Analysis of Connectivity Probability in Platoon-based Vehicular Ad Hoc Networks

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Abstract—Vehicular Ad Hoc Networks (VANETs) can provide safety and non-safety related applications and services to improve the passenger safety and comfort. Due to the space and time dynamics of moving vehicles, network connectivity is an important performance metric to indicate the quality of the network and the user’s satisfaction. Grouping vehicles into platoons in the highway can improve road safety, reduce fuel consumption, and decrease traffic congestion. In this paper, we study the connectivity characteristic of platoon-based VANETs. The connectivity probabilities are analyzed for the Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication scenarios. The relationships between the connectivity probability and the key parameters are investigated, including the traffic density, the coverage of the ordinary vehicle, the coverage of the platoon, the coverage of the Road Side Unit (RSU), the distance between two adjacent RSUs and the ratio of the platoon in the VANET. The results can help the transport system designer to control the traffic on the highway to satisfy the connectivity requirement. Analysis results show that the connectivity probability can be significantly improved when there are platoons in a network.

Keywords—Vehicular Ad Hoc Networks (VANETs); platoon; connectivity probability; Vehicle-to-Vehicle (V2V); Vehicle-to-Infrastructure (V2I)

I. INTRODUCTION

Vehicular Ad Hoc Networks (VANETs) have recently received significant interest from both academia and industry. Through Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communications, VANETs can support safety-related and non-safety-related applications and services among vehicles. Specifically, vehicles on the road can communicate with each other through a multi-hop ad hoc connection. They can also access the Internet and other broadband services through the roadside infrastructure, i.e., Road Side Units (RSUs) or access points (APs) along the road [1]. When a vehicle moves out of the radio coverage area of a RSU, it is located in the coverage gap between two adjacent RSUs and will use its neighboring vehicles (if any) as relays to access the RSU [2].

A platoon is a train of vehicles composed of a head vehicle and a number of followers travelling at highway speeds with only a few meters between them, which is expected to reduce traffic congestion and fuel consumption. The function of such vehicle platoon works as an improved version of adaptive cruise control, matching the movement of a vehicle to the distance, speed, and the direction of the vehicle in front. Grouping vehicles into platoons on the highway can improve road safety and energy efficiency [3]. Moreover, once in a platoon, this allows drivers to relax and do other things like reading or even taking a nap while the platoon drives toward its long distance destination. Through cooperation between the platoon members, the data availability in the VANETs can be improved and the data access delay can be reduced [4].

According to the traffic theory [5], the traffic state on a road can be classified into two different phases. One is the free-flow state when the density of vehicles is low and the other is the forced-flow state where there are a large number of vehicles. In the free-flow state, network connectivity is a very important performance metric of the quality of the network and the user’s satisfaction, since it might be difficult to transfer messages to other vehicles in the case of disconnections [6]. A report from the Department of Transportation has indicated that the platooning probability for vehicles on the highway can be higher than 70% [7]. Especially in the free-flow state, vehicles are more likely to form platoons when they travel in the same direction. So it is essential to study the connectivity in platoon-based VANETs according to the new network characteristic, such as the different transmission ranges of vehicles, the different transmission range of RSUs and the different distribution of vehicles.

Connectivity has been studied a lot for conventional VANETs. In the V2V communication scenario, the study in [8] investigated the $k$-connectivity of one-dimensional linear VANETs. It mainly considered the connectivity between the source node and the destination node but not the whole network connectivity. The authors in [9] presented a new analytical framework for determining the connectivity requirements such as the minimum node spatial density and the
minimum required transmission range for distributing traffic information in VANETs. The probability of connectivity was studied based on the equivalent $M/G/\infty$ queue theory in [10]. The authors found the lower and upper bounds based on the mobility patterns of the network. The study in [11] presented an analysis model for multi-hop connectivity of Inter-Vehicle Communication (IVC) in both uniform and non-uniform traffic. In the V2I communication scenario, connectivity is mainly concerned between the vehicles and the RSUs. The authors in [12] developed a distributed connectivity improvement strategy based on the deployment of RSUs to improve the connectivity of VANETs to a desired level while minimizing the energy consumption and signal confliction. The study in [13] considered the effect of the RSU placement on the connectivity of VANETs and determined the minimum number of RSUs required to cover a straight road. This effect has also been studied in [14] by considering the multichannel features of the recently published IEEE 802.11p/IEEE 1609.4 standards. The authors of [15] presented an analytical model to predict both uplink and downlink multi-hop connectivity probabilities in infrastructure-based VANETs. It is observed that none of the previous studies has considered the connectivity in platoon-based VANETs.

