region of discrimination at about 470 mμ. Similarly, starting at the red end, they make no discrimination in λ until the region at approximately 570 mμ. The position of the sharply rising inner portion on the short-wave side varies much less from time to time and from individual to individual than the one on the long-wave side.

From our measurements it appears that protanopes and deuteranopes show a very similar capacity for λ discrimination. It may be that when many individuals have been studied a consistent difference will appear over the whole spectrum similar to the very small and doubtful difference found by Pitt in the restricted region studied by him with six protanopes and six deuteranopes. However, the individual variation is so great that in terms of λ discrimination, either in a restricted region or over the whole spectrum, it is not possible to classify an individual as a protanope or a deuteranope.

IV

Spectral Brightness Distribution

(*A*) *Normals and Dichromats*

Measurements of brightness distribution in the spectrum became of interest for colorblindness when it was found that the two classes of dichromat distinguished by Seebeck (1837) have a different brightness distribution in the spectrum. The deuteranope's brightness is much like the normal, whereas the protanope's is distinctly depressed in the red (Macé and Nicati, 1879; von Kries and Küster, 1879; Donders, 1881). Actually the protanope's brightness maximum is shifted toward the blue compared to the normal or deuteranope (Brodhun, 1887; Koenig, 1903 *b*; Abney, 1913; Exner, 1921; Kohlrausch, 1931; Pitt, 1935).

Since Langley's (1888) introduction of energy distribution data into spectral brightness determinations, the visual effectiveness of the spectrum for the color-normal has been repeatedly determined, and has become an established datum (Gibson and Tyndall, 1923). Not so for the colorblind. Energy measurements in the spectrum are not easy to make; and investigators have been content to record the relative brightness distribution in a particular spectrum for the colorblind in comparison with the color-normal, sometimes (see especially Pitt, 1935) going to extraordinarily circuitous lengths to find the real shape of the data without making the energy measurements.

We have determined the spectral brightness distribution for the three colorblinds recorded in the previous section, making our own energy measurements in the spectrum, and using a method which does not involve heterochromic photometry.

(*B*) *Method*

Fig. 2 shows that for the two large, end-stretches of the spectrum, the dichromat sees no differences in wavelength. Within these stretches the spectrum may therefore be compared in brightness without introducing any "color" differences.[3] But even in the region between 470 and 570, the just perceptible step $\Delta\lambda$, though

[3] We use the term "color" here to include everything but brightness. Actually, as the following paper shows, the differences between contiguous wavelengths which the colorblind distinguishes are not concerned with hue, but with saturation.

small, is finite; and brightness comparisons can be made between points which differ by less than $\Delta\lambda$. This step-by-step method resembles the procedure of Gibson and Tyndall for the normal eye, but in addition it avoids all color differences for the colorblind.

The Helmholtz Color Mixer was used for these measurements. We set a reference λ in the left collimator, and the subject determined by means of the Nicol prism the relative brightness of the same λ in the right collimator. All further relative brightness measurements were then referred to this λ in the right collimator as a standard. Keeping the reference λ in the left collimator constant, the brightness of a series of wavelengths in the right collimator was then measured by the subject, until a complete match between the two sides became impossible. The reference λ in the left collimator was then changed to a new reference λ, chosen so as to match the last λ measured in the right collimator; the brightness of the new reference was then determined relative to the last measured λ. A new section of the spectrum in the right collimator was then measured against the new reference λ in the left until a complete match on both sides again became impossible. A new reference, chosen as before, was then introduced into the left collimator; its brightness was calibrated as before, and a new section of the spectrum measured against it. The procedure was repeated as often as necessary to cover the spectrum. For those regions where $\Delta\lambda$ is large, one reference λ easily served for 75 or 100 mμ, and determinations were made every 10 mμ. But between 470 and 570 mμ the reference λ had to be changed with increasing frequency, until near 500 mμ it was changed for every other measurement. Usually three, and frequently five readings were made for each determination of relative brightness; for calibrating each new reference λ five and often ten readings were taken. About fifty steps were required to traverse the spectrum.

We calibrated the energy distribution of the spectrum with a Hilger linear thermopile placed at the exit pupil of the telescope, and thus avoided corrections for the transmission of the prism and lenses, and for the dispersion of the spectrum. With the low resistance thermopile, we used a Paschen galvanometer supplied by the Cambridge Instrument Company. The deflection of the galvanometer was carefully calibrated with known voltages. To increase the energy at the exit pupil, both collimators were used at the same time, and the telescope lens was kept at full aperture. The slits were opened to 0.5 mm. Six readings were made at each point at 10 mμ intervals along the spectrum. The results secured were smooth, so that a calibration curve could easily be drawn through them. The values necessary for correcting the measured, relative brightnesses were taken from the curve.

