A New Target Field Method for Optimizing Longitudinal Gradient Coils’ Property

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Abstract — A new approach has been presented in the paper to optimize the longitudinal gradient coil performance. First, traditional spherical harmonic target field method is deduced, the relation between the magnetic field and the current distribution is described by a matrix equation. Then, simulated annealing method is introduced to the optimization procedure, and those high order coefficients which are used to vanish become the variables designed for optimization. Finally, stream function method is used to transform the current density into discrete gradient coils. Comparison between traditional method and the optimized method shows that the inhomogeneity in the region of interest can be reduced from 13.59% to 4.9%, the coil efficiency is increased from $11.56 \text{mT/m/A}$ to $18.07 \text{mT/m/A}$, and the minimum distance of the discrete current wire is raised from 0.95 mm to 2.02 mm.

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In MRI (Magnetic Resonance Imaging) system, the gradient field are used to encode the NMR returned signals from patients. It is very important to design a high-quality gradient coil. Nowadays, there are mainly two main methods have been used in the design of gradient coils: TF (target field) method and SA (simulated annealing) method. TF method is a widely used method for designing gradient coils, which was first put forward by Turner [1, 2] in 1986. In this method, the desired magnetic fields are first specified in the DSV region, and then the current density is evaluated by inverse Fourier transformation. Finally, stream functions [3] are used to change the current density into discrete wires. Terry uses spherical harmonics to represent the target field [4] and provides us a very convenient way to design biplanar gradient and shim coils. However, there are some deficiencies in traditional TF method. One particular drawback is that the current distribution is continuous current density, which can only be approximated by a discrete distribution, and the discrete process may bring some errors to the ideal field. In addition, longitudinal gradient coils designed by traditional TF method tend to concentrate to the periphery of the pole-piece, which increases the inductance and makes gradient coils extremely hard to built. Finally, it is usually a difficult job for TF method to optimize all the coil performance simultaneously. Another way for gradient coil design is stochastic optimization. One of the best tools is SA (Simulated Annealing) method which was first published by Metropolis in 1953 and further developed by Kirkpatrick and co-workers. In 1993, Crozier applied the algorithm to the design of gradient coils [5, 6]. The advantage of this method is that we can make an overall control of coil performance by defining a suitable energy function $E$. However, an obvious shortage is that it needs a large amount of time to make the optimization procedure to converge to the optimal solution. Here we follow Morrone's target field method for the design of transverse gradient coils, a new approach have been introduced to optimize the current geometry with simulated annealing method. A high quality gradient coil has been built using the new optimization method.

1. METHODS

1.1. Target Field Method

The divergence of magnetic induction $\mathbf{B}(\mathbf{r})$ vanish everywhere, $\nabla \cdot \mathbf{B} = 0$. $\mathbf{B}$ can be written by taking the curl of the vector potential $\mathbf{A}(\mathbf{r})$ in the form (1)–(2). $\mathbf{A}(\mathbf{r})$ is defined to describe the
magnetic field at the observation point \( \mathbf{r} \), which is generated by the current density \( \mathbf{J}(\mathbf{r'}) \). The vacuum permeability \( \mu_0 = 4\pi \cdot 10^{-7} \text{N/A}^2 \).

\[
\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}(\mathbf{r'})}{|\mathbf{r} - \mathbf{r'}|} d\tau'
\]

\[
\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \nabla \frac{1}{|\mathbf{r} - \mathbf{r'}|} \times \mathbf{j}(\mathbf{r'}) d\tau'
\]

In biplanar gradient coil design, current distribution is constrained to the pole pieces, and steady state magnetic phenomena are characterized by no change in the net charge density anywhere. Then we can easily get \( \nabla \cdot \mathbf{J}(\mathbf{r'}) = 0 \) and \( \mathbf{J}_z(\mathbf{r'}) = 0 \). So current density can be expressed in terms of the curl of the stream function \( S \) (4–6), and stream function \( S \) is defined by the form of series expansion (3).

