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*Research Article*

# Mean-Square Exponential Input-to-State Stability of Stochastic Fuzzy Recurrent Neural Networks with Multiproportional Delays and Distributed Delays

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We are interested in a class of stochastic fuzzy recurrent neural networks with multiproportional delays and distributed delays. By constructing suitable Lyapunov-Krasovskii functionals and applying stochastic analysis theory, Itô's formula and Dynkin's formula, we derive novel sufficient conditions for mean-square exponential input-to-state stability of the suggested system. Some remarks and discussions are given to show that our results extend and improve some previous results in the literature. Finally, two examples and their simulations are provided to illustrate the effectiveness of the theoretical results.

## 1. Introduction

Since the theory and application of cellular neural networks was proposed by L.O. Chua and L. Yang in 1998, neural networks have become a hot topic. They can be applied to the analysis of static images and signal processing, optimization, pattern recognition, and image processing. Usually, a neural network is an information processing system. Its characteristic is local connections between cells, and its output functions are piecewise linear. Clearly, it is easy to realize large-scale nonlinear analog signals in real time and parallel processing, which improves the running speed. As is well-known, the stability is an important theoretical problem in the field of dynamics systems (e.g., see [1–24]). Thus, it is interesting to investigate the stability of nonlinear neural networks.

On one hand, the switching speed of amplifier is limited and the errors occur in electronic components. As a consequence, delays happen to dynamics systems, and the delays often destroy the stability of dynamics systems, even cause the heavy oscillation (e.g., see [25–35]). So it is significant to study the stability of delayed neural networks. For example, [36] discussed the global stability analysis for a class of Cohen-Grossberg neural network models. A new comparison principle is firstly introduced to study the stability of stochastic delayed neural networks in [37]. Global asymptotic stability analysis for integrodifferential systems modeling neural networks with delays was investigated in [38]. In [39], Zhu et al. considered the robust stability of Markovian jump stochastic neural networks with delays in the leakage terms. For more related results we refer the authors to [25, 40–43] and references therein. It is worthy to point out that all of the works aforementioned were focused on the traditional types of delays such as constant delays, time-varying bounded delays, and bounded distribute delays. However, delays in real lives may be unbounded. In this case, a class of so-called proportional delays can be used to describe the model of human brain, where delays give information of history and the entire history affects the present. Thus, it is interesting to study the stability of neural networks with proportional delays.

On the other hand, all of the works mentioned above were focused on the traditional neural networks models, which did not consider fuzzy logic. But in the factual operations, we always encounter some inconveniences such as the complicity and the uncertainty or vagueness. In fact, vagueness always opposite to exactness. Therefore, vagueness can not be avoided in the human way of regarding the world. So fuzzy theory is regarded as the best suitable setting to take vagueness into consideration. It is reported that there have appeared many results on the stability analysis of fuzzy neural networks in the literatures. For example, Li and Zhu introduced a new way to study the stability of stochastic fuzzy delayed Cohen-Grossberg neural networks [44]. They used Lyapunov functional, stochastic analysis technique and nonnegative semimartingale convergence theorem to solve the problem. In [45], Balasubramaniam and Ali studied the robust exponential stability of uncertain fuzzy Cohen-Grossberg neural networks with time-varying delays. However, to the best of our knowledge, until now, there have been no works on the stability of fuzzy neural networks with proportional delays.

Motivated by the above discussion, in this paper we investigate the problem of the input-to-state stability analysis for a class of the stochastic fuzzy delayed recurrent neural networks with multiproportional delays and distributed delays. Some novel sufficient conditions are derived to ensure the mean-square exponential input-to-state stability of the suggested system based on constructing suitable Lyapunov-Krasovskii functionals and stochastic analysis theory, Itô's formula and Dynkin's formula. Several remarks and discussions are presented to show that our results extend and improve some previous results in the literature. Finally, two examples and their simulations are given to show the effectiveness of the obtained results.

The rest of the paper is as follows. In Section 2, we introduce the model, some necessary assumptions, and preliminaries. In Section 3, we investigate the mean-square exponential stability of the considered model. In Section 4, we provide two examples to illustrate the effectiveness of the obtained results. Finally, we conclude the paper in Section 5.

## 2. Model Formulation and Preliminaries

Let $C([p,1];\mathbb{R}^n)$ denote the family of continuous functions $\phi$ from $[p,1]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\| = \sup_{p\le\theta\le1}|\phi(\theta)|$. Denote by $\mathscr{L}^2_{\mathscr{F}_0}([p,1];\mathbb{R}^n)$ the family of all $\mathscr{F}_0$ measurable, $C([p,1];\mathbb{R}^n)$ valued stochastic variables $\phi = \phi(s) : p \le s \le 1$ satisfying $\int_p^1 \mathbf{E}|\phi(s)|^2 ds < \infty$, and $C([-\tau,1];\mathbb{R}^n)$ valued stochastic variables $\phi = \phi(s) : -\tau \le s \le 0$ satisfying $\int_{-\tau}^0 \mathbf{E}|\phi(s)|^2 ds < \infty$, in which $\mathbf{E}$ stands for the correspondent expectation operator with respect to the given probability measure $P$. $l_\infty$ denotes the class of essentially bounded functions $u$ from $[1,\infty)$ to $\mathbb{R}$ with $\|u\|_\infty = \operatorname{ess\,sup}_{t\ge1}|u(t)| < \infty$, $\mathbb{R}$ denotes real number. $\mathbb{R}^n$ denotes $n$ dimensions Euclidean space.

