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# THERMAL TIMESCALE MASS TRANSFER AND THE EVOLUTION OF WHITE DWARF BINARIES

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## ABSTRACT

The evolution of binaries consisting of evolved main sequence stars ($1 < M_d/\ \mathrm{M}_\odot < 3.5$) with white dwarf companions ($0.7 < M_{wd}/\ \mathrm{M}_\odot < 1.2$) is investigated through the thermal mass transfer phase. Taking into account the stabilizing effect of a strong, optically thick wind from the accreting white dwarf surface, we have explored the formation of several evolutionary groups of systems for progenitors with initial orbital periods of 1 and 2 days. The numerical results show that CO white dwarfs can accrete sufficient mass to evolve to a Type Ia supernova and ONeMg white dwarfs can be built up to undergo accretion induced collapse for donors more massive than about 2 $\mathrm{M}_\odot$. For donors less massive than $\sim 2\ \mathrm{M}_\odot$ the system can evolve to form a He and CO or ONeMg white dwarf pair. In addition, sufficient helium can be accumulated ($\sim 0.1\ \mathrm{M}_\odot$) in systems characterized by $1.6 \lesssim M_d/\ \mathrm{M}_\odot \lesssim 1.9$ and $0.8 \lesssim M_{wd}/\ \mathrm{M}_\odot \lesssim 1$ such that sub Chandrasekhar mass models for Type Ia supernovae, involving off center helium ignition, are possible for progenitor systems evolving via the Case A mass transfer phase. For systems characterized by mass ratios $\gtrsim 3$ the system likely merges as a result of the occurrence of a delayed dynamical mass transfer instability. A semi-analytical model is developed to delineate these phases which can be easily incorporated in population synthesis studies of these systems.

*Subject headings:* binary: close – stars: evolution – stars: mass loss – stars: cataclysmic variables – supernovae: general

## 1. INTRODUCTION

Close binary systems consisting of main sequence-like stars with white dwarf companions have long been recognized as an important stage of evolution for understanding the formation of several diverse classes of objects. Such systems are products of an evolution of a main sequence-like star with a red giant or asymptotic giant branch companion in which significant mass and orbital angular momentum have been lost. The extremely non conservative evolution facilitated the transformation of the system from a wide orbit with an orbital period of $\sim 1$ year to a narrow one with a period of several days via a common envelope phase where the orbital energy released in the spiralling in process is sufficient to eject the common envelope (for reviews see Iben & Livio 1993; Taam & Sandquist 2000). For the orbital periods of post common envelope systems $\lesssim 0.5$ day, angular momentum losses by magnetic braking are effective (see Pylyser & Savonije 1988) in shrinking the orbit further, producing cataclysmic variable systems. On the other hand, for periods $\gtrsim 0.5$ day, the main sequence-like stars with masses $\gtrsim 1\ \mathrm{M}_\odot$ can evolve to the mass transfer stage as a result of envelope expansion induced by the nuclear burning in the main sequence star's interior. The future evolution of the nuclear evolved systems with donors in the mass range of $1 - 3\ \mathrm{M}_\odot$ may significantly contribute to the formation channels for the production of supersoft X-ray sources (massive white dwarfs accreting at rates sufficient for steady hydrogen burning; van den Heuvel et al. 1992), of a class of Type Ia supernovae models (Li & van den Heuvel 1997), of ultra short period (P $\lesssim$ 30 min) interacting double white dwarf AM CVn systems (Podsiadlowski, Han, & Rappaport 2003), and of detached double white dwarf systems (Nelemans et al. 2001).

Common to the evolution of these systems is the occurrence of a phase of mass transfer on a thermal timescale. Until recently, this population of systems was generally neglected in population synthesis studies since it was assumed that the accreting white dwarf would expand to red giant dimensions as a result of reactivation of hydrogen burning at high mass accretion rates ($\gtrsim 10^{-7}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$). This expansion was hypothesized to lead to the formation of a second common envelope phase and to the eventual formation of a double degenerate system. However, it was pointed out by Kato & Hachisu (1994) that an optically thick wind can be driven from the surface of white dwarfs more massive than about 0.5 $\mathrm{M}_\odot$, thereby stabilizing the mass transfer in the system and making new binary evolutionary channels possible. In this case, the photosphere of the accreting white dwarf lies within its corresponding Roche lobe, and the system can be stabilized preventing evolution into the common envelope phase. The existence of such solutions was made possible by a strong peak in the OPAL opacities at temperatures of about $1.6 \times 10^5$ K (Iglesias, Rogers, & Wilson 1987, 1990; Iglesias & Rogers 1991, 1993; Rogers & Iglesias 1992) achieved in the envelope of white dwarfs accreting at high rates. Recently, observational evidence for the accretion wind picture has been suggested by Hachisu & Kato (2003) based on the long term variability of the light curve of the transient supersoft X-ray source RX J0513.9-6951 (see Reinsch et al. 2000).

Our study is, in part, similar to that conducted by Li & van den Heuvel (1997), but differs in that we map out the boundaries delineating the evolutionary channels leading to the formation of double degenerate dwarfs, neutron stars by accretion induced collapse, and near

Chandrasekhar and sub-Chandrasekhar Type Ia supernova models from such progenitor binary systems. As such, we have carried out detailed binary evolutionary calculations of the Roche lobe filling donor at a stage of evolution between the main sequence and the base of the giant branch at orbital periods for which nuclear evolution rather than angular momentum losses dominate. The evolution of the system is carried through the thermal timescale mass transfer phase to determine its ultimate outcome. To obtain a clear picture of the possible evolutionary histories, the mass and evolutionary state of the donor star and the mass of the white dwarf are systematically varied for systems characterized by initial orbital periods of 1 and 2 days. The assumptions and input physics underlying our calculations are described in §2. The detailed binary sequences and numerical results are described and compared with a semi-analytical picture for the boundaries of the various evolutionary channels in §3. Finally, we summarize and discuss the implications of our results in the concluding section.

## 2. FORMULATION

The binary evolutionary sequences calculated in this investigation are based on a stellar evolution code developed by Kippenhahn, Weigert, & Hofmeister (1967) and updated as described in Podsiadlowski, Rappaport, & Pfahl (2002). The stellar models are computed using a reaction network with rates taken from Rauscher & Thielemann (2000, 2001) (see also Thielemann, Truran, & Arnould 1986) and using OPAL opacities (Rogers & Iglesias 1992), supplemented with opacities at low temperatures (Alexander & Ferguson 1994), for a solar metallicity ($Z = 0.02$).