\[
S = S e_z = - \sum_{q=1}^{Q} U_q \sin \left[ q c \left( \rho' - \rho_{\text{min}} \right) \right] e_z
\]

\[
J'_\rho = \frac{1}{\rho'} \frac{\partial S}{\partial \varphi'} = 0
\]

\[
J'_\varphi = - \frac{\partial S}{\partial \rho'} = \sum_{q=1}^{Q} U_q q c \cos \left[ q c \left( \rho' - \rho_{\text{min}} \right) \right]
\]

\[
c = \frac{2\pi}{(\rho_{\text{max}} - \rho_{\text{min}})}
\]

where \( \rho_{\text{max}} \) and \( \rho_{\text{min}} \) are the maximum and minimum winding radius, and variable \( Q \) indicates that there are totally \( Q \) terms of coefficient \( U_q \). Then we will return to (2) and substitute \( |\mathbf{r} - \mathbf{r'}|^{-1} \) by the spherical harmonic expansion.

\[
\nabla \frac{1}{|\mathbf{r} - \mathbf{r'}|} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{4\pi}{2n + 1} \frac{r^n}{r_{n+1}} Y_{nm}(\theta, \varphi) Y^*_{nm}(\theta', \varphi')
\]

\[
= - \text{Re} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{2(n-m)!}{\delta_m (n+m)!} r^n P_m^n (\cos \theta) e^{im\varphi'} \nabla' \left( e^{-im\varphi'} r^{n+1} P_m^n (\cos \theta') \right)
\]

Since we put the original point \( O \) in the center of the main magnet, as is shown in Figure 1, the distance of observation point \( \mathbf{r} \) is smaller than the distance of source point \( \mathbf{r'} \). And let \( x = \cos \theta \),

![Figure 1: Main magnet structure of the micro magnetic resonance imaging system.](image-url)
we can rewrite Legendre function \( P^m_n(x) \) by means of Rodrigues’s formula. Thus, equation (7) is obtained. Longitudinal gradient coil are required to provide a field \( B \) such that the \( z \) component \( B_z \) is in direct proportion to \( z \). And since longitudinal gradient coils are axial symmetric, we can easily find that coefficient \( m = 0 \). For terms \( m \neq 0 \), expression \( r^n P^m_n(x) \) should be canceled in the process of magnetic field integration.

After expand the gradient operator of (7) in cylindrical coordinates, \( z \) component of the the magnetic induction intensity \( B \) is given by the equation (8)

\[
B_z^a(r, \theta, \varphi) = \mu_0 \sum_{n=0}^{\infty} r^n P_n(\cos \theta) \left\{ \sum_{q=1}^{Q} U_q \rho' q_c \cos [qc (\rho' - \rho_{\text{min}})] \right\} \left[ \frac{- (n + 1)}{\rho'^{(n+2)}} \frac{1}{r^{(n+2)}} \frac{dP_n(\cos \theta')}{d\theta'} \right] d\rho'
\]

For biplanar gradient coils, there are two same current sheets fixed at a distance of \( \pm a \) from the origin \( O \). Because of the inherent symmetry associated with the gradient fields, it is convenient to consider symmetric current distribution \( J_{z=a} = -J_{z=-a} \). Therefore, we can write \( z \)-component of the magnetic field as

\[
B_z(r, \theta, \varphi) = B_z^a + B_z^{-a} = \mu_0 \sum_{n=1}^{\infty} r^{2n-1} P_{2n-1}(\cos \theta) \left\{ \sum_{q=1}^{Q} U_q \rho' q_c \cos [qc (\rho' - \rho_{\text{min}})] \right\} \left[ \frac{-2n}{\rho'^{(2n+1)}} \frac{1}{r^{(2n+1)}} \frac{dP_{2n-1}(\cos \theta')}{d\theta'} \right] d\rho'
\]

According to coefficient \( n \) and \( m \), we can rewrite equation (9) into the the form of matrix equation (10). \( C_q \) is the weighting factor of term \( r^{2n-1} P_{2n-1}(x) \). Coefficient \( A_{qn} \) can be calculated from equation (9).

\[
\begin{bmatrix}
A_{11} & \cdots & A_{1Q} \\
\vdots & \ddots & \vdots \\
A_{N1} & \cdots & A_{NQ}
\end{bmatrix}
\begin{bmatrix}
U_1 \\
\vdots \\
U_Q
\end{bmatrix}
= 
\begin{bmatrix}
C_1 \\
\vdots \\
C_N
\end{bmatrix}
\]

\[ (10) \]

So, if coefficient \( C_1, C_2, \ldots, C_N \) is given, coefficient \( U_1, U_2, \ldots, U_Q \) is obtained by equation (10). Then, according to stream function theory, we can transform current density distribution into discrete wires. And the current intensity carried by each wire is written as

\[
I_0 = (S_{\text{max}} - S_{\text{min}}) / N_0
\]

\[ (11) \]

