Curvelet Domain Watermark Detection Using Alpha-Stable Models

Chengzhi Deng, Huasheng Zhu, Shengqian Wang
Department of Computation Science & Technology
Nanchang Institute of Technology
Nanchang, China
dengchengzhi@126.com

Abstract—This paper address issues that arise in copyright protection systems of digital images, which employ blind watermark verification structures in the curvelet domain. First, we observe that statistical distribution with heavy algebraic tails, such as the alpha-stable family, are in many cases more accurate modeling tools for the curvelet coefficients than families with exponential tails such as generalized Gaussian. Motivated by our modeling results, we then design a new processor for blind watermark detection using the Cauchy member of the alpha-stable family. We analyze the performance of the new detector in terms of the associated probabilities of detection and false alarm and we compare it to the performance of the generalized Gaussian detector and the traditional correlation-based detector by performance experiments. The experiments prove that Cauchy detector is superior to the others.

Keywords—watermarking; alpha-stable model; curvelet; locally most powerful

I. INTRODUCTION

In recent years, digital watermarking has attracted increasing interest from many areas as the date security and copyright protection issues are becoming increasingly important [1]. It embeds secrete information into digital multimedia products for copyright notification and protection. Watermarks are ordinarily embedded using linear combinations. And watermark embedding can be realized in space and transform domain. In this work, we focus on multiplicative spread spectrum schemes [2]. Since they automatically implement a simple contrast masking in Watson’s perceptual model and thus achieve a better perceptual quality.

Most current watermark detection methods aim to extract the watermark without use of the original host signal, employing only the secret key. These blind detection methods usually perform correlation detection. However, the correlator is not optimal for non-Gaussian date. Recently, theoretical and more systematic approaches to the problem of watermark detection have been developed. The use of optimal or nearly optimal detectors based on the signal statistics has been proposed and has already been shown to yield significantly better results than the correlator. In DCT domain, detectors based on generalized Gaussian model have been designed in [3-7] and the Cauchy model has been used to improve detection performance in [8, 9]. In wavelet and ridgelet domain, Ng and Xing have proposed maximum likelihood detection using Laplacian model [10, 11].

Within recent years, Candès and Donoho developed a new geometric multiscale transform, named curvelet transform [12-14]. The curvelet transform breaks the limitation of the wavelet transform and provides spares representation for the objects with $C^2$ singularities. It can capture the perceptually important features of image. When watermarks are embedded into those curvelet coefficients, the robustness of watermarks will be enhanced. In this paper, we claim that the alpha-stable family of distributions is sufficiently flexible and rich to characterize the curvelet coefficients at least as accurately as the generalized Gaussian distribution. Motivated by our modeling results, we design a novel statistical watermark detector based on the Cauchy member of the alpha-stable family and we analyze its theoretical performance. We use the Cauchy model as it is the only non-Gaussian alpha-stable distribution with a closed form probability density function (PDF), but also because it leads to improved, robust detection schemes. Finally, we compare the performance of the Cauchy processor with that of the generalized Gaussian detector and the traditional correlation-based detector. We show that it achieves better detection than the generalized Gaussian detector and the traditional correlation-based detector in a wide range of watermark-to-document power levels for real images.

II. CURVELET TRANSFORM AND WATERMARKING

Curvelets were first introduced in [13]. Soon after their introduction, researchers developed numerical algorithms for their implementation. The first generation of discrete curvelet transform is based on ridgelet [12]. It uses a preprocessing step involving a special partitioning of phase-space followed by the ridgelet transform. Due to the complexity of the first generation curvelet and its redundancy, Candès and his coworker proposed a new mathematical architecture of curvelet construction, named the second generation curvelet transform [14]. And they developed two new fast implementations, one based on unequally-spaced fast Fourier transform (USFFT), another based on the wrapping of specially selected Fourier samples. In this paper, we adopt the second generation discrete curvelet transform.
Let \( \mathbf{x} = [x_1, x_2, \ldots, x_N]^T \) and \( \mathbf{y} = [y_1, y_2, \ldots, y_N]^T \) be \( N \)-dimensional vectors representing curvelet coefficients of a host image and the associated watermarked image, respectively. A watermark \( \mathbf{w} = [w_1, w_2, \ldots, w_N]^T \), which is chosen from a set \( \mathbf{M} \), is embedded into \( \mathbf{x} \), giving \( \mathbf{y} \). In this paper, the watermarks are assumed to have a normal PDF \( \mathcal{N}(0,1) \). Then the multiplicative embedding is

\[
y_i = x_i \cdot (1 + \alpha w_i), \quad i = 1, 2, \ldots, N
\]  
(1)

Where \( \alpha \) is a positive scalar representing the embedding strength. The embedding strength is tuned to provide a tradeoff between robustness and imperceptibility of the watermark. Note that \( \alpha \) can be set adaptively for different \( x_i \) by the perceptual analysis, but in this paper, we do not consider it and set it to be the same for all host data.