(C) *Visibility Curves*

Table II gives the data, which are also shown graphically in Fig. 3. It is apparent that with minor exceptions they are regular and smooth.

TABLE II

Brightness Distribution in Spectrum. The Maximum in Each Case is Placed at 100

Deuteranopes				Protanope	
A. W. G.		S. R. F.		H. J.	
λ in mμ	Brightness	λ in mμ	Brightness	λ in mμ	Brightness
422.8	4.72	404.2	0.11	404.3	0.39
433.4	5.94	414.6	0.26	414.5	1.24
442.9	5.91	425.3	0.35	425.3	2.76
453.0	7.52	435.0	0.46	435.2	4.26
463.3	10.82	445.3	0.58	445.4	5.34
470.4	17.33	454.7	0.75	454.6	7.54
476.4	20.72	464.8	1.22	464.7	10.26
479.4	23.56	476.1	2.15	480.0	17.04
482.6	27.29	472.2	1.61	485.0	21.76
484.3	24.85	484.1	2.65	487.4	25.84
485.8	31.02	488.2	3.12	489.8	29.52
487.5	38.28	491.7	4.29	492.4	32.00
489.2	39.93	495.2	5.79	494.0	32.96
490.8	42.14	497.8	7.76	495.5	34.48
494.4	48.44	499.7	7.66	497.6	37.76
497.1	60.19	501.0	7.47	499.6	41.92
498.7	66.63	502.6	8.64	501.2	43.68
501.7	62.30	504.0	10.71	503.3	49.04
505.4	60.19	505.2	12.27	506.3	57.12
509.1	62.80	507.2	13.86	510.2	64.48
514.0	68.94	509.1	17.41	514.2	71.44
519.1	82.27	512.1	20.61	518.2	80.00
527.6	84.05	514.1	23.96	523.7	88.40
536.5	84.74	516.0	29.44	536.4	99.36
546.1	97.48	518.2	34.97	546.2	99.20
556.6	99.00	520.4	39.80	557.9	92.48
567.9	100.25	523.7	47.83	568.1	85.84
579.9	99.55	531.8	66.48	578.6	66.64
589.6	81.05	536.4	83.45	588.5	51.92
599.7	73.49	546.1	84.62	600.5	34.16
608.5	65.54	558.0	93.08	609.6	24.16
619.6	47.19	568.1	89.63	618.6	16.64
629.3	35.61	578.7	100.00	630.3	8.32
639.5	25.77	588.7	96.03	640.4	4.74
650.4	16.50	600.6	78.57	649.4	2.88
662.1	8.88	609.6	81.76	660.5	1.36
674.5	4.69	618.6	53.07	670.0	0.72
690.4	1.91	630.3	36.23	680.5	0.40
701.7	0.92	640.4	21.68	691.5	0.24
		649.4	16.69		
		660.7	8.64		
		670.4	4.80		
		680.6	2.81		
		691.3	1.33		
		700.0	0.80		

