An Optimum Design for 3-D Fixture Synthesis in a Point Set Domain

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Abstract—This article addresses the problem of fixture synthesis for 3-D workpieces with a set of discrete locations on the workpiece surface as a point set of candidates for locator and clamp placement. A sequential optimization approach is presented in order to reduce the complexity associated with an exhaustive search. The approach is based on a concept of optimum experimental design, while the optimization focuses on the fixture performance of workpiece localization accuracy. In using the D-optimality criterion to minimize the workpiece positioning errors, two different greedy algorithms are developed for force-closure fixturing in the point set domain. Both 2-D and 3-D examples are presented to illustrate the effectiveness of the synthesis approach.

Index Terms—Fixture synthesis, form-closure, greedy algorithms, workholding.

I. INTRODUCTION

This article describes a research approach to automatic synthesis of a class of fixtures for three-dimensional (3-D) workpieces. A fixture is represented as a collection of fixels, i.e., a set of point locators and clamping elements. For the class of fixtures considered, the fixels must be applied to the workpiece at the locations from a collection of specified locations on the workpiece surfaces. In general, the set of fixture locations available is assumed to be potentially large; for example, the locations might be generated by discretizing the exterior surfaces of the workpiece. The goal of fixture synthesis is to determine an optimal or suboptimal fixture that satisfies fixturing functional requirements, usually among a vast set of feasible configurations. The fixturing requirements include essentially kinematic localization and total fixturing (i.e., form-closure). The optimization objective is to minimize the workpiece positional errors due to workpiece surface and fixel set-up errors.

Thus, the task of fixture synthesis in the point set domain becomes an optimal selection of six locators and one or more clamps among the given N candidate locations. A direct method is the exhaustive search, but it would become impractical for a large number of candidates. The problem is combinatorial in its complexity, and one needs to use an efficient method for a practical solution.

In the approach presented in the paper, the fixture synthesis problem is described as a “design of experiments” in the framework of statistical analysis. The objective of minimizing the workpiece positioning errors is defined as the so-called D-optimality. This leads to the application of an efficient method of greedy algorithms widely used in optimum experimental design. The “greedy” method allows us to find a satisfactory fixture, often suboptimal, without resorting to a complete search of exhaustive type.

II. RELATED WORK

In general, a study of workpiece fixturing suggests one of two kinds of problems: 1) fixture analysis and 2) fixture synthesis. The problem of fixture analysis is to determine the performance of a given fixturing scheme under a set of fixturing requirements such as force closure. Many efficient solution techniques for this problem are reported in the literature. On the other hand, fixture synthesis requires determining a fixturing layout to meet a given set of performance requirements. The analysis problem seems to have received far more attention than the synthesis problem in the literature [1].

The essential requirement of fixturing is the century-old concept of force closure [2], which has been extensively studied in recent years [3]–[5]. There are several formal methods for fixture analysis based on the classical screw theory [6], [7] or other geometric perturbation techniques [8], [9]. Conventional fixture synthesis procedures such as “3-2-1” rules have been described in traditional design manuals [10]. There has been some progress in automatic fixture design with feature-based, geometric-reasoning, or heuristic approaches [11]–[13]. However, the success of these approaches has been limited to parts with prismatic or other simple features. Fixture synthesis tools are yet to emerge for general 3-D workpieces such as turbine airfoils.

Recently, the problem of designing modular fixtures has gained much attention [9], [14], [15], but the new algorithms developed thus far are only for two-dimensional (2-D) or 2 1/2-D polygonal or polyhedral parts. Modular fixtures are difficult to implement for complex shaped 3-D parts, and this fact is evident in industrial production of turbine airfoils.