Cox has claimed that the watermark should be embedded into the perceptually significant features of image to protect. The rationale behind it relies on the fact that, in order to design a robust watermarking method, the watermarks have to be embedded into perceptually significant features of an image, such as edges. In this fashion, attacks which try to destroy the watermarks, are likely to severely affect the features where the watermarks have been embedded thus making the host data unusable after the attack. Curvelet transform can sparsely capture curve singularities of images. Ordinarily, the more energetic the curvelet directional subband is, the richer the feature is. And the features in such curvelet directional subband are the more perceptually significant. As an embedding criterion, we resort to embed the watermark in the most energetic curvelet subband of the image. Given curvelet coefficient \( c(j, \ell, k) \), the most energetic the curvelet directional subband is selected by

\[
\ell_{\text{optimal}} = \arg \max_{\ell} \left[ \sum \sum (c(j, \ell, k))^2 \right]
\]  
(2)

III. LOCALLY MOST POWERFUL DETECTION

As often mentioned, watermark detection resembles the communications problem of reliably transmitting a weak signal through noisy channel (in our case the host image) and detecting its presence at the receiver. Thus, the watermark detection problem can be formulated as a binary hypothesis test, where the two hypotheses concern the existence of a watermark. The two hypotheses for the test are formulated as follows:

\[
H_0: \quad y_i = x_i; \quad H_1: \quad y_i = x_i \cdot (1 + \alpha w_i)
\]  
(3)

The watermark detector based on the likelihood ratio is formulated as follows:

\[
\ell(y) = f_f(y \mid H_1) / f_f(y \mid H_0) > \lambda
\]  
(4)

In this test, there can be two types of errors, namely the false acceptance of \( H_1 \) when \( H_0 \) is true, more commonly known as a “false alarm”, and the acceptance of \( H_0 \) under \( H_1 \), i.e., the “miss” of the transmitted signal. The threshold \( \lambda \) of (4) can be determined by the Neyman-Pearson (N-P) criterion, which minimizes the probability of missing a watermark for a bounded false alarm probability \( P_{fa} \). The resulting test guarantees that the power of the test, i.e., the probability of detection, will be maximized for a predetermined false alarm. By taking logarithm on both sides of (4), we obtain

\[
\ln\left[ \ell(y) \right] = \ln\left[ f_f(y \mid H_1) / f_f(y \mid H_0) \right] > \ln(\lambda)
\]  
(5)

It is well known from detection theory that a test that maximizes the detection probability for any possible \( w_i \) under \( H_1 \), i.e., a Uniformly Most Powerful (UMP) test, is quite rare and can be found only for very particular noise model (e.g. Gaussian data with signal that takes only positive values). Nevertheless, for the detection of signal in non-Gaussian noise environments, a detector that is optimal in the N-P sense can be designed for weak signal. This detector is known as a Locally Most Powerful (LMP) test, since it achieves asymptotically optimum performance for low signal levels. In the watermarking problem the strength \( \alpha \) of the embedded information \( w_i \) is small, so a LMP test is appropriate for it. For \( \ln\left[ f_f(y \mid H_1) \right] \) of (5), we have the following Taylor series approximation at \( \alpha = 0 \),

\[
\ln\left[ f_f(y \mid H_1) \right] = \ln\left[ f_f(y \mid H_0) \right] + \frac{\partial \ln f_f(y \mid H_1)}{\partial \alpha} \bigg|_{\alpha=0} \cdot \alpha
\]  
(6)

Combining Eq. (5) and Eq. (6) yields,

\[
\ln\left[ \ell(y) \right] = \partial \ln f_f(y \mid H_1) / \partial \alpha \bigg|_{\alpha=0} > \hat{\lambda}
\]  
(7)

where \( \hat{\lambda} = \ln(\lambda) / \alpha \). The locally most powerful test (LMP) is

\[
T_{\text{LMP}}(y) = \partial \ln f_f(y \mid H_1) / \partial \alpha \bigg|_{\alpha=0} > \lambda'
\]  
(8)

Where \( I(\alpha) = -E[\partial^2 f_f(y) / \partial \alpha^2] \) is the Fisher information. The LMP is derived from the optimal detector, likelihood ratio test, its performance is optimal for small value of \( \alpha \). By the central limit theorem, the PDF of the LMP statistic in (8) for large data records (when \( N \) is large) is Gaussian model under each hypothesis.