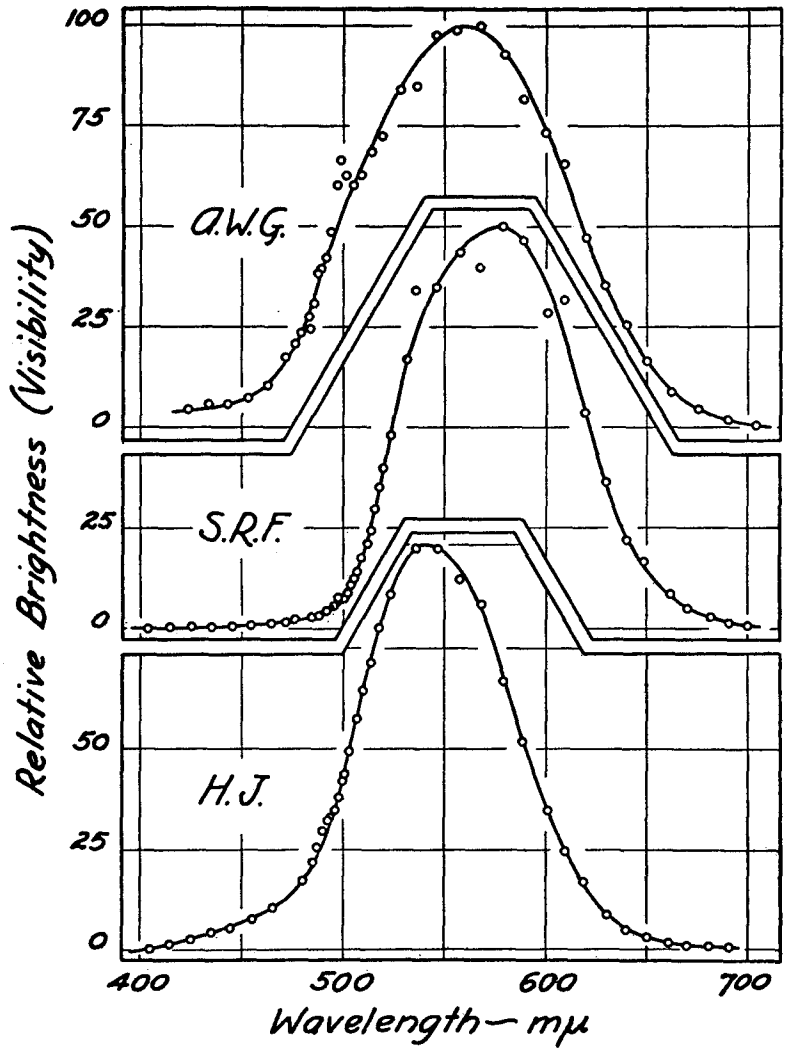

FIG. 3. Brightness distribution in the spectrum for two deuteranopes (A. W. G. and S. R. F.) and one protanope (H. J.). The maximum for each has been arbitrarily placed at 100.

Perhaps the best way of examining the visibility curves is to view them against the normal background. In Fig. 4 the stippled area represents the range of measurements for the 52 color-normals investi-

gated by Gibson and Tyndall. The protanope H. J. barely falls within the normal range on the blue side of his maximum, and is definitely outside the normal range on the red side. On the other hand, the deuteranope S. R. F. barely falls within the normal range on the red side and is distinctly outside on the blue side. The deuteranope A. W. G. has a rather wide visibility curve which falls within the normal range on both sides. Pitt's recent averages of six deuteranopes and six protanopes are included in Fig. 4. H. J. is an almost

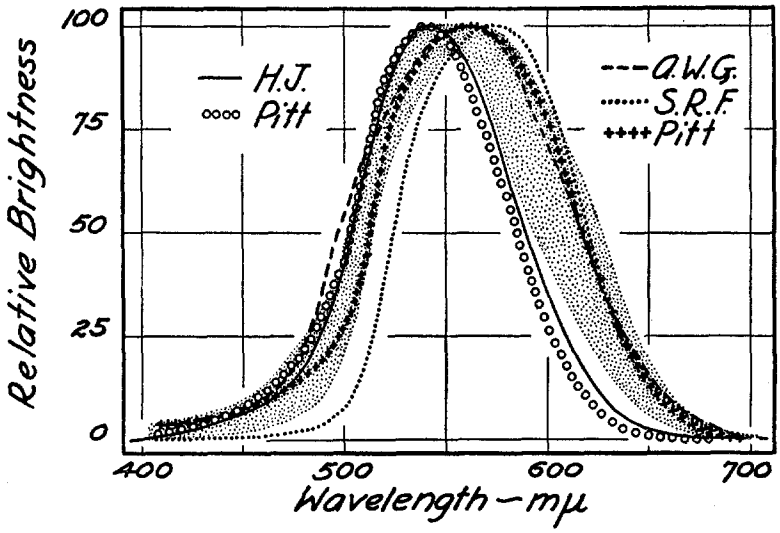

FIG. 4. Brightness distribution in the spectrum. The stippled background represents the range for fifty-two color-normals measured by Gibson and Tyndall (1923). A. W. G. and S. R. F. are our deuteranopes, and the crosses give the average of Pitt's (1935) deuteranopes. H. J. is our protanope, and the circles give the average of Pitt's protanopes.

perfect replica of the averaged protanope; his curve and the average curve are narrower than the normal and deuteranope curves. A. W. G. and S. R. F. coincide with the averaged deuteranope on the red side, but lie to either side of the average on the blue side.