### 2.1. *Mass transfer*

During phases of the evolution when the donor filled its Roche lobe, the mass transfer rate, $\dot{M}_{\rm tr}$, was calculated in an implicit manner. In this case, $\dot{M}_{\rm tr}$ is found such that the radius of the donor is equal to its Roche lobe radius, $R_{\rm L}$ approximated as (see Eggleton 1983)

$$R_{\rm RL} = \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} A, \qquad (1)$$

where $q$ is the ratio the mass of the donor, $M_d$, to the mass of the white dwarf, $M_{\rm wd}$, and $A$ is the orbital separation. We consider the radius-mass exponents of the Roche lobe $\zeta_{\rm RL} = d \ln R_{\rm L} / d \ln M$ and of the star itself $\zeta = d \ln R / d \ln M$ in our solution method. The response of the Roche lobe to the mass transfer (MT) is solely a function of the mass ratio, whereas the response of the stellar radius to the MT is the function of the MT rate. For a given model, we tabulate values of $\zeta(\dot{M})$. By equating the Roche lobe radius and the predicted stellar radius, one can find the MT rate for further models. The MT rate solution is not necessarily unique since it can be multi-valued. In this case, we accept the smallest value of the MT rate. While the star evolves, the response of the stellar radius evolves as well, and we recalculate the table of $\zeta(\dot{M})$ if the predicted stellar radius differs from the calculated one by $\delta \ln R = 10^{-4}$.

### 2.2. *Growth of the white dwarf*

The ultimate fate of the material transferred to the white dwarf depends on the mass and composition of the white dwarf and the rate of mass transfer. In this study, we consider CO white dwarfs with initial masses between 0.7 $M_\odot$ and 1.15 $M_\odot$ and ONeMg white dwarfs more massive than 1.15 $M_\odot$. The actual growth of the white dwarf is determined by the amount of accreted hydrogen which is eventually converted to helium and heavier elements. In the case of rapid mass transfer, the white dwarf accumulates matter at the rate

$$\dot{M}_{\rm cr} \sim 7.5 \times 10^{-7} (M_{\rm wd}/\ {\rm M}_\odot - 0.4)\ {\rm M}_\odot\ {\rm yr}^{-1}, \qquad (2)$$

with the remaining matter radiatively driven away in a strong optically thick superwind from the white dwarf surface (Hachisu, Kato, & Nomoto 1999). The accumulation ratio, $\eta_H = \dot{M}_{\rm cr} / \dot{M}_{\rm tr} \leq 1$ . For transfer rates less than $\dot{M}_{\rm cr}$, but greater than about $\dot{M}_{\rm low} \sim 10^{-7}$ $M_\odot$ yr$^{-1}$, hydrogen burns steadily to helium ($\eta_H \sim 1$). If the mass transfer rate varies in the range between $\sim 3 \times 10^{-8}$ $M_\odot$ yr$^{-1}$ and $\dot{M}_{\rm low}$, the white dwarf experiences mild recurrent hydrogen shell flashes. For the mass transfer rates below $\sim 3 \times 10^{-8}$ $M_\odot$ yr$^{-1}$, strong unstable hydrogen shell flashes result in nova explosions, and we assume that the evolution is fully non conservative with all the transferred material ejected from the system. In this regime, the white dwarf mass is constant (i.e., $\eta_H = 0$).

In our calculations, $\eta_H$ is based on the work of Prialnik & Kovetz (1995) who considered the accretion of hydrogen-rich matter onto white dwarfs of varying mass and central temperature. Since the accumulation ratios do not sensitively depend on the thermal state of the white dwarf, fits of their results were used for accretion rates ranging from $10^{-9} M_\odot {\rm yr}^{-1}$ to $10^{-6} M_\odot {\rm yr}^{-1}$ for their hot white dwarf models.

In the mass accretion rate regime where hydrogen-rich matter is retained, helium will naturally be accumulated. For a sufficiently massive helium layer the helium will be ignited. The critical mass necessary for helium ignition is dependent on the rate at which helium is processed in the hydrogen burning shell $\dot{M}_{\rm He}$ (Kato & Hachisu 1999). For rates less than $1.3 \times 10^{-6}$ $M_\odot$ yr$^{-1}$, helium shell burning is unstable in the white dwarf envelope. The accumulation ratio, defined as the ratio of the processed material remaining after one cycle of a helium flash to the ignition mass, has been estimated by Kato & Hachisu (1999) as

$$\eta_{\rm He} = \begin{cases} 1, & \\ & -5.9 \leq \log \dot{M}_{\rm He} \lesssim -5 \\ -0.175(\log \dot{M}_{\rm He} + 5.35)^2 + 1.05), & \\ & -7.3 < \log \dot{M}_{\rm He} < -5.9 \end{cases} \qquad (3)$$

where $\dot{M}_{\rm He}$ is units of $M_\odot\ yr^{-1}$. We note that the He shell is always unstable whenever a significant amount of hydrogen is processed to helium.

If $\dot{M}_{\rm He}$ falls below the lower range in Eq. 3, the helium layer is not sufficiently hot to ignite helium shell burning in a thin mass layer. In this case, helium is accumulated in the layer until the increasing density leads to the helium ignition in a thick mass layer of about 0.1 $M_\odot$. This ignition of the helium layer likely leads to the ignition of the CO or ONeMg core disrupting the star as a sub-Chandrasekhar mass Type Ia supernova (see Taam 1980; Livne & Glasner 1991; Woosley & Weaver 1994; Garcia-Senz, Bravo, & Woosley 1999).

### 2.3. *Orbital evolution*

The evolution of the orbital separation is dependent on the prescription for mass and angular momentum loss. Since we consider systems characterized by orbital periods ($P \gtrsim 1$ day) exceeding the bifurcation period (Pylyser & Savonije 1988) the only loss of orbital angular momentum from the system reflects that carried by the ejected matter. In this study we assume that the mass lost in the radiatively driven wind carries the specific orbital angular momentum of the white dwarf. Hence, the rate of angular momentum loss is given by

$$\dot{J} = \frac{M_d}{M_{wd}} \frac{J}{M_d + M_{wd}} \dot{M}_{wind} \tag{4}$$

where the orbital angular momentum, $J$, of the system is

$$J = \frac{M_d M_{wd}}{M_d + M_{wd}} A^2 \frac{2\pi}{P} \tag{5}$$

and $\dot{M}_{wind}$ is the mass loss rate from the system.