In this paper, we focus on the connectivity probability in platoon-based VANETs. The major contribution of this paper is to investigate the connectivity in platoon-based VANETs consisting platoons and ordinary vehicles that are not involved in any platoon. The connectivity probability in platoon-based V2V and V2I communication scenarios are presented. The relationships between the connectivity probability of the VANETs and the key parameters are studied, including the traffic density, the coverage of the ordinary vehicle, the coverage of the platoon, the coverage of the RSU, the distance between two adjacent RSUs and the ratio of the platoon in the VANET.

The rest of the paper is organized as follows. A platoon-based VANET model is derived in Section II. Section III analyzes the connectivity probability of VANETs. Numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. PLATOON-BASED VANET MODEL

We consider an unidirectional and uninterrupted one-way vehicle traffic highway. The VANET under consideration consists of $N$ vehicles, which are randomly distributed along the highway segment. It is assumed that there are $K$ ordinary vehicles and $M$ platoons. Each platoon is seen as a single vehicle in this context and platoon members in each platoon assumed to be connected with each other and can connect with their platoon leader directly. Let $p$ denote the ratio of the platoon in the network, which means that the probability that a vehicle on the highway segment is a platoon. We have

$$p = \frac{M}{N} = \frac{M}{(K + M)}$$  \hspace{1cm} (1)

Then, we can find that the probability that a vehicle on the highway segment is an ordinary vehicle is $1 - p$.

Let $R_1$ and $R_2$, ($R_1 < R_2$), denote the transmission ranges of the ordinary vehicles and the platoon leaders, e.g. trucks with higher-placed antennas. In the paper, we consider $R_1$ and $R_2$ as the coverage of the ordinary vehicle and the platoon. Moreover, it is assumed that $R_2$ is large enough to cover all the platoon members in a platoon and the length of the platoon is smaller than $R_2 - R_1$.

We consider the network scenario where the vehicles are distributed on the highway according to a Poisson distribution. Let $\rho$ be the traffic density in terms of vehicles per meter. Hence, the probability that $k$ vehicles are found in a distance of $x$ meters is expressed by

$$f(k, x) = \frac{(\rho x)^k e^{-\rho x}}{k!}, k \geq 0$$  \hspace{1cm} (2)

Then, the probability that there are no vehicles in an interval with length $x$ on the highway segment is given by

$$g(x) = f(0, x) = e^{-\rho x}$$  \hspace{1cm} (3)

In addition, let $X$ represent the inter-vehicle distance between two consecutive vehicles. We can obtain the probability that the distance between two vehicles is smaller than $x$, which also means that there are at least one vehicle in the interval with length $x$. The probability is given by

$$P_X\{X \leq x\} = h(x) = 1 - e^{-\rho x}$$  \hspace{1cm} (4)

Then, we can find that $X$ is independent identically distributed (i.i.d) and obeys an exponential distribution.

III. ANALYSIS OF THE CONNECTIVITY PROBABILITY

In the free-flow state, connectivity becomes the main performance metric for inter-vehicle communications since a vehicle may have difficulties to deliver messages to other vehicles at the light load. In this section, the connectivity in platoon-based VANETs is studied. Two network scenarios are considered. One is V2V communication scenario where...
the VANET only consists of vehicles. The other one is V2I communication scenario in which we consider both vehicles and RSUs. Then, we study the network connection status in terms of the connectivity probability of VANETs.

