Adaptive Neural Network Tracking Control of MIMO Nonlinear Systems With Unknown Dead Zones and Control Directions
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Abstract—In this paper, adaptive neural network (NN) tracking control is investigated for a class of uncertain multiple-input–multiple-output (MIMO) nonlinear systems in triangular control structure with unknown nonsymmetric dead zones and control directions. The design is based on the principle of sliding mode control and the use of Nussbaum-type functions in solving the problem of the completely unknown control directions. It is shown that the dead-zone output can be represented as a simple linear system with a static time-varying gain and bounded disturbance by introducing characteristic function. By utilizing the integral-type Lyapunov function and introducing an adaptive compensation term for the upper bound of the optimal approximation error and the dead-zone disturbance, the closed-loop control system is proved to be semiglobally uniformly ultimately bounded, with tracking errors converging to zero under the condition that the slopes of unknown dead zones are equal. Simulation results demonstrate the effectiveness of the approach.

Index Terms—Adaptive control, dead zone, neural network (NN) control, Nussbaum function, sliding mode control.

I. INTRODUCTION

In recent years, adaptive control system design using either neural networks (NNs) or fuzzy logic systems to parameterize the unknown nonlinearities has received much attention [1]–[5]. Direct adaptive tracking control was proposed for a class of continuous-time nonlinear systems using radial basis function NNs in [1]. By utilizing modulation function and fuzzy systems, adaptive fuzzy control was investigated in [2]. Multilayer-neural-network-based indirect adaptive control was developed for feedback linearization of a class of nonlinear systems in [3]. Using a families of novel integral Lyapunov functions for avoiding the possible controller singularity problem without using projection, adaptive neural controls were presented for a class of nonlinear systems in a Brunovsky form [4], and for a class of multiple-input–multiple-output (MIMO) nonlinear systems with a triangular structure in the control inputs in [5] and [6]. Furthermore, two design schemes of adaptive controller for single-input–single-output (SISO)/MIMO uncertain nonlinear systems were proposed in [7] and [8], and ensured tracking error converging to zero. Based on the principle of sliding mode control and the approximation capability of fuzzy systems, decentralized indirect and direct adaptive fuzzy control schemes were proposed for a class of nonlinear systems with constant control gains and unknown function control gains in [9] and [10].

When there is no a priori knowledge about the signs of control gains, adaptive control of such systems becomes much more difficult. The first solution was given in [11] for a class of first-order linear systems, where the Nussbaum-type gain was originally proposed. When the high-frequency control gains and their signs are unknown, gains of Nussbaum-type [11] have been effectively used in controller design in solving the difficulty of unknown control directions [12] and [13].

Nonsmooth nonlinear characteristics such as dead zone, backlash, and hysteresis are common in actuator and sensors such as mechanical connections, hydraulic actuators, and electric servomotors. Dead zone is one of the most important nonsmooth nonlinearities in many industrial processes, which can severely limit system performance; and its study has been drawing much interest in the control community for a long time [14]–[20]. To handle the systems with unknown dead zones, adaptive dead-zone inverses were proposed in [14]–[16]. In [14] and [15], adaptive dead-zone inverses were built for linear systems with unmeasurable dead-zone outputs. Asymptotical adaptive cancellation of an unknown dead zone was achieved analytically under the condition that the output of a dead zone was measurable in [16]. A compensation scheme was presented for general nonlinear actuator dead zone of unknown width in [18]. In [19], by a given matching condition to the reference model, adaptive control with adaptive dead-zone inverse has been introduced. By using a new description of a dead zone with equal slopes, robust adaptive control was developed for a class of nonlinear systems in [20] without constructing the inverse of the dead zone. In [21], decentralized variable structure control was proposed for a class of uncertain large scale linear systems with state time-delay and dead-zone input. However, the parameters \( u_{-\lambda}, u_{+\lambda} \) of the dead zones [21] need to be known, and the disturbances satisfy the matching condition. In [22], a servomechanism problem of controlling a scalar output variable to \( \lambda \)-track was addressed for a class of special SISO linear system with nonlinear actuator characteristics. Adaptive output feedback control using backstepping and smooth inverse
function of the dead zone was proposed for a class of SISO nonlinear systems with unknown dead zone [23].

In this paper, we consider a class of uncertain MIMO nonlinear systems with both unknown dead zones and unknown gain signs. By employing analytic extension for the functions outside the dead band and introducing characteristic functions, the dead-zone output is represented as a simple linear system with a static time-varying gain and bounded disturbance. Based on the proposed novel description of dead-zone model, robust adaptive NN control is developed without necessarily constructing a dead-zone inverse and knowing some parameter bounds of dead zones. In addition, the problem of both unknown control direction and unknown control gain functions is solved by using Nussbaum-type functions and employing integral-type Lyapunov function. Moreover, all signals involved in the closed-loop system are proved to be semiglobally uniformly ultimately bounded, with tracking errors converging to zero under the condition that the slopes of unknown dead zones are equal.

This paper is organized as follows. The problem formulation and preliminaries are given in Section II. Section III presents multilayer NNs used in the controller design, and an adaptive NN control is developed for SISO systems using an integral-type Lyapunov function. Furthermore, this scheme is extended to MIMO systems. The closed-loop system stabilities are analyzed as well. In Section IV, adaptive NN control is investigated for a class of nonlinear MIMO systems with the dead zones of equal slopes. Simulation results are performed to demonstrate the effectiveness of the approach in Section V. Section VI contains the conclusions.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Problem Formulation

Consider a class of uncertain MIMO nonlinear systems in triangular control structure with dead zones in the following form:

\[
\begin{align*}
\dot{x}_{1j} &= x_{1j+1}, \quad j = 1, \ldots, n_1 - 1 \\
\dot{x}_{ij} &= x_{i+1, j}, \quad j = 1, \ldots, n_1 - 1 \\
\dot{x}_{im} &= f_i(x, u_1, \ldots, u_{m-1}) + b_i(x_1, \ldots, x_{i-1})u_i, \quad i = 2, \ldots, m \\
y_1 &= x_{11}, \ldots, y_m = x_{m1}
\end{align*}
\]

(1)

Dead zone:

\[
\begin{align*}
u_i = D_i(v_i) &= \begin{cases} 
g_i(v_i), & \text{if } v_i \geq b_{ir} \\
0, & \text{if } b_{il} < v_i < b_{ir} \\
g_i(v_i), & \text{if } v_i \leq b_{il}
\end{cases}
\end{align*}
\]

(2)

where \( x = [x_1^T, x_2^T, \ldots, x_m^T]^T \in \mathbb{R}^n \) is the state vector \( x_i = [x_{i1}, \ldots, x_{in_i}]^T, \ i = 1, \ldots, m, \ n = \sum_{i=1}^{m} n_i \); \( f_1(x), f_2(x, u_1), \ldots, f_m(x, u_1, \ldots, u_{m-1}) \) are the unknown continuous functions; \( b_1(x_1), b_2(x_2), \ldots, b_m(x_m) \) are the unknown differentiable control gains; \( y_i \in \mathbb{R}^n \) denotes the \( i \)th subsystem output; \( u_i \in \mathbb{R} \) is the output of the \( i \)th dead zone (and the input to the \( i \)th subsystem); \( v_i(t) \in \mathbb{R} \) is the input to the \( i \)th dead zone; \( b_{il} \) and \( b_{ir} \) are the unknown parameters of the \( i \)th dead zone; and the functions \( g_{ir}(v_i) \) and \( g_{il}(v_i) \) are smooth nonlinear functions. The dead zone with the input \( v_i \) is shown in Fig. 1. This describes a very general class of \( D_i(v_i) \). As \( g_{ir}(v_i), g_{il}(v_i), b_{ir}, \) and \( b_{il} \) are all unknown, control system design becomes difficult.

To facilitate control system design, we need to make the following assumptions that first appeared in [24].

Assumption 1: The dead-zone outputs \( u_1, \ldots, u_m \) are not available.

Assumption 2: The dead-zone parameters \( b_{ir} \) and \( b_{il} \) are unknown bounded constants, but their signs are known, i.e., \( b_{ir} > 0 \) and \( b_{il} < 0 \), \( i = 1, \ldots, m \).

Assumption 3: The functions \( g_{il}(v_i) \) and \( g_{ir}(v_i) \) are smooth, and there exist unknown positive constants \( k_{il0}, k_{ir0}, k_{il1}, k_{ir1} \) such that

\[
\begin{align*}
0 < k_{il0} \leq g_{il}(v_i) &\leq k_{il1} \quad \forall v_i \in (-\infty, b_{il}) \\
0 < k_{ir0} \leq g_{ir}(v_i) &\leq k_{ir1} \quad \forall v_i \in [b_{ir}, +\infty)
\end{align*}
\]

(3)

where \( g_{il}(v_i) = \frac{d g_{il}(z)}{d z} |_{z=v_i} \) and \( g_{ir}(v_i) = \frac{d g_{ir}(z)}{d z} |_{z=v_i} \).

For notational convenience, we will extend the definition for functions \( g_{il}(v_i) \) and \( g_{ir}(v_i) \) as follows:

\[
\begin{align*}
g_{il}(v_i) &= g_{il}(b_{il})(v_i - b_{il}) \quad \forall v_i \in (b_{il}, b_{ir}) \\
g_{ir}(v_i) &= g_{ir}(b_{ir})(v_i - b_{ir}) \quad \forall v_i \in (b_{ir}, +\infty)
\end{align*}
\]

(4)

Therefore, (3) is true for \( v_i \in (-\infty, b_{ir}) \), and (4) is also true for \( v_i \in [b_{ir}, +\infty) \).