The orbital and stellar evolution coupled to the stellar response to mass loss implicitly provides the constraint imposed on the system for determining the rate of mass transfer as described above.

## 3. RESULTS

In order to facilitate an understanding of the detailed numerical results we, first, approximately estimate the boundaries delineating the possible outcomes of the evolution in the next subsection. A description of the detailed binary evolutionary sequences follows in the subsequent subsections.

### 3.1. *Semi-analytical picture*

The evolutionary fate of a given initial binary system is critically dependent on the rate of mass transfer during the Roche lobe overflow phase. As a result, an estimate for the average mass transfer rate is required to determine the approximate boundaries separating the possible outcomes. In the following, we take the average mass transfer rate as

$$\overline{\dot{M}} = \Delta M / t_{th}, \tag{6}$$

where $t_{th}$ is the thermal time-scale, on which the mass transfer operates, and $\Delta M$ is the mass lost by the donor during this mass transfer event. The thermal mass transfer from a MS star or a giant star takes place when the accretor is more massive star and terminates when $M_{\rm d}(t) = M_{\rm d} - \Delta M = M_{\rm wd} + \eta_{\rm H}(\dot{M})\eta_{\rm He}(\dot{M}_{\rm He})\Delta M$. In this description, only $\Delta M = (M_{\rm d} - M_{\rm wd})/(1 + \eta_{\rm H}(\dot{M})\eta_{\rm He}(\dot{M}_{\rm He})\eta_{\rm He})$ can be lost during the thermal mass transfer phase. In the case of the star evolving through the Hertzsprung Gap (HG), the thermal time mass transfer is driven also by the star's own expansion, and complete envelope can be lost ($\Delta M = M_{env}$) during this phase, however we do not treat it separately in our simplified picture.

The thermal timescale $t_{\rm th}$ on which $\Delta M$ is lost, is approximately given as

$$\begin{aligned} t_{\rm th} &= t_{\rm th,\odot} \frac{M_{\rm d}/M_\odot \Delta M/M_\odot}{R_{\rm d}/R_\odot \;\; L_{\rm d}/L_\odot} \\ &= t_{\rm th,\odot} \frac{(M_{\rm d}/M_\odot)^{-2.25} \Delta M/M_\odot}{R_{\rm d}/\ {\rm R}_\odot}. \end{aligned} \tag{7}$$

Here we used the mass-luminosity relation for main sequence stars, $L \propto M^{3.25}$, (this is also correct for stars at HG, since they maintain the same luminosity as at the end of the MS), $t_{\rm th,\odot}$ is the thermal time-scale of the Sun. In this simplified estimate, we do not follow the evolution of the star through the MT, and therefore use only the value of $t_{\rm th}$ at the onset of the MT. The stellar radius, $R$, is identified then with the radius of the Roche lobe and is a function of the orbital period of the binary and the mass ratio of the system. This yields a very simple description for the average mass transfer rate as

$$\overline{\dot{M}} = k \frac{R_{\rm d}/\ {\rm R}_\odot (M_{\rm d}/M_\odot)^{2.25}}{t_{\rm th,\odot}}, \tag{8}$$

where k is some factor of the order of unity, which can be taken from the detailed calculations.

The final mass of the accretor is $M_{wd,f} = M_{wd} + \eta_{\rm H}(\dot{M})\eta_{\rm He}(\dot{M}_{\rm He})\overline{\dot{M}} t_{\rm th}$ (for the complete description for $\eta_{\rm H}$ and $\eta_{\rm He}$ see §2.2). This simple approximate approach permits an exploration of the parameter space for the final product of the binary evolution as a function of initial donor mass, white dwarf mass, and orbital period. As representative examples we illustrate the boundaries for the possible evolutionary fates of systems characterized by initial orbital periods of 1 and 2 days in Figs. 1 and 2 respectively. For $P = 1$ day, where the thermal mass transfer occurs near the main sequence we adopt $t_{\rm th,\odot} \approx 3 \times 10^7$ yr. On the other hand, a 1 $M_\odot$ star in the HG has a thermal timescale of $1.8 \times 10^7$ yr. This corresponds to $t_{\rm th,\odot} \approx 6 \times 10^7$ yr in equation (7). For both Fig. 1 and 2 a value of $k = 1.5$ was used based on the results of our detailed calculations

The parameter range for which a white dwarf can accumulate sufficient mass to either evolve to a 1.4 $\rm M_\odot$ white dwarf required for a Type Ia supernova event (SN Ia) or an accretion induced collapse (AIC) is enclosed within the solid boundaries. The initial mass of the white dwarf is the distinguishing characteristic in these evolutionary channels since CO white dwarfs with initial masses $\lesssim 1.15\ \rm M_\odot$ explode upon the ignition of carbon, whereas ONeMg white dwarfs with initial masses $\gtrsim 1.15\ \rm M_\odot$ collapse to form neutron stars as a result of electron capture processes. The mass and composition of the initial white dwarf component of the system is necessary, but not sufficient to ensure these evolutionary fates, since the donor must supply matter at a rate such that the white dwarf can grow significantly. Only donors of an intermediate mass range can satisfy such a condition since the evolution is truncated for massive donors by the onset of a delayed dynamical mass transfer instability (Webbink 1985; Hjellming 1989) and for low masses by the occurrence of the nova phenomenon. Specifically, the upper limit to the donor's mass is a consequence of the change in the donor's response when the steep entropy profile of the outer envelope layers is removed. In this case stellar contraction is replaced by stellar expansion as the flat entropy profile of the interior layers is exposed when sufficient matter is removed on a thermal timescale. Since the Roche lobe is contracting during this phase, the mass transfer process becomes dynamically unstable. Based on the results of our detailed calculations we adopt a critical mass ratio of 2.9 (for $P = 1$ day) and 3.1 (for $P = 2$ days) above which the delayed dynamical instability (DDI) is found. For systems which enter the DDI phase, it is highly likely that