The data show clearly that just as the protanope's curve is shifted toward the blue compared to the normal, so the deuteranope's curve is shifted toward the red, but not so much. This has already been noted by Pitt.

The least variable part of the visibility curves seems to be the red side of the maximum, and this furnishes the only reliable means of telling when a given individual is a protanope or a deuteranope (Donders, 1884; von Kries, 1897). We have adopted this as a diagnostic routine test, making a brightness comparison between 550 and 650 mμ. The comparison is made in two steps, first measuring 550 against 600, and then 600 against 650, and thus avoids a heterochromic match. The ratio of 550/650 mμ is large, near 35 for the protanope, and small, near 4 for the deuteranope. Of the ten protanopes and twelve deuteranopes whose diagnostic brightness ratio we have measured, we have found no case falling far from these values.

V

Neutral Point

The neutral point of a dichromat is that point in the spectrum which he can match with white light. Using white of 5000° K we have determined the position of the neutral point for the twenty-two dichromats just referred to. The measurements were made exactly as with wavelength discrimination. Half the field of the Color Mixer contained white, while the other had a succession of wavelengths whose brightness the subject could control. He was required to state whether he could match the two halves or not. The band in the spectrum which the dichromat can match with white is usually about 1 mμ wide.

The measurements are shown in Fig. 5, which includes not only our own data, but those of Koenig (1884) and of Pitt. The figure shows that the position of the neutral point for the deuteranope is more widely scattered than for the protanope. The extreme position of one neutral point at 525 mμ is authentic; we naturally questioned it, and repeated the determination.

The average position of the neutral point for the twenty-one protanopes in Fig. 5 is 496.5 mμ; for the twenty-five deuteranopes it is 504.3 mμ. The averages for our own cases are protanopes 498.2 mμ, and deuteranopes 510.2 mμ. In spite of this distinct difference between the averages of the two types, the individual variation is so large that the neutral point of any single person cannot be used to

identify the type of dichromat he is. This is possible only in terms of his brightness distribution in the spectrum.

With our protanope H. J. we have carefully investigated the effect of brightness on the position of the neutral point. In the brightness range between 25 and 5000 photons we could find no change in its position. Obviously our lowest intensity was well above that for which Koenig (1884) had found a gradual shift in the neutral point.

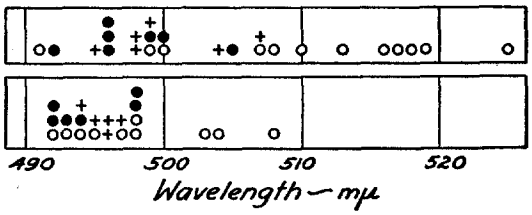

FIG. 5. Distribution of neutral points for twenty-one protanopes (lower box) and twenty-five deuteranopes (upper box). The open circles are our own measurements; the crosses are from Pitt (1935) while the solid circles are from Koenig (1884).

VI

Color Mixture

(*A*) *Uniqueness of Mixtures*

The most revealing characteristic of the dichromat is his capacity to match the spectrum with mixtures of only two primaries. The common formulation of this fact has been that just as a color-normal can match the spectrum with unique mixtures of three primaries, so a dichromat can match the spectrum with unique mixtures of two primaries. Our experience, however, has shown that the dichromat cannot give *unique* values in gauging the spectrum with two primaries.

The reason for this lies in the data of wavelength discrimination as already shown in Fig. 2. For the dichromat between 460 and 520 mμ, Δλ varies from 1 to 6 mμ; but for the rest of the spectrum it becomes rapidly larger. For the spectrum below 450 and above 550 mμ the interval Δλ varies from 10 to 50 mμ. Thus, for example, since the dichromat cannot distinguish between 420 and 450 mμ, a mixture of two primaries made to match 450 mμ will also match 420 mμ provided

brightness differences are eliminated. The match for 450 mμ therefore cannot be unique. This is obvious for spectral regions where $\Delta\lambda$ is large, but it is equally true where $\Delta\lambda$ is small.[4] The dichromat cannot usually discriminate 510 from 515 mμ. Hence a mixture of two primaries which matches 510 also matches 515 and cannot be considered unique for either.

These considerations were forced upon us when we set out to gauge the spectrum of our dichromats with two primaries by the usual procedure which permits the subject to vary the combined brightness as well as the proportions of the two primaries. The results secured in this way were frequently indeterminate, depending on the brightness level, and forced us to adopt a wholly different procedure. We set a specified mixture of two primaries, and the subject determined the limits of λ which he could match with it by varying only the brightness of the mixture.