According to the differential mean value theorem, we know that there exist \( \xi_{il}(v_i) \in (-\infty, b_{il}) \) and \( \xi_{ir}(v_i) \in (b_{ir}, +\infty) \) such that

\[
\begin{align*}
g_{il}(v_i) &= g_{il}(b_{il})(v_i - b_{il}) \quad \forall v_i \in (-\infty, b_{il}) \\
g_{ir}(v_i) &= g_{ir}(b_{ir})(v_i - b_{ir}) \quad \forall v_i \in (b_{ir}, +\infty)
\end{align*}
\]

(5)

Based on Assumption 3, the dead zone (2) can be rewritten as follows:

\[
u_i = K_i^T(v_i(t)) \Phi_i(v_i(t))v_i(t) + d_i(v_i(t)) \quad \forall t \geq 0
\]

(6)
where \( |d_i(v_i(t))| \leq p_i^+ \), \( p_i^+ \) is an unknown positive constant with \( p_i^+ = (k_{ir} + k_{id}) \max \{b_{ir}, -b_{id}\} \), and

\[
K_i(v_i(t)) = [K_{ir}(v_i(t)), K_{id}(v_i(t))]^T \tag{10}
\]

\[
K_{ir}(v_i(t)) = \begin{cases} 0, & \text{if } v_i(t) \leq b_{id} \\ g_{fr}(b_{ir}), & \text{if } b_{id} < v_i(t) < b_{ir} \\ g_{fr}(v_i(t)), & \text{if } b_{ir} \leq v_i(t) \leq +\infty \end{cases} \tag{11}
\]

\[
K_{id}(v_i(t)) = \begin{cases} g_{fd}(b_{id}), & \text{if } v_i(t) < b_{ir} \\ g_{fd}(b_{ir}), & \text{if } b_{id} < v_i(t) < b_{ir} \\ 0, & \text{if } v_i(t) \geq b_{ir} \end{cases} \tag{12}
\]

\[
\Phi_i(v_i(t)) = \left[\varphi_{ir}(v_i(t)), \varphi_{id}(v_i(t))\right]^T \tag{13}
\]

\[
\varphi_{ir}(v_i(t)) = \begin{cases} 1, & \text{if } v_i(t) > b_{id} \\ 0, & \text{if } v_i(t) \leq b_{id} \end{cases} \tag{14}
\]

\[
\varphi_{id}(v_i(t)) = \begin{cases} 1, & \text{if } v_i(t) < b_{ir} \\ 0, & \text{if } v_i(t) \geq b_{ir} \end{cases} \tag{15}
\]

\[
d_i(v_i(t)) = \begin{cases} -\frac{\alpha_i}{\mu_i}(\xi_{ir}(v_i(t)))b_{id}, & \text{if } v_i(t) \geq b_{id} \\ -\frac{\alpha_i}{\mu_i}(b_{id}) + \frac{\alpha_i}{\mu_i}(b_{ir})v_i(t), & \text{if } b_{id} < v_i(t) < b_{ir} \\ -\frac{\alpha_i}{\mu_i}(\xi_{id}(v_i(t)))b_{id}, & \text{if } v_i(t) < b_{id} \end{cases} \tag{16}
\]

Remark 1: For the case of linear dead zone outside the dead-band, control system design has been extensively studied [14, 15, 20, 25]. To the best of our knowledge, (9) is to capture the most realistic situation, and as such it is different from the existing idealized description [14, 16, 17, 19, 20, 23]. As shown in [24], we know that \( K_i(v_i(t))\Phi_i(v_i(t)) \in [\min\{k_{id}b_{ir}\}, k_{id} + k_{ir}] \subseteq (0, +\infty) \).

The control objective is to design an adaptive controller \( v_i(t) \) for system (1) such that the output \( y_i(t) \) follows the specified desired trajectory \( y_{id}, i = 1, \ldots, m \).

Define \( x_{id} \) and \( e_i \) as

\[
\begin{align*}
x_{id} &= \left[y_{id}, \dot{y}_{id}, \ldots, y_{id}^{(n_{id}-1)}\right]^T \\
e_i &= x_i - x_{id} = [e_1, e_2, \ldots, e_{m_1}]^T
\end{align*}
\]

and the filtered tracking error \( s_i \) as

\[
s_i = \left(\frac{d}{dt} + \lambda_i\right)^{n_i-1} e_{i1} = \sum_{k=0}^{n_i-1} C_{n_{id} - 1}^{k} e_{i1-k} e_{ik+1} = \sum_{j=1}^{n_i-1} C_{n_{id} - 1}^{j-1} e_{ij} e_{ij} + e_{mi} = \sum_{j=1}^{n_i-1} C_{n_{id} - 1}^{j-1} e_{ij} e_{ij} + e_{mi} \tag{17}
\]

where \( e_{ij} = C_{n_{id} - 1}^{j-1} e_{ij} \), \( j = 1, \ldots, n_i, \lambda_i > 0, i = 1, \ldots, m \) are positive constants, specified by designer.

Assumption 4: Smooth functions \( b_i(\tau_i) \), and their signs are unknown, and there exist constants \( b_0 \) and \( b_1 \) such that \( 0 < b_0 \leq |b_i(\tau_i)| \leq b_1, \forall \tau_i \in R^{n_i} \) with \( n_i = \sum_{j=1}^{i} n_j, i = 1, \ldots, m \).

Assumption 5: The desired trajectory vectors are continuous and available, and \( \Omega_{id} = [x_{id}^T, \dot{y}_{id}^{(n_{id})}]^T \subseteq \Omega_{id} \subseteq R^{n_{id}+1} \) with \( \Omega_{id} \) known compact set, \( i = 1, \ldots, m \).

B. Nussbaum Function Properties

In order to deal with the unknown control gain sign, the Nussbaum gain technique is employed in this paper. A function \( N(\zeta) \) is called a Nussbaum-type function if it has the following properties:

1. \( \lim_{\zeta \to +\infty} \sup \frac{1}{\zeta} N(\zeta) \zeta = +\infty \)

2. \( \lim_{\zeta \to -\infty} \inf \frac{1}{\zeta} N(\zeta) \zeta = -\infty \)

Commonly used Nussbaum functions include: \( \zeta^2 \cos(\zeta) \), \( \zeta^2 \sin(\zeta) \), and \( \exp(\zeta^2) \cos((\pi/2)\zeta) \) [11, 13, 22, 26]. For clarity, the even Nussbaum function \( N(\zeta) = \epsilon^2 \cos((\pi/2)\zeta) \) is used throughout this paper.

Lemma 1 [12]: Let \( V(\cdot) \) and \( \zeta(\cdot) \) be smooth functions defined on \( [0, t_f] \) with \( V(t) \geq 0, \forall t \in [0, t_f] \), and \( N(\cdot) \) be an even smooth Nussbaum-type function. If the following inequality holds:

\[
V(t) \leq c_0 + \epsilon^{-c_1 t} \int_0^t g(x(\tau)) N(\zeta) \zeta e^{c_1 \tau} d\tau \tag{18}
\]

then \( c_0 \) represents some suitable constant, \( c_1 \) is a positive constant, and \( g(x(\tau)) \) is a time-varying parameter, which takes values in the unknown closed intervals \( I = [\bar{r}, \bar{r}^+], \) with \( \bar{r} \not\in I \), then \( V(t), \zeta(t), \int_0^t g(x(\tau)) N(\zeta) \zeta e^{c_1 \tau} d\tau \) must be bounded on \( [0, t_f] \).

Lemma 2 [13]: Let \( V(\cdot), \zeta(\cdot) \) be smooth functions defined on \( [0, t_f] \) with \( V(t) \geq 0, \forall t \in [0, t_f] \), and \( N(\cdot) \) be an even smooth Nussbaum-type function. If the following inequality holds:

\[
V(t) \leq c_0 + \epsilon^{-c_1 t} \int_0^t g(x(\tau)) N(\zeta) \zeta e^{c_1 \tau} d\tau + \epsilon^{-c_1 t} \int_0^t \zeta e^{c_1 \tau} d\tau, \quad \forall t \in [0, t_f] \tag{19}
\]

where \( c_0 \) represents some suitable constant, \( c_1 \) is a positive constant, and \( g(x(\tau)) \) is a time-varying parameter, which takes values in the unknown closed intervals \( I = [\bar{r}, \bar{r}^+], \) with \( \bar{r} \not\in I \), then \( V(t), \zeta(t), \int_0^t g(x(\tau)) N(\zeta) \zeta e^{c_1 \tau} d\tau \) must be bounded on \( [0, t_f] \).

Lemma 3 [26]: For any given positive constant \( t_f > 0 \), if the solution of the resulting closed-loop system is bounded on the interval \( [0, t_f] \), then \( t_f \) is finite.

In this paper, \( \| \cdot \| \) denotes the 2-norm, \( \| \cdot \|_F \) denotes the Frobenius norm, \( ||A|| = \sum_{k=1}^{n} |a_k| \) with \( A = [a_1, \ldots, a_i] \in R^i, \) and \( \lambda_{\min}(B) \) and \( \lambda_{\max}(B) \) denote the smallest and largest eigenvalues of a square matrix \( B \), respectively.

III. CONTROL SYSTEM DESIGN AND STABILITY ANALYSIS

A. Multilayer Neural Networks (MNNs)

NNs have been widely used in modeling and control of nonlinear systems because of their good capabilities of nonlinear function approximation, learning, and fault tolerance. In this paper, three-layer NNs will be used to approximate a continuous function \( h(z) : R^n \mapsto R \) as described by [27, 28]

\[
h_{mn}(z, W, V) = W^TS(V^Tz) \tag{20}
\]
where $z = \begin{bmatrix} z_1, \ldots, z_p \end{bmatrix}^T$, $\bar{z} = \begin{bmatrix} z^T, 1 \end{bmatrix}^T$, $V = \begin{bmatrix} v_1, \ldots, v_l \end{bmatrix} \in R^{(p+1) \times l}$ and $W = \begin{bmatrix} w_1, \ldots, w_l \end{bmatrix} \in R^p$ are the first-to-second layer and the second-to-third layer weights, respectively, $S(V^T \bar{z}) = \begin{bmatrix} s(v_1^T \bar{z}), \ldots, s(v_l^T \bar{z}) \end{bmatrix}^T$ with $s(\lambda) = 1/(1 + e^{-\gamma \lambda})$ and constant $\gamma > 0$, and the NN node number $l > 1$.

Let

$$h(z) = h_{mn}(z, W^*, V^*) + \varepsilon(z) \quad \forall z \in \Omega \subset R^p \quad (21)$$

where $W^*, V^*$ are ideal NN weights, $\Omega \subset R^p$ is a compact set, and $\varepsilon(z)$ is the NN approximation error.

The ideal weights $W^*$ and $V^*$ are “artificial” quantity required for analytical purposes. $W^*$ and $V^*$ are defined as follows:

$$\begin{bmatrix} W^*, V^* \end{bmatrix} = \arg \min_{(W, V)} \left[ \sup_{z \in \Omega} \| h_{mn}(z, W, V) - h(z) \| \right].$$

(22)

It is clear that $W^*$ and $V^*$ are usually unknown and need to be estimated in controller design. Let $\hat{W}$ and $\hat{V}$ be the estimates of $W^*$ and $V^*$, respectively, and $\hat{z} = \hat{S}(\hat{V} \hat{z})$.