Extensive research on machining fixture designs is reported recently [16], [17]. A nonlinear programming method is used in [16], similar to an approach to robotic grasping [5]. The method cannot deal with problems of combinatorial nature and is computationally inefficient. A mixed integer programming method with a finite element workpiece model is proposed in [17]. Unfortunately, the method is demonstrated to be effective only for a small scale design, such as additional support considerations. There have also been extensive research activities in the areas of tool accessibility and path clearance, and process planning. Additional results in fixture analysis and design are reviewed in [12] and [9].

A fundamental aspect of fixture performance is the positioning accuracy of workpiece localization. The positioning accuracy is subject to positional variability of the locators and geometric variability of the workpiece. In general, the locator positional variability depends on the dimensioning and tolerancing scheme assigned to the fixture assembly and its components. A complete model of the variability is usually not available at the early stage of fixture design. However, the impact of a locating scheme can be predicted based on a statistical characterization of the positional variability. With this basis, an optimal locating scheme would minimize workpiece positional variations due to the locator positioning errors. When also satisfying the clamping and force-closure conditions, such a fixture is referred to as an optimal fixture. This is the focus of the synthesis approach proposed in this article.

A. Fixture Model

In order to facilitate deriving the optimal synthesis method, we need an appropriate fixture model. This has been well studied in the literature [8], [6], [16]. We adapt the geometric model of but with an algebraic interpretation. For clarity, we describe the model for fixtures of point contacts without friction and for rigid workpiece. At each contact, a surface normal is assumed to be well defined. At the end of the paper, extensions to include finite contact regions, friction, and contact compliance are discussed.

Following the geometric perturbation analysis presented in [8], the workpiece geometric boundary is represented by a piecewise differentiable function \( g \) in terms of body-embedded coordinates \( r \) as \( g(r) = 0 \). Suppose that the \( i \)th locator is in contact with the workpiece, then it would satisfy the workpiece surface function \( g(x_i) = 0 \).

Let's introduce a small perturbation in the location of the workpiece \( \delta g_i = \{\delta \theta_i, \delta \tau_i\} \), including both the orientation and the position. This perturbation results in a perturbation in the function value \( g(r_i) \)

\[
\delta g_i = \delta g(r_i) = -[\nabla g_i \cdot (r_i \times \nabla g_i)^T] \delta q
\]

where \( \nabla g_i = \nabla g(r_i) = \partial g/\partial r_i \) is the gradient vector of function \( g(r) \). Here, \( \delta g_i \) is known as the algebraic distance of a point separated from a geometric surface [20]. For a piecewise differentiable workpiece surface, the algebraic distance can be normalized with respect to the norm \( ||\nabla g_i|| \) of the gradient vector to yield the following perturbation equation:

\[
\delta g_i/||\nabla g_i|| = -[n_i^T (r_i \times n_i)^T] \delta q
\]

where \( n_i = \nabla g_i/||\nabla g_i|| \), defining the unit normal at the \( i \)th contact point on the surface. We rewrite the equation as

\[
\delta y_i = h_i^T \delta q \quad \text{where} \quad h_i^T = -[n_i^T (r_i \times n_i)^T]
\]

with the normalized algebraic distance \( \delta y_i = \delta g_i/||\nabla g_i|| \). It is also easy to show that

\[
\delta y_i = n_i^T \delta r_i
\]

where \( \delta r_i \) is the resulting perturbation of the locator position vector \( r_i \) with respect to the workpiece. Thus, \( \delta y_i \) represents the projection of the locator positional error along the normal direction of the workpiece surface.

For a fixture of \( n \) locators, a collection of their individual perturbation equations is defined by

\[
\delta y = G^T \delta q
\]
where \( \delta \mathbf{y} = [\delta y_1 \ \delta y_2 \ \cdots \ \delta y_n]^T \) and \( \mathbf{G} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \cdots \ \mathbf{h}_n] \). Here, matrix \( \mathbf{G} \) is called the locator matrix and it is defined entirely by the locator locations specified on the workpiece surface.