In this section, we consider the following class of stochastic fuzzy delayed recurrent neural networks with multiproportional delays and distributed delays:

$$\begin{aligned} dx_i(t) = \Bigg[ & -d_i x_i(t) + \bigwedge_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ & + \bigwedge_{j=1}^{n} b_{ij} g_j(x_j(p_j t)) + \bigwedge_{j=1}^{n} c_{ij} \int_{t-\tau(t)}^{t} h_j(x_j(s))\, ds \\ & + \bigvee_{j=1}^{n} d_{ij} f_j(x_j(t)) + \bigvee_{j=1}^{n} e_{ij} g_j(x_j(p_j t)) \\ & + \bigvee_{j=1}^{n} f_{ij} \int_{t-\tau(t)}^{t} h_j(x_j(s))\, ds + u_i(t) \Bigg] dt \\ & + \sum_{j=1}^{n} \sigma_{ij}(x_j(t), x_j(p_j t))\, dw_j(t), \end{aligned} \tag{1}$$

$$x_i(t) = \varphi_i(t), \quad p \le t \le 0, \tag{2}$$

for all $t \ge 0, i = 1,2,\ldots,n$, where $x_i(t)$ represents the state variable of the $i$th neuron at time $t$; $d_i$ is the self-feedback connection weight strength. The constants $a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}$, and $f_{ij}$ are the connection weights of the $j$th neuron to the $i$th neuron at time $t$ or $p_j t$. $f_j(x_j(t))$, $g_j(x_j(p_j t))$, and $h_j(x_j(t))$ are the $j$th neuron activation functions at time $t$ or $p_j t$. $u_i(t)$ is the control input of the ith neuron at time $t$, and $u = (u_1(t), u_2(t), \ldots, u_n(t)) \in l_\infty$. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. The noise perturbation $\sigma_{ij} : \mathbb{R}\times\mathbb{R}$ is a Borel measurable function, and $w_j(t)$, $j \ge 0$, $j = 1,2,\ldots,n$ are scalar standard Brownian motions defined on a complete probability space $(\Omega, \mathscr{F}, \mathbf{P})$ with a natural filtration $\{\mathscr{F}_t\}_{t\ge1}$. The constants $p_j$, $j = 1,2,\ldots,n$ are proportional delay factors and satisfy $0 < p_j t = t - (1-p_j)t$, where $(1-p_j)t$ are time-varying continuous functions that satisfy $(1-p_j)t \longrightarrow +\infty$ as $t \longrightarrow +\infty$ and $p_j \ne 1$. $\tau(t)$ is the time-varying delay, which satisfies $0 \le \tau(t) \le \tilde{\tau}$ and $\dot{\tau}(t) \le \delta < 1$.

Throughout this paper, we assume that the following conditions are satisfied.

*Assumption 1.* There exist positive constants $L_i$ and $M_i$ such that

$$\begin{aligned} |f_i(u) - f_i(v)| &\le L_i |u - v|, \\ |g_i(u) - g_i(v)| &\le M_i |u - v|, \end{aligned} \tag{3}$$

for all $u, v \in \mathbb{R}$ and $i = 1,2,\ldots,n$.

*Assumption 2.* There exist nonnegative constants $\mu_{ij}$, $\nu_{ij}$, and $\delta_{ij}$ such that

$$\begin{aligned} & [\sigma_{ij}(x,y,z) - \sigma_{ij}(x',y',z')]^2 \\ & \quad \le \mu_{ij}(x-x')^2 + \nu_{ij}(y-y')^2 + \delta_{ij}(z-z')^2 \end{aligned} \tag{4}$$

for all $x, x', y, y', z, z' \in \mathbb{R}$ and $i, j = 1,2,\ldots,n$.