**Lemma 4** [28]: For NN (20), the NN estimation error can be expressed as

$$\hat{W}^T S(\hat{V} \hat{z}) - W^* T S(V^* \hat{z}) = \hat{W}^T (\hat{S} - \hat{S}^T \hat{V}) + \hat{W}^T \hat{S}^T \hat{z} + d_u \quad (23)$$

where $\hat{S} = S(\hat{V} \hat{z})$, $\hat{S}^T = \text{diag}\{\hat{s}_1, \ldots, \hat{s}_{l-1}, 0\}$ with $\hat{s}_k = (\hat{S}(\hat{z}))_{kl} / d_{z_{l-1} = \hat{z}}$, $k = 1, \ldots, l - 1$, and the residual term $d_u$ is bounded by

$$\|d_u\| \leq \|V^*\|_F \|\hat{S}^T \hat{z}\|_F + \|W^*\|_F \|\hat{S}^T \hat{z}\| + \|W^*\|_1.$$  

(24)

According to (21) and (23), we obtain

$$h(z) = \hat{W}^T S(\hat{V} \hat{z}) - \hat{W}^T (\hat{S} - \hat{S}^T \hat{V} \hat{z}) - \hat{W}^T \hat{S}^T \hat{z} - d_u + \varepsilon(z).$$

(25)

**B. Adaptive NN Control for SISO System ($m = 1$)**

To illustrate the design methodology clearly, we first consider the SISO system ($m = 1$).

From (1), (9), and (16), we obtain

$$\begin{align*}
\dot{s}_1 &= f_1(x_1) + \nu_1 + b_1(x_1)K_1^T (v_1(t)) \Phi_1 \\
& \quad \times (v_1(t)) \dot{v}_1(t) + b_1(x_1) d_1(v_1(t))
\end{align*}$$

(26)

where $\nu_1 = \sum_{j=1}^{n_1} c_{1j} e_{1,j+1} - y_{1d}$.

To avoid control singularity, we employ integral Lyapunov function [4]. In this paper, we define a smooth scalar function as follows:

$$V_{s1} = \int_0^{s_1} \frac{\sigma}{b_1(\bar{x}_1^+, \sigma + \beta_1)} d\sigma$$

(27)

where $\beta_1 = \frac{y_{1d}}{I_{1d}} - \sum_{j=1}^{n_1} c_{1j} e_{1,j}$ and $\bar{x}_1^+ = [x_1, \ldots, x_{1, n_1-1}]^T$.

By second mean value theorem for integrals, $V_{s1}$ can be rewritten as

$$V_{s1} = \frac{1}{b_1(\bar{x}_1^+, \lambda_{s1}, \sigma + \beta_1)} \int_0^{s_1} \sigma d\sigma$$

$$= 2 \frac{s_1^2}{b_1(\bar{x}_1^+, \lambda_{s1}, \sigma + \beta_1)}$$

(28)

with $\lambda_{s1} \in (0, 1)$. Because $0 < b_{10} \leq b_1(x_1)_1$, it is shown that $V_{s1}$ is positive definite with respect to $s_1$.

Differentiating $V_{s1}$ with respect to time $t$, we obtain

$$\dot{V}_{s1} = \frac{s_1}{b_1(x_1)} \dot{s}_1 + \frac{\nu_1}{s_1} \frac{1}{b_1(x_1)} s_1 \int_0^{s_1} \sigma \left[ \sum_{k=1}^{n_{1-1}} \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{x_{1,k+1}}}{\partial x_{1,k}} \right] dx_{1,k+1} \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{x_{1,k+1}}}{\partial \beta_1} d\sigma.$$  

(29)

Because $\dot{\beta}_1 = -\nu_1$ and $\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{\sigma} = \partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{\beta_1}$, it is shown that

$$\int_0^{s_1} \sigma \left[ \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{\beta_1}}{\partial \sigma} \right] d\sigma$$

$$= -\nu_1 s_1 \frac{s_1}{b_1(x_1)} + \int_0^{s_1} \frac{\nu_1}{b_1(x_1)} d\sigma.$$  

(30)

Substituting (26) and (30) into (29), and applying (16), we obtain

$$\dot{V}_{s1} = \frac{s_1}{b_1(x_1)} \dot{s}_1$$

$$+ \frac{1}{s_1} \int_0^{s_1} \sigma \left[ \sum_{k=1}^{n_{1-1}} \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{x_{1,k+1}}}{\partial x_{1,k}} \right] dx_{1,k+1} \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{x_{1,k+1}}}{\partial \beta_1} d\sigma$$

$$- \nu_1 s_1 + \nu_1 \int_0^{s_1} \frac{\nu_1}{b_1(x_1)} d\sigma.$$  

(31)

where

$$g_1(t) = \frac{b_1(x_1)}{b_1(x_1)} K_1^T (v_1(t)) \Phi_1 (v_1(t))$$

(32)

$$h_1(z_1) = \frac{f_1(x_1)}{b_1(x_1)}$$

$$+ \frac{1}{s_1} \int_0^{s_1} \sigma \left[ \sum_{k=1}^{n_{1-1}} \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{x_{1,k+1}}}{\partial x_{1,k}} \right] dx_{1,k+1} \frac{\partial [b_1^{-1}(x_1^+, \sigma + \beta_1)]_{x_{1,k+1}}}{\partial \beta_1} d\sigma$$

$$\leq s_1 g_1(t) v_1(t) + s_1 h_1(z_1) + |s_1| b_1^s.$$  

(33)
and we have

\[ 0 \sum_{k=1}^{n-1} \partial \Theta_{i k} \theta_{i 1} + \beta_i) \right] \left( x_{1, k+1} \right) + \left[ b_i \left( \Theta_{i 1} + \beta_i \right) \right] d\theta \]  

(33)

with \( z_1 = [x^T, s_1, \nu_1, \beta_1]^T \in \mathbb{R}^n \) and \( p_1 = n_1 + 3 \).

Define a compact set

\[ \Omega_1 = \left\{ [x^T, s_1, \nu_1, \beta_1]^T \mid x_1 \in \Omega_1, x_{1d} \in \Omega_{1d} \right\} \]  

(34)

where \( \Omega_1 \subset \mathbb{R}^n \) is a sufficiently large compact set satisfying \( \Omega_1 \supset \Omega_0 \), which will be defined later in step 2 in Theorem 1.

Let \( \hat{W}_i^T S_i (\hat{V}_i^T z_1) \) be the approximation of the three-layer NNs, which are discussed in Section III-A, on the compact \( \Omega_1 \) to \( h_1(z_1) \), then we have

\[ h_1(z_1) = \hat{W}_i^T S_i \left( \hat{V}_i^T z_1 \right) - W_i^T \left( \hat{S}_i - \hat{S}_i \hat{V}_i^T z_1 \right) \]  

(35)

where \( z_1 = [z_{11}, \ldots, z_{1n}]^T = [x^T, s_1, \nu_1, \beta_1]^T, \) \( z_{1i} = [x^T, s_1, \nu_1, \beta_1]^T \) \( \hat{V}_i \in \mathbb{R}^{(n+1) \times n} \) and \( \hat{W}_i \in \mathbb{R}^{(n+1) \times n} \) denote the estimates of \( V_i^T \) and \( W_i^T \), respectively. \( \hat{S}_i \) and \( \hat{V}_i^T \) are ideal constant weights, \( \hat{S}_i = S_i (\hat{V}_i^T z_1) = [s_i^{(1T) z_1}, \ldots, s_i^{(nT) z_1}]^T \) with \( s_i(z) = 1 / (1 + e^{-\gamma z}) \), and constant \( \gamma_0 > 0 \), \( \hat{S}_i = \text{diag} \{ \hat{S}_{i1}, \ldots, \hat{S}_{i n_{1d}} \} \) with \( \hat{S}_{ik} = s_i^{(kT) z_1} = d[s_i(z)] / d(z_{ik}) \). \( \kappa = 1, \ldots, n_{1d} - 1 \), the NN node number \( l_1 > 1 \) the residual term \( d_{11} \) is bounded by

\[ \left| d_{11} \right| \leq \| V_i \|_F \left| x_1 \hat{W}_i \hat{S}_i \right|_F + \| W_i \|_F \left| \hat{S}_i \hat{V}_i^T z_1 \right| + \| W_i \|_F \]  

(36)

and the approximation error \( \varepsilon_1(z_1) \) in (35) satisfies \( \| \varepsilon_1(z_1) \| \leq \varepsilon_i, \forall z_1 \in \Omega_1 \) with constant \( \varepsilon_i > 0 \).

Consider the following control law:

\[ v_1(t) = N(\xi_1) \left[ k_{10} s_1 + \hat{W}_i^T S_i (\hat{V}_i^T z_1) \right] + \delta_i \xi_1(z_1) \tan(h(s_1 \xi_1(z_1) / \rho_i)) \]  

(37)

\[ \dot{\xi}_1 = k_{10} s_1 + \hat{W}_i^T S_i (\hat{V}_i^T z_1) s_1 + \delta_i \xi_1(z_1) \tan(h(s_1 \xi_1(z_1) / \rho_i)) \]  

(38)

where \( N(\xi_1) = e^{\xi_1} \cos((\pi/2) \xi_1), k_{10}, \) and \( \rho_i > 0 \). \( \delta_i \) is the estimate of \( \delta_i^* \) with \( \delta_i^* = \max \{ \| V_i \|_F, \| W_i \|_F, \| W_i \|_F \} + \varepsilon_i + \rho_i \) at time \( t \), and

\[ \varepsilon_1(z_1) = \left| z_1 \hat{W}_i \hat{S}_i \right|_F + \left| \hat{S}_i \hat{V}_i^T z_1 \right| + 1. \]  

(39)

The adaptive laws are employed as follows:

\[ \dot{W}_1 = \Gamma_{11} \left( \left( \hat{S}_1 - \hat{S}_i \hat{V}_i^T z_1 \right) s_1 \right. \]  

(40)

\[ \dot{V}_1 = \Gamma_{12} \left( \hat{W}_1 \hat{S}_i \hat{V}_i^T z_1 - \sigma_1 \hat{V}_1 \right) \]  

(41)

\[ \dot{\delta}_i = \eta_i \left( s_1 \xi_1(z_1) \tan(h(s_1 \xi_1(z_1) / \rho_i)) - \sigma_1 \delta_i \right) \]  

(42)

where \( \Gamma_{11} > 0, \Gamma_{12} > 0, \sigma_{u1}, \sigma_1, \eta_i, \) and \( \delta_i \) are strictly positive constants.

**Theorem 1:** Consider the closed-loop system consisting of the plant (1), the adaptive control given by (37) and (40)–(42). Under Assumptions 1–5, for bounded initial conditions, the overall closed-loop neural control system is semiglobally stable in the sense that all of the signals in the closed-loop system are bounded, the parameter estimates

\[ (\hat{W}_1, \hat{V}_1, \hat{\delta}_1) \in \Omega_{11} \]  

(43)

and \( \forall x_1(0) \in \Omega_{10} \), the state vector

\[ x_1 \in \Omega_{1c} = \left\{ x_1 \| x_1 - x_{1d} \| \leq c_{10} (1 + \| A_1 \|) \| x_1 (0) \| \right\} \]  

(44)

where \( \mu_i > 0 \) and \( c_{10} > 0 \), and the compact set \( \Omega_{10} \) and the vector \( A_1 \) will be defined later in the proof.