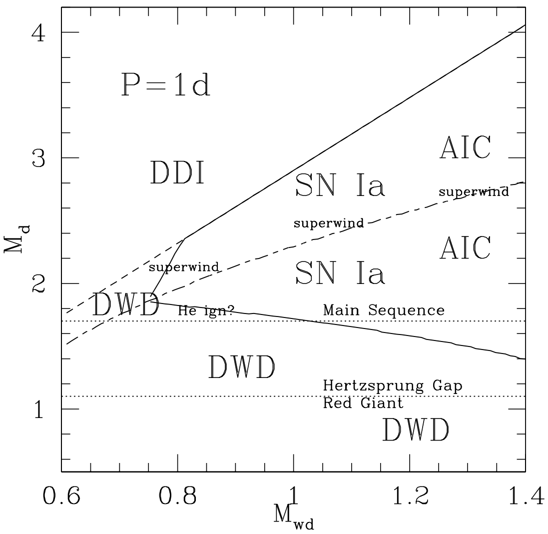

FIG. 1.— Possible evolutionary fates of systems characterized by donors of initial mass $M_d$ and white dwarfs of initial mass $M_{wd}$ in units of $M_\odot$ for an orbital period of 1 day based on a semi-analytical model (see text). The solid curve delineates the area where the white dwarf can accrete sufficient mass for near Chandrasekhar mass models of SN Ia and AIC models of neutron star formation. The dashed curve represents the condition of the occurrence of a DDI (mass ratio q=3.1). Double white dwarf systems are formed at locations marked DWD. For reference, donors which undergo mass transfer above the upper dotted line lie close to the main sequence, those below are in the Hertzsprung gap, and those below the lower dotted line are red giants. Above the short dashed - long dashed line the mass transfer leads to the development of a superwind from the white dwarf. For stars that initiate mass transfer near the main sequence, but for which the rate is insufficient to lead to the superwind or to the SN Ia phase, a sub-Chandrasekhar SN is a possible outcome.

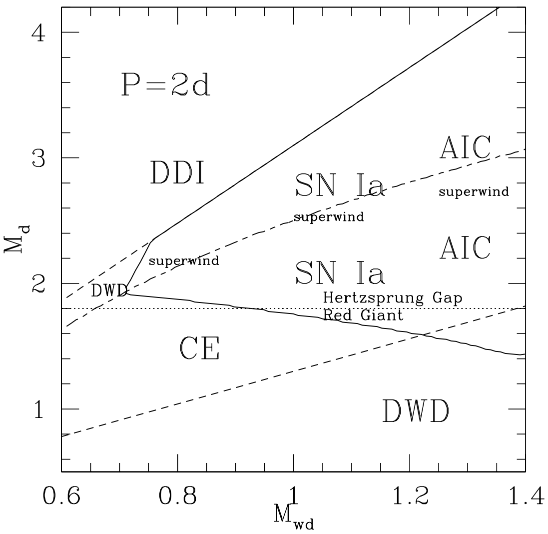

FIG. 2.— Possible evolutionary fates of systems characterized by donors of initial mass $M_d$ and white dwarfs of initial mass $M_{wd}$ in units of $M_\odot$ for an orbital period of 2 days based on a semi-analytical model (see text). In addition to the boundaries described in Fig. 1, a lower dashed curve separates systems which enter into the common envelope phase (corresponding to mass ratios $\gtrsim 1.3$) from those systems which evolve to form double white dwarf systems without requiring a common envelope phase.

the system will merge, leaving behind a rapidly rotating remnant. This upper limit, based on the DDI criterion, overestimates the donor's mass for systems characterized by white dwarfs less massive than 1.05 $M_\odot$ since donors in the mass range between 1.8 $M_\odot$ and 3.3 $M_\odot$ either are not sufficiently massive to build up the white dwarf to 1.4 $M_\odot$ (even assuming an accumulation efficiency of unity) or because the efficiency of accretion is strongly reduced by the radiatively driven wind. A lower limit on the donor mass for the SN Ia and AIC evolutionary scenario is determined by the occurrence of strong unstable hydrogen shell flashes in the envelope of the white dwarf. In particular, the average mass transfer rate as determined by the thermal timescale of the donor is insufficiently high for stars $\lesssim 1.7$ $M_\odot$ to prevent the ejection of significant mass from the system via nova explosions.

The remaining combinations of donor and white dwarf masses primarily lead to the formation of a double white dwarf (DWD) systems. In these cases the mass transfer rate is insufficient to lead to the growth to the SN Ia or AIC phase. We note that there exists a narrow range of parameter space ($M_d \sim 1.7$ $M_\odot$ and $M_{wd} \sim 0.8 - 1$ $M_\odot$) for which a significant layer of helium may be accumulated, giving rise to an evolutionary channel in which a sub-Chandrasekhar mass supernova model may be viable.

The range in donor and white dwarf masses delineating the evolutionary fates of systems in Fig. 2 for an initial orbital period of 2 days at the onset of mass transfer are similar to those described in Fig. 1, allowing for the possibility that white dwarfs can be significantly built up. For this greater orbital period, the main sequence like donors are evolved to a greater extent and the onset of mass transfer occurs after the formation of a helium core when the donor is in the Hertzsprung gap or at the base of the giant branch. As is evident from Fig. 2 there exists a region where the system can enter into a common envelope phase as a result of evolution to the giant stage. The lower dashed line denotes this region corresponding to the condition that the mass ratio equals 1.3 as based on our detailed numerical calculations. For mass ratios greater than 1.3 the system enters into the common envelope phase. In this case the donor has a well defined core-envelope structure typical of red giant stars. A double degenerate system consisting of a He white dwarf with a CO or ONeMg white dwarf in a short period orbit ($P \lesssim 1$ hour) may result depending on the particular evolutionary state of the donor at onset of the common envelope stage (Sandquist, Taam, & Burkert 2000). An additional difference between the systems depicted in Fig. 1 and in Fig. 2 is the absence of systems which can evolve to a sub-Chandrasekhar mass SN. This reflects the fact that the regime where a sufficient helium mass layer accumulates on the white dwarf occurs for systems in which the mass transfer takes place only when the donor is close to the main sequence. For donors of more advanced evolutionary stages in mass transferring systems at initial orbital periods of 2 days, the timescale for evolution is so short that significant accumulation of helium required for a sub Chandrasekhar model is not found.

