A fuzzy linguistic algorithm for adaptive test in Intelligent Tutoring System based on competences

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A B S T R A C T
The Computerized Adaptive Tests (CAT) are common tools for the diagnosis process in Intelligent Tutor System based on Competency education (ITS-C). The item selection process to form a CAT plays a key role because it must ensure the selection of the item that best contributes to student assessment at any time. The item selection mechanisms proposed in the literature present some limitations that decrease the efficiency of CAT and its adaptation to the student profile. This paper introduces a new item selection algorithm, based on a multi-criteria decision model that integrates experts’ knowledge modeled by fuzzy linguistic information that overcomes previous limitations and enhances the accuracy of diagnosis and the adaptation of CAT to student’s competence level. Finally, such an algorithm is deployed in a mobile tool for an ITS-C.

1. Introduction

An ITS provides direct customized instruction or feedback to students in their learning processes by means of Artificial Intelligence (AI) techniques, being mainly applied to knowledge representation, managing an instruction strategy as an expert both in the teaching and pedagogical issues in order to diagnose properly the student learning status at any time. To fulfill its objective, an ITS is organized by an architecture (see Fig. 1) composed by a domain model (what is taught?), student model (who is taught?), diagnosis of the student (Wenger, 1987), instructional model (how is it taught?) (Bourdeau & Grandbastien, 2010) and the interface (man–machine interaction) (Nkambou, 2010; Wenger, 1987). It is worthy to highlight that the pedagogical model of reference guides the design of each component of the architecture of an ITS, so the knowledge representation in the domain model, student model and the diagnosis process will depend on the adopted model.

In Badaracco and Martínez (2011) has been designed a novel architecture for an Intelligent Tutoring System based on competency education (ITS-C) in which the diagnosis process estimates, updates and stores, in the student’s model, the knowledge achieved by the student during the learning process. The quality of instruction provided by an ITS depends on the amount and accuracy of the information stored and updated in the student model. Tests are tools widely accepted for evaluation by its generality, ease of deployment and automatic correction. During the 80s emerged the Computer-Administered Tests (Weiss, 1982) that have evolved to the current sophisticated Computerized Adaptive Tests (CAT), which are used for evaluation in most of ITS (Guzmán, Concejó, & Pérez de la Cruz, 2007; Jeremić, Jovanović, & Gašević, 2012). A CAT evaluates and updates the student model by displaying an item/question to the student at each time, that is selected according to his/her level of knowledge and other criteria such as minimum entropy, maximum information, difficulty, etc. (Guzmán et al., 2007; Neira, 2002; Rudner, 2009).

The item selection process in a CAT plays a key role for the usefulness and adaptation of the diagnosis process. In ITS-C is defined a domain model of competency, a curriculum domain model, a student model based on competences and a diagnosis process based on CAT whose performance improved classical ITS (Badaracco & Martínez, 2011). However the current performance of CAT avoids a better efficiency and adaptation to the student’s profile because it still needs the calibration of characteristic curves that is intractable in small institutions and whose parameters are uncertain and hard to understand. The item selection algorithm uses one single criterion, is time consuming and does not take into account dynamics of the process.

To overcome previous drawbacks this paper proposes a new fuzzy linguistic item selection algorithm implemented for the diagnosis process in ITS-C. Such an algorithm models the item selection process as an Multi-Criteria Decision Making (MCDM) problem by
using experts’ knowledge linguistically modeled (Herrera & Martínez, 2000; Zadeh, 1975a, 1975b, 1975c) to avoid calibration, and it includes a dynamic component in the selection process. Finally the algorithm is implemented in a ITS-C with a mobile client.

The paper is organized as follows, Section 2 reviews the ITS-C architecture and the diagnosis by means CAT. Section 3 presents the fuzzy linguistic dynamic algorithm for CAT in ITS-C. Section 4 shows an ITS-C with a mobile client that runs the previous algorithm. Finally some concluding remarks are pointed out.

2. Intelligent tutoring systems based on competency-based education. Architecture and diagnosis process

An ITS-C extends an ITS by linking the latter and the pedagogical model based on Competency-based Education (CBE) using the architecture showed in Fig. 2 (Badaracco & Martínez, 2011). The domain model, student model are briefly reviewed and then a further detailed revision of the diagnosis of an ITS-C (Badaracco & Martínez, 2011) is presented to facilitate the understanding of the proposal introduced in section 3.

2.1. Domain model of ITS-C

The representation of the domain model in an ITS-C is based on the descriptors utilized in CBE (Badaracco & Martínez, 2011) that reflect good professional practices to guide the development of the competency associated with an occupational role or profile (Catalano et al., 2004; Europe Tuning, 2000; Zalba, 2006). Such a set of descriptors are:

- Competency unit (cu): It is a main function that describes and groups the different activities concerning the role or profile chosen.
- Competency element (ce): It is the disaggregation of a main function (cu) that aims to specify some critical activities. A function (cu) can be specified by one or more competency elements (ce), according to its complexity or variety.
- Evidence of performance (evp): It checks if a process is performed according to best practices.
- Evidence of product (evp): It is a descriptor about scientific-technologic knowledge that allows the user understands, reflects and justifies competent performance.