**Proof:** The proof includes two steps. We will first assume that \( x_1(t) \in \Omega_1, \forall t \geq 0 \), on which NN approximation (35) is valid, and construct stable adaptive NN control over \( \Omega_1 \). Then, we will show that there exists nonempty initial set \( \Omega_{10} \) such that the state \( x_1(t) \) indeed remains in the compact set \( \Omega_1 \) for all \( t \geq 0 \) if initial state \( x_1(0) \) initiates from \( \Omega_{10} \).

We will first assume that \( x_1(t) \in \Omega_1 \) holds for all time, and find the upper bounds of system states. Later, for the appropriate initial condition \( x_1(0) \) and the adaptive controller parameters, we prove that state \( x_1(t) \) indeed remains in the compact set \( \Omega_1 \) for all \( t \geq 0 \).

**Step 1:** Suppose that \( x_1(t) \in \Omega_1, \forall t \geq 0 \), then NN approximation (35) is valid. Consider the Lyapunov function candidate

\[ V_1(t) = V_{s_1}(t) + \frac{1}{2} \hat{W}_1^T W_1 + \frac{1}{2} \left( \hat{V}_1^T V_1 \right) + \frac{1}{2 \eta_i} \hat{\delta}_1^2 \]  

(45)

Differentiating \( V_1(t) \) with respect to time \( t \) leads to

\[ \dot{V}_1(t) = \dot{V}_{s_1}(t) + \hat{W}_1^T W_1 + \hat{V}_1^T \hat{V}_1 + \frac{1}{\eta_i} \hat{\delta}_1 \dot{\delta}_1 \]  

(46)

Substituting inequality (31) into (46), noting (35) and (36), and using control law (37) and (38), it follows that

\[ \dot{V}_1(t) \leq g_1(t) N(\xi_1) \dot{\xi}_1 + \dot{\xi}_1 - k_{10} \xi_1 - \hat{W}_1^T S_i (\hat{V}_i^T z_1) s_1 \]  

(47)
Using adaptive tuning laws (40)–(42), and the inequality 0 ≤ |r| − r tanh(r/ε0) ≤ 0.2785ε0, for ε0 > 0, r ∈ R, and the fact that $W_{1}^{T}\hat{S}_{1}^{T}S_{1} = tr\{\hat{V}_{1}^{T}S_{1}W_{1}^{T}\}$, we obtain

$$
\hat{V}_{1}(t) ≤ g_{1}(t)N(\xi_{1})\hat{\xi}_{1} + \hat{\xi}_{1} - k_{10}s_{1}^{2} + 2\delta^{2}_{1}N(\xi_{1})s_{1}\frac{1}{\mu_{1}} - \sigma_{u_{1}}W_{1}^{T}\hat{S}_{1}
$$

$$
- \sigma_{v_{1}}|\hat{V}_{1}| \leq \sigma_{v_{1}}\hat{\delta}_{1}\hat{\delta}_{1}
$$

By completion of squares, the following inequalities hold:

$$
- \sigma_{u_{1}}W_{1}^{T}\hat{W}_{1} \leq - \sigma_{u_{1}}\frac{||\hat{W}_{1}||^{2}}{2} + \sigma_{u_{1}}\frac{||\hat{W}_{1}||^{2}}{2}
$$

$$
- \sigma_{v_{1}}|\hat{V}_{1}| \leq - \sigma_{v_{1}}\frac{||\hat{V}_{1}||}{\sigma_{v_{1}}} + \sigma_{v_{1}}\frac{||\hat{V}_{1}||}{\sigma_{v_{1}}}
$$

$$
- \sigma_{v_{1}}\hat{\delta}_{1}\hat{\delta}_{1} \leq - \sigma_{v_{1}}\frac{\hat{\delta}_{1}}{2} + \sigma_{v_{1}}\frac{\hat{\delta}_{1}}{2},
$$

We obtain

$$
\hat{V}_{1}(t) ≤ - k_{10}s_{1}^{2} + g_{1}(t)N(\xi_{1})\hat{\xi}_{1} + \hat{\xi}_{1} + 2\delta^{2}_{1}N(\xi_{1})s_{1}\frac{1}{\mu_{1}} - \sigma_{u_{1}}W_{1}^{T}\hat{\xi}_{1} - \sigma_{v_{1}}|\hat{V}_{1}| + \sigma_{u_{1}}\frac{||\hat{W}_{1}||^{2}}{2} + \sigma_{v_{1}}\frac{||\hat{V}_{1}||}{2} + \sigma_{v_{1}}\frac{\hat{\delta}_{1}\hat{\delta}_{1}}{2}.
$$

Define the following constants:

$$
\lambda_{10} = \min\{2k_{10}b_{10}, \sigma_{u_{1}}/\lambda_{\max}(\Gamma_{u_{1}}), \sigma_{v_{1}}/\lambda_{\max}(\Gamma_{u_{1}}), \sigma_{v_{1}}\eta_{1}\},
$$

$$
\mu_{10} = 0.2785\delta^{2}_{1}\rho_{1} + \sigma_{u_{1}}\frac{||\hat{W}_{1}||^{2}}{2} + \sigma_{v_{1}}\frac{||\hat{V}_{1}||}{2} + \sigma_{v_{1}}\frac{\hat{\delta}_{1}\hat{\delta}_{1}}{2}.
$$

Thus, we have

$$
\hat{V}_{1}(t) ≤ - \lambda_{10}V_{1}(t) + \mu_{10} + g_{1}(t)N(\xi_{1})\hat{\xi}_{1} + \hat{\xi}_{1}.
$$

Multiplying inequality (55) by $e^{\lambda_{10}t}$ yields

$$
\frac{d}{dt}\left(V_{1}(t)e^{\lambda_{10}t}\right) ≤ e^{\lambda_{10}t}\mu_{10} + e^{\lambda_{10}t}\left(g_{1}(t)N(\xi_{1})\hat{\xi}_{1} + \hat{\xi}_{1}\right).
$$

Integrating (56) over $[0, t]$, we have

$$
0 ≤ V_{1}(t) ≤ \mu_{10} \frac{t}{\lambda_{10}} + V_{1}(0) - \mu_{10} \frac{t}{\lambda_{10}} + e^{\lambda_{10}t}
$$

$$
+ e^{\lambda_{10}t}\int_{0}^{t}(g_{1}(\tau)N(\xi_{1}) + 1)\hat{\xi}_{1}e^{\lambda_{10}\tau}d\tau
$$

$$
≤ \mu_{10} \frac{t}{\lambda_{10}} + V_{1}(0) + \mu_{10} \frac{t}{\lambda_{10}}
$$

$$
+ e^{\lambda_{10}t}\int_{0}^{t}(g_{1}(\tau)N(\xi_{1}) + 1)\hat{\xi}_{1}e^{\lambda_{10}\tau}d\tau.
$$

From (10)–(15) and Assumption 3, we know that $g_{1}(t) \in [-\min\{k_{10}, k_{11}\}, k_{11} + k_{1r1}] \subset (0, +\infty)$ or $g_{1}(t) \in [-k_{1r1} + k_{1r1}, -\min\{k_{10}, k_{120}\}] \subset (-\infty, 0)$. According to Lemma 2, we have that $\int_{0}^{t}g_{1}(\tau)N(\xi_{1})\hat{\xi}_{1}e^{\lambda_{10}\tau}d\tau$ are bounded in $[0, t]$. According to Lemma 3, we know that the above conclusion is true for $\tau_{f} = +\infty$. Therefore, $\delta_{1}$, $\delta_{2}$, $\delta_{4}$, and $\delta_{5}$ are bounded in $L_{\infty}$. Let $C_{Q_{1}}$ be the upper bound of $e^{-\lambda_{10}t}\int_{0}^{t}g_{1}(\tau)N(\xi_{1}) + 1)\hat{\xi}_{1}e^{\lambda_{10}\tau}d\tau$ in $[0, \infty)$, and

$$
\mu_{1} = \frac{\mu_{10}}{\lambda_{10}} + V_{1}(0) + C_{Q_{1}}
$$

then $\delta_{1} ≤ \frac{\mu_{10}}{2\mu_{1}/\lambda_{10}}$, $\delta_{2} ≤ \frac{\mu_{10}}{2\mu_{1}/\lambda_{10}}$, $\delta_{1} ≤ \frac{\mu_{10}}{2\mu_{1}/\lambda_{10}}$, and $\delta_{5} ≤ \frac{\mu_{10}}{2\mu_{1}/\lambda_{10}}$.

Define $\omega = (\xi_{1}, \xi_{2}, \cdots, \xi_{m-1})^{T} \in R^{m-1}$. From Equation (17), we know that there is a state-space representation for $\omega_{1} = [\Lambda_{1}^{T}]c_{1}$, where $\omega_{1} = A_{1}^{T}b_{1} + b_{1}^{T}s_{1}$ with $A_{1}$ = $[A_{11}, \cdots, A_{1n-1}, I_{1}]$, $b_{1} = [0, \cdots, 0]$, $s_{1}$ = $A_{1}b_{1}$ as a stable matrix, 2) there is a positive constant $C_{1}$ such that $|e^{-\lambda_{10}t}| ≤ C_{1}e^{-\lambda_{10}t}$, and 3) the solution for $\omega_{1}$ is

$$
\omega_{1}(t) = e^{\lambda_{10}t}\omega_{1}(0) + \int_{0}^{t}e^{\lambda_{10}(t-\tau)}s_{1}(\tau)d\tau.
$$

Accordingly, it follows that

$$
||\omega_{1}(t)|| ≤ C_{1}||\omega_{1}(0)||e^{\lambda_{10}t} + C_{10}t + \int_{0}^{t}e^{-\lambda_{10}(t-\tau)}s_{1}(\tau)d\tau.
$$

Therefore, we have

$$
||\omega_{1}(t)|| ≤ C_{10}||\omega_{1}(0)|| + \frac{C_{10}\sqrt{2b_{1}m_{1}}}{\lambda_{1}}.
$$

Noting $s_{1} = \Lambda_{1}^{T}\omega_{1} + c_{1}$, and $c_{1} = [\omega_{1}^{T}, c_{1m-1}]^{T}$, we obtain

$$
||e_{1}|| ≤ ||\omega_{1}|| + ||c_{1m-1}|| ≤ (1 + ||A_{1}||)||s_{1}|| + ||s_{1}||.
$$

Substituting inequality (59) into the above inequality leads to

$$
||e_{1}|| ≤ C_{10}(1 + ||A_{1}||)||s_{1}|| + \frac{1 + (1 + ||A_{1}||)c_{20}}{\lambda_{1}}\sqrt{2b_{1}m_{1}}.
$$

Since $C_{10}$, $||A_{1}||$, and $\lambda_{1}$ are positive constants, and $\omega_{1}(0)$ and $s_{1}(0)$ depend on $x_{1}(0) = x_{1d}(0)$, we conclude that there exists a positive constant $R_{1} = R_{1}(c_{1}, x_{1}(0), \hat{\xi}_{1}(0), \delta_{1}(0))$ for $c_{1}, x_{1}(0), \hat{\xi}_{1}(0), \delta_{1}(0)$ such that

$$
||e_{1}|| ≤ R_{1}(c_{1}, x_{1}(0), \hat{\xi}_{1}(0), \delta_{1}(0)) = C_{10}(1 + ||A_{1}||)||s_{1}|| + \frac{1 + (1 + ||A_{1}||)c_{20}}{\lambda_{1}}\sqrt{2b_{1}m_{1}}.
$$

Noting $x_{1} = e_{1} + x_{1d}$ and Assumption 5, we obtain

$$
||x_{1}|| ≤ ||e_{1}|| + ||x_{1d}|| ≤ C_{10}(1 + ||A_{1}||)||s_{1}|| + \frac{1 + (1 + ||A_{1}||)c_{20}}{\lambda_{1}}\sqrt{2b_{1}m_{1}} + ||x_{1d}|| \in L_{\infty}.
$$
Then, we can conclude that all the closed-loop signals are semiglobally uniformly ultimately bounded for bounded initial conditions.