A. Connectivity analysis in the V2V communication scenario

Fig. 1 shows the platoon-based VANET communication scenario without RSUs. In the figure, $X_i$ ($i = 1, 2, \ldots, N - 1$) represents the random variable denoting the inter-vehicle distance between two consecutive vehicles. In this scenario, the VANET will be connected if there is a path connecting any pair of vehicles. This shows that the distance between any two consecutive vehicles should be smaller than the transmission range of the vehicles $R$, i.e., $X_i \leq R$. Let $P_c$ be the connectivity probability of the VANET. Then, we have

$$P_c = Pr\{X_1 \leq R, X_2 \leq R, \ldots, X_{N-1} \leq R\} \quad (5)$$

Since $X_i$ is i.i.d random variable, we have

$$P_c = \prod_{i=1}^{N-1} P_i \{X_i \leq R\} = \prod_{i=1}^{N-1} [(1 - p) * P_i \{X_i \leq R_1\} + p * P_i \{X_i \leq R_2\}] \quad (6)$$

Formula (6) describes the relationship among the key parameters, i.e., the connectivity probability ($P_c$), the transmission range of the vehicles ($R_1$ and $R_2$), and the ratio of the platoon in the network ($p$). When the vehicles are distributed on the highway according to a Poisson distribution, according to formula (4), the connectivity probability of the VANET can be given by

$$P_c = [(1 - p)(1 - e^{-\rho R_1}) + p(1 - e^{-\rho R_2})]^{N-1} \quad (7)$$

B. Connectivity analysis in the V2I communication scenario

In the V2I communication scenario, vehicles can access the Internet and communicate with other vehicles through RSUs. Hence, we mainly have interest in the connection between the vehicles and the RSUs. In this section, we consider the communication between the vehicles and a RSU within two hops. We study the one hop direct access and two hops via a relay situation, where an arbitrary vehicle can use a neighboring vehicle located in the coverage of either RSU as its relay to access the RSU.

Fig. 2 shows a platoon-based VANET bounded by two adjacent RSUs. The distance between the two adjacent RSUs is denoted as $L$. Let $R_{RSU}$ denote the transmission range of the RSU. Vehicles within the coverage range of their neighboring RSU can directly access to the RSU in one hop, while other vehicles can use a neighboring vehicle located in the coverage of a RSU as their relay to access the RSU. For instance, $V_j$ is the vehicle located in the coverage gaps between $RSU_1$ and $RSU_2$ and it cannot directly communicate with none of the RSUs. However, it can use a neighboring vehicle ($V_{j-1}$ or $V_{j+1}$) as its relay to access the RSUs.

In this section, let $P_r$ denote the connectivity probability that an arbitrary vehicle can connect with the RSU within two hops. Due to the different values of the transmission ranges of the RSU and the vehicles, the overlapping area between them will have different situations. Therefore, the analysis results will be different based on the relationships between the transmission ranges. Without loss of generality, we discuss five communication scenarios.

1) $0 < L \leq 2R_{RSU}$: In this case, all the vehicles are under the coverage of two adjacent RSUs and can directly communicate with their neighboring RSU. Consequently, the connectivity probability of the network is one, i.e., $P_r = 1$.