Therefore the domain model contains the expert’s competences profile about a knowledge domain, hence for an ITS-C it will consist of four components briefly detailed below, further description see (Badaracco & Martínez, 2011):

(i) A domain model of competency (DMCo): It is represented by a semantic network whose nodes are competence units (cu), competence elements (ce), descriptors (evd, evp, evk) and their relations.
(ii) A curriculum domain model (CuDM): It deploys the DMCo according to a teaching strategy that defines the competences associated to a professional profile to perform a training proposal in different situations. The CuDM based on the CBE takes a modular structure, in which each module (Mi) contains competency elements (ce) belonging to the DMCo.
(iii) A set of descriptors: The descriptors associated with the ce of the didactic modules are evd, evp, and evk, that belong to a bank of items.
(iv) Test specifications: They are provided by the teachers and associated with the diagnosis process considering the scope of application and the rules that the system should follow to propose adaptive tests according to the student’s necessities of learning.

2.2. Student model of ITS-C

In an ITS-C the student model of competence (SMC) stores student’s information, whose data are updated through a diagnosis process. For the representation of the student’s knowledge and learning process, the SMC uses an overlay model in the semantic network of the CuDM (Badaracco & Martínez, 2012).

In such a semantic network the nodes evp, evd and evk store a probability distribution P(θevp = k|ui), P(θevd = k|ui), and P(θevk = k|ui), regarding the student’s level of competency k in the corresponding node, k can take values from 1 to the maximum number of level of competency on which the student is evaluated. Being θ the student’s level of technical-scientific knowledge about a descriptor for a response pattern ui obtained from the responses provided by the student in the test T (see Fig. 3) during the diagnosis process.

2.3. Diagnosis for ITS-C based on CAT

Our interest in the diagnosis for ITS-C is because its key role in ITS due to the fact that the quality of instruction offered by an ITS depends on it. In ITS-Competency based Education (ITS-C) (Badaracco & Martínez, 2011), the diagnosis process follows the pedagogical model of reference for achieving a greater efficiency. The diagnosis process estimates and updates the level of competency achieved by the student in the nodes of the SMC. To carry out the diagnosis of an ITS-C, it was adapted and extended the Computerized Adaptive Test (CAT) based on the Item Response Theory (IRT) (Badaracco & Martínez, 2012; Guzmán et al., 2007). In CAT systems the relationship between student outcomes in the test and its response to a certain item can be described by a monotone increasing function called the Item Characteristic Curve (ICC). The ICC of an ITS-C coincides with the correct response option of the characteristic curve of option (CCO). Its main components are:

- A response model associated to the items: It describes the student’s expected performance according to his/her estimated knowledge. An ITS-C uses a discrete and non parametric response model based on the Item Response Theory (IRT) (van der Linden & Hambleton, 1997) able to evaluate multiple choice answers (Guzmán et al., 2007).
- Bank of items: Each item i, is associated to its descriptors (evd, evp or evk) and each option of i, corresponds to a characteristic curve of option (CCO) obtained by a calibration process based on the Ramsay algorithm (Ramsay, 1991).

Each CCO is
represented by a probability distribution, \( P(\bar{u}_i|u_i) \), where each component represents the probability that the student selects the response pattern \( \bar{u}_i \), given her level of competence \( \theta \).

To develop a test the teachers must provide test specifications considering the scope of application and the student's necessities of learning, namely:

(i) **Initial level of Knowledge:** The initial knowledge estimation is crucial because determines the length of the CAT for each student. It may be estimated by using different models based on previous information.

(ii) **Criterion for selecting descriptor (evp, evd or evk):** The algorithm selects the descriptor that has the level of knowledge associated with lower probability (Guzmán et al., 2012; Owen, 2007):

\[
\min(\theta_{ex}) = \min(\text{MAP}(P(\theta_{ex}|\bar{u}_i)))
\]

(iii) **Criterion for selecting items:** The adaptive CAT mechanism uses different methods to select the items for the test, a common method is the maximum information (Guzmán et al., 2007; Owen, 1969) that selects the item which maximizes the information in the provisional distribution of student's knowledge. The information function for the item, \( I_j \), is calculated as follows:

\[
P(I_j) = \left( \frac{P_j(\theta_i)}{P_j(\theta_i)} \right)^2
\]

Being \( \theta_i \) the knowledge level of the student \( i \), \( P_j(\theta_i) \) the value of the CCO for the student's level, and \( P(J) \) the function derived from the CCO at that point. Other selection criteria were proposed in (Owen, 1969; Guzmán et al., 2007).

(iv) **Stop criterion:** The test should stop when the student achieves a level of knowledge fixed a priori, though there are other criteria.