1.2. Simulated Annealing Method

SA method is a global optimization technique, the basis of which is rooted from Monte Carlo iteration. The SA program begins at a high temperature, and the energy function of the system is optimized step by step through the process of annealing. In order to circumvent being trapped in local energy minima, we use Metropolis accept rule which allows the system to make positive energy move with a probability linked to Boltzman statistics. It is very convenient for SA to make an overall control of the gradient-coil properties by defining energy function \( E \),

\[
E = k_1 \frac{I_0}{MD} + k_2^{(\text{per}-5\%)}
\]

\[ (12) \]

\[
\text{per} = \frac{2 (\text{grad}_{\text{max}} - \text{grad}_{\text{min}})}{(\text{grad}_{\text{max}} + \text{grad}_{\text{min}})}
\]

\[ (13) \]

where \( MD \) is minimum distance of the discrete gradient coil. \( \text{per} \) is an index of inhomogeneity. \( \text{grad}_{\text{max}} \) and \( \text{grad}_{\text{min}} \) are the maximum and minimum gradient values in DSV (Diameter of spherical volume) region. \( k_1 \) and \( k_2 \) are weighting factors for efficiency and uniformity.
1.3. Optimization

We found a way to optimize the property of the gradient coil. In traditional target field method, we only consider the term $n = 1$ to design longitudinal gradient coils, and the other coefficients $C_2, \ldots, C_N$ are used to design high order shim coils. However, take into consideration of the error brought by current density discretization, Coefficients $C_2, \ldots, C_N$ can be used to reduce the error and improve the coil efficiency. Simulated annealing method has been used to optimize these coefficients, and the energy function can be written as equation (12).

2. RESULTS AND DISCUSSION

Gradient coils of the Micro-MRI (Magnetic Resonance Imaging) system are fixed on two pole pieces of the permanent magnet, as shown in Figure 1. $a = 0.0236$ m, the longitudinal distance between the current plane and the origin $O$; $d = 0.025$ m, the distance between the current plane and the pole plate; $\rho_{\text{max}} = 0.065$, maximum radius of the gradient coils; $\rho_{\text{min}} = 0$ m, minimum radius of the gradient coils; $N_0 = 12$, coil turns; In SA method, we use geometric rule as the annealing schedule rule, $T(i) = T_0 \times 0.95^i$, where $T_0 = 15.3$ is the initial optimization temperature; $M = 50$, maximum sample steps at each temperature; $S = 100$ annealing steps, so the total number of iterations is 5000; Since the existence of image interference, its 4rd-order influence must be taken into account in the model. Other parameters are listed: $k_1 = 10^{-4}$ and $k_2 = 20$, weighting factors for efficiency and homogeneity respectively; $DSV = 0.03$ m, diameter of spherical volume.

<table>
<thead>
<tr>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>per Effi(mT/m/A)</th>
<th>MD(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13.95%</td>
<td>11.56</td>
</tr>
<tr>
<td>0.06</td>
<td>37.50</td>
<td>$-6.64 \times 10^4$</td>
<td>$-2.25 \times 10^6$</td>
<td>4.91%</td>
<td>18.07</td>
</tr>
</tbody>
</table>

In traditional target field method only the first term of coefficient $C$ is used to design $z$-gradient coil, the results have been displayed in the first line of Table 1. However, during the process of optimization, we will use coefficient $C_2, C_3, \ldots, C_N$ as the variable optimized in SA method. The results have been shown in the second line of Table 1. The inhomogeneity in the DSV region is reduced from 13.59% to 4.91%, the efficiency of the gradient coil is increased from 11.56 mT/m/A to 18.07 mT/m/A, and the minimum distance is allowed to be adjusted from 0.95 mm to 2.02 mm.

Figure 2 is the gradient coil designed by traditional target field method, the current distribution often focus to a location which makes it hard to build. Figure 3 illustrates the final gradient wire optimized by simulated annealing method.
3. CONCLUSIONS
A new approach to optimize the longitudinal gradient performance has been proposed in this paper. Those high order coefficients are used as the variables optimized in simulated annealing method. Using the optimization method, the coil efficiency, uniformity and minimum distance have been considerably improved. In view of these test results, it appears that this new method is worthy and convenient to be used in the application of shim and gradient coils design.

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