Optimizing Toll Enforcement in Transportation Networks: a Game-Theoretic Approach

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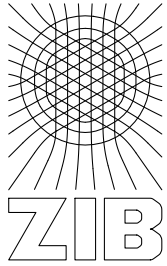

Truck toll in Germany

A Huge and original system

- Started in 2005
- Tax based on distance and truck category (avg price: 0,17 Euro/km)
- Total revenue around 3 Billion Euros / year
- No toll barriers

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Toll Enforcement

- Automatic controls: 300 toll checker gantries (Kontrollbrücke)
- Mobile control units: 300 vehicles - 540 officers from BAG (Federal Office of Goods Transport)

Optimizing Toll Enforcement

Cooperation project between ZIB and BAG

Development of a tool to generate

- feasible duty rosters
- optimal tour planning

Specifications

Mobile inspection teams must carry out a network-wide **control whose intensity is proportional to** given spatial and time dependent **traffic distributions**.

Outline

1 Problem formulation
- Players and Payoffs
- Model examples

2 Computation of Equilibria
- Nash Equilibria for MAXPROFIT
- a MIP formulation for Stackelberg Equilibria

3 Numerical Results

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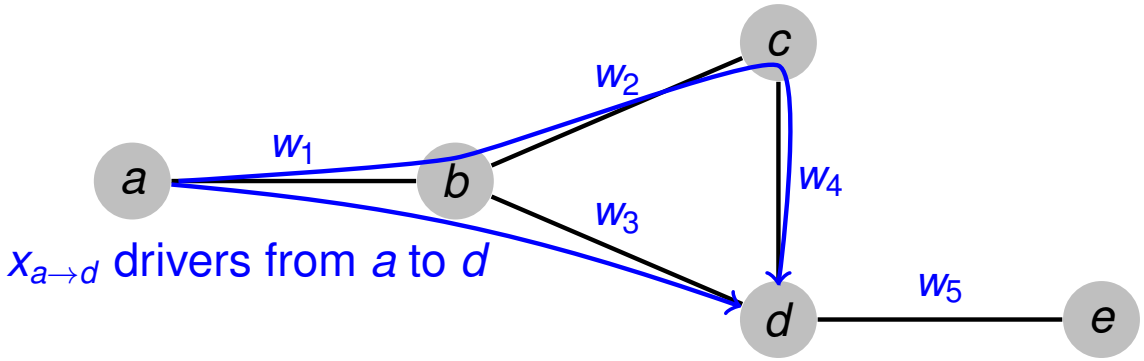
A simple, general model



Notation

$G(V, E)$	:	Graph of the network
$\mathcal{K}$	:	set of commodities (OD pairs) on the network
x_k	:	nb of users (trucks) of commodity $k \in \mathcal{K}$
w_e	:	cost for taking $e \in E$ (travel charge + toll fare)

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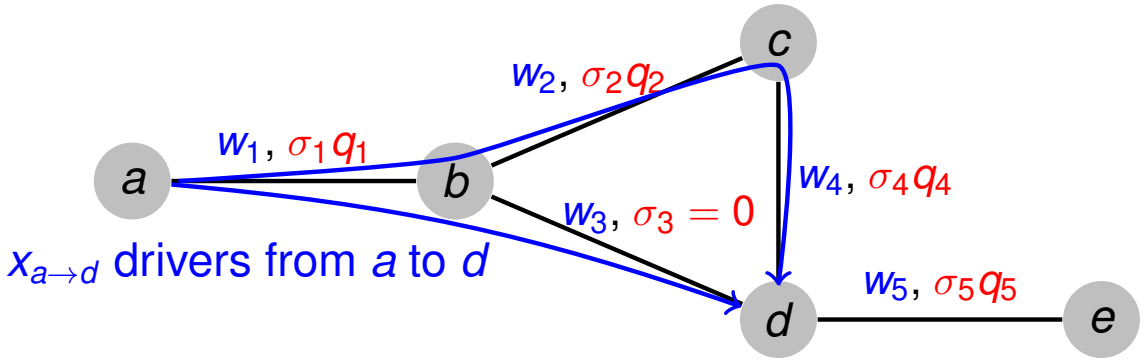

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w_e	:	cost for taking $e \in E$ (travel charge + toll fare)
q_e	:	Probability that an inspector is on section $e \in E$
σ_e	:	Probability to be controlled on e, conditionally to the presence of an inspector

A network spot-cheking game

Given:

DiGraph $G(V, E)$ with costs and control risks $(w_e, \sigma_e)_{e \in E}$,
commodities $\mathcal{K} = \{k_1, \ldots, k_m\}$ with traffic $(x_k)_{k \in \mathcal{K}}$,
amount P of the penalty if a toll evader is caught.

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Assuming:

- stationary regime: users of commodity k have learnt the probabilities π_r to be controlled on each path $r \in \mathcal{R}_k$.
- selfish, rational drivers: Choose their path $r \in \mathcal{R}_k$ so as to minimize their expected loss $\sum_{e \in r} w_e + \pi_r P$.