During the management of a test, the student's knowledge is estimated every time that he/she answers a question, by updating the student's knowledge distribution (Owen, 1969), as:

\[
P(\theta_{ex}|\bar{u}_1, \ldots, \bar{u}_i) = \left\{ \begin{array}{ll} P(\theta_{ev}|\bar{u}_1, \ldots, \bar{u}_{i-1}) & \text{if } Q_i \text{ assesses evd}, \\
\text{P}(\theta_{ev}|\bar{u}_i, \bar{u}_{i-1}) & \text{in other case.} \end{array} \right.
\]

Being \( P(\theta_{ex}|\bar{u}_1, \ldots, \bar{u}_{i-1}) \) the a priori student's knowledge estimation on evd, evp or evk, and \( P(\bar{u}_i|u_i) \) the CCO for the option of the response pattern.

The algorithm 1 graphically showed in Fig. 5. It is clear that the less efficient the item selection procedure the less performance the CAT and hence the diagnosis. Some drawbacks of Algorithm 1 (Guzmán et al., 2007; Neira, 2002; Rudner, 2009; Suarez-Cansino & Hernandez-Gomez, 2007) are pointed out below:

- Only one attribute of the item has influence in your choices.
- The characteristic curves are obtained by calibration processes that are intractable in small institutions and do not use the teachers' knowledge.
- Parameters of characteristic curves are difficult to understand.
- Obtaining the criterion values is time-consuming.
- The item selection process is dynamic, nevertheless CAT uses static values.

These drawbacks causes that diagnosis may lose effectiveness regarding the adaptively and length of the CAT and accuracy in the evaluation. So in the following section it is presented a new
3. A linguistic multi-criteria item selection algorithm for CAT applied to ITS-C

Once it is clear the importance of the diagnosis and item selection processes to achieve the learning objectives in an ITS generally and in an ITS-C specifically. Here, it is presented a new approach for item selection process in CAT applied to ITS-C, in order to overcome the limitations pointed out previously, such an approach is characterized by:

- Reformulating the item selection problem as a multi-criteria decision making (MCDM) problem for choosing the items.
- Modeling experts’ knowledge regarding the usefulness of items by means of fuzzy linguistic information.
- Including dynamism in the decision problem.

Therefore, across this section first it is briefly revised the type of decision model used in our proposal and some concepts about linguistic decision making and computing with words (Martínez, Ruan, & Herrera, 2010; Zadeh, 1996), due to its necessity in the item selection process. Second it is introduced the new item selection algorithm (see Fig. 6) based on a linguistic multi-criteria decision method which contemplates the dynamics of change in the student model. Such a selection algorithm increases the performance and adaptability of the CAT in ITS-C.

3.1. Linguistic decision making and computing with words

Decision making (DM) is a core area of different research fields such as artificial intelligence both theory and practice, planning, psychology and alike. Because the information that will manage the proposal for the item selection process, our interest is focused on decision problems defined under circumstances with vague, imprecise and uncertain information (Bellman & Zadeh, 1970; Herrera & Martínez, 2000; Zimmermann, 1987) in which expert usually provides linguistic information. In such cases the uncertainty may be of non-probabilistic nature. Among the tools to deal with such a type of uncertainty, are fuzzy logic and fuzzy linguistic approach. The use of linguistic information enhances the reliability and flexibility of classical decision models (Martínez, Ruan, Herrera, Herrera-Viedma, & Wang, 2009). Additionally, it implies the need of Computing with words (CW) to operate with words or sentences defined in a natural or artificial language instead of numbers, it emulates human cognitive processes to improve solving processes of problems dealing with uncertainty. Therefore, CW has been applied as computational basis to linguistic decision making, because it provides tools close to human beings reasoning processes related to decision making, which improve the resolution of decision making under uncertainty as linguistic decision making.

3.1.1. ITS-C decision model for item selection in CAT

The reformulation of the item selection process for the ITS-C is based on a Multi-Criteria Decision Making (MCDM) model
(Pedrycz et al., 2011) that evaluates alternatives according to a set of criteria. Such a model is established by:

- **A decision framework**: that defines the problem structure, the criteria and the preference modeling that will use the experts to express their assessments for each criterion about alternatives.

- **The solving process**: there exist different solving processes for MCDM problem a general model consists of two following phases (see Fig. 7) (Roubens, 1997):

  1. **Aggregation Phase**: It aim is to obtain a collective preference value for each unit of representation, from the individual values.
of preference provided by the experts or criteria involved in the problem, using an aggregation operator of information.

2. **Exploitation Phase**: The input information are collective values obtained in the previous phase. Its aim is to select the best alternative from the collective values. This is done by using functions or selection criteria that allow us to sort and select the best alternative from vectors of utility or preference relations.

### 3.1.2. Linguistic computational model for CW in linguistic DM

The use of fuzzy tools to deal with DM under uncertainty has provided successful results. Different linguistic computing models have been applied to accomplish processes of CW in linguistic Decision Making problems (Martínez & Herrera, 2012; Pedrycz et al., 2011).