Step 2: In the following, we will find the conditions such that \( x(t) \in \Omega_1 \), \( \forall t \geq 0 \). First, define a set

\[
\Omega_{10} = \left\{ x(0) \mid \| x(t) - x_{ud}(t) \| < R_1(0, x(0), 0, 0, 0), \forall t \geq 0 \right\} \subset \Omega_1, x(0) \in \Omega_{1d}, x_{ud} \in \Omega_{ud}, \tag{63}
\]

It is easy to see that for all \( x(0) \in \Omega_{10} \) and \( x_{ud} \in \Omega_{ud} \), we have \( x(t) \in \Omega_1, \forall t \geq 0 \). Then, for the system with \( x(0) \in \Omega_{10} \), bounded \( \hat{W}_1(0), \hat{V}_1(0), \hat{\delta}_1(0) \), and \( x_{ud} \in \Omega_{ud} \), the following constant \( c_1^0 \) can be determined by:

\[
c_1^0 = \sup_{c \in \mathbb{R}^+} \{ c \| x(t) \| x_{ud}(t) \| \} \leq R_1(\varepsilon, x(0), \hat{W}_1(0), \hat{V}_1(0), \hat{\delta}_1(0), 0, 0, 0), \forall t \geq 0 \} \subset \Omega_1, x(0) \in \Omega_{1d}, x_{ud} \in \Omega_{ud}. \tag{64}
\]

From (53) and (54), we know that if the adaptive control parameters \( \sigma_{u1}, \sigma_{1d}, \sigma_1, \) and \( \rho_1 \) are chosen to be sufficiently small and \( k_{10}, \eta_1, \lambda_{\min}(\Gamma_{u1}) \), and \( \lambda_{\min}(\Gamma_{1d}) \) are taken to be sufficiently large, then the constant \( c_1 = \mu_{10}/\lambda_{10} \) can be made arbitrarily small. Therefore, for the initial condition \( x(0) \in \Omega_{10} \), bounded \( \hat{W}_1(0), \hat{V}_1(0), \hat{\delta}_1(0), \) and \( x_{ud} \in \Omega_{ud} \), if the adaptive control parameters are appropriately chosen such that \( c_1 = \mu_{10}/\lambda_{10} \leq c_1^0 \), then system state \( x(t) \) indeed stays in \( \Omega_1 \) for all time \( t \).

Through the process of the proof, it is clear that there is a nonempty initial compact set \( \Omega_{10} \); as long as initial state \( x(0) \) starts from \( \Omega_{10} \), the state \( x(t) \) will never escape out of the conservative compact set \( \Omega_{1d} \), belonging to the chosen compact set \( \Omega_1 \) for all time \( t \). Because NN approximation is only valid on a compact set, we have to present the idea in the above manner, and at the same time avoid the so-called circular argument as commonly understood in the classical-model-based control. To help understand the above proof, the defined three compact sets in Theorem 1 are shown in Fig. 2 as discussed in [29].

Remark 2: Because the output of an unknown dead zone is not available, controller design based on the inverse of a dead zone usually needs to estimate some unknown parameters related to the dead zone [14], [19], [23], [30]. However, for controller design based on (9), there is no need to estimate the parameters related to dead zones, and can avoid the need for parameter bounds of dead zones (for example, the parameters \( b_l \) and \( b_r \) of the dead zone need to be known in [19, eq. (22)]; the parameters \( u_{l}, u_{r} \) of the dead zones also need to be known in [21, eq. (14)]). In addition, since \( b_l \) and \( b_r \) are unknown, \( \varphi_{ur}(v_i(t)) \) and \( \varphi_{dl}(v_i(t)) \) are unknown as well, i.e., both \( \Phi_{ur}(v_i(t)) \) and \( \Phi_{dl}(v_i(t)) \) cannot be computable. However, the particular description (9) makes the control system design possible without necessarily constructing a dead-zone inverse by utilizing Lemmas 1–3.

C. Adaptive NN Control for MIMO System (\( m \geq 2 \))

In this section, the design in Section III-B is extended to MIMO system (1), which contains \( m \) interconnected subsystems. For the \( i \)th subsystem

\[
\begin{align*}
\dot{x}_{ij} &= x_{i,j+1}, \quad j = 1, \ldots, n_i - 1, \\
\dot{x}_{in_i} &= f_i(x_i, u_{i1}, \ldots, u_{i(n_i-1)}) + b_i(x_i, \ldots, x_i) u_{i}, \quad i = 2, \ldots, m \\
y_i &= x_{i1} 
\end{align*}
\]

(65)

the filtering tracking error \( s_i \) is determined by (17). From (9), (16), and (65), we obtain

\[
\dot{s}_i = f_i(x_i, u_{i1}, \ldots, u_{i(n_i-1)}) + b_i(x_i, \ldots, x_i) u_{i} + b_i(x_i) d_i(x_i(t)) \tag{66}
\]

where \( d_i = \sum_{j=1}^{n_i-1} c_{ij} g_{ij, j+1} - y_{il}^{(n_i)} \).

Define a smooth scalar function as follows:

\[
V_{si} = \frac{1}{2} \sigma_{i} \left[ b_i(x_i, x_{i1}, x_{i2}, \ldots, x_{i(n_i-1)}, s_i, \beta_i) \right] d\sigma \tag{67}
\]

where \( \beta_i = y_{il}^{(n_i-1)} - \sum_{j=1}^{n_i-1} c_{ij} \sigma_{i} \) and \( \hat{x}_{i} = [x_{i1}, \ldots, x_{i(n_i-1)}] \).

By second mean value theorem for integrals, \( V_{si} \) can be rewritten as

\[
V_{si} = \frac{1}{2} \sigma_{i} \left[ b_i(x_i, x_{i1}, x_{i2}, \ldots, x_{i(n_i-1), s_i, \beta_i}) \right] d\sigma \tag{68}
\]

with \( \lambda_{si} \in (0, 1) \). Because \( 0 < b_{il} \leq b_i(x_i) \), it is shown that \( V_{si} \) is positive definite with respect to \( s_i \).

Differentiating \( V_{si} \) with respect to time \( t \) and applying (16) and (66), we obtain

\[
\dot{V}_{si} = \frac{s_i}{b_i(x_i)} \dot{s}_i + \frac{s_i}{2} \sigma_{i} \left[ \frac{1}{b_i(x_i)} \left( \sum_{j=1}^{n_i-1} \frac{\partial [b_i^{-1}(x_i, \sigma + \beta_i)]}{\partial x_{ij}} x_{i,k+1} + \sum_{k=1}^{n_i-1} \frac{\partial [b_i^{-1}(x_i, \sigma + \beta_i)]}{\partial x_{ik}} x_{i,k+1} + \frac{\partial [b_i^{-1}(x_i, \sigma + \beta_i)]}{\partial \beta_i} \beta_i \right) d\sigma \right]
\]

\[
= \frac{s_i}{b_i(x_i)} \dot{s}_i
\]
We may conclude that

and with constant

where

and

are ideal constant weights,

and

is with

and constant

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are with

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Remark 3: Since

we may conclude that the right-hand side of (71) is a function of variable

Define the compact sets

and

as follows:

where

is a sufficiently large compact set satisfying

\( \Omega_j \supset \Omega_{j-1} \) and

will be defined later in Theorem 2. \( l = 1, \ldots, i - 1, i = 2, \ldots, m. \)

Let \( \tilde{W}_i^T S_i (\tilde{V}_i^T z_i) \) be the approximation of the three-layer NNs, which are discussed in Section III-A, on the compact \( \Omega_{z_i} \) to \( h_i(z_i) \); then we have

\[
\begin{align*}
    h_i(z_i) &= \tilde{W}_i^T S_i (\tilde{V}_i^T z_i) - \tilde{W}_i^T (\tilde{S}_i - \tilde{S}_i' \tilde{V}_i^T z_i) \\
    &= \tilde{W}_i^T S_i' \tilde{V}_i^T z_i - d_{HI} + \varepsilon_i(z_i) \quad (75)
\end{align*}
\]

where

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Remark 3: Since

we may conclude that the right-hand side of (71) is a function of variable

Define the compact sets

and \( \Omega_{z_i} \) as follows:

\[
\begin{align*}
    \Omega_{z_i} &= \{ [x^T, s_i, \nu_i, \beta_i, \nu_i, v_i, \ldots, v_{i-1}]^T \mid x_j \in \Omega_j, j = 1, \ldots, i, m, \} \quad (73) \\
    \Omega_{z_i} &= \{ [x^T, s_i, \nu_i, \beta_i, \nu_i, v_i, \ldots, v_{i-1}]^T \mid x_j \in \Omega_j, j = 1, \ldots, i, m, \} \quad (73)
\end{align*}
\]

Remark 3: Since

we may conclude that the right-hand side of (71) is a function of variable

Define the compact sets

and \( \Omega_{z_i} \) as follows:

\[
\begin{align*}
    \Omega_{z_i} &= \{ [x^T, s_i, \nu_i, \beta_i, \nu_i, v_i, \ldots, v_{i-1}]^T \mid x_j \in \Omega_j, j = 1, \ldots, i, m, \} \quad (73) \\
    \Omega_{z_i} &= \{ [x^T, s_i, \nu_i, \beta_i, \nu_i, v_i, \ldots, v_{i-1}]^T \mid x_j \in \Omega_j, j = 1, \ldots, i, m, \} \quad (73)
\end{align*}
\]

where

and

will be defined later in Theorem 2. \( l = 1, \ldots, i - 1, i = 2, \ldots, m. \)