2) $2R_{RSU} - L < 2R_{RSU} + R_j$: In this case, Fig. 2 shows that there is a coverage gap between the two RSUs. The probability that a vehicle is located in the coverage of either RSU and can connect to either RSU with one hop is $2R_{RSU}/L$. Moreover, the coverage gap is smaller than the transmission range of the platoon leader. If the vehicle located in the coverage gap is an ordinary vehicle, then $L \leq (2R_{RSU} + R_1)$, the radio coverage of the ordinary vehicle has overlaps with the two RSUs. According to formula (4), the probability that the ordinary vehicle can find at least one vehicle in its vicinity as a relay and connect with either RSU via the relay is

$$p_{10} = \frac{L - 2R_{RSU}}{L} (1 - p)[1 - e^{-\rho (2R_{RSU} + 2R_1 - L)}] \quad (8)$$

Similarly, when $L \leq (2R_{RSU} + R_2)$, if the vehicle located in the coverage gap is a platoon, its radio coverage overlaps with the two RSUs. In this case, the probability that the platoon can connect with either RSU via a relay is

$$p_{20} = \frac{L - 2R_{RSU}}{L} p[1 - e^{-\rho (2R_{RSU} + 2R_2 - L)}] \quad (9)$$
The connectivity probability $P_c$ can be expressed as

$$P_c = \frac{2R_{RSU}}{L} + p_{11} + p_{22}$$

$$= 1 - \frac{2R_{RSU}}{L}[1 - e^{-\rho(2R_{RSU}+2R_1+L)}]$$

$$+ \frac{pe^{-\rho(2R_{RSU}+2R_2-L)}}{\rho} \int_0^{R_1} (1 - p)[e^{-\rho x} - e^{-\rho(2R_{RSU}+2R_2-L)}]dx$$

$$+ (1 - p)[e^{-\rho R_2} - e^{-\rho(2R_{RSU}+2R_2-L)}]$$

$$- \frac{1}{L}(2R_{RSU} + 2R_2 - L)e^{-\rho(2R_{RSU}+2R_2-L)}$$

$$+ (1 - p)(2R_{RSU} + 2R_1 - L)e^{-\rho(2R_{RSU}+2R_1-L)}$$

3) $2R_{RSU} + R_2 < L \leq 2R_{RSU} + 2R_1$: In this case, we consider the vehicle either as an ordinary vehicle or a platoon. If the vehicle located in the coverage gap is an ordinary vehicle, the ordinary vehicle has an overlapping area with both RSUs with the probability $(2R_{RSU} + 2R_1 - L)/L$. If the ordinary vehicle can find at least one vehicle in its vicinity as a relay, it can connect with the RSUs. The connectivity probability between the ordinary vehicle and the RSUs is given by

$$p_{11} = (1 - p)\frac{2R_{RSU} + 2R_1 - L}{L}[1 - e^{-\rho(2R_{RSU}+2R_1-L)}]$$

In addition, with the probability $(2L-4R_{RSU}-2R_1)/L$, the ordinary vehicle can overlap with one RSU. The connectivity probability between the ordinary vehicle and one RSU is

$$p_{12} = (1 - p)\frac{2R_1}{L}\int_{2R_{RSU}+2R_1-L}^{R_1} (1 - e^{-\rho x})dx$$

$$= \frac{2(1-p)[R_{RSU} - R_1]}{L} + \frac{2(1-p)[e^{-\rho R_1} - 1]}{\rho L}$$

If the vehicle located in the gap is a platoon, we have the connectivity probability between the platoon and both RSUs as

$$p_{21} = p\frac{2R_{RSU} + 2R_2 - L}{L}[1 - e^{-\rho(2R_{RSU}+2R_2-L)}]$$

Moreover, the connectivity probability between the platoon and one RSU is given by

$$p_{22} = \frac{2}{L}\int_{2R_{RSU}+2R_2-L}^{R_2} (1 - e^{-\rho x})dx$$

$$= \frac{2p[R_{RSU} - R_2]}{L} + \frac{2p[e^{-\rho R_2} - e^{-\rho(2R_{RSU}+2R_2-L)}]}{\rho L}$$