The use of natural language by the experts in decision situations under uncertainty is quite common in real-world problems. This type of information is usually modeled by linguistic information, originating linguistic decision making (Martínez et al., 2010). The fuzzy linguistic approach presents a direct way of representing qualitative aspects as linguistic values by means of linguistic variables. (Zadeh, 1975a, 1975b, 1975c). Zadeh introduced the concept of linguistic variable as “a variable whose values are not numbers but words or sentences in a natural or artificial language”. A linguistic value is less precise than a number but it is closer to human cognitive processes used to solve successfully problems dealing with uncertainty.

A linguistic value is defined by its syntax and semantics, the latter is usually represented by a fuzzy membership function (see Fig. 8). One crucial aspect to determine the validity of a CW approach is the selection of the membership functions for the linguistic term set. There exist different approaches to choose the linguistic descriptors and different ways to define their semantics (Zadeh, 1975a; Zadeh, 1975b; Zadeh, 1975c).

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined (Herrera, Herrera-Viedma, & Verdegay, 1995; Yager, 1995).

The semantics of the terms are given by fuzzy numbers defined in the [0,1] interval, which are described by membership functions. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function (Bonissone & Decker, 1986). Since the linguistic assessments given by the users

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Fig. 6. Item selection based on a MCDM model.
The Symbolic Translation of a linguistic term consists of adding a value 0 as symbolic translation: \( s_i \in S \rightarrow (s_i, 0) \)

This model has developed a computational technique based on \( \Delta \) and \( \Delta^{-1} \) functions, for further details see (Martínez & Herrera, 2012).

3.2. A multicriteria decision model for item selection in diagnosis process of ITS-C

Here it is proposed and developed a novel fuzzy linguistic multi-criteria decision algorithm for item selection in CAT during the diagnosis process of ITS-C whose main aim is to overcome the drawbacks pointed out in section 2.

Therefore, this new approach uses experts’ knowledge, linguistically modeled, to evaluate the usefulness of an item by multiple criteria avoiding calibration and the item selection process is then reformulated by means of a multi-criteria decision model including dynamics in the solving process.

During the development of this approach, first it is defined its specific decision framework, second it is introduced a dynamic resolution process to select items in CAT during the diagnosis process of ITS-C. Eventually it is detailed the algorithm for item selection process. Fig. 9 shows the integration of the linguistic MCDM model into the diagnosis process.

3.2.1. MCDM framework for item selection in CAT during diagnosis process

The first step of the MCDM model for the item selection process consists of developing a framework to establish the decision model, alternatives, criteria, the expression domains and structures utilized to evaluate criteria, etc. It means that the framework defines the domains and scales in which experts provide their assessments.

For our linguistic decision model the framework is based on a MCDM structure with:

- \( E = \{I_1, I_2, \ldots, I_n\} \) set of items (alternatives) to be evaluated.
- \( C = \{c_1, c_2, \ldots, c_m\} \) set of criteria that characterizes each evaluated item \( I_i \). The most likely criteria mentioned in the literature on item selection processes within CAT (Guzmán et al., 2007; Neira, 2002; Rudner, 2009) that will use for our proposal are the following ones:
  - **Discrimination** \((c_1)\): Contributes to differentiate levels of knowledge in probability distributions bit scattered.
  - **Information** \((c_2)\): Contributes to increase the accuracy of the estimate of the level of skills, obtaining pointed distributions.
  - **Closeness** \((c_3)\): Contributes to select items with a difficulty level close to student’s competences evaluated.
  - **Difficulty** \((c_4)\): Contributes to determine levels of competences for which the probability of answering an item correctly or incorrectly is the same, so if the level is high, the condition the item difficulty is too.
Regarding these criteria, experts express their opinions by means of linguistic labels in a linguistic term set, \( S \), by means of vectors of linguistic assessments:

- Assessed in \( S = \{ s_0: \text{Very Low}; s_1: \text{Low}; s_2: \text{Medium}; s_3: \text{High}; s_4: \text{Very High}\} \) linguistic term set.

\[ X_d^j = \{x_0^j, x_1^j, \ldots, x_m^j\} \] utility vectors, being \( x_i^j \in S \) the experts’ evaluation concerning the contribution of item \( I_i \) in the evaluation of the competency level, \( k \), according to the criterion \( c_j \).

The criteria are weighted according to their relevance across the selection process. Such relevance is provided by a weighting vector, \( W \), that it is modified across the time by a dynamic process.

\[ W = \{w_1, w_2, \ldots, w_m\} \] set of weight associated to each criterion \( c_j \).

### 3.2.2. Item selection process

Once it has been defined the decision framework for the decision model that will be applied in the item selection process of the diagnosis phase of the ITS-C. Here, it is detailed the item selection process that consists of the following steps:

1. Gathering experts’ knowledge: The teachers involved in the ITS-C provide their knowledge about the contribution of an item \( I_i \) in the evaluation of the competency level, \( k \), according to each criterion \( c_j \) by means of a linguistic value, \( x_i^j \), in \( S \).