(*B*) *Procedure*

The primaries were 458.7 and 570.0 mμ, and were located in the right collimator of the Helmholtz Color Mixer, the latter by the position of the collimator as a whole, and the former by the position of the Rochon prism in it. The Nicol prism in the right collimator determined the value of the mixture which appeared in half of the field. In the other half, one wavelength after another was tested to define the boundaries λ_1 and λ_2 between which the dichromat could match the mixture of primaries merely by controlling their combined brightness.

Essentially this is the technic of λ discrimination, except that the standard in half the field is a mixture of two primaries. Moreover the range of λ_1 and λ_2 obviously includes two steps in λ discrimination, because we measured the extreme matching positions to the short-wave end and to the long-wave end for each mixture. The width, however, is not twice that of a single step since the steps to either side are rarely equal.

In this way we tested a series of mixtures, sufficient to cover the spectrum. We maintained a roughly uniform brightness of between 2000 and 5000 photons throughout the spectrum by controlling the slit width and the Nicol prism of the homogeneous light in the left collimator. The collimator slit was no greater than 0.5 mm.; the exit slit at the telescope was 0.4 mm. The energy content of the two primaries was determined as before with the Hilger thermopile and Paschen galvanometer; their relative brightness was then computed from the respective visibility curves in Fig. 3.

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[4] It deserves to be pointed out that the uniqueness of trichromatic matches for color-normals is subject to the same limitations in spectral regions where $\Delta\lambda$ is larger than 2 or 3 mμ.

(C) *Mixture Data*

Table III gives the data for the deuteranope A. W. G. and for the protanope H. J. We measured A. W. G. twice, 1 month apart. The two series are so similar that it would serve no useful purpose to print

TABLE III

Color Mixture of Deuteranope A. W. G. and of Protanope H. J. Spectral Limits λ_1 and λ_2 Matched by Mixtures of Primaries 458.7 and 570.0 mμ

Energy ratio of primaries 458.7 mμ/570.0 mμ	A. W. G.		H. J.	
	λ_1	λ_2	λ_1	λ_2
91.5	415.2	473.8	436.6	469.1
			439.1	478.5
67.1	422.9	472.0		
51.4	436.1	475.4	431.5	469.2
			438.2	470.4
32.8	447.9	475.4	440.2	468.0
22.8	466.1	477.4	446.6	469.1
	464.8	478.6		
12.7	474.6	482.2	451.9	474.2
8.08	481.6	485.4	454.4	474.8
3.99	489.9	492.5		
3.50			473.0	478.6
1.90	496.8	498.7	479.5	482.2
0.754	504.6	506.9	489.5	490.4
0.357	512.6	514.2	496.1	497.4
			495.1	496.8
0.243			502.0	504.1
0.177	519.1	520.3		
0.0837	525.2	528.8	509.3	511.4
0.0546	528.9	532.4	513.5	517.0
0.0333	531.4	541.0	518.5	522.7
0.0181	538.7	568.8	524.0	528.4
0.0114	537.1	606.1	527.5	532.8
0.00782	542.5	615.5	533.2	536.8
0.00497			533.2	542.4
0.00378			534.0	566.4
0.00277			535.6	581.3

them both. The data here given are from the second series. The table records the energy ratio of the two primaries and the limits between which the dichromat matches each of the mixtures. The relative brightnesses of the two primaries 458.7 and 570.0 mμ are 8.6

and 79.5 for A. W. G., and 9.4 and 99.0 for H. J. as taken from their visibility curves in Fig. 3. Therefore to convert energy ratios into brightness ratios in Table III, A. W. G.'s ratios are to be multiplied by 0.0950, and H. J.'s by 0.108,—in both cases by very nearly 1/10.

The data are plotted in Fig. 6, with the logarithm of the energy ratio as ordinates, and the limits λ_1 and λ_2 as abscissas. Because of the log plot, the shape of the relationship remains the same whether the ratio of the primaries is in terms of energy, or of brightness, or of arbitrary units. For rough conversion into brightness ratios subtract 1 from the log values of the ordinates.

The data for the two subjects are not very different. In both there is a stretch between about 480 and 530 mμ where the mixture changes quite sharply with λ, and where the wavelength band corresponding to a specific mixture is very small. This is in keeping with the small values of $\Delta\lambda$ found here in measurements of λ discrimination. To either side of this central stretch the matching band widens very rapidly, and this also is in harmony with the data of λ discrimination.[5] The region of rapidly changing and sharply defined mixtures extends over about 2 log units of ratio of the primaries.