\[
T_{\text{LMP}}(y) = \begin{cases} \mathcal{N}(0,1) & \text{under } H_0 \\ \mathcal{N}(\sqrt{I(\alpha)} \bigg|_{\alpha=0},1) & \text{under } H_1 \end{cases}
\]  
(9)

When the mean and variance of the normally distributed LMP known, the detection and false alarm probabilities are respectively given by

\[
P_{\text{fa}} = P(T_{\text{LMP}}(y) > \lambda') = \int_{\lambda'}^\infty f_{\text{LMP}}(y) \, dT_{\text{LMP}}(y) = Q(\lambda') - \alpha \cdot I(\alpha)^{1/2} \bigg|_{\alpha=0}
\]  
(10)

\[
P_{\text{fa}} = P(T_{\text{LMP}}(x) > \lambda') = \int_{\lambda'}^\infty f_{\text{LMP}}(x) \, dT_{\text{LMP}}(x) = Q(\lambda')
\]  
(11)
where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-t^2/2\right) dt \).

Given the false alarm probability, the threshold \( \lambda' \) can be derived.

\[
\lambda' = Q^{-1}\left(P_{fa}\right)
\]

By predefining this threshold we can find the relation between \( P_{fa} \) and \( P_{det} \), which leads to the Receiver Operation Character (ROC)

\[
P_{det} = Q\left(Q^{-1}\left(P_{fa}\right) - \alpha \sqrt{I(\alpha)_{\alpha=0}}\right)
\]

IV. ANALYSIS OF DETECTION PERFORMANCE

The symmetric alpha-stable family \((S\alpha S)\) is often used to describe non-Gaussian signals characterized by an impulsive nature \([15]\), which exhibit heavy-tailed PDF. There exist closed form expressions for the PDF of \( S\alpha S \) random variables. One is Gaussian, the other is Cauchy. The PDF of Cauchy model is given by

\[
f_x(x) = \gamma \sqrt{\frac{\pi}{\gamma^2 + x^2}}
\]

The Cauchy PDF gives a good enough fit to the heavy-tailed data to ensure that samples in its tails are not “missed”.

Using i.i.d. assumption for the watermarked curvelet coefficients, we can express conditional probability as

\[
f_y(y | H_i) = \frac{1}{\pi} \prod_{i=1}^{N} \gamma \left(1 + \alpha w_i\right) \frac{1}{\gamma^2 + \left(x^2 + \gamma^2\right) (1 + \alpha w_i)}
\]

After taking the derivation of logarithm of \((15)\) at \( \alpha = 0 \), we obtain

\[
\frac{\partial \ln\left[f_y(y | H_i)\right]}{\partial \alpha}_{\alpha=0} = \sum_{i=1}^{N} w_i \frac{x^2 - \gamma^2}{x^2 + \gamma^2}
\]

Further, the Fisher information at \( \gamma = 0 \) is computed as

\[
I(\alpha)_{\alpha=0} = N + 2 \gamma \sum_{i=1}^{N} E\left[\left(x^2 - \gamma^2\right)\left(x^2 + \gamma^2\right)^2\right]
\]

Where we approximate the expectation by its experimental value. Consequently, the locally most powerful test statistic is

\[
T_{Cauchy}^{LMP} = \sqrt{\frac{\sum_{i=1}^{N} w_i \frac{x^2 - \gamma^2}{x^2 + \gamma^2}}{N + 2 \gamma \sum_{i=1}^{N} E\left[\left(x^2 - \gamma^2\right)\left(x^2 + \gamma^2\right)^2\right]}}
\]

\[
P_{det} = Q\left(Q^{-1}\left(P_{fa}\right) - \alpha \sqrt{I(\alpha)_{\alpha=0}}\right)
\]

V. SIMULATION RESULTS

To measure the modeling performances of Laplacian, generalise Gaussian and Cauchy distribution, simulations are carried out. Firstly, we show results on modeling data obtained by applying curvelet transform to the tested natural images. Fig.1 (a) and Fig. 1 (b) show the estimated and the observed densities of the curvelet coefficients of Lena and Peppers image on log scale. From the figure, it can be seen that the histogram is clearly non-Gaussian whereas the Cauchy model is closer to the actual histogram.
respectively. The ROC curves indicate that the Cauchy detector outperforms the GGD detector and Laplacian detector. This is in agreement with the modeling results in Fig. 1, where it was shown that the Cauchy model is better for curvelet coefficients.

VI. CONCLUSIONS

In this paper, we study the statistical model of curvelet coefficients and find that the Cauchy model is more appropriate than generalized Gaussian model and Laplacian model. We analyze the Cauchy locally mostly powerful detector giving the formula of the ROC. In the experiments, the high asymptotical performance of Cauchy detector is verified.

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