Theorem 2: Consider the closed-loop system consisting of the plant (1), control law (77), and adaptation laws (80)-(82). If Assumptions 1–5 hold, then for bounded initial conditions, the overall closed-loop neural control system is semiglobally stable in the sense that all signals in the closed-loop system are bounded, the parameter estimates

\[
\begin{align*}
    \hat{W}_i, \hat{V}_i, \hat{\delta}_i \in \Omega_{w_i} \\
    \left\{ (\hat{W}_i, \hat{V}_i, \hat{\delta}_i) \mid ||\hat{W}_i||_F \leq \frac{2\mu_i}{\lambda_{\min} (\Gamma_{w_i})}, \right. \\
    \left. ||\hat{V}_i||_F \leq \frac{2\mu_i}{\lambda_{\min} (\Gamma_{v_i})}, ||\hat{\delta}_i||_F \leq 2\eta_i \mu_i \right\} \quad (83)
\end{align*}
\]
and \( \forall x_i(0) \in \Omega_0 \), the state vector

\[
x_i \in \Omega_{\omega} = \left\{ x_i \mid \| x_i - x_{id} \| \leq c_{i0} (1 + \| A_i \|) \| \omega_i(0) \| + \left[ 1 + \left( 1 + \| A_i \| \right) \frac{c_{i0}}{\lambda_i} \right] \sqrt{2 \eta_i \mu_i}, x_{id} \in \Omega_{id} \right\} \subset \Omega_i
\]

(84)

where \( \omega_i = [\epsilon_{i3}, \cdots, \epsilon_{i,n_i-1}]^T \in \mathbb{R}^{n_i-1}, \omega_i = A_i \omega_i + b_i, s_i \) is one state–space representation for mapping \( s_i = [\alpha_i, \epsilon_i, \cdots, \epsilon_{i,n_i-1}]^T \) with \( A_i = [\lambda_i, \cdots, \lambda_{i,n_i-1}]^T \), \( b_i = [0, \cdots, 0] \in \mathbb{R}^{n_i-1}, A_i \) being a stable matrix, and \( c_{i0} \) being a positive constant satisfying \( \| e^{-\lambda_i t} \| \leq c_{i0} e^{-\lambda_i t} \), \( i = 1, \ldots, m \), and

\[
\Omega_0 = \{ x_i(0) \mid \{ x_i(t) \mid \| x_i(t) - x_{id}(t) \| < R_i(0, x_i(0), 0, 0, 0), \forall t \geq 0 \} \subset \Omega_i, x_i(0) \in \Omega_{\omega}, x_{id}(t) \in \Omega_{id} \}
\]

(85)

with \( C_i = \mu_i/\lambda_i \), and

\[
R_i \left( C_i x_i(0), \dot{W}_i(0), \ddot{V}_i(0), \dot{\delta}_i(0) \right) = c_{i0} (1 + \| A_i \|) \| \omega_i(0) \| + \left[ 1 + \left( 1 + \| A_i \| \right) \frac{c_{i0}}{\lambda_i} \right] \sqrt{2 \eta_i \mu_i}.
\]

(86)

**Proof:** The proof includes two steps. We first suppose that \( x_i(t) \in \Omega_i \) holds for all time, and find the upper bounds of system states. Later, for the appropriate initial condition \( x_i(0) \) and the adaptive controller parameters, we prove that state \( x_i(t) \) indeed remains in the compact set \( \Omega_i \) for all \( t \geq 0 \).

Suppose that \( x_i(t) \in \Omega_i, \forall t \geq 0 \), then NN approximation (75) is valid. Consider the Lyapunov function candidate

\[
V_i(t) = V_{s_i}(t) + \frac{1}{2} \dot{W}_i^T T_{ui}^{-1} \dot{W}_i + \frac{1}{2} \left\{ \dot{V}_i^T T_{vi}^{-1} \dot{V}_i \right\} + \frac{1}{2 \eta_i} \dot{\delta}_i^2.
\]

(87)

Differentiating \( V_i(t) \) with respect to time \( t \) leads to

\[
\dot{V}_i(t) = \dot{V}_{s_i}(t) + \frac{1}{2} \ddot{W}_i^T T_{ui}^{-1} \dot{W}_i + \dot{\delta}_i(0) \frac{1}{\eta_i} \dot{\delta}_i + \frac{1}{\eta_i} \dot{\delta}_i.
\]

(88)

Substituting inequality (69) into (88), and using (75) and (76) and control law (77) and (78), we have that

\[
\dot{V}_i(t) \leq s_i (g_i(t) v_i(t) + s_i \dot{h}_i(z_i) + |s_i| k_i \dot{z}_i + \dot{W}_i^T T_{ui}^{-1} \dot{W}_i + \dot{\delta}_i \left\{ \dot{V}_i^T T_{vi}^{-1} \dot{V}_i \right\} + \frac{1}{\eta_i} \dot{\delta}_i.
\]

(89)

Using adaptive tuning laws (80)–(82), and the inequality \( 0 \leq |r| - r \tan h(r/c_\alpha) \leq 0.2785 c_\alpha \), for \( c_\alpha > 0 \) and \( r \in R \), and the fact that \( \dot{W}_i^T S_i \dot{W}_i = tr \{ \dot{V}_i^T S_i \dot{V}_i \} \), we obtain

\[
\dot{V}_i(t) \leq g_i(t) (N(\dot{\tilde{z}}_i) \dot{\tilde{z}}_i) + \dot{\tilde{z}}_i - k_i \dot{z}_i + \frac{1}{\eta_i} \dot{\delta}_i.
\]

(90)

By completion of squares, the following inequalities hold:

\[
-\sigma_{ui} \dot{W}_i^T \dot{W}_i \leq -\frac{\sigma_{ui} \| \dot{W}_i \|^2}{2} + \frac{\sigma_{ui} \| W_i^* \|^2}{2}
\]

(91)

\[
-\sigma_{vi} \dot{V}_i^T \dot{V}_i \leq -\frac{\sigma_{vi} \| \dot{V}_i \|^2}{2} + \frac{\sigma_{vi} \| V_i^* \|^2}{2}
\]

(92)

\[
-\sigma_{\dot{\delta}} \dot{\delta}_i \leq -\frac{\sigma_{\dot{\delta}} \dot{\delta}_i^2}{2} + \frac{\sigma_{\dot{\delta}} \dot{\delta}_i^2}{2}.
\]

(93)

We obtain

\[
\dot{V}_i(t) \leq -k_i \dot{z}_i + g_i(t) N(\dot{\tilde{z}}_i) \dot{\tilde{z}}_i + \frac{1}{\eta_i} \dot{\delta}_i.
\]

(94)

Define the following constants:

\[
\lambda_0 = \min \{ 2k_i b_i, \sigma_{ui}/\lambda_{\max} (T_{ui}^{-1}), \sigma_{vi}/\lambda_{\max} (T_{vi}^{-1}), \sigma_{\dot{\delta}} \}
\]

(95)

\[
\mu_0 = 0.2785 \frac{\sigma_{ui} \| W_i^* \|^2}{2} + \frac{\sigma_{vi} \| V_i^* \|^2}{2} + \sigma_{\dot{\delta}} \dot{\delta}_i^2
\]

(96)

Thus, we have

\[
\dot{V}_i(t) \leq -\lambda_0 \dot{V}_i(t) + \mu_0 + g_i(t) N(\dot{\tilde{z}}_i) \dot{\tilde{z}}_i + \frac{1}{\eta_i} \dot{\delta}_i.
\]

(97)

Multiplying inequality (97) by \( e^{\lambda_0 t} \) yields

\[
\frac{d}{dt} (V_i(t) e^{\lambda_0 t}) \leq e^{\lambda_0 t} \mu_0 + e^{\lambda_0 t} \left( g_i(t) N(\dot{\tilde{z}}_i) \dot{\tilde{z}}_i + \frac{1}{\eta_i} \dot{\delta}_i \right).
\]

(98)

Integrating (98) over \([0, t] \), we have that

\[
0 \leq V_i(t) \leq \frac{\mu_0}{\lambda_0} + \left[ V_i(0) - \frac{\mu_0}{\lambda_0} \right] e^{-\lambda_0 t} + e^{\lambda_0 t} \int_0^t (g_i(\tau) N(\dot{\tilde{z}}_i) + 1) \dot{\tilde{z}}_i e^{\lambda_0 \tau} d\tau
\]

\[
\leq \frac{\mu_0}{\lambda_0} + V_i(0) + e^{-\lambda_0 t} \int_0^t (g_i(\tau) N(\dot{\tilde{z}}_i) + 1) \dot{\tilde{z}}_i e^{\lambda_0 \tau} d\tau.
\]

(99)

From (10)–(15) and Assumption 3, we know that \( g_i(t) \in \min \{ k_i b_i, k_i \} \subset (0, +\infty) \) or \( g_i(t) \in \left[ -(k_i b_i + k_i \epsilon_i), -\min \{ k_i b_i, k_i \} \right] \subset (0, +\infty) \). According to Lemma 2, we have that \( V_i(t), \dot{\tilde{z}}_i(t), \) and \( \int_0^t g_i(\tau) N(\dot{\tilde{z}}_i) d\tau \)
are bounded in $[0, t_f]$. According to Lemma 3, we see that the above conclusion holds for $t_f = +\infty$. Therefore, $\delta_i$, $||\hat{\delta}_i||$, and $||\hat{V}_i||_{\infty}$ are unknown positive constants with $\delta_i, \hat{\delta}_i, \hat{V}_i \geq 0$. Let $C_{C_i}$ be the upper bound of $e^{-\lambda_0 t_0} \int_0^t g_i(\tau) N(\zeta_i) + 1 \delta_i \epsilon_{\hat{\delta}_i} d\tau$ in $[0, \infty)$, and

$$\mu_i = \frac{H_0}{\lambda_0} + V_i(0) + C_{C_i} \tag{100}$$

then $s_i^2 \leq 2b_i V_i(t) \leq 2b_i \mu_i$. Similarly, $||\hat{V}_i||^2 \leq 2\mu_i / \lambda_{\text{min}}(\Gamma_{\hat{V}_i}^{-1})$, and $||V_i||^2 \leq 2\mu_i / \lambda_{\text{max}}(\Gamma_{V_i}^{-1})$, and $|\hat{\delta}_i|^2 \leq 2\mu_i \hat{\delta}_i$.

Furthermore, similar to the discussion in Theorem 1, we can conclude that all the closed-loop signals are semiglobally uniformly ultimately bounded.