*Assumption 3.*

$$f_j(0) = g_j(0) = h_j(0) = u_j(0) = 0, \qquad \sigma_{ij}(0,0) = 0, \quad i, j = 1, 2, \ldots, n. \tag{5}$$

Obviously, under Assumptions 1–3 we see that there exists a unique solution of system (1)-(2). Let $x(t, \varphi)$ denote the solution from the initial data $x(s) = \varphi(s)$ on $s \in [p, 1]$ in $\mathscr{L}^2_{\mathscr{F}_0}([p, 1]; \mathbb{R}^n)$. It is clear that system (1)-(2) has a trivial solution or zero solution $x(t; 0) \equiv 0$ corresponding to the initial data $\varphi(s) = 0$. By applying the following variable transformations $y_i(t) = x_i(e^t), v_i(t) = u_i(e^t), \widetilde{w}_j(t) = w_j(e^t), \phi_i(t) = \varphi_i(e^t)$, then system (1)-(2) is equivalently transformed into the following stochastic recurrent neural networks with constant delays and time-varying coefficients

$$\begin{aligned} dy_i(t) = e^t \Bigg[ & -d_i y_i(t) + \bigwedge_{j=1}^{n} a_{ij} f_j(y_j(t)) \\ & + \bigwedge_{j=1}^{n} b_{ij} g_j(y_j(t-\tau_j)) + \bigwedge_{j=1}^{n} c_{ij} \int_{t-\tau(t)}^{t} h_j(y_j(s))\, ds \\ & + \bigvee_{j=1}^{n} d_{ij} f_j(y_j(t)) + \bigvee_{j=1}^{n} e_{ij} g_j(y_j(t-\tau_j)) \\ & + \bigvee_{j=1}^{n} f_{ij} \int_{t-\tau(t)}^{t} h_j(y_j(s))\, ds + v_i(t) \Bigg] dt + \sum_{j=1}^{n} \sigma_{ij} \\ & \cdot (y_j(t), y_j(t-\tau_j))\, d\widetilde{w}_j(t), \quad t \geq 0, \end{aligned} \tag{6}$$

$$y_i(t) = \phi_i(t), \quad -\tau \leq t \leq 0, \tag{7}$$

where

$$\begin{aligned} & \tau_j = -\log p_j, \\ & \qquad j = 1, 2, \ldots, n, \ \tau = \max\left\{\widetilde{\tau}, \max_{1\leq j\leq n} \tau_j\right\}, \\ & \phi_i(t) \in C([-\tau, 0]; \mathbb{R}), \\ & \phi(t) = (\phi_1(t), \phi_2(t), \ldots, \phi_n(t))^T \in C([-\tau, 0]; \mathbb{R}^n). \end{aligned} \tag{8}$$

## 3. Main Results

In this section, we will discuss the mean-square exponential input-to-state stability of the trivial solution for system (1)-(2) under Assumptions 1–3.

**Theorem 4.** *Let Assumptions 1–3 hold. The trivial solution of system (1)-(2) is mean-square exponentially input-to-state stable, if there exist positive scalars $\eta_i$, $\alpha_i$, $\beta_i$, $(i = 1, 2, \ldots, n)$, $\lambda > 1$ such that*

$$\begin{aligned} 2\eta_i d_i \geq{} & (1+\lambda)\eta_i + \alpha_i + \tau\beta_i + \sum_{j=1}^{n} \eta_j \mu_{ji} + \sum_{j=1}^{n} \eta_j |a_{ji}| L_i \\ & + \eta_i \sum_{j=1}^{n} |a_{ij}| L_j + \eta_i \sum_{j=1}^{n} |b_{ij}| M_j \\ & + \eta_i \sum_{j=1}^{n} |c_{ij}| N_j + \sum_{j=1}^{n} \eta_j |d_{ji}| L_i \\ & + \eta_i \sum_{j=1}^{n} |d_{ij}| L_j + \eta_i \sum_{j=1}^{n} |e_{ij}| M_j \\ & + \eta_i \sum_{j=1}^{n} |f_{ij}| N_j, \end{aligned} \tag{9}$$

$$\begin{aligned} \alpha_i \geq{} & \sum_{j=1}^{n} e^{\lambda\tau} \eta_j \nu_{ji} + \sum_{j=1}^{n} e^{\lambda\tau} \eta_j |b_{ji}| M_i \\ & + \sum_{j=1}^{n} e^{\lambda\tau} \eta_j |e_{ji}| M_i, \end{aligned} \tag{10}$$

$$\beta_i \geq \frac{\tau}{(1-\delta)} \left( \sum_{j=1}^{n} \eta_j |c_{ji}| N_i + \sum_{j=1}^{n} \eta_j |f_{ji}| N_i \right), \tag{11}$$

*Proof.* Since system (6)-(7) is equivalent to system (1)-(2), we only need to prove that the trivial solution of system (6)-(7) is mean-square exponentially input-to-state stable. To this end, we let $\sigma(t) = (\sigma_{ij}(t))_{n\times n}, \sigma_{ij} = \sigma_{ij}(y_j(t), y_j(t-\tau_j))$, for the sake of simplicity.

$$\begin{aligned} V(t, y(t)) = {} & e^{(\lambda-1)t} \sum_{i=1}^{n} \eta_i y_i^2(t) + \int_{t-\tau_i}^{t} e^{\lambda s} \sum_{i=1}^{n} \alpha_i y_i^2(s)\, ds \\ & + \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} e^{\lambda s} \sum_{i=1}^{n} \beta_i y_i^2(s)\, ds. \end{aligned} \tag{12}$$