2. Item contribution: Once it is known the contribution of the item according to each criterion, an overall contribution, \( x_i^j \), must be computed taking into account the contribution of each criterion, \( x_i^j \). Such item contribution is computed by aggregating individual contributions (see Fig. 7) by using Eq. (9).

3. Item selection: The next step is to obtain the most suitable item according to the previous overall contributions. Therefore, applying an exploitation process which selects the item with maximum utility, \( I_i = \max(x_i^j) \) being \( k \) the current level of student evaluated.

This is the item selection process, but considering that it is utilized in the diagnosis process of the ITS-C in the CAT. Once the item has been chosen still there are some extra steps that should be taken into account.

4. Student diagnosis: the item is removed from the item bank and presented to the student. Afterwards it is calculated the new distribution \( P(\theta = k|\bar{u}) \) of student level of competence and the level of competence \( \theta = k \).

5. Stop criterion: If the threshold of accuracy required in the evaluation of the student has been reached then the CAT ends, otherwise a dynamic process to recalculate the weighting vector, \( W \), associated to the criteria it is applied and then the process goes back to step 2.

![Fig. 9. Linguistic MCDM item selection in the diagnosis process.](image-url)
From the previous process we are going to describe in further detail its main novelties: the aggregation step (step 2) and the dynamic process of the stop criterion (step 5). Because the others has been further detailed in the literature (Badaracco & Martínez, 2012; Guzmán et al., 2007; Wainer, 1990):

- **Aggregation process**: It obtains a linguistic collective assessment, \( x^t_k \), for each item, \( k \), by aggregating the expert's linguistic assessments, \( x^l_k \). Such a collective value will represent the contribution of the item \( k \) in the evaluation of the competence level, \( t \). To accomplish the aggregating process a linguistic aggregation operator must be chosen (Martínez et al., 2010). It has been pointed out that the 2-tuple model provides easy, accurate and understandable results in CW. Therefore, we propose the use of the weighted average operator extended to 2-tuple fuzzy information linguistic proposed by (Herrera & Martínez, 2000):

\[
X^t_k = \Delta \left( \sum_{j=1}^{m} \Delta^{-1} (s_j, z_j) \cdot w_j \right) = \Delta \left( \sum_{j=1}^{m} \beta_j \cdot w_j \right)
\]  

Being \( \{(s_1, z_1), \ldots, (s_m, z_m)\} \) the linguistic assessments to be aggregated, \( \{w_1, \ldots, w_m\} \) the weighting vector associated to the set of criteria \( C \) and \( \beta_j \) the numerical value obtained by function \( \Delta \) (see Eq. (8)).

- **Dynamic weighting computation**: During the diagnosis process it is logic that the weights assigned to the criteria used in the item selection process change across the time (Badaracco & Martínez, 2012), this change occurs according to the approximation of the level of competence inferred \( (\theta^t_{e_{k}} = k_{n-1}) \) in a time or iteration \( t = n-1 \). And the current level of competence at time \( t = n \) inferred after answering the \( n^{th} \) item of the CAT. Therefore, in the item selection process we propose the following dynamic process to manage the change of the weighting vector across the time.

(a) Initially, \( t = 1 \), the weights \( \{w_t\} \) for the criteria are equally distributed, i.e., \( n = 4 = > w_j = 0.25 \).

(b) It is defined, \( \delta_k \), as the increment of the probability for the current competency level \( \theta^t_{e_{k}} = k_n \) regarding to the probability of \( \theta^{t-1}_{e_{k}} = k_{n-1} \):

\[
\delta_k = P(\theta^t_{e_{k}} = k_n | u_s) - P(\theta^{t-1}_{e_{k}} = k_{n-1} | u_{s-1})
\]

Noted as indicator of convergence to a positive real number \( \varepsilon \). The values \( \delta_k \) and \( \varepsilon \) will have a preformance that affect the weights of the criteria. It is based on the principles proposed in the literature:

- Information increases with the difficulty of way relating to student's level of competence (Neira, 2002; Guzmán et al., 2007);

- Entropy is a measure of the uniformity of distribution. The aim is to obtain a pointed shape distribution and select the item that represents the largest decrease of entropy (Rudner, 2009);

(c) The indicator of convergence \( \varepsilon \) is defined as a small number, e.g., \( 0 < \varepsilon < 0.05 \). Such that, according to the relation between \( \delta_k \) and \( \varepsilon \) the weighting vector for next iteration, \( t = n + 1 \), is recalculated based on the principles proposed by (Neira, 2002; Guzmán et al., 2007; Rudner, 2009):

- Information increases with the difficulty of way relating to student's level of competence.

- Entropy is a measure of the uniformity of distribution. The objective is, therefore, have a pointed shape distribution and select the item that represents the largest decrease in entropy.