**B. Optimal Localization**

From (5), we have

\[
\| \delta \mathbf{y} \| = \delta \mathbf{y}^T \delta \mathbf{y} = \mathbf{eq}^T (\mathbf{G} \mathbf{G}^T) \mathbf{eq}
\]

which shows that the fixture localization errors \( \delta \mathbf{y} \) are on the same order of magnitude as the locator positioning errors \( \delta \mathbf{y} \) and, clearly, they depend on locations of the locators as defined by the locator matrix \( \mathbf{G} \).

The primary objective of our optimal fixture synthesis is to minimize the localization errors \( \delta \mathbf{y} \).

In general, the locator positioning errors \( \delta \mathbf{y} \) depend on the dimensioning and tolerancing scheme assigned to the fixture assembly and its components. In the early stage of fixture design, only variations or tolerances of the error sources may be known. Therefore, the locator errors may be considered as statistical variables. If it is assumed that the tolerances of the error sources may be known. Therefore, the locator errors follow independent normal distribution \( N(0, \sigma^2) \), then variances of the localization error \( \delta \mathbf{y} \) are given as

\[
\text{var}(\delta \mathbf{y}) = \sigma^2 (\mathbf{G} \mathbf{G}^T)^{-1}.
\]

Here, from a statistical viewpoint matrix \( \mathbf{G} \mathbf{G}^T \) completely characterizes the total variance in the workpiece position and orientation. We shall denote that \( \mathbf{M} = \mathbf{G} \mathbf{G}^T \) and shall refer to it as contact information matrix (also known as Fisher information matrix in statistics).

**C. D-Optimality**

The task now is to choose a suitable design criterion to achieve the design objective of high localization accuracy such that the other fixture requirements are satisfied. The optimal criterion chosen is the determinant of the locator information matrix \( \mathbf{M} = \mathbf{G} \mathbf{G}^T \). This is known as D-Optimality [21], and the D-optimal synthesis using this criterion is defined as

\[
\max \det(\mathbf{M}) \quad \text{where} \quad \mathbf{M} = \mathbf{G} \mathbf{G}^T.
\]

The D-optimality criterion has three important and suitable features. First, the D-optimal design minimizes the variance of the workpiece positional parameters \( \delta \mathbf{y} \), and thus reduces the overall effect of the locator positional errors on the localization accuracy. Second, the D-optimal design reduces the interactions between locators, so the six degrees of rigid-body freedom of the workpiece are best separated among the given locators [22, p. 124]. From a geometric point of view, the determinant criterion implies that the columns of locator matrix \( \mathbf{G} \) are made to be as orthogonal as possible. This makes particular locators more sensitive to particular workpiece positional parameters. Third, a D-optimal synthesis can be constructed efficiently with greedy algorithms to be discussed next.

**V. SEQUENTIAL OPTIMIZATION FOR FIXTURE SYNTHESIS**

As we mentioned in Section III, the major difficulty of fixture synthesis in the point set domain is the combinatorial complexity in a complete search for the global optimal fixture. For practical purpose, we may have to resort to a more efficient method with a tradeoff to obtain a suboptimal solution. In our case of optimal localization, if the localization errors are reduced within a specified range, the resulting fixture design would be considered acceptable.

For the D-optimal synthesis, an efficient technique is made possible by exploiting an incremental change of the locator information matrix with respect to a sequential deletion of locators from the candidate set. Consider the effect of deleting one location from \( N \) candidates \( \mathbf{h}_i \) (\( i = 1, 2, \ldots, N \)). The locator information matrix \( \mathbf{M} \) before any deletion can be written as

\[
\mathbf{M} = \mathbf{G} \mathbf{G}^T = \sum_{i=1}^{n} \mathbf{h}_i \mathbf{h}_i^T.
\]

Now suppose the \( j \)th location is to be deleted and let \( \mathbf{G}_{(j)} \) and \( \mathbf{M}_{(j)} \) denote the respective locator matrix \( \mathbf{G} \) and the information matrix \( \mathbf{M} \) with the \( j \)th location deleted. Clearly, we have

\[
\mathbf{M}_{(j)} = \mathbf{G}_{(j)} \mathbf{G}_{(j)}^T - \mathbf{h}_j \mathbf{h}_j^T.
\]