Then by Itô's formula, we have the following stochastic differential equation:

$$\begin{aligned} dV(t, y(t)) = {} & \mathscr{L}V(t, y(t))\, dt \\ & + V_y(t, y(t))\, \sigma(t)\, dw(t), \end{aligned} \tag{13}$$

where $V_y(t, y(t)) = (\partial V(t, y(t))/\partial y_1, \ldots, \partial V(t, y(t))/\partial y_n)$ and $\mathscr{L}$ is the weak infinitesimal operator such that

$$\begin{aligned} \mathscr{L}V(t, y(t)) = {} & (\lambda - 1) e^{(\lambda-1)t} \sum_{i=1}^{n} \eta_i y_i^2(t) \\ & + 2e^{(\lambda-1)t} \sum_{i=1}^{n} \eta_i y_i(t) e^{\lambda t} \Bigg[ -d_i y_i(t) \end{aligned}$$

$$+ \bigwedge_{j=1}^{n} a_{ij} f_j\left(y_j(t)\right) + \bigwedge_{j=1}^{n} b_{ij} g_j\left(y_j\left(t-\tau_j\right)\right) + \bigwedge_{j=1}^{n} c_{ij}$$

$$\cdot \int_{t-\tau(t)}^{t} h_j\left(y_j(s)\right) ds + \bigvee_{j=1}^{n} d_{ij} f_j\left(y_j(t)\right)$$

$$+ \bigvee_{j=1}^{n} e_{ij} g_j\left(y_j\left(t-\tau_j\right)\right) + \bigvee_{j=1}^{n} f_{ij} \int_{t-\tau(t)}^{t} h_j\left(y_j(s)\right) ds$$

$$\left. + \nu_i(t) \right] + e^{(\lambda-1)t} \sum_{i=1}^{n} \eta_i \sum_{j=1}^{n} \sigma_{ij}^2\left(y_j(t), y_j\left(t-\tau_j\right)\right)$$

$$+ e^{\lambda t} \sum_{i=1}^{n} \alpha_i y_i^2(t) - e^{\lambda(t-\tau)} \sum_{i=1}^{n} \alpha_i y_i^2\left(t-\tau_i\right) + \tau(t)$$

$$\cdot e^{\lambda t} \sum_{i=1}^{n} \beta_i y_i^2(t) - (1-\dot{\tau}(t)) e^{\lambda t} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \beta_i y_i^2(s) ds$$

$$\leq \lambda e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i^2(t) - 2e^{\lambda t} \sum_{i=1}^{n} \eta_i d_i y_i^2(t) + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t)$$

$$\cdot \bigwedge_{j=1}^{n} a_{ij} f_j\left(y_j(t)\right) + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t)$$

$$\cdot \bigwedge_{j=1}^{n} b_{ij} g_j\left(y_j\left(t-\tau_j\right)\right) + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t) \bigwedge_{j=1}^{n} c_{ij}$$

$$\cdot \int_{t-\tau(t)}^{t} h_j\left(y_j(s)\right) ds + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t)$$

$$\cdot \bigvee_{j=1}^{n} d_{ij} f_j\left(y_j(t)\right) + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t)$$

$$\cdot \bigvee_{j=1}^{n} e_{ij} g_j\left(y_j\left(t-\tau_j\right)\right) + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t) \bigvee_{j=1}^{n} f_{ij}$$

$$\cdot \int_{t-\tau(t)}^{t} h_j\left(y_j(s)\right) ds + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i y_i(t) \nu_i(t)$$

$$+ e^{\lambda t} \sum_{i=1}^{n} \eta_i \sum_{j=1}^{n} \mu_{ij} y_j^2 + e^{\lambda t} \sum_{i=1}^{n} \eta_i \sum_{j=1}^{n} \nu_{ij} y_j^2\left(t-\tau_j\right)$$

$$+ e^{\lambda t} \sum_{i=1}^{n} \alpha_i y_i^2(t) - e^{\lambda(t-\tau)} \sum_{i=1}^{n} \alpha_i y_i^2\left(t-\tau_i\right)$$

$$+ \tau e^{\lambda t} \sum_{i=1}^{n} \beta_i y_i^2(t) - (1-\delta) e^{\lambda t} \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \beta_i y_i^2(s) ds$$

$$\leq e^{\lambda t} \sum_{i=1}^{n} \left( -2\eta_i d_i + \lambda \eta_i + \sum_{j=1}^{n} \eta_j \mu_{ji} + \alpha_i + \tau \beta_i \right) y_i^2(t)$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \bigwedge_{j=1}^{n} \left|a_{ij}\right| \left|f_j\left(y_j(t)\right) - f_j(0)\right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \bigwedge_{j=1}^{n} \left|b_{ij}\right| \left|g_j\left(y_j\left(t-\tau_j\right)\right) - g_j(0)\right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \bigwedge_{j=1}^{n} \left|c_{ij}\right| \left| \int_{t-\tau(t)}^{t} h_j\left(y_j(s)\right) ds \right.$$