(d) Therefore the dynamic computation of the weighting vector, \( W \), is based on the following cases:

- If \( \theta^{t-1}_{e_{k}} = \theta_{e_{k}} \) and \( \delta_k > \varepsilon \) then level converges to \( k_n \). Therefore the weighting vector should change as:
  - Increasing the weight of \( w_3 \)
  - Decreasing weights \( w_1 \) and \( w_4 \)
  - Keep the value of \( w_2 \)

The items with maximum contribution will be those whose difficulty level is close to the student’s current competence, thereby improving the adaptation and so the accuracy of the assessment.

\[
\begin{align*}
  w_2 &= 0.50 \\
  w_1 &= 0.25 \\
  w_1 &= 0.125 \\
  w_4 &= 0.125
\end{align*}
\]  

Increasing the accuracy in estimating the level and accelerates the convergence of the evaluation.

\[
\begin{align*}
  w_2 &= 0.50 \\
  w_1 &= 0.25 \\
  w_1 &= 0.125 \\
  w_4 &= 0.125
\end{align*}
\]  

- If the level \( \theta^{t-1}_{e_{k}} = \theta_{e_{k}} \) and \( 0 < \delta_k < \varepsilon \) then level does not converge to \( k_n \) and loses precision. It is necessary that the algorithm selects items which enhances discriminative \( \delta_k \) and thus the convergence of the evaluation. Therefore the weighting vector should change as:

\[
\begin{align*}
  w_1 &= 0.50 \\
  w_2 &= 0.25 \\
  w_4 &= 0.125 \\
  w_3 &= 0.125
\end{align*}
\]  

- If the level \( \theta^{t-1}_{e_{k}} \neq \theta_{e_{k}} \) has been a change in the level competence estimate. Therefore the weighting vector should change as:
  - Increasing the weight of \( w_3 \)
  - Decreasing weights \( w_1 \) and \( w_4 \)
  - Keep the value of \( w_2 \)

The items will be most useful by those with equally likely to be answered correctly or incorrectly, thus the algorithm to prefer the level most likely competence in the distribution.

\[
\begin{align*}
  w_4 &= 0.50 \\
  w_1 &= 0.25 \\
  w_1 &= 0.125 \\
  w_2 &= 0.125
\end{align*}
\]

### 3.2.3. Algorithm for item selection process

Within the operation of a CAT in a STI-C, the steps of item selection algorithm are the following one (see Algorithm 2) (see Fig. 10):
Algorithm 2. Multi criteria item selection algorithm.

```
INT
FOR j=1 TO m
w_j = 0.25
ENDFOR
DO WHILE NOT stop criteria
evk_node ← MAP(\(P(\theta_{evk} = k|\theta_{evk})\))
ev_d ← MAP(\(P(\theta_{evd} = k|\theta_{evd})\))
exp_node ← MAP(\(P(\theta_{exp} = k|\theta_{exp})\))
min_node ← Equation(evk_node, ev_d, exp_node)
\(N^* = 6\)
\(r = 0.65\)
FOR i, item_bank of min_node
IF \(N^* < \text{Equation}(\theta_{evk})\)
\(N^* = \text{Equation}(\theta_{evk})\)
selected_item ← i
ENDFOR
ENDFOR
SHOW selected item
item_response ← student_response
END
DO CASE
CASE min_node = evk_node
FOR k = 1 TO l
\(P(\theta_{evk} = k|\theta_{evk}) = P(\theta_{evd} = k|\theta_{evd})\)
\(P(\theta_{evd} = k|\theta_{evd}) = P(\theta_{exp} = k|\theta_{exp})\)
\(P(\theta_{exp} = k|\theta_{exp}) = \text{Equation}(\theta_{exp} = k|\theta_{exp}), \text{item_response}\)
\(P(\theta_{evk} = k|\theta_{evk}) = P(\theta_{evd} = k|\theta_{evd})\)
ENDFOR
\(\theta_{evd} = \text{Equation}(\theta_{evd} = k|\theta_{evd})\)
\(\theta_{exp} = \text{Equation}(\theta_{exp} = k|\theta_{exp})\)
\(\theta_{evk} = \text{Equation}(\theta_{evk} = k|\theta_{evk})\)
CASE min_node = ev_d
FOR k = 1 TO l
\(P(\theta_{evd} = k|\theta_{evd}) = P(\theta_{exp} = k|\theta_{exp})\)
\(P(\theta_{exp} = k|\theta_{exp}) = \text{Equation}(\theta_{exp} = k|\theta_{exp}), \text{item_response}\)
\(P(\theta_{evd} = k|\theta_{evd}) = P(\theta_{exp} = k|\theta_{exp})\)
ENDFOR
\(\theta_{exp} = \text{Equation}(\theta_{exp} = k|\theta_{exp})\)
\(\theta_{evd} = \text{Equation}(\theta_{evd} = k|\theta_{evd})\)
\(\theta_{evk} = \text{Equation}(\theta_{evk} = k|\theta_{evk})\)
CASE min_node = exp_node
FOR k = 1 TO l
\(P(\theta_{exp} = k|\theta_{exp}) = P(\theta_{evk} = k|\theta_{evk})\)
\(P(\theta_{evk} = k|\theta_{evk}) = \text{Equation}(\theta_{evk} = k|\theta_{evk}), \text{item_response}\)
\(P(\theta_{exp} = k|\theta_{exp}) = P(\theta_{evk} = k|\theta_{evk})\)
ENDFOR
\(\theta_{evk} = \text{Equation}(\theta_{evk} = k|\theta_{evk})\)
\(\theta_{exp} = \text{Equation}(\theta_{exp} = k|\theta_{exp})\)
\(\theta_{evd} = \text{Equation}(\theta_{evd} = k|\theta_{evd})\)
ENDCASE
IF \(\theta_{evd} > \theta_{exp}\)
\(\theta_{evd} = P(\theta_{evd} = k|\theta_{evd})\)
DO CASE
CASE \(\theta_{evd} > \theta_{exp}\)
FOR j = 1 TO m
\(w_j = \text{casistico}10\)
ENDFOR
CASE \(0 < \theta_{evd} < \theta_{exp}\)
FOR j = 1 TO m
\(w_j = \text{casistico}1\)
ENDFOR
CASE \(\theta_{exp} < \theta_{evk}\)
FOR j = 1 TO m
\(w_j = \text{casistico}2\)
ENDFOR
ELSE
FOR j = 1 TO m
\(w_j = \text{casistico}13\)
ENDFOR
ENDCASE
\(\theta_{evd} = \text{Equation}(\theta_{evd} = k|\theta_{evd}), P(\theta_{evd} = k|\theta_{evd}), P(\theta_{exp} = k|\theta_{exp})\)
RETURN
```