For both subjects the center of this sharp region is about the same distance to the right of the neutral point; the center is at 507 mμ for A. W. G., and at 503 mμ for H. J. For the deuteranope A. W. G. the energy ratio corresponding to this point is 0.661, while for the protanope H. J. it is 0.191. In other words, for the protanope H. J. much less (in energy) of the 458.7 primary and much more of 570.0 primary are required than for the deuteranope A. W. G. to match the region of sharpest discrimination. This difference persists when relative energy

[5] Near the two extremes of the mixture data where the interval $\Delta\lambda$ is large, there are frequently found small patches of the spectrum perhaps 3 mμ wide, which the dichromat cannot match with the specific mixture of primaries used to match the spectrum to the right and left of the non-matching patch. The position of these non-matching islands is quite certain at any time, but is very variable from day to day. The data for these regions as given therefore represent the edges of the matching bands as found by starting from those wavelengths which the dichromat could not match, and working closer and closer until wavelengths were found which he could match. These non-matching islands are not due to the apparatus or method because we did not eliminate them even after many variations in technic and apparatus.

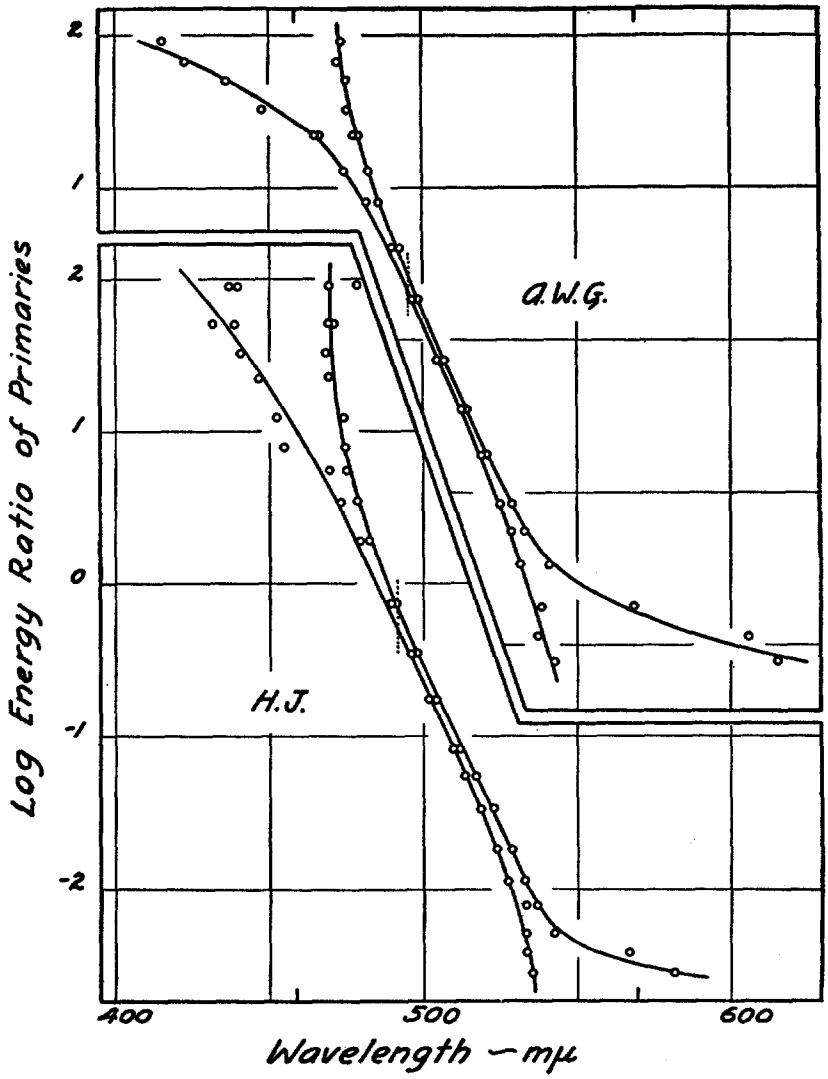

FIG. 6. Color Mixture. The ordinates give the ratio of the amounts of the two primaries 458.7 and 570.0 mμ (shown on the abscissas) which are matched by regions of the spectrum included between the points. A. W. G. is the deuteranope; H. J. the protanope.

is replaced by relative brightness, because the conversion factors are very nearly the same in the two cases, and may indicate a specific difference between the two types of dichromat. This difference is not to be confused with the minor difference in mixture ratios which Wright (1929) first failed to find and Pitt (1935) in his laboratory later did find between the two types of dichromat; Pitt's difference probably depends on the method of measurement which assumes the uniqueness already referred to.