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Goal:

Distribute the controls q over the edges of the network, to

- maximize the expected revenue (fares + penalties)
- **OR** minimize the number of evaders (or evaded kilometers)

Players of the spot-checking game

- "Player k" represents all users of commodity k.
  - Commits to a mixed strategy $\hat{\boldsymbol{p}}^{\boldsymbol{k}}$, i.e. chooses path r with probability $\hat{p}_r^k$.

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- "The Inspector" represents the set of controllers.
  - Commits to a strategy $\boldsymbol{q} \in \mathcal{Q} \subset [0,1]^E$.

Game Theory Background

Player	Strategy	Payoff
Inspector	$\mathbf{q} \in \mathcal{Q}$	$\Pi_C(\mathbf{p}, \mathbf{q})$, $\mathbf{p} := \{\mathbf{p}_k, k \in \mathcal{K}\}$
$\forall k \in \mathcal{K}$, Player k	$\mathbf{p}_k \in \mathcal{P}_k$	$\Pi_k(\mathbf{p}_k, \mathbf{q})$

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Best response strategies

$$\mathbf{p}^* \in BR(\mathbf{q}) \iff \forall k \in \mathcal{K}, \forall \mathbf{p}'_k \in \mathcal{P}_k, \quad \Pi_k(\mathbf{p}^*_k, \mathbf{q}) \geq \Pi_k(\mathbf{p}'_k, \mathbf{q});$$

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Nash Equilibrium

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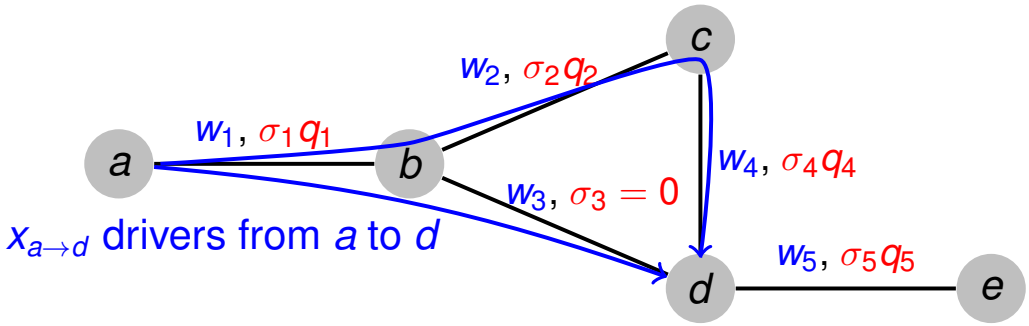
Stackelberg equilibrium

$(\mathbf{p}^*, \mathbf{q}^*)$ is a *Stackelberg equilibrium* iff $(\mathbf{p}^*, \mathbf{q}^*) \in \underset{\mathbf{p} \in BR(\mathbf{q})}{\arg\max} \ \Pi_c(\mathbf{p}, \mathbf{q})$.

Users' best response strategies

Probability to be controlled on path r



$$\pi_r = 1 - \prod_{e \in r} (1 - q_e \sigma_e)$$

$$\simeq \sum_{e \in r} q_e \sigma_e$$

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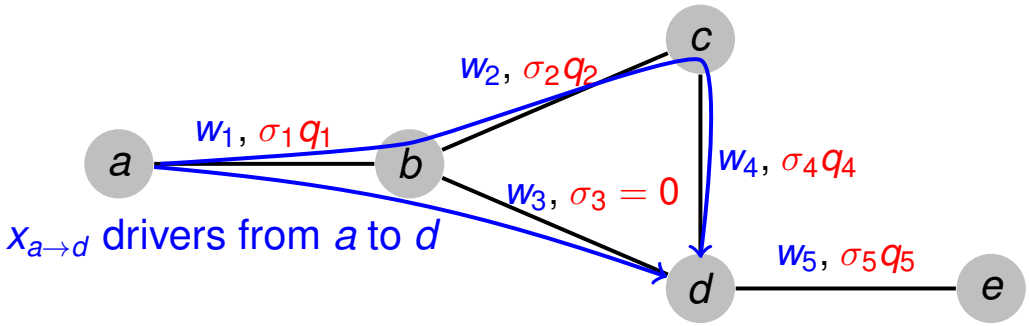

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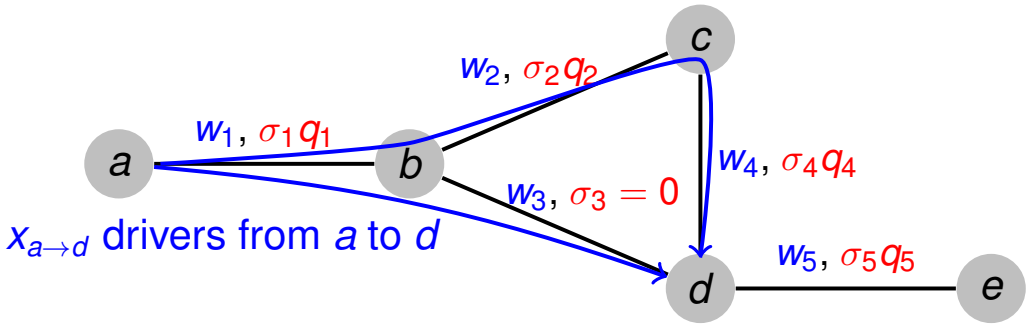

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Expected loss of Player k when choosing path r

$$\Pi_k(r) = -\left(\sum_{e \in r} w_e + q_e \sigma_e P \right)$$

Best strategy for Player k given $\boldsymbol{q} \in \mathcal{Q}$: Choose a shortest path in $G(V, E)$ with augmented weights $w'_e := w_e + q_e \sigma_e P$.