As a result, the overall connectivity probability that the vehicles connect with the RSUs can be expressed by

$$P_c = \frac{2R_{RSU}}{L} + p_{11} + p_{12} + p_{21} + p_{22}$$

$$= 1 + \frac{2}{\rho L}[p[e^{-\rho R_2} - e^{-\rho(2R_{RSU}+2R_2-L)}]$$

$$+ (1 - p)[e^{-\rho R_1} - e^{-\rho(2R_{RSU}+2R_1-L)}]$$

$$- \frac{1}{L}(2R_{RSU} + 2R_2 - L)e^{-\rho(2R_{RSU}+2R_2-L)}$$

$$+ (1 - p)(2R_{RSU} + 2R_1 - L)e^{-\rho(2R_{RSU}+2R_1-L)}$$

4) $2R_{RSU} + 2R_1 < L \leq 2R_{RSU} + 2R_2$: In this case, when the vehicle located in the coverage gap is an ordinary vehicle, it can only overlap with one RSU and $p_{11} = 0$. The ordinary vehicle has an overlapping area with one RSU with the probability of $2R_1/L$ and the connectivity probability between the platoon and one RSU is given by

$$p_{12} = (1 - p)\frac{2}{L}\int_0^{R_1} (1 - e^{-\rho x})dx$$

$$= \frac{2(1-p)[R_{RSU} + 2R_1] + 2(1-p)[e^{-\rho R_1} - 1]}{\rho L}$$

If the vehicle located in the gap is a platoon, the platoon can either have an overlapping area with both RSUs or one RSU. The connectivity probability analysis results between the platoon and the RSUs will be the same as formula (13) and formula (14).

The overall connectivity probability that the vehicles connect with the RSUs can be given by

$$P_c = \frac{2R_{RSU}}{L} + p_{12} + p_{21} + p_{22}$$

$$= p + \frac{2}{\rho L}[p[e^{-\rho R_2} - e^{-\rho(2R_{RSU}+2R_2-L)}]$$

$$+ (1 - p)[e^{-\rho R_1} - 1] + \frac{2}{L}(2R_{RSU} + 2R_1)]$$

$$- \frac{1}{L}(2R_{RSU} + 2R_2 - L)e^{-\rho(2R_{RSU}+2R_2-L)}$$

5) $L > 2R_{RSU} + 2R_2$: In this case, the vehicles located in the coverage gap can only overlap with one RSU. Hence, $p_{11} = 0$ and $p_{21} = 0$. Similarly, we consider the vehicle either as an ordinary vehicle or a platoon. If the vehicle located in the gap is an ordinary vehicle, the ordinary vehicle has an overlapping area with one RSU with the probability of $2R_1/L$. The connectivity probability analysis result between the ordinary vehicle and one RSU will be the same as it is in Section 4) and will be equal to $p_{12}$.

If the vehicle located in the gap is a platoon, the platoon has an overlapping area with one RSU with the probability of $2R_2/L$. We have the connectivity probability between the platoon and one RSU as

$$p'_{22} = \frac{2}{L}\int_0^{R_2} (1 - e^{-\rho x})dx$$

$$= \frac{2R_2}{L} + \frac{2p[e^{-\rho R_2} - 1]}{\rho L}$$

The overall connectivity probability that the vehicles connect with the RSUs is given by

$$P_c = \frac{2R_{RSU}}{L} + p_{12} + p'_{22}$$

$$= \frac{2R_{RSU}}{L} + 2(1-p)R_1 + 2pR_2$$

$$+ 2[(1-p)e^{-\rho R_1} + pe^{-\rho R_2} - 1]}{\rho L}$$

Based on the proposed model derived in Section II and the above formulas, the relationships between the connectivity probability of the VANET with the key parameters can
be given, which can help the transport system designer to control the traffic on the highway to satisfy the connectivity requirement, e.g., determining the appropriate traffic density, the suitable transmission ranges and the befitting ratio of the platoon. Furthermore, the difference of the connectivity probability between a platoon-based network and a network without platoons can be compared.