With some straightforward linear algebra manipulations [21], the determinant of matrix \( \mathbf{M}_{(j)} \) is found to be

\[
\det \mathbf{M}_{(j)} = (1 - p_{jj}) \det \mathbf{M}.
\]

Further, the inverse of matrix \( \mathbf{M}_{(j)} \) can be updated as

\[
\mathbf{M}_{(j)}^{-1} = \mathbf{M}^{-1} + (\mathbf{M}^{-1} \mathbf{h}_j)(\mathbf{M}^{-1} \mathbf{h}_j)^T / (1 - p_{jj}).
\]

In fact, \( p_{jj} \) is a diagonal element of the so-called prediction matrix \( \mathbf{G}^T (\mathbf{G} \mathbf{G}^T)^{-1} \mathbf{G} \), and it is easy to show that \( 0 \leq p_{jj} \leq 1 \). Clearly, the determinant is nonincreasing with respect to any deletion. Equations (10)–(13) define the recursive formula for updating \( \mathbf{M}_{(j)}^{-1} \) and \( \det \mathbf{M}_{(j)} \). Particularly, the inverse needs to be calculated only once for the initial \( \mathbf{M} \). The technique is rooted to the concept of optimal design of experiments [21] and has been employed for parameter identification problems of similar nature [23, 24].

It should be noted that the sequential optimization is a greedy method for a discrete optimization problem. It does not guarantee that the global optimum will be reached. However, by deleting only the location of the least impact, the loss of localization accuracy is always kept to be minimum at each deletion iteration. If more than one location is deleted in each iteration, the number of cases to be considered at a time will be combinatorially exploded [21]. Further, more rapid reduction of \( \det \mathbf{M} \) tends to result in a lesser optimal solution.
At this juncture, we describe the condition of form-closure for complete fixture solutions with a consideration of clamp selection. The form-closure condition is well understood [8], [4], [6], and it is sufficient to describe it for the case of a single clamp. The clamp is in point contact at location \( r_c \), perpendicular to the workpiece surface. For frictionless contact, the clamping force exerted on the workpiece is given as

\[
F_c = - (n_c^T (r_c \times n_c))^T \lambda_c = h_c \lambda_c
\]

where \( \lambda_c \) denotes the clamping force intensity and \( \lambda_c > 0 \). Following the analysis of [6], it is easy to show that the fixture has form-closure if and only if

\[
G \alpha + F_c = 0 \quad \text{for} \quad \lambda_c > 0, \quad \alpha > 0
\]

where \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T \) with \( \alpha_i \) indicating the magnitude of the generalized reaction force \( f_i \) of the \( i \)th locator, i.e., \( f_i = \alpha_i h_i \). A general form for the solution of this equation for \( n \geq 6 \) can be described as

\[
\alpha = -G^+ h_c \lambda_c + (I - P) \gamma
\]

where \( G^+ \) is the pseudoinverse of \( G \) (or \( G^+ = G^T (GG^T)^{-1} \)), \( P \) is the so-called prediction matrix \( (P = G^+ G = G^T (GG^T)^{-1} G) \), and \( \gamma \) is an arbitrary \( n \times 1 \) vector.