### 3.2. *Detailed calculations*

We have computed the evolution of binary systems initially consisting of main sequence-like stars of masses 1 -

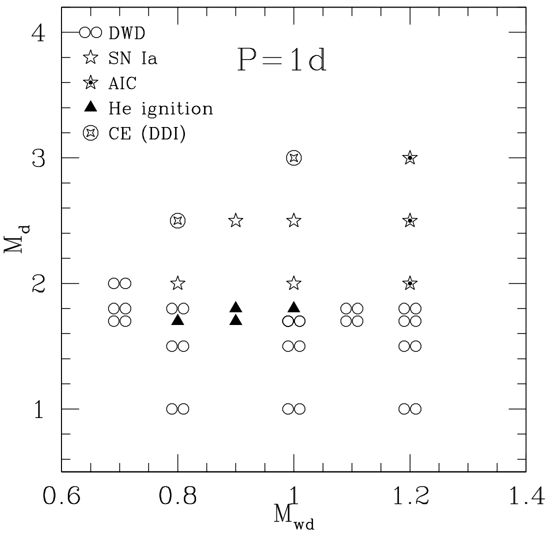

FIG. 3.— The evolutionary fates of the binary sequences calculated for an initial orbital period of 1 day and donors of initial mass $M_d$ and white dwarfs of initial mass $M_{wd}$ in units of $\mathrm{M}_{\odot}$. The symbols representing the fate of the system as a double white dwarf (DWD), a near Chandrasekhar mass supernova model (SN Ia), a helium ignition sub Chandrasekhar model (He ignition), accretion induced collapse (AIC), and an evolution into the common envelope stage via a delayed dynamical mass transfer instability (CE DDI) are shown.

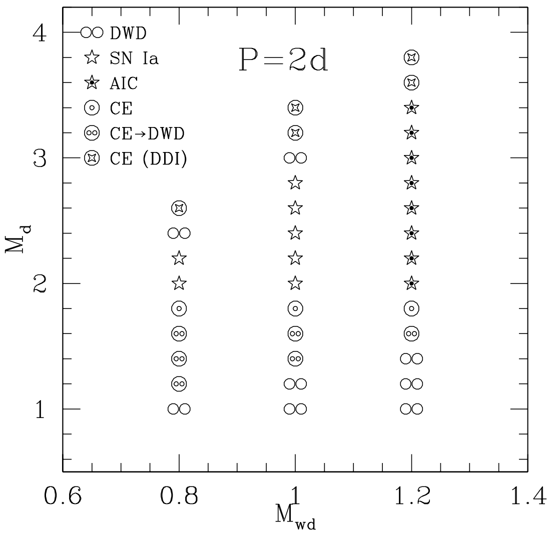

FIG. 4.— The evolutionary fates of the binary sequences calculated for an initial orbital period of 2 days and donors of initial mass $M_d$ and white dwarfs of initial mass $M_{wd}$ in units of $\mathrm{M}_{\odot}$. The symbols representing the fate of the system as a double white dwarf (DWD), a near Chandrasekhar mass supernova model (SN Ia), accretion induced collapse (AIC), and an evolution into the common envelope stage via a dynamical instability (CE) leading to merger and to possibly a DWD system (CE → DWD), and delayed dynamical mass transfer instability (CE DDI) are shown.

3.8 $\mathrm{M}_{\odot}$ and white dwarfs of masses $0.7-1.2$ $\mathrm{M}_{\odot}$ through the thermal mass transfer phase. The evolutionary state of the donor stars was chosen such that the Roche lobe overflow phase was initiated at either an orbital period of 1 or 2 days. To sample the parameter space adequately, a total of 65 evolutionary sequences were calculated with 28 and 37 sequences calculated for systems at initial orbital periods of 1 and 2 days respectively. The systems were evolved until the accreting white dwarf had reached 1.4 $\mathrm{M}_{\odot}$, the donor had evolved to a point where it was clear that the system would evolve into a DWD system, or the system enters into a common envelope phase as a result of either a dynamical or delayed dynamical mass transfer instability.

A visual summary of the fate of all the calculated evolutionary sequences is presented in Figs. 3 and 4 for initial orbital periods of 1 and 2 days respectively. Upon inspection of the boundaries displayed in Figs. 1 - 4, it is clear that the results based on the semi-analytical approach compare very favorably with the detailed binary evolutionary computations. The quantitative results of representative evolutionary sequences are listed in Table 1 and 2 for initial orbital periods of 1 and 2 days respectively. Here, the initial mass of the white dwarf, $M_{wd;0}$, mass of the white dwarf at the end of the calculations, $M_{wd;f}$, the initial mass of the donor, $M_{d;0}$, mass of the donor at the end of the calculations, $M_{d;f}$, the minimum orbital period that the binary system has reached during the computed evolution, $P_{\min}$, the orbital period at the end of the calculations, $P_f$, the evolution time from the onset of the mass transfer phase, $t_{tr}$, and the average mass transfer rate, $\dot{M}_{tr}$ are listed.

In the following, we divide the description of the detailed numerical results into those sequences for which growth of the accreting white dwarf is sufficient to lead to a near Chandrasekhar or sub-Chandrasekhar mass model for SN Ia and to an AIC phase and sequences leading to the formation of double degenerate systems or to a merged object.

### 3.2.1. *Growth of white dwarf to SN Ia or AIC phase*

The progenitor systems which favor the growth of the white dwarf to a near Chandrasekhar mass are characterized by massive white dwarfs ($M_{\mathrm{wd;0}} \gtrsim 0.8$ $\mathrm{M}_{\odot}$). It can be inferred from Tables 1 and 2 that the efficiency of mass accretion by the white dwarf, as defined by the ratio of the mass accreted by the white dwarf to the mass lost by the donor, is a strong function of the mass and evolutionary state of the donor. Specifically, the efficiency of white dwarf growth ranges from about 10% to 70% over the calculated grid with the highest efficiencies obtained for donors with masses $\sim 2$ $\mathrm{M}_{\odot}$. Clearly the mass transfer rates for these donors are in the regime where steady hydrogen burning takes place, but yet the superwind is not so effective in driving a significant amount of matter from the system. That is, the thermal timescale mass transfer associated with donors of $\sim 2$ $\mathrm{M}_{\odot}$ favors a higher efficiency of material accumulation on the white dwarf in comparison to other donors. In contrast, the progenitor systems which lose a significant fraction of their mass via the radiatively driven wind from the white dwarf surface during the evolution generally are characterized by systems with massive donors $\gtrsim 2.5$ $\mathrm{M}_{\odot}$. In fact, the degree to which systems lose mass is exemplified by the systems which evolve to the SNIa or AIC phase. For these systems, the total mass loss from the system can lead to the presence of $\sim 0.4-1.9$ $\mathrm{M}_{\odot}$ surrounding the system at the time the white dwarf has been built up to 1.4 $\mathrm{M}_{\odot}$.