$$\left. - h_j(0) \right| + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \bigvee_{j=1}^{n} \left|d_{ij}\right| \left| f_j\left(y_j(t)\right) \right.$$

$$\left. - f_j(0) \right| + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \bigvee_{j=1}^{n} \left|e_{ij}\right| \left| g_j\left(y_j\left(t-\tau_j\right)\right) \right.$$

$$\left. - g_j(0) \right| + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \bigvee_{j=1}^{n} \left|f_{ij}\right|$$

$$\cdot \left| \int_{t-\tau(t)}^{t} h_j\left(y_j(s)\right) ds - h_j(0) \right| + 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right|$$

$$\cdot \left|\nu_i(t)\right| + e^{\lambda t} \sum_{i=1}^{n} \left( -e^{-\lambda \tau} \alpha_i + \sum_{j=1}^{n} \eta_j \nu_{ji} \right) y_i^2\left(t-\tau_i\right)$$

$$+ e^{\lambda t} (\delta - 1) \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \beta_i y_i^2(s) ds$$

$$\leq e^{\lambda t} \sum_{i=1}^{n} \left( -2\eta_i d_i + \lambda \eta_i + \sum_{j=1}^{n} \eta_j \mu_{ji} + \alpha_i + \tau \beta_i \right) y_i^2(t)$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \sum_{j=1}^{n} \left|a_{ij}\right| L_j \left|y_i(t)\right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \sum_{j=1}^{n} \left|b_{ij}\right| M_j \left|y_i\left(t-\tau_j\right)\right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \sum_{j=1}^{n} \left|c_{ij}\right| N_j \left| \int_{t-\tau(t)}^{t} \left(y_j(s)\right) ds \right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \sum_{j=1}^{n} \left|d_{ij}\right| L_j \left|y_i(t)\right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \sum_{j=1}^{n} \left|e_{ij}\right| M_j \left|y_i\left(t-\tau_j\right)\right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \sum_{j=1}^{n} \left|f_{ij}\right| N_j \left| \int_{t-\tau(t)}^{t} \left(y_j(s)\right) ds \right|$$

$$+ 2e^{\lambda t} \sum_{i=1}^{n} \eta_i \left|y_i(t)\right| \left|\nu_i(t)\right| + e^{\lambda t} \sum_{i=1}^{n} \left( -e^{-\lambda \tau} \alpha_i \right.$$

$$\left. + \sum_{j=1}^{n} \eta_j \nu_{ji} \right) y_i^2\left(t-\tau_i\right) + e^{\lambda t} (\delta - 1) \int_{t-\tau(t)}^{t} \sum_{i=1}^{n} \beta_i y_i^2$$