(*D*) *Uniqueness and Brightness*

Examination of Fig. 6 shows that in the central stretch between 480 and 530 mμ the matching range is small, and a reasonable uniqueness may be claimed for a certain mixture matching a given λ. Beyond this region no approach to uniqueness is possible, and the two edges of the matching band become more and more separated. For example, 475 mμ is near the best λ discrimination of H. J. Yet as Fig. 6 shows, the upper and lower limits for the matching mixture differ by over 0.3 log unit, that is by 100 per cent. So rapid is the change here, that for 470 mμ the upper and lower limits already differ by over 0.75 log unit, that is by 600 per cent.

This whole point has previously been overlooked and has resulted in the growth of certain false notions. For example, both Koenig (1887) and Brodhun (1893) were led astray by it and concluded that Newton's law for the addition of brightness was not valid for dichromats. They found, as had van der Weyde (1882) that for dichromats under certain conditions, color matches (mixtures *vs.* homogeneous light) did not remain valid at all intensities. This, if true, is surely a startling situation that must be interpreted and understood.

Brodhun's work will illustrate the situation. With the two primaries 460 and 615 mμ Brodhun, who was a deuteranope, gauged parts of the spectrum at different brightnesses. For 480 and 490 mμ he found that the ratios of the primaries remained constant regardless of the illumination, while for 540 and 560 mμ the ratios varied strikingly with the illumination.

We have repeated these experiments precisely as Brodhun made them, and there is no question of their truth. The only difficulty with them is that they are meaningless. They depend for their exist-

ence first on the non-uniqueness of color matches for dichromats except for very restricted regions of the spectrum, and second on the purely irrelevant fact that in most spectra the two primaries differ considerably in brightness.

The method nearly always employed is to place homogeneous light in one side of a field, and two spectral primaries in the other side, and then to ask the subject to vary the relative amounts of the two primaries as well as their combined brightness in order completely to match the homogeneous light. The dichromat thus has two problems. First, he must select a combination of the two primaries which will resemble the homogeneous light, and second, he must change their combined brightness to match it in brightness as well. The subject does first the one, and then the other, repeating the procedure until a match is secured.

It is apparent from Fig. 6 that at 500 mμ the vertically recorded mixture range is very limited and, therefore, the ratio setting will be almost unique because a small change in the relative amounts of the two primaries will change the recognizable appearance of the mixture. At 540 mμ, however, the mixture range which will produce a match is tremendous. Since the relative brightness of the two primaries is very different, varying the ratio of the primaries also varies the total brightness, and a match can be achieved by this means alone. At low intensities, the match will be made mainly by reducing the brightness of the brighter long-wave primary, and the resulting ratio of long to short-wave primaries will be small. At high intensities the match will be made mainly by increasing the brighter primary, and the ratio of long to short-wave primaries will be large. This is what Brodhun found; but it is due basically to the great range of mixtures which can match 540 mμ, and not to the failure of the third law of color mixture.

We have repeated Brodhun's measurements with our procedure, and the results are as expected. His primaries (416 and 615 mμ) were in the right collimator, and 560 mμ in the left. With the right Nicol prism we set a specific ratio of the two primaries which we knew easily matched 560 mμ for the dichromat, and this ratio remained undisturbed throughout the experiment. The brightness of 560 mμ was then set at a specified value by means of the left Nicol prism.