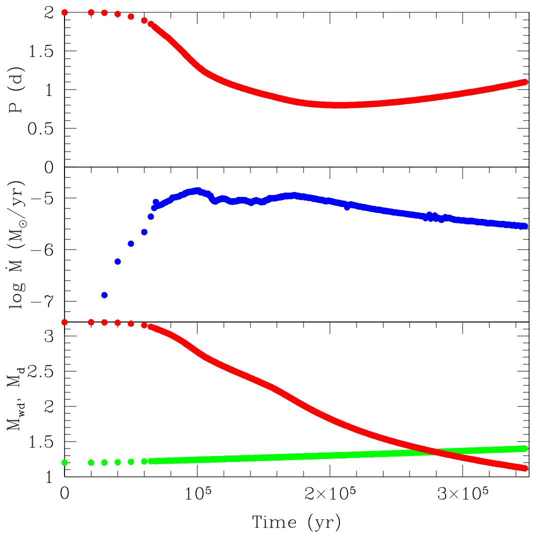

FIG. 5.— Evolution of a binary characterized by $M_{d;0} = 3.2$ M$_\odot$, $M_{wd;0} = 1.2$ M$_\odot$ at an initial orbital period of 2 days. Upper panel: The temporal evolution of the orbital period. Middle panel: The temporal evolution of the mass transfer rate. Lower panel: The variation of the mass of the donor and white dwarf during the evolution.

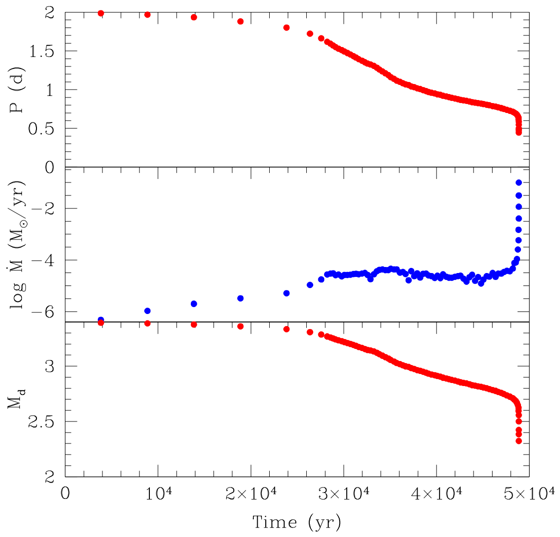

FIG. 6.— Evolution of a binary characterized by $M_{d;0} = 3.4$ M$_\odot$, $M_{wd;0} = 1$ M$_\odot$ at an initial orbital period of 2 days for a system that evolves to a delayed dynamical mass transfer instability. Upper panel: The temporal evolution of the orbital period. Middle panel: The temporal evolution of the mass transfer rate. Lower panel: The variation of the mass of the donor during the evolution. The white dwarf mass is not shown since it increases by only 0.02 M$_\odot$.

An example of such an evolution characterized by an ONeMg white dwarf of $M_{wd;0} = 1.2$ M$_\odot$ and a donor of $M_{d;0} = 3.2$ M$_\odot$ at an initial orbital period of 2 days is illustrated in Fig. 5. In this case the mass transfer rates rise rapidly to $\sim 10^{-5}$ M$_\odot$ yr$^{-1}$ within $10^5$ yrs and averages about $6.8 \times 10^{-6}$ M$_\odot$ yr$^{-1}$ over the entire evolution. The efficiency of the mass accretion onto the white dwarf is low in this sequence amounting to $\sim 10\%$. The orbital period initially decreases by more than a factor of 2 reaching a minimum period of 0.8 days after $2 \times 10^5$ yrs before increasing to a period of 1.1 day at which point the white dwarf has increased to 1.4 M$_\odot$ and the donor has decreased to 1.1 M$_\odot$. The mass loss in such a sequence has been extensive with about 1.9 M$_\odot$ enveloping the system. It is expected that the system would undergo an accretion induced collapse further widening the orbit as the gravitational mass of the compact object is reduced by about $0.1 - 0.2$ M$_\odot$. With the additional expansion of the donor, the system will evolve to longer orbital periods and enter the low mass X-ray binary phase.

We have also found that systems with donors and white dwarfs in a narrow range ($1.6 \lesssim M_{d;0}/$ M$_\odot \lesssim 1.9$, $0.8 \lesssim M_{wd;0}/$ M$_\odot \lesssim 1$) can lead to the accumulation of a sufficient layer of helium mass on the CO white dwarf required for the initiation of an off center helium detonation. The conditions required for the growth of the helium layer are a function of the evolutionary state of the donor since the strong helium flash regime was found to only occur for donors which undergo the MT phase near the main sequence at short orbital periods ($P \sim 1$ day).

### 3.2.2. *Formation of DWD or single merged object*

Although the evolutionary sequences were carried out only through the thermal timescale mass transfer phase, the outcome of an evolution as a DWD system can be inferred once the mass transfer rates in the system have decreased to the point when strong hydrogen shell flashes are expected to occur in the white dwarf envelope. Figures 3 and 4 illustrate that DWD systems can form from progenitor systems with either a low mass ($\lesssim 1.4$ M$_\odot$) or high mass ($\gtrsim 2.6$ M$_\odot$) donor. For the lower mass donors, the temporal evolution of the mass transfer rates lead to nova explosions over part of the evolution such that the white dwarf does not build up to 1.4 M$_\odot$. On the other hand, there is a narrow range for high mass donors where, although the white dwarf can be built up significantly, the donor becomes less massive than the white dwarf during the evolution such that the mass transfer rates have decreased to the extent that accretion is prevented by the occurrence of nova explosions. The DWD systems formed via this channel are expected to have long orbital periods ($P \gtrsim 1$ days).