IV. NUMERICAL RESULTS

In this section, the network connectivity probability is evaluated in the V2V communication scenario and the V2I communication scenario. We study the impacts of the transmission range of the ordinary vehicle, the transmission range of the platoon, the transmission range of each RSU, the distance between two adjacent RSUs, the traffic density and the ratio of the platoon in the network on the connectivity probability. The results show that with the platoon located in the network, the connectivity probability will be increased both in the V2V communication scenario and in the V2I communication scenario.

A. Analysis results in the V2V communication scenario

The connectivity in the V2V communication scenario denotes that any pair of vehicles in the VANETs can be connected directly or through a multi-hop path.

Fig. 3. Connectivity probability with different traffic density in V2V scenario

Fig.3 shows the connectivity probability of the network with different values of the traffic density ($\rho$) in the V2V communication scenario. It is clear that the connectivity probability increases when there are platoons in the network. For example, when $\rho = 0.02$ vehicles/meter, $R_2 = 800m$ and there are 60 ordinary vehicles and 40 platoons in the network, the connectivity probability in the platoon-based VANET is 0.3349, while it is 0.1604 in a VANET without platoons. When the traffic density is smaller than 0.006 vehicles/meter, the connectivity probabilities in both platoon-based VANETs and VANETs without platoons are very small and close to 0. When the traffic density is larger than 0.006 vehicles/meter, the connectivity probabilities in both VANETs are significantly increased with the traffic densities increase. However, the connectivity probabilities in a platoon-based VANET are always larger than those in a VANET without platoons.

Fig. 4. Connectivity probability with different ratio of platoon in V2V scenario

Fig.4 shows the connectivity probability in terms of different ratio of platoon ($p$). It can be found that, when the ratio of platoon increases in the network, the connectivity probability will improve. The connectivity probability becomes larger as the traffic density increases. It is clear that the network will be fully connected ($P_c = 1$) when the vehicles are all platoons for the case with 800m transmission range.

B. Analysis results in the V2I communication scenario

In the V2I communication scenario, we consider the connectivity probability as the probability that the vehicles on the highway can access to arbitrary RSU beside the road within two hops. The vehicles can connect with the RSUs directly or through a vehicle under the coverage of the RSU as a relay.

Fig. 5. Connectivity probability with different $L$ in V2I scenario

Fig.5 shows the connectivity probability of the network in terms of distance between two adjacent RSUs. It can be found that the connectivity probability will decrease when the
distance $L$ increases. The connectivity probability is higher in platoon-based networks compared to networks without platoons. For example, when the distance $L = 3000m$ and the traffic density are 0.1 vehicles/meter or 0.005 vehicles/meter, the connectivity probabilities in the platoon-based VANET are 0.9133 or 0.8089, respectively, which are larger than those in the case without platoons.

![Connectivity probability in V2I Scenario](image)

**Fig. 6.** Connectivity probability with different traffic density in V2I scenario

Fig.6 shows the connectivity probability in terms of different traffic density with different ratio of the platoon. The connectivity probability increases with higher traffic density or larger ratio of platoon in the VANET. When the traffic density is 0.03 vehicles/meter, compared with the case without platoons ($p = 0$), the connectivity probability can be improved with 15.79% in the platoon-based VANET ($p = 1$) in the V2I communication scenario.

V. CONCLUSION

In this paper, we have analyzed the connectivity probability of platoon-based VANETs where vehicles in the network are Poisson distributed with different traffic density. Our analysis has taken the V2V communication scenario and V2I communication scenario into consideration, respectively. We study the relationships between the connectivity probability and the key parameters, including the traffic density, the transmission range of the ordinary vehicle, the transmission range of the platoon, the transmission range of the RSU, the distance between two adjacent RSUs and the ratio of the platoon in the VANET. These results can help the transport system designer to control the traffic on the highway to satisfy the network connectivity requirement. The numerical results show that the connectivity probabilities in platoon-based VANETs are larger than them in the VANETs without platoons both in the V2V communication scenario and in the V2I communication scenario. In the future, we will study the simulation of this work and extend it to a two-way street scenario.

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