The first term is the particular solution and the second term is the homogeneous solution. In the case of six locators, \( G^+ \) becomes \( G^{-1} \) and the homogeneous solution disappears, i.e., \( P = I \). For \( n > 6 \), the homogeneous solution, \( \alpha_h = (I - P) \gamma_c \), corresponds to the internal forces among the locators [5]. However, when considering the fact that the locators are passive elements, the internal forces should never arise physically. Thus, the homogeneous solution must be ignored, and the feasible solution is given as

\[
\alpha = -G^T (GG^T)^{-1} h_c \lambda_c \quad \text{with} \quad \lambda_c > 0.
\]

Therefore, the locator force intensities can be written as

\[
\alpha_i = -h_i^T M^{-1} h_c \lambda_c \quad (i = 1, 2, \ldots, n)
\]

and the form-closure condition is given as

\[
\frac{\alpha_i}{\lambda_c} = - (h_i^T M^{-1} h_c) > 0 \quad (i = 1, 2, \ldots, 6).
\]

With this analysis, we now present two different greedy algorithms for the complete fixture specification based on the sequential optimization concept.

A. Algorithm 1: Sequential Optimal Localization

In the first algorithm, only the requirement of optimal localization of locators is considered in the process of the D-optimal synthesis. A clamp is determined for the form-closure requirement after the six optimal locators are found.

Algorithm 1:

1. With the initial \( N > 6 \) candidate locations \( h_i[i = 1, 2, \ldots, N] \), let \( n = N \) and calculate \( M \) and \( M^{-1} \).

2. Find and delete \( j \)th candidate location such that

\[
\min(h_i M^{-1} h) \quad (j = 1, 2, \ldots, n).
\]

3. Decrease \( n \) by 1, update \( M \) and \( M^{-1} \) according to (10) and (13), and repeat the deletion process until \( n = 6 \), yielding the final six optimal locators.

4. For each clamp candidate \( c \) among the remaining \( N = 6 \) locations, a suitable clamp location \( h_c \) is selected if it satisfies (20).

Here, we present a simple 2-D example to illustrate the algorithm. A 3-D example is presented in the following section.

Example 1: A Hexagon: The first example is a hexagonal object. A set of 62 initial locator points are shown in Fig. 1, and the final three optimal locator locations obtained are shown in Fig. 2. The values of \( \log(\det M) \) of each deletion iteration are plotted in Fig. 3. Note that during the deletion process, the determinant always shows a decreasing trend.
trend, and the rate of decrease increases as the number of the candidate locators reduces toward 3.

In this example, it is apparent that there are a number of potential clamp locations after finding the three locators. It is not difficult to determine an adequate clamping location for form-closure.

B. Algorithm 2: Sequential Optimal Localization with Form-Closure

The first algorithm is a two-step process, separating clamping scheme from the optimal locating scheme. In general, however, it is possible that, for the resulting optimal (or suboptimal) locators, few or none clamping locations would exist for form-closure. This could be a common problem of optimum fixture design schemes that focuses on the locators alone, such as the one reported in [16].

Therefore, it is more suitable to consider the form-closure requirements simultaneously with an optimum locating scheme. It is sufficient to describe the second algorithm for the case of a single clamp. Essentially, we modify the first algorithm with a preference in deletion. In a deletion iteration, it first targets those locator candidates with a nonpositive reaction force in the presence of clamping. Among these locators, the D-optimality is sought. In the end when \( n = 6 \), it arrives at a D-optimal locating scheme with form-closure. Naturally, the final value of the determinant of the contact information matrix \( M \) would not be greater than that obtained with the first algorithm.

The second algorithm is summarized and illustrated below with two examples.

**Algorithm 2:**

1. With the initial \( N \) candidate locations \( h_i (i = 1, 2, \ldots, N) \), let \( n = N \) and calculate \( M \) and \( M^{-1} \).

   For the given clamp \( h_c \), calculate

   \[
   \frac{\alpha_i}{\lambda_i} = -h_i^T M^{-1} h_i \quad (i = 1, 2, \ldots, n).
   \]

2. If there exist locators with \( \alpha_i \leq 0 \), delete \( j \)th locator such that

   \[
   \min(p_{jj} = h_j^T M^{-1} h_j) \quad (j = 1, 2, \ldots, n).
   \]

   Decrease \( n \) by 1, update \( M \) and \( M^{-1} \) according to (10) and (13), and repeat the deletion process until \( n = 6 \), yielding the final 6 optimal locators.