The evolutionary channel involving a CE phase may also produce DWD systems. The calculations reveal that this evolutionary channel is restricted to the mass transfer taking place after the donor has evolved through the Hertzsprung gap. The response of the star with a deep convective envelope leads to a mass transfer instability and to the evolution into the CE phase. The outcome of this evolution is not well understood, but if the system survives it is likely to have a short orbital period ($P \lesssim 1$ hr). The common envelope calculations carried out by Sandquist, Taam, & Burkert (2000) indicate that evolution into the CE near the base of the giant branch may lead to the successful ejection of the common envelope if the degenerate helium core is more massive than $\sim 0.2 - 0.25$ M$_\odot$.

For more massive donors, the thermal timescale mass transfer phase enters into a period where the mass transfer rate accelerates rapidly, indicating the onset of the delayed dynamical mass transfer instability. For example, a sequence characterized by $M_d = 3.4$ M$_\odot$, $M_{wd} = 1$ M$_\odot$

at an initial orbital period of 2 days is illustrated in Fig. 6. The mass transfer rate accelerates by three orders of magnitude after $\sim 4.9 \times 10^4$ years with the rapid increase occurring over a timescale of $\lesssim$ 200 years. In these sequences, the mass transfer initially takes place on a thermal timescale of the donor until the outer radiative envelope layer has been removed. Further mass loss leads to the exposure of the layers characterized by a flat entropy profile and expansion of the donor results. Since the Roche lobe during this phase is contracting, the system evolves into a CE phase. In this case, merger of the system is likely. The occurrence of this instability occurs at a mass ratio of about 3.1 for donors in the Hertzsprung gap and about 2.9 for donors near the main sequence, confirming earlier estimates by Hjellming (1989).

## 4. SUMMARY

The fate of binary systems which undergo a phase of mass transfer on a thermal timescale has been investigated for binaries composed of main sequence-like donors with white dwarf companions. Allowing for the possibility of an optically thick wind driven from the white dwarf surface gives rise to evolutionary channels in which the white dwarf accretes sufficient mass as required for either a near Chandrasekhar mass or sub-Chandrasekhar mass model for Type Ia supernovae or the formation of neutron stars via an accretion induced collapse model. The range for donor and white dwarf masses delineating the formation of these objects from those producing a pair of degenerate dwarfs has been determined by detailed computations, and reproduced by a simple semi-analytic model, for progenitor systems characterized by initial orbital periods of 1 and 2 days.

Specifically, we have found that CO white dwarfs can accumulate sufficient matter to produce conditions ripe for initiation of a central carbon deflagration/detonation supernova explosion provided that their initial masses are greater than about 0.8 $\mathrm{M}_\odot$ and their donor is more massive than about 2 $\mathrm{M}_\odot$. An upper limit to the donor mass is set by the onset of a delayed dynamical instability which occurs at a mass ratio of $\sim 3$. In addition, the ONeMg white dwarfs (with masses $\gtrsim 1.15$ $\mathrm{M}_\odot$) may accrete sufficient matter to collapse to form a neutron star by electron capture processes for binary systems with donors in the same mass range. During these phases, the mass transfer rates are sufficiently high that hydrogen burning provides the bulk of the energy generation such that the sources are likely to be observed as supersoft X-ray sources (van den Heuvel et al. 1992; Kahabka & van den Heuvel 1997). These evolutionary results are similar to those reported by Li & van den Heuvel (1997) and extend the lower bound of the progenitor white dwarf masses from 0.9 $\mathrm{M}_\odot$ to about 0.8 $\mathrm{M}_\odot$ and less, confirming similar results obtained by Langer et al. (2000) [1]. Although Li & van den Heuvel (1997) did not consider accretion induced collapse in their study, their results are also applicable for this neutron star formation path as well. In these cases, the amount of matter surrounding the system as a result of the wind loss from the white dwarf surface may be as high as $1 - 2$ $\mathrm{M}_\odot$. The existence of such circumbinary material is not dissimilar to that expected for complementary evolutionary channels leading to a SN Ia involving mass transfer from an asymptotic giant branch star to its massive white dwarf companion. We note that additional hydrogen rich matter may also be lost from the system resulting from the interaction of the supernova shell with the donor (Marietta, Burrows, & Fryxell 2000). In the past, one of the arguments put forth against such accreting models is the lack of hydrogen emission in the spectra of SN Ia. However, the recent discovery of hydrogen emission in the observations of the type Ia supernova SN2002ic by Hamuy et al. (2003) provides some support for such models. Although Hamuy et al. (2003) attribute the hydrogen emission from circumstellar matter surrounding an asymptotic giant branch star, the wind present in the accreting white dwarf model in the short period systems investigated here suggests that sufficient matter surrounding the system may be a characteristic of these models as well.

Our detailed systematic investigation has also led to an identification of a channel where a sufficient helium-rich mass layer accumulates ($\sim 0.1$ $\mathrm{M}_\odot$) below the hydrogen-rich layer such that an off center helium detonation may be ignited in the white dwarf envelope. The propagation of the nuclear burning front into the core region, enhanced by geometrical focusing, may lead to the incineration of the entire white dwarf (Woosley & Weaver 1994) for a sub-Chandrasekhar mass star. The range of masses of the progenitor systems as well as the evolutionary state of the donor which follow this evolutionary channel are narrow in the parameter range studied in this paper. That is, only donors in the mass range between 1.6 and 1.9 $\mathrm{M}_\odot$ which transfer matter to their white dwarf companion (with masses in the range $0.8 \lesssim M_{wd}/\ \mathrm{M}_\odot \lesssim 1$) while close to the main sequence are viable. Such limited properties of the progenitor systems may be consistent with the fact that such models are expected to be minor contributors to the SN Ia rate since these models are subluminous compared to models based on the carbon deflagration/detonation near Chandrasekhar mass models.

For donors less massive than $\sim 2$ $\mathrm{M}_\odot$ less matter is transferred to the white dwarf, leading to systems composed of a He white dwarf with a CO or ONeMg white dwarf companion. Of these systems, those that undergo a dynamical mass transfer instability evolve into the common envelope phase. For those systems which survive, the systems are likely to emerge at short orbital periods ($P \lesssim 1$ h), providing an alternative evolutionary channel for the formation of AM CVn binary systems in addition to the channel involving the stable mass transfer evolution of evolved secondaries in cataclysmic variable systems (see Podsiadlowski, Han, & Rappaport 2003). On the other hand, the systems which do not enter into the common envelope phase produce double degenerate systems characterized by orbital periods which are greater than about 1 day.