4. An ITS-C based on a diagnosis process based on a linguistic multi-criteria decision model

Here an illustrative example for a CAT in an ITS-C based on the Algorithm 2 proposed before is developed by using an Android platform, so-called Micro Computerized Adaptive Test M-Cat (see Fig. 11).

We briefly describe the features of the M-Cat application: firstly show the architecture, technologies and profiling of the users used for its implementation. Eventually, it is described the functionality of the M-Cat and its performance in a concrete example.

4.1. M-Cat application

We have chosen to develop M-Cat on a mobile platform (Android) for the following reasons:

- The advent of mobile devices with wide age ranges in population, especially in young people that study on secondary and higher education levels.
- The modularity of the domain model for building and distributing M-Cat is suitable to run on various mobile devices.
Android is a mobile operating system based on Linux, together with applications middleware is aimed to be used in mobile devices such as smartphones, tablets, Google TV and other devices. It is developed and supported by the Open Handset Alliance, which is led by Google.

The Android application development does not require complex programming languages to learn. All that is needed is an acceptable knowledge of Java and access the software development kit (SDK) provided by Google which can be downloaded free. All applications are compressed into File Application Package (APK package is a variant of the JAR format Java and is used for distributing and installing bundled components for the Android platform), which can be easily installed from anywhere in the file browser most devices.

**Architecture and Technology.** M-Cat was developed to run on mobile devices (smartphones and tablets) on the Android platform, the main features of the architecture are:

- The structure of the Android operating system consists of applications running on a Java framework for object-oriented applications on the core Java libraries on a Dalvik virtual machine with runtime compilation. Libraries written in C include a GUI manager (surface manager), an OpenCore framework, a relational database SQLite, a programming interface API OpenGL ES 2.0 3D graphics, a WebKit rendering engine, SGL graphics engine, SSL and Bionic C standard library.
- Most applications are written in Java, there is not a Java virtual machine on the platform. The Java bytecode is executed, but is first compiled into a Dalvik executable and run on Dalvik Virtual Machine Dalvik. Dalvik is a specialized virtual machine designed specifically for Android.
- We have used the Java language and for persistent storage of data we have implemented a SQLite database.
- The development of generic design of M-Cat was made on Android using the SDK. The result is an APK that can be downloaded by the expert teacher for parameterization and distribution (Fig. 12). Or a project package that can be worked through by Android Development Tools (ADT) plugin for the Eclipse integrated development environment (IDE).

**User Interface.** M-Cat chases two main aims, first to facilitate expert teacher designing M-Cat on some domain of knowledge, including the construction of the item bank and then distribute it.
for students use. Second to allow students access on their mobile devices to M-Cat able to assess competencies modularly with feedback to fingertips. There are two types of users:

- Expert teacher. He is responsible for designing parameterize M-Cat, provides all the information necessary for implementation:
  - Parameters of the M-Cat, after downloading the M-Cat the teacher has two alternatives of parameterization:
    - From a mobile device: using the functionality of the interfaces provided by the application (Fig. 13), which have to enter data for the modules (quantity, description), evaluation criteria and ending, etc. Finally, should configure the item bank, item quantity, description, answer choices, etc. As well as to assess the usefulness of each items in accordance the criteria established in the model.
    - The other possibility is to work directly on the project package (through integrated development environment that includes ADT, such as Eclipse) and incorporate directly the SQLite database respecting the prescribed format for M-Cat.