$$\cdot (s) ds$$

$$
\begin{aligned}
&\le e^{\lambda t}\sum_{i=1}^{n}\left(-2\eta_i d_i+\lambda\eta_i+\sum_{j=1}^{n}\eta_j\mu_{ji}+\alpha_i+\tau\beta_i\right)y_i^2(t)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}\eta_i\left|a_{ij}\right|L_j\left(\left|y_i(t)\right|^2+\left|y_j(t)\right|^2\right)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}\eta_i\left|b_{ij}\right|M_j\left(\left|y_i(t)\right|^2+\left|y_j\left(t-\tau_j\right)\right|^2\right)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}\eta_i\left|c_{ij}\right|N_j\left(\left|y_i(t)\right|^2+\left|\int_{t-\tau(t)}^{t}y_j(s)\,ds\right|^2\right)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}\eta_i\left|d_{ij}\right|L_j\left(\left|y_i(t)\right|^2+\left|y_j(t)\right|^2\right)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}\eta_i\left|e_{ij}\right|M_j\left(\left|y_i(t)\right|^2+\left|y_j\left(t-\tau_j\right)\right|^2\right)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n}\eta_i\left|f_{ij}\right|N_j\left(\left|y_i(t)\right|^2+\left|\int_{t-\tau(t)}^{t}y_j(s)\,ds\right|^2\right)\\
&\quad+e^{\lambda t}\sum_{i=1}^{n}\eta_i(\left|y_i(t)\right|^2+\left|\nu_i(t)\right|^2+e^{\lambda t}\sum_{i=1}^{n}\left(-e^{-\lambda\tau}\alpha_i\right.\\
&\quad\left.+\sum_{j=1}^{n}\eta_j\nu_{ji}\right)y_i^2\left(t-\tau_i\right)+e^{\lambda t}(\delta-1)\\
&\quad\cdot\int_{t-\tau(t)}^{t}\sum_{i=1}^{n}\beta_i y_i^2(s)\,ds\\
&\le -e^{\lambda t}\min_{1\le j\le n}\left[2\eta_j d_j-\lambda\eta_j-\sum_{k=1}^{n}\eta_k\mu_{kj}-\alpha_j-\tau\beta_j\right.\\
&\quad-\eta_j\sum_{k=1}^{n}\left|a_{jk}\right|L_k-\sum_{k=1}^{n}\eta_k\left|a_{kj}\right|L_j-\sum_{k=1}^{n}\eta_j\left|b_{jk}\right|M_k\\
&\quad-\sum_{k=1}^{n}\eta_j\left|c_{jk}\right|N_k-\eta_j\sum_{k=1}^{n}\left|d_{jk}\right|L_k-\sum_{k=1}^{n}\eta_k\left|d_{kj}\right|L_j\\
&\quad\left.-\sum_{k=1}^{n}\eta_j\left|e_{jk}\right|M_k-\sum_{k=1}^{n}\eta_j\left|f_{jk}\right|N_k-\eta_j\right]\sum_{i=1}^{n}y_i^2(t)\\
&\quad-e^{\lambda t}\min_{1\le j\le n}\left[e^{-\lambda\tau}\alpha_j-\sum_{k=1}^{n}\eta_k\nu_{kj}-\sum_{k=1}^{n}\eta_k\left|b_{kj}\right|M_j\right.\\
&\quad\left.-\sum_{k=1}^{n}\eta_k\left|e_{kj}\right|M_j\right]\sum_{i=1}^{n}y_i^2\left(t-\tau_i\right)-e^{\lambda t}\min_{1\le j\le n}(1-\delta)\\
&\quad\cdot\int_{t-\tau(t)}^{t}\sum_{k=1}^{n}\beta_k y_k^2(s)\,ds+e^{\lambda t}\min_{1\le j\le n}\left(\sum_{k=1}^{n}\eta_k\left|c_{kj}\right|N_j\right.\\
&\quad\left.+\sum_{k=1}^{n}\eta_k\left|f_{kj}\right|N_j\right)\sum_{k=1}^{n}\left(\int_{t-\tau(t)}^{t}y_k(s)\,ds\right)^2\\
&\quad+e^{\lambda t}\max_{1\le j\le n}\eta_j\left\|\nu\right\|_\infty^2.
\end{aligned}
\tag{14}
$$

Letting $\beta=\min\beta_k$, then we obtain

$$
\begin{aligned}
\mathscr{L}V(t,y(t))&\le -e^{\lambda t}\min_{1\le j\le n}\left[2\eta_j d_j-\lambda\eta_j-\sum_{k=1}^{n}\eta_k\mu_{kj}\right.\\
&\quad-\alpha_j-\tau\beta_j-\eta_j\sum_{k=1}^{n}\left|a_{jk}\right|L_k-\sum_{k=1}^{n}\eta_k\left|a_{kj}\right|L_j\\
&\quad-\sum_{k=1}^{n}\eta_j\left|b_{jk}\right|M_k-\sum_{k=1}^{n}\eta_j\left|c_{jk}\right|N_k-\eta_j\sum_{k=1}^{n}\left|d_{jk}\right|L_k\\
&\quad-\sum_{k=1}^{n}\eta_k\left|d_{kj}\right|L_j-\sum_{k=1}^{n}\eta_j\left|e_{jk}\right|M_k-\sum_{k=1}^{n}\eta_j\left|f_{jk}\right|N_k\\
&\quad\left.-\eta_j\right]\sum_{i=1}^{n}y_i^2(t)-e^{\lambda t}\min_{1\le j\le n}\left[e^{-\lambda\tau}\alpha_j-\sum_{k=1}^{n}\eta_k\nu_{kj}\right.\\
&\quad\left.-\sum_{k=1}^{n}\eta_k\left|b_{kj}\right|M_j-\sum_{k=1}^{n}\eta_k\left|e_{kj}\right|M_j\right]\sum_{i=1}^{n}y_i^2\left(t-\tau_i\right)\\
&\quad-e^{\lambda t}\min_{1\le j\le n}(1-\delta)\beta\int_{t-\tau(t)}^{t}\sum_{k=1}^{n}y_k^2(s)\,ds\\
&\quad+e^{\lambda t}\min_{1\le j\le n}\left(\sum_{k=1}^{n}\eta_k\left|c_{kj}\right|N_j+\sum_{k=1}^{n}\eta_k\left|f_{kj}\right|N_j\right)\\
&\quad\cdot\sum_{k=1}^{n}\left(\int_{t-\tau(t)}^{t}1ds\cdot\int_{t-\tau(t)}^{t}y_k^2(s)\,ds\right)\\
&\quad+e^{\lambda t}\max_{1\le j\le n}\eta_j\left\|\nu\right\|_\infty^2\\
&\le -e^{\lambda t}\min_{1\le j\le n}\left[2\eta_j d_j-\lambda\eta_j-\sum_{k=1}^{n}\eta_k\mu_{kj}-\alpha_j-\tau\beta_j\right.\\
&\quad-\eta_j\sum_{k=1}^{n}\left|a_{jk}\right|L_k-\sum_{k=1}^{n}\eta_k\left|a_{kj}\right|L_j-\sum_{k=1}^{n}\eta_j\left|b_{jk}\right|M_k\\
&\quad-\sum_{k=1}^{n}\eta_j\left|c_{jk}\right|N_k-\eta_j\sum_{k=1}^{n}\left|d_{jk}\right|L_k-\sum_{k=1}^{n}\eta_k\left|d_{kj}\right|L_j\\
&\quad\left.-\sum_{k=1}^{n}\eta_j\left|e_{jk}\right|M_k-\sum_{k=1}^{n}\eta_j\left|f_{jk}\right|N_k-\eta_j\right]\sum_{i=1}^{n}y_i^2(t)\\
&\quad-e^{\lambda t}\min_{1\le j\le n}\left[e^{-\lambda\tau}\alpha_j-\sum_{k=1}^{n}\eta_k\nu_{kj}-\sum_{k=1}^{n}\eta_k\left|b_{kj}\right|M_j\right.\\
&\quad\left.-\sum_{k=1}^{n}\eta_k\left|e_{kj}\right|M_j\right]\sum_{i=1}^{n}y_i^2\left(t-\tau_i\right)-e^{\lambda t}\min_{1\le j\le n}\left[(1-\delta)\beta\right.
\end{aligned}
$$