The dichromat (A. W. G.) was then asked to make the two fields match merely by adjusting the slit of the right collimator which controls the combined brightness of the two primaries. Three readings were made. The brightness of 560 mμ was then increased by changing the left Nicol prism, and A. W. G. again matched it by increasing the common slit of the two primaries. In this way we varied

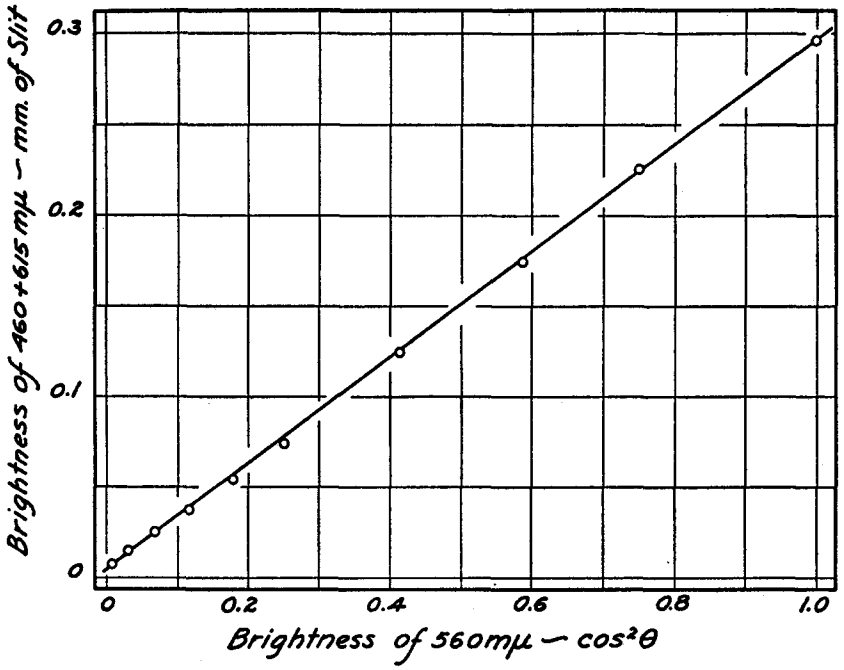

FIG. 7. Brightness and color mixture. There is a linear relation between the combined brightness of the mixture of the two primaries and the brightness of the homogeneous light of 560 mμ which matches them.

the brightness of 560 mμ in ten steps over a range of 1 to 14, the dichromat making three separate readings at each intensity.

The data are in Fig. 7. The abscissas give the intensity of λ560 as $\cos^2\theta$ of the Nicol angle, while the ordinates give the combined intensity of the primaries in terms of slit widths in millimeters. It is obvious that the two are related linearly; that as the brightness of 560 mμ is increased it is necessary to make a corresponding increase

in the brightness of the primaries without changing their relative proportions. We made this type of experiment three times using different primaries and different homogeneous lights, and the results all showed the same thing. In one experiment we even increased the intensity range by a factor of 10 without finding any different result. Therefore, a given mixture will match a given homogeneous light regardless of brightness.[6]

SUMMARY

1. Protanopes and deuteranopes show one maximum of wavelength discrimination which occurs near their neutral point in the region of 500 mμ (blue-green for color-normal). The value of the just discriminable wavelength interval $\Delta\lambda$ is about 1 mμ at this point and is much like the normal. To either side of this, $\Delta\lambda$ rises. It increases rapidly on the short-wave side, and slowly on the long-wave side, rising to about 50 mμ at the two ends of the spectrum.

2. The brightness distribution in the spectrum for dichromats falls only partly outside the range established for color-normals. The protanope curve is narrower than normal, and its maximum lies nearly 15 mμ to the left of it. The deuteranope curves are about the same width as the normal, and their maxima lie slightly but definitely to the right of it. The main difference between protanope and deuteranope spectrum sensitivity lies on the red side of brightness curves, where the deuteranope is strikingly higher. This difference furnishes the only reliable diagnostic sign which may be applied to an individual dichromat for separating the two types.

3. The average position for the neutral point of twenty-one protanopes is 496.5 mμ; of twenty-five deuteranopes 504.3 mμ. The range of variation in the position of neutral point is twice as great for the deuteranope as for the protanope.

4. Dichromatic gauging of the spectrum cannot yield unique mixture values for any wavelength because of the large stretches of poor wavelength discrimination. Data have therefore been secured which locate the spectral ranges that can match specific mixtures of two primaries when brightness differences are eliminated. The form of

[6] Obviously this is true only for strictly foveal fields where the Purkinje phenomenon is avoided, as has been done in all the measurements recorded here.

the data is much the same for a protanope and for a deuteranope; the only difference is in the relative brightness of the primaries.

5. Previously accepted anomalies in the spectral matching of dichromats which have led to the rejection of the third law of color mixture for them, have been eliminated. They are shown to have been due to the non-uniqueness of color matches and the usually disparate brightnesses of the primaries. Color mixture matches for dichromats are valid at all brightnesses.

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