Inspector' s Payoff

- Reward β_e each time a user takes edge e
- Fraction α of all collected Penalties

$$\Pi_C(\boldsymbol{p}, \boldsymbol{q}) = \sum_{k \in \mathcal{K}} x_k \left(\sum_{e \in E} p_e^k (\beta_e + \alpha \sigma_e q_e P) \right)$$

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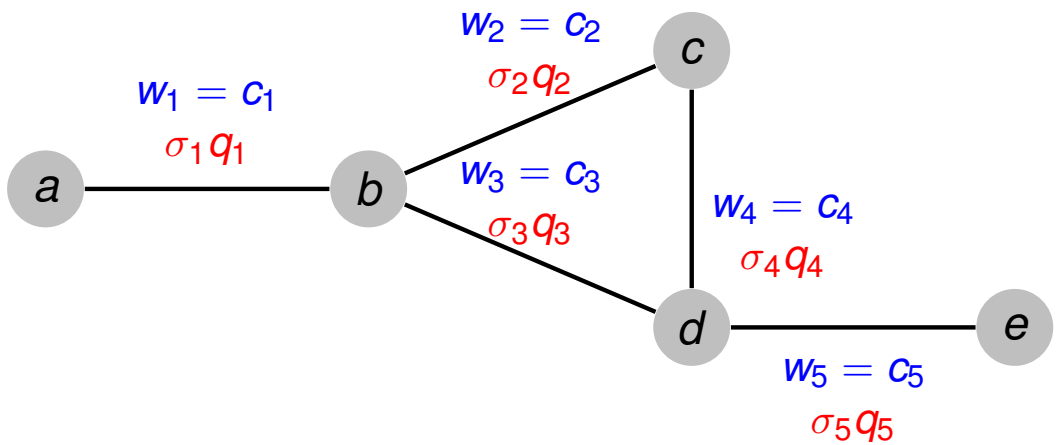
Particular cases

- MAXPROFIT: $\alpha = 1$, $\beta_e =$ toll fare for edge e.
- MAXTOLL: $\alpha = 0$, $\beta_e =$ toll fare for edge e.
- MINEVADERS: $\alpha = 0$, $\beta_e = 1$ when e is an edge representing the option to pay the toll for the whole trip through a commodity.

Model examples: underlying Graph

1. Transit system with fixed access fare T.



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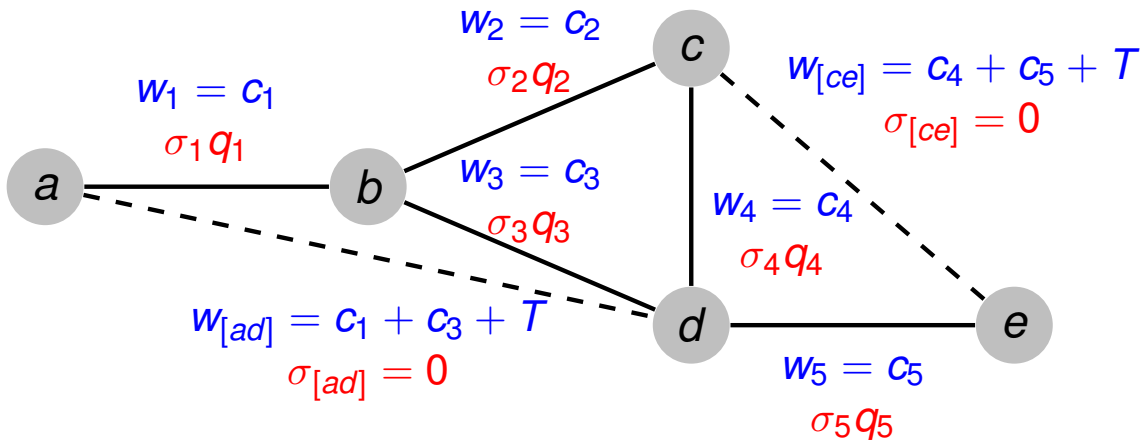
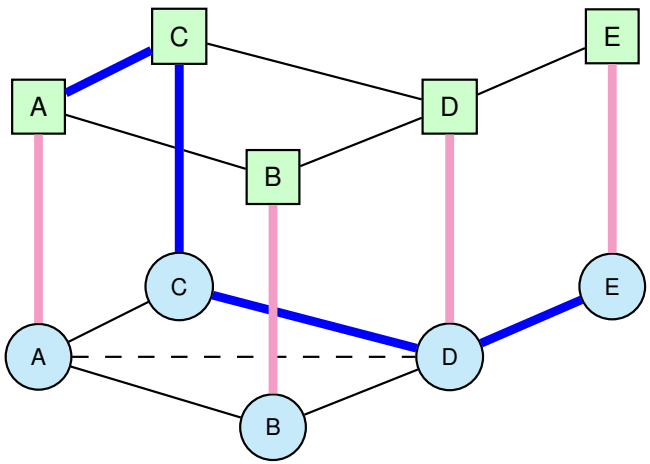


commodity set: $\mathcal{K} = \{a \to d, c \to e\}$

Model examples: underlying Graph

2. Motorway Network with alternative, toll-free trunk roads



TOLL EVASION LAYER

TRANSITION EDGES ($\sigma = 0$, transition cost τ)