Example 2: Hexagon (Continued): The hexagon example is further used to demonstrate the second algorithm. The candidate locations remain the same. Three different clamping cases are used separately. Figs. 4–6 show the respective clamp location and the optimal locators obtained, all with form-closure. For comparison, Fig. 7 shows the determinant values \( \log(\det M) \) for all three cases together with those of the first algorithm previously shown in Fig. 3.

The following two observations are made from Fig. 7. 1) By considering the form-closure condition in the second algorithm, the entire
The sequential deletion process is divided into two phases. In the first phase, nonpositive clamping reaction forces exist among the candidate locators. After about 30 deletions, the reaction forces remain positive for the remaining steps of deletion. 2) The first phase of the second algorithm results in a noticeable drop in the determinant value as compared to the first algorithm. While this drop is gradually reduced in the second phase, the final selection of the locators of the second algorithm may yield a smaller determinant and, thus, a less optimal locator set (e.g., Case 2). These features of the algorithms are further evident in a 3-D example shown next.

**Example 3: A Turbine Airfoil:** Turbine airfoils are good examples of 3-D workpieces with complex geometry. As discussed in Section III, an airfoil may be required to be fixtured only at some specific surface locations for precision manufacturing or inspection. Fig. 8 shows an example airfoil with 1546 predetermined candidate locations and the surface normals.

The first algorithm finds final six optimum locators among these candidates as shown in Fig. 9. The values of $\log(\det \mathbf{M})$ are plotted in Fig. 11. Based on the optimum locators, one has to find a clamp location among the remaining 1540 candidates to satisfy the form-closure condition. This is a linear search and it reveals that there exist only two clamping locations possible for form-closure. This case illustrates the “bias” of the first algorithm. It results in a better set of D-optimal locators, but it leaves with a limited potential for form-closure clamping.

Next, the second algorithm is applied with a given clamp near the middle of the airfoil. This algorithm finds the D-optimal locators with form-closure as shown in Fig. 10. The determinant values are plotted together with those of the first algorithm in Fig. 11 for comparison. As expected, the locators determined by the second algorithm is less optimal than those found by the first algorithm.

**VI. DISCUSSIONS**

With the above description of the synthesis approach, there are a number of interesting aspects of the fixture synthesis algorithms worthy of further discussions.
example, it takes about 4 s of run time, while the airfoil example requires about 12 min of run time. Although not conclusive, these examples show that a purpose-built implementation of the greedy algorithms might be reasonable for practical problems of a modest number of candidate locations, for example, $N = O(1000)$.

The greedy algorithms are inherently not complete and they may not find the global optimum which, in the point set domain, demands an exhaustive search. The completeness is traded off for efficiency gained. For the hexagon example, the global optimum is fortunately obtained for both algorithms except for Case 2 of Algorithm 2. For the airfoil example, the situation is uncertain since it is computationally prohibitive to search for the global optimum given its huge complexity. Of course, one may wish to resort to a more sophisticated scheme such as the genetic algorithm or simulated annealing, but such an approach may not be necessarily practical. To the best of our knowledge, no other more efficient approaches to the fixture synthesis problem seem to be reported and, thus, for us to compare with.

**Optimal Criteria:** It should be pointed out that various other criteria exist for optimization of the Fisher information matrix, including the trace of $M$ known as A-optimality, the condition number, and the minimum eigenvalue (known as E-optimality) respectively [21]. However, D-optimality has several advantages. For example, the D-optimal designs are invariant to change of coordinate system, which is not in general the case for other design criteria [21].

A common problem of using a single criterion for minimizing the localization error with optimization of matrix $M$ is the mixed use of the rotation and translation dimensions which are not comparable. Artificial scales in either dimension may be chosen and they will change the optimality metric [1]. Therefore, such a criterion is used only as a relative measure of the fixture positioning performance. It is not suitable for comparing the performance of fixtures for different workpieces. A possible solution is to separate the two dimensions and define two separate metrics for rotation and translation dimensions respectively. Ultimately, fixture performance should be defined in close relation to the tasks for which the fixture is designed for, e.g., manufacturing or assembly.