The systems which undergo an accretion induced collapse likely evolve to a low mass X-ray binary phase in which a main sequence-like star with an evolved core transfers mass to its newly formed neutron star com-

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[1] In a very recent study on SN Ia progenitors an even lower bound for WD masses was reported – 0.67 $M_\odot$ (Han & Podsiadlowski, astro-ph/0309618). This value, however, was based upon the full accretion of hydrogen rich material even for $\dot{M} < \dot{M}_{\rm low}$.

panion. Investigations of this phase have recently been carried out by Sutantyo & Li (2000), who considered the accretion induced collapse scenario, as well as the intermediate mass X-ray binary scenario (also studied by Podsiadlowski et al. 2002). The system evolves to become a binary millisecond pulsar with a helium white dwarf companion in a short period system. Our results suggest that such systems which form via AIC process are likely to have orbital periods greater than about 1 day.

Finally, we point out that the semi-analytical model for the white dwarf binaries that we have developed in this paper can be easily incorporated into population synthesis investigations. We plan to carry out such calculations in the future in order to assess the importance of the formation channels associated with the thermal timescale mass transfer phase in these systems.

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Table 1. The binary parameters and outcomes of representative model sequences for systems in which the onset of mass transfer occurs at a orbital period of 1 day.

| $M_{\rm wd;0}$ | $M_{\rm d;0}$ | $M_{\rm wd;f}$ | $M_{\rm d;f}$ | $P_{\rm min}$ $[d]$ | $P_{\rm f}$ $[d]$ | $\Delta t_{\rm tr}$ $[yr]$ | $\dot{M}_{\rm tr}$ $[M_\odot/yr]$ | outcome |
|---|---|---|---|---|---|---|---|---|
| 0.7 | 2.0 | 1.34 | 0.89 | 0.42 | 0.5 | $5.5{\cdot}10^6$ | $2\cdot10^{-7}$ | DWD |
| 0.8 | 1.5 | 0.81 | 0.75 | 0.82 | 1.02 | $7.9{\cdot}10^7$ | $9.5\cdot10^{-9}$ | DWD |
| 0.8 | 1.7 | 0.9 | 1.25 | 0.69 | 0.69 | $9.5{\cdot}10^6$ | $4.7\cdot10^{-8}$ | subCh |
| 0.8 | 2.0 | 1.4 | 1.17 | 0.52 | 0.55 | $3.9{\cdot}10^6$ | $2.2\cdot10^{-7}$ | SNIa |
| 1.0 | 1.0 | 1.02 | 0.90 | 1.0 | 1.02 | $1.1{\cdot}10^7$ | $9\cdot10^{-9}$ | DWD |
| 1.0 | 2.5 | 1.4 | 1.37 | 0.51 | 0.51 | $1.7{\cdot}10^6$ | $6.6\cdot10^{-7}$ | SNIa |
| 1.2 | 2.0 | 1.4 | 1.70 | 0.86 | 0.86 | $3.3{\cdot}10^6$ | $9.1\cdot10^{-8}$ | AIC |
| 1.2 | 3.0 | 1.4 | 1.64 | 0.5 | 0.5 | $2.9{\cdot}10^6$ | $4.7\cdot10^{-7}$ | AIC |

Note. — The columns denote: the mass of the white dwarf, $M_{\rm wd;0}$ and of the donor $M_{\rm d;0}$ at the onset of the MT (in $M_\odot$); the mass of the white dwarf $M_{\rm wd;f}$ (in $M_\odot$), the mass of the donor $M_{\rm d;f}$ (in $M_\odot$), and the period $P_{\rm f}$ (in days) at the end of the computation; $P_{\rm min}$ (in days) is the minimum period of the binary system during the computed interval of evolution, $\Delta t_{\rm tr}$ $[yr]$ is the time of the MT phase, and $\dot{M}_{\rm tr}$ (in $M_\odot$ yr$^{-1}$) is the corresponding average MT rate. The possible outcomes are: DWD: double white dwarf system, SNIa - supernova type Ia, subCh - sub-Chandraskehar supernova, AIC - accretion-induced collapse.

Table 2. The binary parameters and outcomes of representative model sequences for systems in which the onset of mass transfer occurs at an orbital period of 2 days.

| $M_{\rm wd;0}$ | $M_{\rm d;0}$ | $M_{\rm wd;f}$ | $M_{\rm d;f}$ | $P_{\rm min}$ $[d]$ | $P_{\rm f}$ $[d]$ | $\Delta t_{\rm tr}$ $[yr]$ | $\dot{M}_{\rm tr}$ $[M_\odot/yr]$ | outcome |
|---|---|---|---|---|---|---|---|---|
| 0.8 | 2.0 | 1.4 | 1.25 | 1.05 | 1.08 | $2.{\cdot}10^6$ | $3.8\cdot10^{-7}$ | SNIa |
| 0.8 | 2.4 | 1.32 | 0.32 | 0.58 | 4.2 | $9.1{\cdot}10^6$ | $2.3\cdot10^{-7}$ | DWD |
| 1.0 | 1.4 | 1.0 | 0.2 | 2.0 | 0.02[a] |  |  | CE→DWD |
| 1.0 | 2.0 | 1.4 | 1.46 | 1.44 | 1.44 | $2.6{\cdot}10^6$ | $2.1\cdot10^{-7}$ | SNIa |
| 1.0 | 2.4 | 1.4 | 1.4 | 1.1 | 1.1 | $8.4{\cdot}10^5$ | $1.2\cdot10^{-6}$ | SNIa |
| 1.0 | 2.8 | 1.4 | 0.6 | 0.7 | 1.7 | $7.7{\cdot}10^5$ | $2.9\cdot10^{-6}$ | SNIa |
| 1.2 | 2.0 | 1.4 | 1.67 | 1.7 | 1.7 | $2.5{\cdot}10^6$ | $1.3\cdot10^{-7}$ | AIC |
| 1.2 | 2.6 | 1.4 | 1.97 | 1.4 | 1.4 | $3.6{\cdot}10^5$ | $1.7\cdot10^{-6}$ | AIC |
| 1.2 | 3.2 | 1.4 | 1.1 | 0.8 | 1.1 | $3.1{\cdot}10^5$ | $6.8\cdot10^{-6}$ | AIC |

Note. — The columns are the same as for Table 1; the additional possible outcome, CE→DWD, corresponds to systems that possibly survive the common envelope phase and form a short period double white dwarf system.

[a] determined by equating the orbital energy to the binding energy of the donor's envelope