$$
\begin{aligned}
&- \left( \sum_{k=1}^{n} \eta_k \left| c_{kj} \right| N_j + \sum_{k=1}^{n} \eta_k \left| f_{kj} \right| N_j \right) \tau \Bigg] \\
&\cdot \int_{t-\tau(t)}^{t} \sum_{k=1}^{n} y_k^2 (s)\, ds + e^{\lambda t} \max_{1 \le j \le n} \eta_j \| \nu \|_\infty^2 .
\end{aligned} \tag{15}
$$

Now we define a Markov time as follows:

$$
\rho := \inf \{ s \ge 0 : |x(s)| \ge k \} . \tag{16}
$$

By using the Dynkin formula, we have

$$
\begin{aligned}
&\mathbf{E} V \left( t \wedge \rho_k, y \left( t \wedge \rho_k \right) \right) - \mathbf{E} V \left( 0, y(0) \right) \\
&\quad = \mathbf{E} \left[ \int_0^{t \wedge \rho_k} \mathscr{L} V \left( s, y(s) \right) ds \right],
\end{aligned} \tag{17}
$$

which implies

$$
\begin{aligned}
&\mathbf{E} V \left( t \wedge \rho_k, y \left( t \wedge \rho_k \right) \right) \\
&\quad = \mathbf{E} V \left( 0, y(0) \right) + \mathbf{E} \left[ \int_0^{t \wedge \rho_k} \mathscr{L} V \left( s, y(s) \right) ds \right].
\end{aligned} \tag{18}
$$

Letting $k \longrightarrow \infty$ on both sides (18), it follows from the monotone convergence theorem, (9), (10), and (12) that

$$
\begin{aligned}
\mathbf{E} V \left( t, y(t) \right) &\le \mathbf{E} V \left( 0, x(0) \right) + \| v \|_\infty^2 \max_{1 \le j \le n} \eta_j \int_0^t e \lambda s \, ds \\
&\le \mathbf{E} V \left( 0, x(0) \right) + \frac{1}{\lambda} \| \nu \|_\infty^2 \max_{1 \le j \le n} \eta_j \left( e^{\lambda t} - 1 \right) \\
&= \sum_{i=1}^{n} \eta_i \mathbf{E} y_i^2 (0) + \int_{-\tau_i}^{0} \mathbf{E} e^{\lambda s} \sum_{i=1}^{n} \alpha_i y_i^2 (s)\, ds \\
&\quad + \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} \mathbf{E} e^{\lambda s} \sum_{i=1}^{n} \beta_i y_i^2 (s)\, ds \\
&\quad + \frac{1}{\lambda} \| \nu \|_\infty^2 \max_{1 \le j \le n} \eta_j \left( e^{\lambda t} - 1 \right) \\
&\le \left( \max_{1 \le j \le n} \eta_j + \tau \max_{1 \le j \le n} \alpha_j + \tau \max_{1 \le j \le n} \beta_j \right) \mathbf{E} \| \phi \|^2 \\
&\quad + \frac{1}{\lambda} \| \nu \|_\infty^2 \max_{1 \le j \le n} \eta_j \left( e^{\lambda t} - 1 \right) .
\end{aligned} \tag{19}
$$