MOTORWAY & SHORTCUTS ($\sigma = 0$)

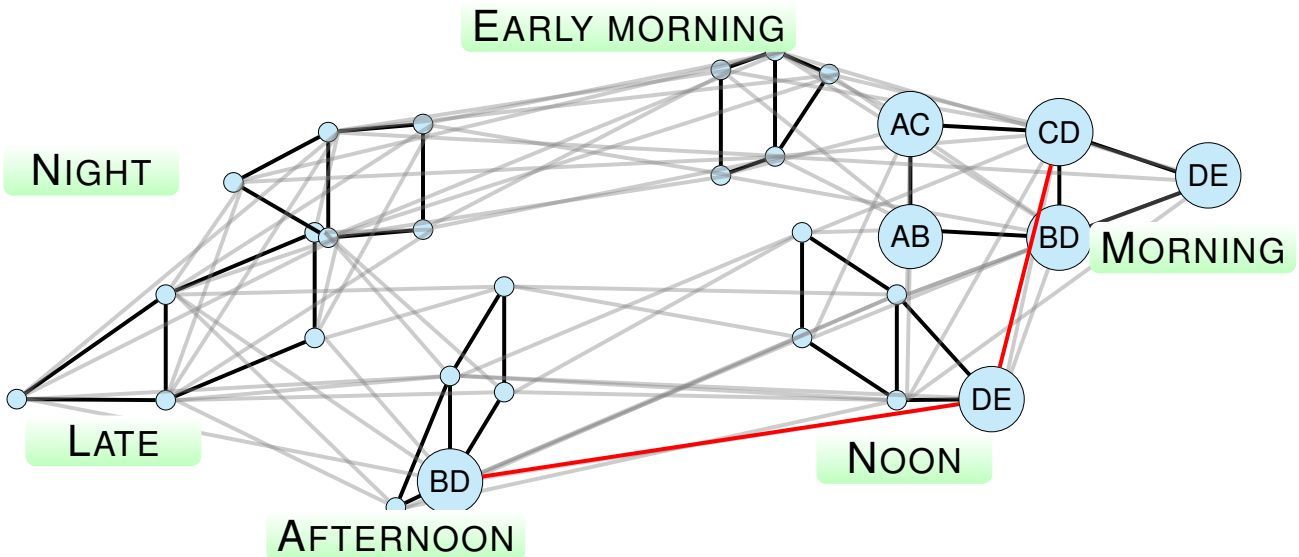
Set of Inspector's strategies $\mathcal{Q}$

1.Ideal distrubution of γ controllers

$$\mathcal{Q} = \{\boldsymbol{q} \in [0,1]^E : \sum_e q_e = \gamma\}$$

2.Feasible Duty rosters

Define $\mathcal{Q}$ as the set of unit flows in a cyclic duty graph:



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Nash Equilibria for MAXPROFIT

Theorem

Although non zero-sum, a Nash equilibrium of the MAXPROFIT spot-checking game can be found by Linear Programming:

$$\max_{\boldsymbol{q},\boldsymbol{\lambda},\boldsymbol{y}} \quad \sum_k x_k \lambda_k \tag{1a}$$

$$\text{s.t.} \quad y^s_v - y^s_u \leq w_{(u,v)} + \sigma_{(u,v)} q_{(u,v)} P, \quad \forall s \in V,\ \forall (u,v) \in E; \tag{1b}$$

$$y^s_s = 0, \quad \forall s \in V; \tag{1c}$$

$$\lambda_k \leq y^{\mathrm{src}(k)}_{\mathrm{dst}(k)} \quad \forall k \in \mathcal{K} \tag{1d}$$

$$\boldsymbol{q} \in \mathcal{Q} \tag{1e}$$

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a MIP for Stackelberg Equilibria

Theorem

A Stackelberg equilibrium of the spot-checking game can be found by solving the following MIP

$$\max_{\boldsymbol{q},\boldsymbol{y},\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\rho}} \quad \sum_{k} \alpha x_k \lambda_k + \sum_{s \in V} \sum_{e \in E} \rho_e^s (\beta_e - \alpha w_e)$$

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$$0 \le w_{(u,v)} + \sigma_{(u,v)} q_{(u,v)} P - (y_v^s - y_u^s) \qquad \forall s \in V, \forall (u,v) \in E;$$

$$y_s^s = 0, \qquad \forall s \in S;$$

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$$0 \le w_{(u,v)} + \sigma_{(u,v)} q_{(u,v)} P - (y_v^s - y_u^s) \le M(1-\mu_{(u,v)}^s), \qquad \forall s\in V, \forall (u,v)\in E;$$

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$$\mu_e^s \in \{0,1\}, \qquad \forall (s,e) \in S \times E.$$