**Remark 4:** Since in control law $v_i(t)$ determined by (77), $\hat{C}_i$ is an adaptive bounding for unknown parameter $\delta_i^e$ including probably large values of $\epsilon_{\hat{\delta}_i}$, $||V_i^e||_{\infty}$, and $||\hat{V}_i^e||$, then large magnitude switching could occur in the control signal. In order to deal with probably large values of $\epsilon_{\hat{\delta}_i}$, $||V_i^e||_{\infty}$, and $||\hat{V}_i^e||$, a modified control scheme is presented as follows.

Consider the following control law:

$$v_i(t) = N(\zeta_i) \left[ k_i(t) s_i + \hat{W}_i^T \hat{S}_i \left( \hat{V}_i^{T} \hat{S}_i \right) \right] \tag{101}$$

$$\hat{\zeta}_i = k_i(t) s_i^2 + \hat{W}_i^T \hat{S}_i \left( \hat{V}_i^{T} \hat{S}_i \right) s_i \tag{102}$$

where $N(\zeta_i) = e^{\frac{\zeta_i}{2}} \cos((\pi/2) \zeta_i)$, $k_i(t) = k_i(t) + (\hat{\zeta}_i \zeta_i)/2\sigma_i$, $k_i(t)$, and $\sigma_i$ are positive constants.

The adaptive laws are employed as follows:

$$\hat{W}_i = \Gamma_{\hat{w}_i} \left\{ \hat{S}_i - \hat{S}_i^T \hat{V}_i \right\} s_i - \sigma_{wi} \hat{W}_i \tag{103}$$

$$\hat{V}_i = \Gamma_{\hat{v}_i} \left\{ \hat{S}_i - \hat{S}_i^T \hat{V}_i \right\} s_i - \sigma_{vi} \hat{V}_i \tag{104}$$

where $\Gamma_{\hat{w}_i} > 0$, $\Gamma_{\hat{v}_i} > 0$, $\sigma_{wi}$, and $\sigma_{vi}$ are strictly positive constants.

Consider the Lyapunov function candidate

$$V_i(t) = V_i(s_i) + \frac{1}{2} \hat{W}_i^T \Gamma_{\hat{w}_i}^{-1} \hat{W}_i + \frac{1}{2} \sigma_{wi} \hat{W}_i^T \hat{V}_i \tag{105}$$

Noting that $\delta_i^e \hat{\zeta}_i(s_i) \leq (\sigma_i \delta_i^e \hat{\delta}_i) s_i^2 + (\hat{\zeta}_i \zeta_i)/2\sigma_i$, it yields the following inequality:

$$\dot{V}_i(t) \leq g_i(t) N(\zeta_i) \hat{\zeta}_i + \dot{\hat{\zeta}}_i - k_i(t) s_i^2 + \delta_i^e \delta_i \zeta_i |s_i|$$

$$\leq g_i(t) N(\zeta_i) \hat{\zeta}_i + \dot{\hat{\zeta}}_i - k_i \sigma_i s_i^2 - \sigma_{vi} \hat{W}_i^T \hat{V}_i$$

$$\leq g_i(t) N(\zeta_i) \hat{\zeta}_i + \dot{\hat{\zeta}}_i - k_i \sigma_i s_i^2 - \sigma_{vi} \hat{W}_i^T \hat{V}_i$$

$$\leq g_i(t) N(\zeta_i) \hat{\zeta}_i + \dot{\hat{\zeta}}_i - k_i \sigma_i s_i^2 - \sigma_{vi} \hat{W}_i^T \hat{V}_i$$

$$\leq g_i(t) N(\zeta_i) \hat{\zeta}_i + \dot{\hat{\zeta}}_i - k_i \sigma_i s_i^2 - \sigma_{vi} \hat{W}_i^T \hat{V}_i$$

$$\leq g_i(t) N(\zeta_i) \hat{\zeta}_i + \dot{\hat{\zeta}}_i - k_i \sigma_i s_i^2 - \sigma_{vi} \hat{W}_i^T \hat{V}_i$$

Therefore, employing the argument in Theorem 2, the similar conclusion can be obtained as well.

From the above two design schemes, we can see that large control efforts are required in this paper using either large magnitude switching or high gains in order to deal with probably large values of $\epsilon_{\hat{\delta}_i}$, $||V_i^e||_{\infty}$, and $||\hat{V}_i^e||_{\infty}$.

**IV. ADAPTIVE NN CONTROL WITH DEAD ZONES OF EQUAL SLOPES**

Note that in Section III, adaptive neural control is designed for the system (1) with general nonlinear dead-zone inputs. In this section, we consider a special case of the system (1) with the dead zones of equal slopes. We make the following assumption.

**Assumption 6:** Suppose that the dead zones are described as follows:

$$u_i = D_i(v_i) = \begin{cases} k_i v_i - b_i, & \text{if } v_i \geq b_i \\ 0, & \text{if } b_i < v_i < b_i \\ k_i v_i - b_i, & \text{if } v_i \leq b_i. \end{cases} \tag{106}$$

The dead-zone parameters $b_i$, $b_i$, $k_i$, and $k_i$ are unknown bounded constants, but their signs are known, i.e., $b_i > 0$, $b_i < 0$, $k_i > 0$, $k_i > 0$, and $k_i = k_i = k_i$, $i = 1, \ldots, m$.

In order to make full use of Assumption 6, the $i$th dead-zone output $u_i$ can be denoted as follows:

$$u_i = k_i v_i + d_i(v_i) \tag{107}$$

where

$$d_i(v_i) = \begin{cases} -k_i b_i, & \text{if } v_i \geq b_i \\ -k_i v_i, & \text{if } b_i < v_i < b_i \\ -k_i b_i, & \text{if } v_i \geq b_i \end{cases} \tag{107}$$

and $|d_i(v_i)| \leq p_i^e$, $p_i^e$ is an unknown positive constant with $p_i^e = k_i \max\{b_i, -b_i\}$.

For the $i$th subsystem of the MIMO nonlinear system (1), we design for each subsystem a full state feedback controller as follows. The closed-loop stability can be proved as a whole, which is similar to the proof of Theorem 2.

Consider the following control law:

$$v_i = N(\zeta_i) \left[ k_i s_i + \hat{W}_i^T \hat{S}_i \left( \hat{V}_i^{T} \hat{S}_i \right) \right] \tag{108}$$

$$\dot{\hat{\zeta}}_i = k_i \sigma_i s_i^2 + \hat{W}_i^T \hat{S}_i \left( \hat{V}_i^{T} \hat{S}_i \right) s_i + \hat{\zeta}_i \zeta_i \tag{109}$$

where $N(\zeta_i) = e^{\zeta_i/2} \cos((\pi/2) \zeta_i)$, $k_i(s_i)$ is a positive constant, $\hat{\zeta}_i$ is the estimate of $\zeta_i$ with $\hat{\zeta}_i = \max\{||V_i^e||_{\infty}, ||\hat{V}_i^e||_{\infty}, ||V_i^e||_{\infty}, ||\hat{V}_i^e||_{\infty}, ||V_i^e||_{\infty}, ||\hat{V}_i^e||_{\infty}\}$ at time $t$, and the sign function is defined as

$$\text{sgn}(z) = \begin{cases} 1, & \text{if } z > 0 \\ 0, & \text{if } z = 0 \\ -1, & \text{if } z < 0 \end{cases} \tag{110}$$

The adaptive laws are employed as follows:

$$\dot{\hat{V}}_i = \Gamma_{\hat{V}_i} \left\{ \hat{S}_i - \hat{S}_i^T \hat{V}_i \right\} s_i \tag{111}$$

$$\dot{\hat{V}}_i = \Gamma_{\hat{V}_i} \left\{ \hat{S}_i - \hat{S}_i^T \hat{V}_i \right\} s_i \tag{112}$$

$$\dot{\hat{\zeta}}_i = \eta_i \hat{\zeta}_i \delta_i \tag{113}$$

where $\Gamma_{\hat{V}_i} > 0$, $\Gamma_{\hat{V}_i} > 0$, and $\eta_i$ is a strictly positive constant.

From (69) and (106), we know that

$$\dot{V}_i(t) \leq \frac{b_i(\hat{V}_i)}{|b_i(\hat{V}_i)|} k_i s_i v_i + s_i b_i(\hat{z}_i) + |s_i| p_i^e \tag{115}$$
Substituting (115) into (88), and using (75) and (76) and control law (108) and (109), it follows that

\[
\dot{V}_i(t) \leq \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)}k_i q_i + s_i b_i h_i(z_i) + |s_i|p_i^2 + \tilde{W}_i^T T \tilde{W}_i
\]

\[+ \frac{1}{\eta_k} \tilde{z}_i \dot{\tilde{z}}_i - \tilde{W}_i^T S_i \left\{ \tilde{W}_i^T S_i \tilde{z}_i - d_{\text{ini}} + \epsilon(z_i) s_i \right\} s_i
\]

\[+ |s_i| p_i^2 + \tilde{W}_i^T T \tilde{W}_i + \frac{1}{\eta_k} \tilde{z}_i \dot{\tilde{z}}_i. \tag{116}
\]

Using adaptive tuning laws (112)–(114), and the fact that \( \tilde{W}_i^T S_i \tilde{W}_i = \text{tr} \{ \tilde{W}_i^T S_i \tilde{W}_i^T S_i \}, \) we obtain

\[\dot{V}_i(t) \leq \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)}k_i N(\tilde{z}_i) + \tilde{z}_i - k_i s_i^2, \quad z_i \in \Omega_{\tilde{z}_i}. \tag{117}\]

Now, we will show that \( s_i^2 \) is uniformly continuous. Consider the resulting closed-loop system. There are three modes for its right-hand side corresponding to three cases: \( s_i > 0, s_i < 0, \) and \( s_i = 0 \) (sliding mode). Changing the sign of \( s_i \) implies changes in the motion of the system and it does not make a jump in the system state \( x_i \), therefore the system state is continuous, though the right-hand side of the differential equation is changed. In each mode, the system state \( x_i \) is continuous, and hence \( s_i \) is continuous in view of (17).