The next step consists in the generation of the M-Cat (APK application) for distribution.

- Packaging and distribution:
  - According the specifications set by the teacher, the packaging can build the application ready to be installed on a device.
Since $X_k$ is presented and the aggregation Table 1 is the expert assessment of items for level 4, 2-tuples represent values of recalculation of $e_{vk}$ evidence node 4.2. Performance

Step 2: Based on the student response, it is computed new distribution of level of competence assuming that selects option shown to the student (see Fig. 14).

Step 3: Since no one has achieved threshold, proceed to step 4.

Step 4:

$\delta_k = 0.72 - 0.40 = 0.32$

Since $\theta_{vk}^{-1} = \theta_{vk}$ and $0 \leq \delta_k \leq \varepsilon$ then apply the casuistic (11);

Table 2

| $l_x$ | $P(l_x|\epsilon_l)$ | $l_y$ | $P(l_y|\epsilon_l)$ |
|-------|-----------------|-------|-----------------|
| 0.05(k = 1) | 0.05(k = 1) | 0.05(k = 2) | 0.05(k = 3) |
| 0.05(k = 3) | 0.05(k = 4) | 0.05(k = 4) | 0.05(k = 5) |

Table 3

<table>
<thead>
<tr>
<th>$l_x$</th>
<th>Criteria</th>
<th>$X_k^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125+C1</td>
<td>(VL,0)</td>
<td>(L,0)</td>
</tr>
<tr>
<td>0.25+C2</td>
<td>(L,0)</td>
<td>(H,0)</td>
</tr>
<tr>
<td>0.25+C3</td>
<td>(H,0)</td>
<td>(VL,0)</td>
</tr>
<tr>
<td>0.125+C4</td>
<td>(VL,0)</td>
<td>(H,0)</td>
</tr>
</tbody>
</table>

– The layout provides the application. It can be done via Android Market, download site or any means of distribution.

• Student. It is the ultimate receiver of the M-Cat, download and install the application, perform the proposed test and interaction (Fig. 14). He is admitted to:

  • Check status of student model.
  • Select element of competition to assess.
  • Set up system options.
  • Perform the M-Cat.

4.2. Performance

Let us suppose that is being evaluating a student by CAT in evidence node $e_{vk}$, whether the probability distribution of competence levels as follows:

$P(\theta_{vk} = 1|\epsilon_l) = 0.10$

$P(\theta_{vk} = 2|\epsilon_l) = 0.10$

$P(\theta_{vk} = 3|\epsilon_l) = 0.30$ \Rightarrow $\theta_{vk}^{-1} = k = 4$

$P(\theta_{vk} = 4|\epsilon_l) = 0.40$

$P(\theta_{vk} = 5|\epsilon_l) = 0.10$

Table 1 is the expert assessment of items for level 4, 2-tuples represent the aggregation $X_k^i$ of the vector of preferences:

Let CCO values of $l_x$ and $l_y$ items bank for level 4 (see Table 2):

Step 1: It is selected item with $\max(X_k^i)$ i.e., $l_x$ and the item is shown to the student (see Fig. 14).

Step 2: Based on the student response, it is computed new distribution of level of competence assuming that selects option 4:

$P(\theta_{vk} = 1|\epsilon_l) = 0.10 = 0.00$

$P(\theta_{vk} = 2|\epsilon_l) = 0.10 = 0.00$

$P(\theta_{vk} = 3|\epsilon_l) = 0.30 = 0.05$

$P(\theta_{vk} = 4|\epsilon_l) = 0.40 = 0.90$

$P(\theta_{vk} = 5|\epsilon_l) = 0.10 = 0.05$

\Rightarrow $\theta_{vk}^i = k = 4$

Step 3: Changes the weights of the criteria $c_j$ are recalculated $X_k^i$ are obtained the values shown in Table 3:

Observe that in the previous iteration $X_k^i < X_k^i$, but with the change of the weights $w_j$ and the recalculation of $X_k^i$ now $X_k^i > X_k^i ((M, 0) > (M, -0.25))$ and although both have the same label is possible to select the most useful item, reflecting the dynamics present in the selection process. Moreover, the representation of information with linguistic 2-tuples, allows to select an alternative from among several that have the same label value. It is continues with step 1.

5. Conclusions

We have presented a new fuzzy linguistic dynamic algorithm in selection of items in a CAT for ITS-C where we have exploited the idea of transforming the selection of items in a multi-criteria decision problem also contemplates the dynamics that can affect to the criteria, well as the representation of the contribution of the items by expert criteria using 2-tuple fuzzy linguistic information to achieve expressiveness, flexibility and intelligibility. The computational model has proven effective in a CAT for STI-C to select the item with maximum utility in the evaluation of the student. Moreover, the modular architecture of the domain model on an ITS-C is particularly useful for micro CAT applications for mobile devices.

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References