**Bottom-Up Approaches:** We have noticed that the rules of recursively updating $(\det M)$, $M$, and $M^{-1}$ are similar for adding a locator instead. This suggests that an alternative to the “top-down” technique of sequential deletion is a “bottom-up” approach. In the initial step, one may select six locators randomly or using a sequential addition procedure [25]. Then, an exchange process can be performed as a sequence of alternating additions and deletions of locator and clamp elements for optimization. Promising preliminary results of such an exchange algorithm and a seesaw algorithm were reported in [25] and [26] for the turbine airfoil example with even greater computational efficiency. It also appears that the algorithms of force optimization developed for robotic grasping in [27] and [28] may be useful in a bottom-up approach.

**Contact Friction and Compliance:** Contact friction and contact compliance can be further incorporated into the optimal fixture synthesis. It should be noted that a reliable workpiece localization can only be accomplished by the kinematic constraints imposed by the locators, not by their frictional contact forces. Thus, the kinematic model of (5) remains unchanged. In the form-closure analysis of (16), however, a frictional locator generates two more contact forces in the tangent directions. Two similar frictional forces may also arise at the clamping contact. Thus, in solving for all unknown intensities of the passive reaction forces, we will obtain an extended form for the contact information matrix $M$ [29]. The entire greedy algorithms remain similar.

Further, using the results of the well-studied problem of contact mechanics, the elastic deformations of locator/clamp and workpiece con-

**Complexity and Completeness:** In the point set domain with a given number $N$ of pixel candidates, the complexity of fixture synthesis is combinatorial. In the case of a single clamp, a complete search of the best set of six locators and one clamp would require an evaluation of combinations with a complexity of $O(N^7)$. Further, for each combination, one has to calculate $\det M$ and solve for the form-closure equations (16) or its equivalent $M^{-1}$. This is clearly impractical even for a modest number of candidates, for example, $N = 100$.

The greedy algorithms presented in the article are primarily motivated for a practical solution to the problem. Based on the sequential optimization (or deletion) principle, the algorithms perform a linear search in each iteration with a total of $(N + 7)(N - 6)/2$ cases. Thus, the complexity of the greedy algorithms is $O(N^2)$ or $O(N^3)$ if the clamp search is enumerated. In addition, except for the initialization, terms $(\det M)$, $M$, and $M^{-1}$ are recursively updated without explicit computations. The combined reduction in computational complexity offers a real potential for practical use.

The two greedy algorithms are implemented on a Pentium-II personal computer within MATLAB software system. For the hexagon
tact can also be considered in the model. Thus, the effects of workpiece curvatures and the locator/clamp sizes can be fully accounted for, resulting in a more accurate determination of the fixture performance. Details of these considerations will be reported separately.

VII. CONCLUSIONS

This article presents an investigation to the problem of fixture synthesis in a point-wise 3-dimensional domain. The synthesis approach is based on the concept of optimum experimental design, while the optimization focuses on the fixture performance of localization accuracy and form-closure. Two greedy algorithms based on sequential optimal deletion are described. Both 2-D and 3-D examples are presented to show the synthesis procedures and their computational complexity.

To our best knowledge, the work described here represents a primary effort toward fixture synthesis with optimal localization performance in the point set domain. The D-optimality analysis reveals that more efficient algorithms may be constructed based on a principle of alternating deletions and additions. When combined with a random initialization, such a procedure may result in a practical design tool capable of finding optimal or near-optimal solutions effectively [26]. Relevant issues such as frictional and area contact, elastic compliance, and locator contact force balance can also be addressed readily [29]. Other areas of application of the approach may include optimal CMM measurement locations and adaptive workpiece localization with on-line measurements.

REFERENCES