Integrating (117) over \([0, t]\), we have

\[
0 \leq V_i(t) \leq - \int_0^t k_i s_i^2(\tau) d\tau + V_i(0)
\]

\[+ \int_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) \tilde{z}_i d\tau
\]

\[\leq V_i(0) + \int_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) \tilde{z}_i d\tau. \tag{118}\]

According to Lemma 1, we have that \( V_i(t), \tilde{z}_i(\tau) \), \( \int_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) \tilde{z}_i d\tau \) are bounded in \([0, t_f]\). Note that \( I_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) \tilde{z}_i d\tau = \int_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) d\tilde{z}_i \), and \( \int_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) \tilde{z}_i d\tau \) is bounded in \([0, t_f]\) for bounded initial value \( \tilde{z}_i(0) \). According toLemma 1, we see that the above conclusion is true for \( t_f = +\infty \). Therefore, we know that \( V_i(t), \tilde{z}_i, ||W_i||, ||\tilde{W}_i||, \) and \( ||V_i||_F \) \( \in L_{\infty}, s_i \in L_2 \). Furthermore, it is easy to obtain \( x_i, \epsilon_i, \) and \( \tilde{z}_i \) \( \in L_{\infty} \). This means that \( s_i^2 \) is uniformly continuous in the interval \([0, \infty)\). According to Barbalat’s lemma, we conclude that \( \lim_{t \to \infty} s_i = 0 \). From the discussion in [31], we have \( \lim_{t \to \infty} e_{\text{ini}} = 0 \). Let \( C_{\tilde{z}_i} \) be the upper bound of \( \int_0^t \left( \frac{b_i(\tilde{z}_i)}{b_i^{\ast}(\tilde{z}_i)} k_i N(\tilde{z}_i) + 1 \right) d\tau \) in \([0, \infty)\).

\[
\mu_i = V_i(0) + C_{\tilde{z}_i}, \text{ then } s_i^2 \leq 2h_i V_i(t) \leq 2h_i \mu_i. \text{ Similarly, we obtain } ||\tilde{W}_i||^2 \leq 2\mu_i / \lambda_{\text{min}}(\Omega_{\tilde{z}_i}^{-1}), ||\tilde{W}_i||_F \leq 2\mu_i / \lambda_{\text{min}}(\Omega_{\tilde{z}_i}^{-1}), \text{ and } ||\tilde{z}_i||^2 \leq 2\eta_k \mu_i. \text{ The following theorem shows the stability and control performance of the closed-loop adaptive system.}
\]

**Theorem 3:** Consider the closed-loop system consisting of the plant (1) under Assumptions 1 and 4–6, control law (108), and adaptation laws (112)–(114). For bounded initial conditions, the overall neural control system is semiglobally stable in the sense that all of the signals in the closed-loop system are bounded, the parameter estimates

\[
(\tilde{W}_i, \tilde{z}_i, \tilde{z}_i) \in \Omega_{\tilde{z}_i} \Rightarrow \left\{ (\tilde{W}_i, \tilde{z}_i, \tilde{z}_i) ||\tilde{W}_i||^2 \leq \frac{2\mu_i}{\lambda_{\text{min}}(\Omega_{\tilde{z}_i}^{-1})}, \right\}
\]

\[
||\tilde{W}_i||^2 \leq \frac{2\mu_i}{\lambda_{\text{min}}(\Omega_{\tilde{z}_i}^{-1})}, ||\tilde{z}_i||^2 \leq 2\eta_k \mu_i \tag{119}\]

the state vector

\[
x_i \in \Omega_{\text{ad}} \Rightarrow \left\{ x_i ||x_i - x_{\text{ad}}|| \leq \epsilon_0 (1 + ||x_i||) ||\omega_i(0)|| + \frac{1 + ||x_i|| \epsilon_0}{\lambda_i} \right\} \sqrt{2h_i \mu_i}, x_{\text{ad}} \in \Omega_{\text{ad}} \tag{120}\]

and the tracking errors converge to zero asymptotically, i.e., \( \lim_{t \to \infty} e_{\text{ini}} = 0 \).}

**Remark 5:** Although the stability proofs in Sections III-C and IV is similar except the use of the sign function and the tanh function, and \( \sigma \)-modification terms, the asymptotic stability cannot be obtained with different slopes of the dead zone. In fact, if we adopt the same design scheme for the systems with general nonlinear dead zones similar to the adaptive control (108)–(114) without \( \sigma \)-modification and with the \text{sgn}(.) function, which is used in the control design, then we can obtain the following inequality:

\[
0 \leq V_i(t) \leq - \int_0^t k_i s_i^2(\tau) d\tau + V_i(0) + \int_0^t (g_i(\tau) N(\tilde{z}_i) + 1) \tilde{z}_i d\tau
\]

\[
\leq V_i(0) + \int_0^t (g_i(\tau) N(\tilde{z}_i) + 1) \tilde{z}_i d\tau. \tag{118}\]

Since \( g_i(\tau) \) is a function of time \( t \) in the above equation, we know that the stability result cannot be obtained from the above inequality and Lemma 1 as well as Lemma 2. Therefore, for the dead zones with equal slopes, we can obtain asymptotic tracking result using adaptive control given by (108)–(114) from Lemma 1 and Theorem 3. However, we can only obtain the semiglobally stable result using adaptive control given by (77)–(82) for the general dead zones by Theorem 2 and Theorem 2.

**Remark 6:** Under weak conditions, we can only achieve ultimate boundedness of system trajectory as detailed in Theorems 1 and 2. However, under more strict conditions, we can achieve the asymptotic stability as stated in Theorem 3. Though we have obtained the asymptotic stability of the tracking error in Theorem 3 by introducing discontinuous sign function \text{sgn}(.) function, the discontinuous control law (108) will lead to the “chattering”
phenomenon in control. This is also verified in the simulations: good tracking performance, but chattering control signals. Chattering may be undesirable in practice. This phenomenon can be eliminated using controller (77) or (101). But the asymptotic stability will disappear.

V. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed approach, the developed adaptive NN tracking controller is applied to the following nonlinear system:

\[
\begin{align*}
    \dot{x}_{11} &= x_{12} \\
    \dot{x}_{12} &= x_{21} - 0.3\sin(x_{12}) + b_1(x_{11})u_1 \\
    \dot{x}_{21} &= x_{22} \\
    \dot{x}_{22} &= x_{21}u_1 + (x_{22} + x_{11} + 0.5\cos(x_{21}))u_2^3 + b_2(x_{22})u_2 \\
    y_1 &= x_{11}, y_2 = x_{21}
\end{align*}
\]

(121)

where \(b_1(x_{11}) = 2 - \sin^2(x_{11})\) and \(b_2(x_{22}) = 3 + \sin(x_{22})\), \(u_i (i = 1, 2)\) are the outputs of the dead zones. The control objective is to make the system outputs \(y_i (i = 1, 2)\) follow the desired trajectory \(y_{id} (i = 1, 2)\).

Both NNs \(\hat{W}_i^T S_i(V_i^T z_i)\), \(i = 1, 2\), contain ten hidden nodes (i.e., \(l_1 = l_2 = 10\)) and the coefficients in activation function \(s(\cdot)\) are taken as \(\gamma_{10} = \gamma_{20} = 3.5\). The desired tracking trajectories are \(y_{id}(t) = 0.5[\sin(t) + \sin(0.5t)]\) and \(y_{bd} = \sin(0.5t) + 0.5\sin(1.5t)\). The parameters of the dead zones are \(k_{d1} = 0.5, k_{d1} = 1.5, k_{d2} = 1.5, k_{d2} = 2.5, b_{d1} = -0.5, b_{d1} = 0.5, b_{d2} = -2.5, \) and \(b_{d2} = 2\). The control law is given by \((77)\), and the adaptive laws are determined by \((80)\)–\((82)\). The design parameters of the above adaptive control are chosen as \(\lambda_1 = 1.5, \lambda_2 = 2, k_{d1} = 2, k_{d2} = 5, \eta_1 = \eta_2 = 0.1, \rho_1 = \rho_2 = 0.2, \sigma_1 = \sigma_2 = \sigma_{u1} = \sigma_{u2} = \sigma_{v1} = \sigma_{v2} = 0.01, \Gamma_{u1} = \Gamma_{u2} = \text{diag}(2.5), \) and \(\Gamma_{v1} = \Gamma_{v2} = \text{diag}(10)\). The
initial conditions are $x_{11}(0) = 0.4$, $x_{12}(0) = 0$, $x_{21}(0) = -0.6$, $x_{22}(0) = 0$, $\delta_1(0) = 0.5$, and $\delta_2(0) = 0.5$, $i = 1, 2$. $\bar{W}_1(0)$ and $\bar{W}_2(0)$ are taken as zeros; $\bar{V}_1(0)$ and $\bar{V}_2(0)$ are randomly taken in the intervals $[-1.0, 1.0]$ and $[-0.5, 0.5]$, respectively. $\delta_1(0)$ and $\delta_2(0)$ are randomly taken in the interval $[0, 0.5]$. The simulation results are shown in Figs. 3–6.

If we employ control law (101), adaptive laws (103) and (104), $\lambda_1 = \lambda_2 = 0.5$, $k_1 = k_2 = 0.1$, $\sigma_1 = 0.5$, $\sigma_2 = 0.2$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}(2.5)$, and $\Gamma_{v1} = \Gamma_{v2} = \text{diag}(2.5)$, and all other conditions being the same as in Case 1, then the simulation results are shown in Figs. 7–10.

If we choose $b_1(x_{11}) = -(2 - \sin^2(x_{11}))$, $b_2(x_{22}) = -(3 + \sin(x_{22}))$, control law (101), adaptive laws (103) and (104), $\sigma_1 = 5$, $\sigma_2 = 2$, $\Gamma_{w1} = \Gamma_{w2} = \text{diag}(0.1)$, and all other conditions being the same as in Case 2, then the simulation results are shown in Figs. 11–14.
If we select $k_1 = k_2 = 0.5$, $k_{21} = k_{22} = 1.5$, control law (108), adaptive laws (112)-(114), $\lambda_1 = \lambda_2 = 2.5$, $k_{30} = 4$, $k_{20} = 5$, $\eta_1 = \eta_2 = 0.1$, $\Gamma_{u1} = \Gamma_{u2} = \text{diag}\{0.1\}$, $\Gamma_{v1} = \Gamma_{v2} = \text{diag}\{2.5\}$, and all other conditions being the same as in Case 1, the simulation results are shown in Figs. 15–18.

Figs. 3–18 clearly show that the system outputs and control signals are bounded for system (121) with the dead zones. From Figs. 3, 4, 7, 8, 11, 12, 15, and 16, it can be seen that fairly good tracking performance is obtained. Fig. 17 shows that the control signal $u_1$ happens to chattering phenomenon due to using discontinuous sign function $\text{sgn}(s_1)$.

VI. CONCLUSION

Three adaptive NN tracking control schemes have been presented for a class of uncertain MIMO nonlinear systems with unknown dead zones and control directions. Controller singularity problems have been solved by employing integral Lyapunov function. Based on the intuitive concept and piecewise description of a dead zone and the principle of sliding mode control, the developed controller can guarantee that all signals involved are semiglobally uniformly ultimately bounded. The proposed approach does not require the bounds of the dead-zone parameters to be known. Moreover, the functions outside the dead band may be nonlinear functions.

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