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$$\sum_{\{u:(v,u)\in E\}} \rho_{(v,u)}^s - \sum_{\{u:(u,v)\in E\}} \rho_{(u,v)}^s = \begin{cases} \sum_v x_{(s\to v)} & \text{if } s = v; \\ -x_{(s\to v)} & \text{if } (s\to v)\in\mathcal{K}; \\ 0 & \text{otherwise,} \end{cases} \qquad \forall (s,v)\in S\times V;$$

$$0 \le \rho_e^s \le M\mu_e^s, \qquad \forall (s,e)\in S\times E;$$

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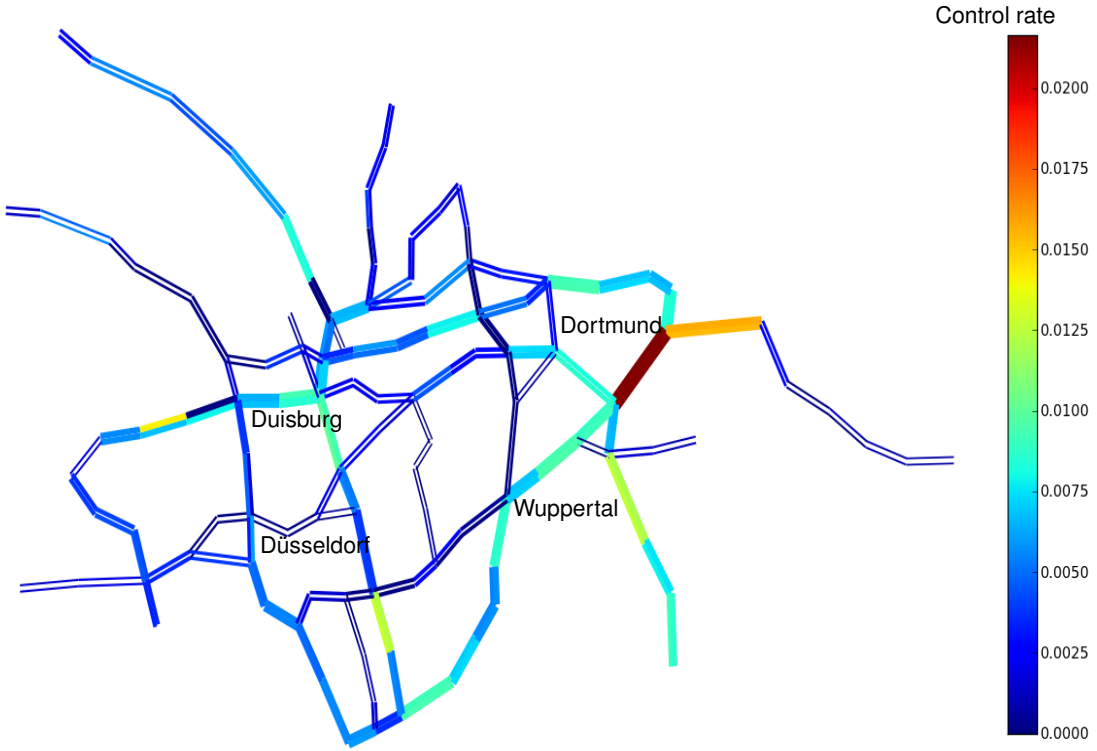
Experimental settings

- The Motorway network is divided into control regions. We solve the problem independently on each region.
- Real traffic data (averaged over time).
- All instances solved with CPLEX.
- We compare the results to the *naive* solution, where controls are proportional to the traffic volume on each edge.
- All the results are presented for the model with one toll-edge for each commodity, and $\mathcal{Q} = \{\boldsymbol{q} \in [0,1]^E : \sum_e q_e = \gamma\}$.

Optimal control strategies

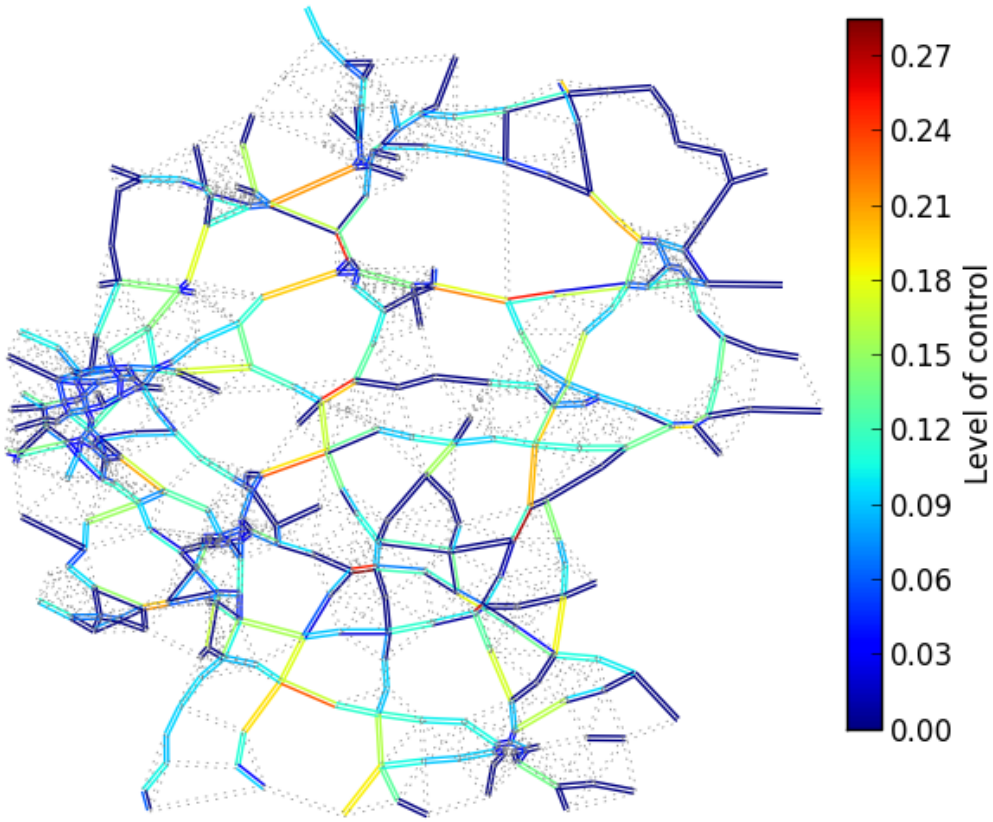
Optimum of the Nash LP,
MAXPROFIT setting, $\gamma = 3$ inspectors:



Region of North Rhine-Westfalia:
111 nodes, 264 directed edges, 4905 commodities.

Optimal control strategies

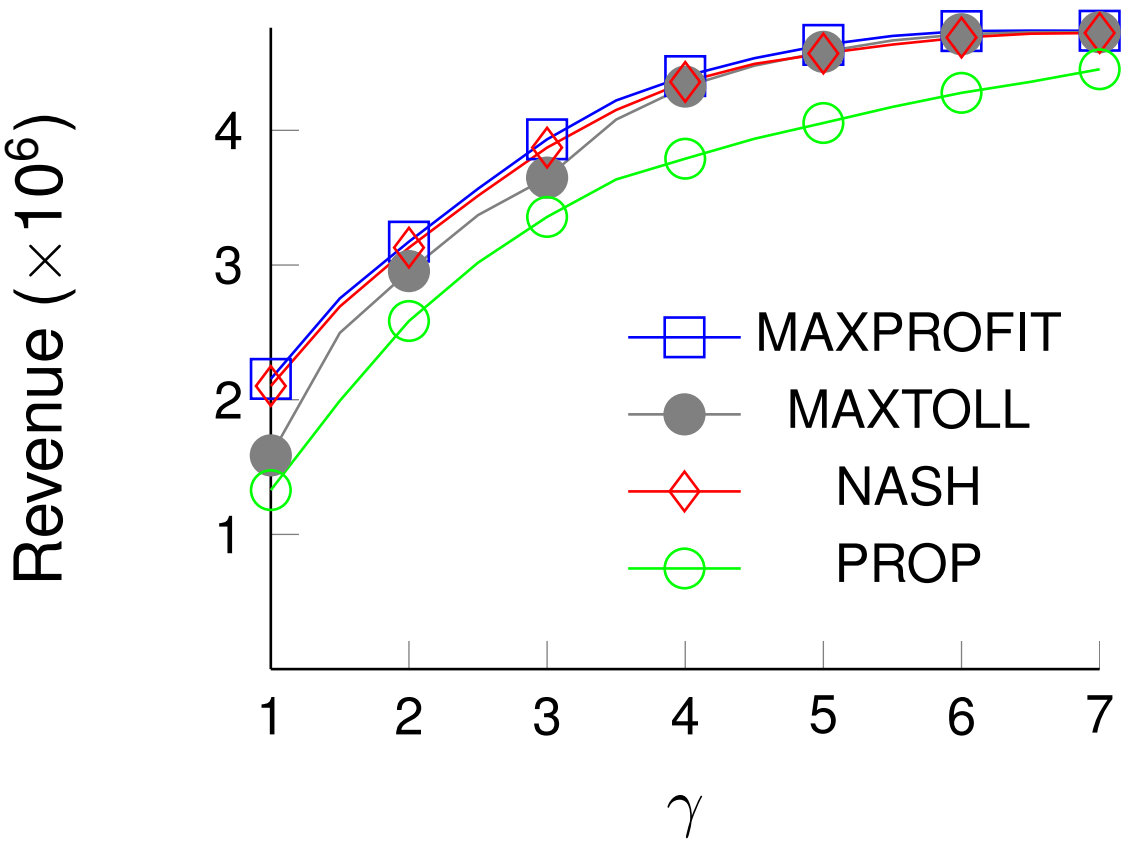
Near optimum of the Stackelberg MIP (gap = 1.5%),
MAXPROFIT setting, $\gamma = 50$ inspectors:



Whole Germany:
319 nodes, 2948 edges, 5013 commodities

Comparison of different equilibria

Region of Berlin-Brandenburg ($|V| = 45$, $|E| = 130$, $|\mathcal{K}| = 596$)



Revenue vs. Number of controllers

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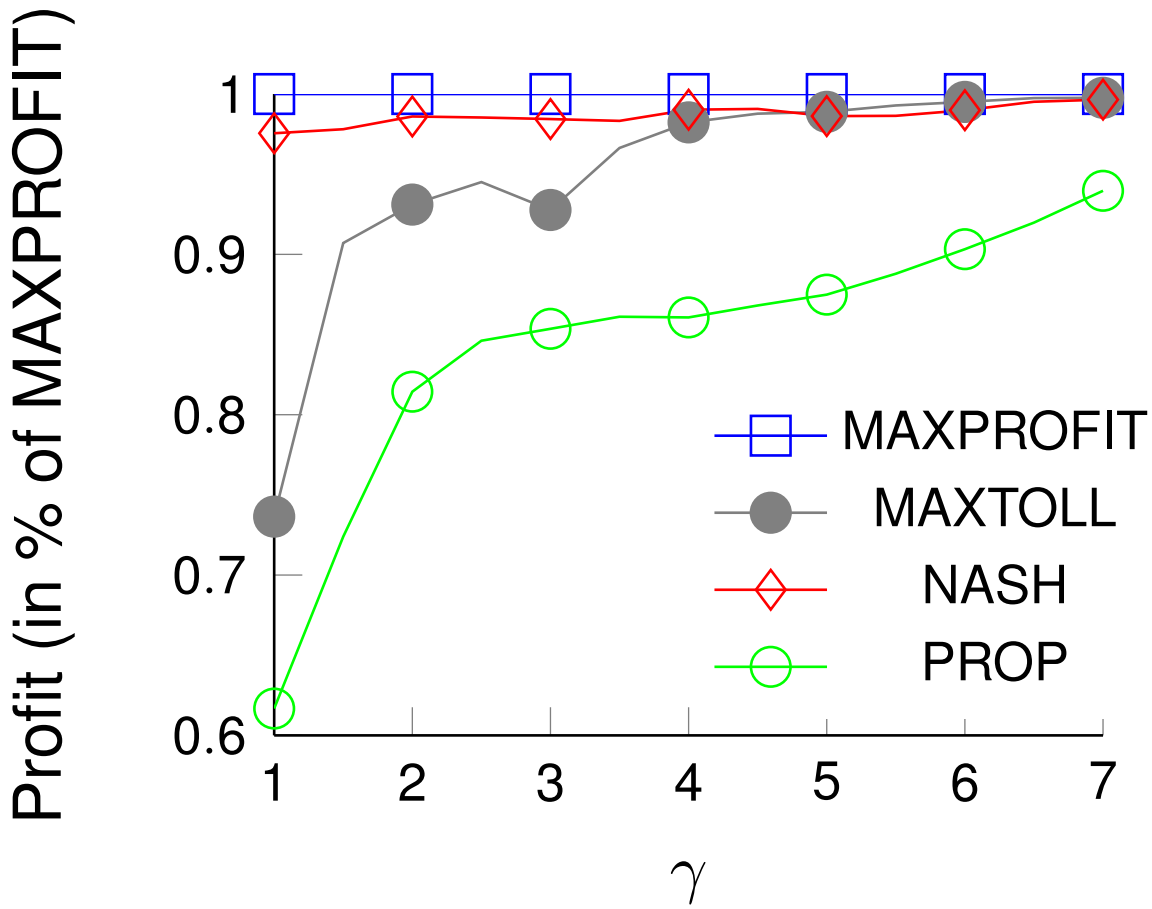

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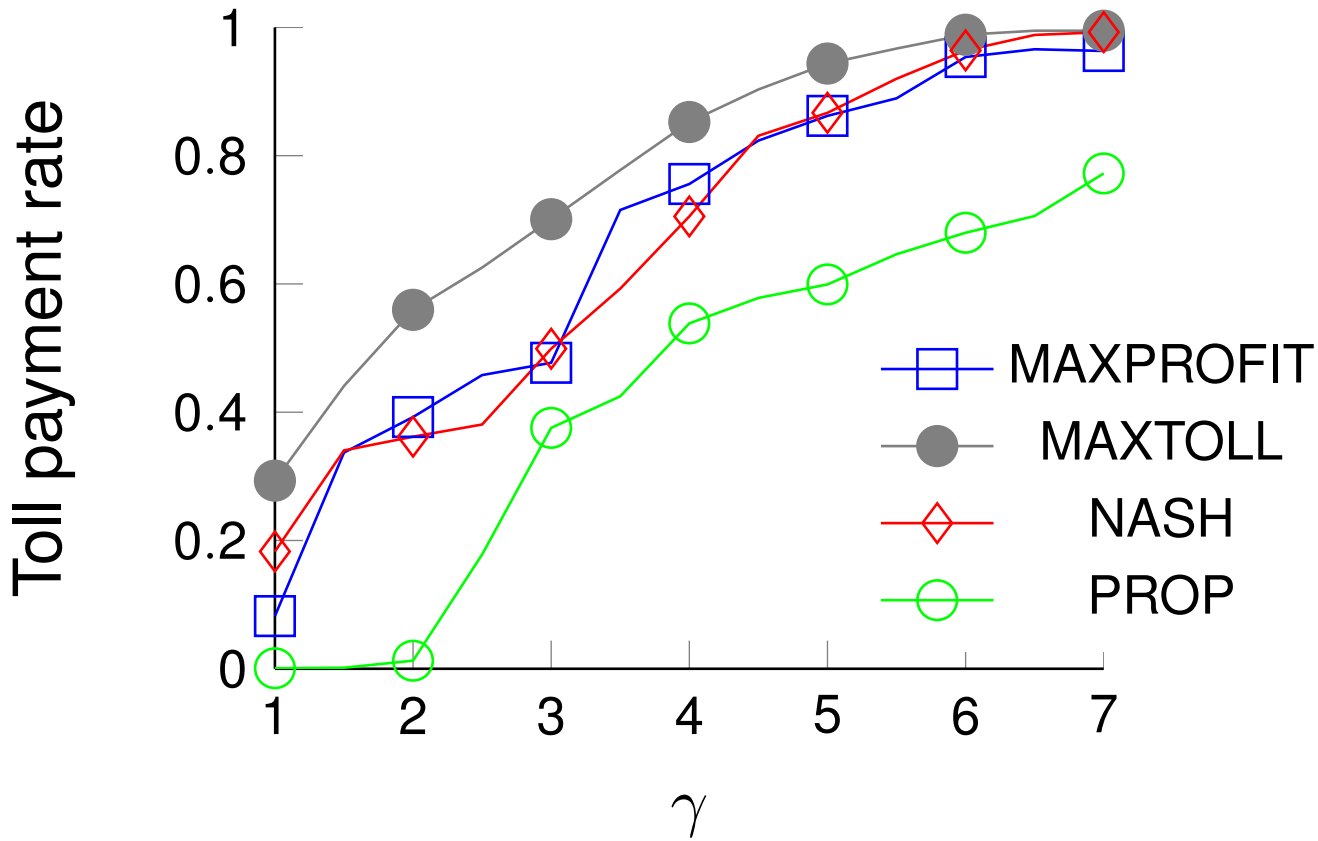

Toll Payment rate vs. Number of controllers

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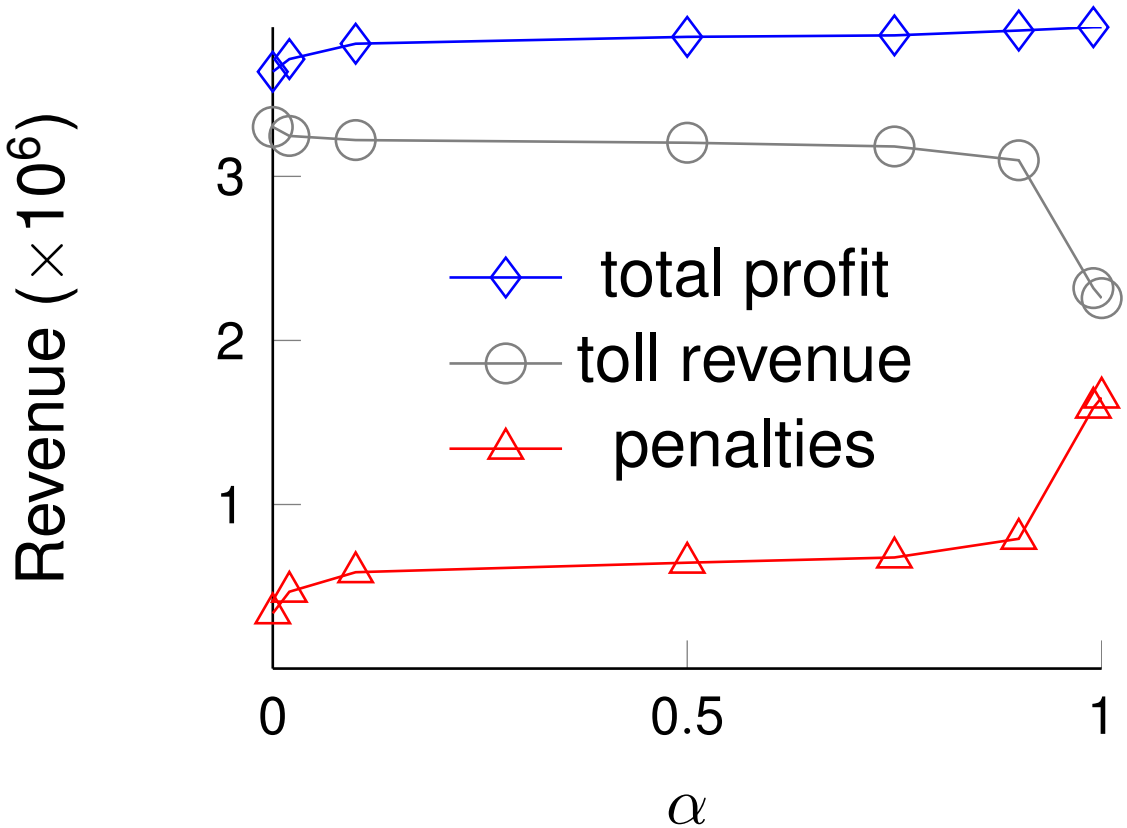

Effect of α on the fraction of toll in Revenue

Conclusion and perspectives

Conclusion

- A flexible framework to model toll enforcement problems in transportation networks
- LP and MIP formulation to compute Nash and Stackelberg strategies
- The Nash LP remains tractable for very large instances and seems to approximate the Stackelberg strategy that optimizes the total revenue.

Future work

- Integrate this approach in our duty roster optimizer
- Investigate theoretical bounds on the gap between Nash and Stackelberg Strategies (for MAXPROFIT)