On the other hand, from the definition of $V(t, x(t)$, we have

$$
\begin{aligned}
\mathbf{E} V \left( t, y(t) \right) &\ge \mathbf{E} e^{(\lambda - 1)t} \sum_{i=1}^{n} \eta_i y_i^2 (t) \\
&\ge e^{(\lambda - 1)t} \min_{1 \le i \le n} \eta_i \mathbf{E} \left| y(t) \right|^2 .
\end{aligned} \tag{20}
$$

Combining (19) and (20), the following inequation holds:

$$
\begin{aligned}
&\mathbf{E} \left| y(t) \right|^2 \\
&\quad \le \frac{\max_{1 \le i \le n} \eta_i + \tau \max_{1 \le i \le n} \alpha_i + \tau \max_{1 \le i \le n} \beta_i}{\min_{1 \le i \le n} \eta_i} \\
&\quad \cdot e^{-(\lambda - 1)t} \mathbf{E} \| \phi \|^2 + \frac{\max_{1 \le i \le n} \eta_i}{\lambda \max_{1 \le i \le n} \eta_i} \| v \|_\infty^2 .
\end{aligned} \tag{21}
$$

From (21) we see that the trivial solution of system (6)-(7) is mean-square exponentially input-to-state stable. The proof of Theorem 4 is completed. □

**Corollary 5.** *Assume that all the conditions of Theorem 4 hold. Then the trivial solution of system (1)-(2) with $u(t) \equiv 0$ is mean-square exponentially stable.*

*Remark 6.* If we ignore the effects of delays, then system (1)-(2) becomes a stochastic recurrent neural network without delays. The results obtained in this paper are also applicable to the case of stochastic recurrent neural networks without delays.

*Remark 7.* Compared with the result in [44], our model is more general than that in [44]. In fact, multiproportional delays and distributed delays are considered in this paper and they yield much difficulty in the proof of our result, whereas only a simple constant delay was discussed in [44].

*Remark 8.* Compared with the result in [46], our model is also more general than that in [46] since distributed delays and fuzzy factor in this paper were ignored in [46].

## 4. Illustrative Examples

In this section, we will use two examples to show the effectiveness of the obtained result.

*Example 1* (2-dimension case). Consider the case of 2-dimension stochastic recurrent neural networks with multiproportional delays

$$
\begin{aligned}
dx_i(t) = \Bigg[ &-d_i x_i(t) + \bigwedge_{j=1}^{2} a_{ij} f_j \left( x_j(t) \right) \\
&+ \bigwedge_{j=1}^{2} b_{ij} g_j \left( x_j \left( p_j t \right) \right) + \bigwedge_{j=1}^{2} c_{ij} \int_{t-\tau(t)}^{t} h_j \left( x_j(s) \right) ds \\
&+ \bigvee_{j=1}^{2} d_{ij} f_j \left( x_j(t) \right) + \bigvee_{j=1}^{2} e_{ij} g_j \left( x_j \left( p_j t \right) \right) \\
&+ \bigvee_{j=1}^{2} f_{ij} \int_{t-\tau(t)}^{t} h_j \left( x_j(s) \right) ds + u_i(t) \Bigg] dt + \sum_{j=1}^{2} \sigma_{ij} \\
&\cdot \left( x_j(t), x_j \left( p_j t \right) \right) dw_j(t),
\end{aligned} \tag{22}
$$

$$
x_i(t) = \varphi_i(t), \quad p \le t \le 1, \ i = 1, 2, \tag{23}
$$

where

$$f(x) = g(x) = h(x) = \begin{cases} 0.1x, & \text{if } x \le 0, \\ 0.1\tan(x), & \text{if } x > 0, \end{cases} \tag{24}$$
$$u(t) = 0.08\sin(t),$$

and

$$\begin{aligned} &\left(\sigma_{ij}\left(x_j(t)\right), \left(p_j t\right)\right)_{2\times 2} \\ &\quad = \begin{pmatrix} 0.4x_1(t) & 0.2\left(x_2(t) + x_2(p_2 t)\right) \\ 0.2x_1(p_1 t) & 0.1\left(x_2(t) + x_2(p_2 t)\right) \end{pmatrix}. \end{aligned} \tag{25}$$

Other parameters of system (22)-(23) are given as follows:

$$\begin{aligned} \left(a_{ij}\right)_{2\times 2} &= \begin{pmatrix} 0.5 & 0.3 \\ 0.4 & 0.8 \end{pmatrix}, \\ \left(b_{ij}\right)_{2\times 2} &= \begin{pmatrix} 0.6 & 0.3 \\ 0.8 & 0.2 \end{pmatrix}, \end{aligned} \tag{26}$$

$$\begin{aligned} \left(c_{ij}\right)_{2\times 2} &= \begin{pmatrix} 0.4 & 0.3 \\ 0.2 & 0.5 \end{pmatrix}, \\ \left(d_{ij}\right)_{2\times 2} &= \begin{pmatrix} 0.5 & 0.7 \\ 0.3 & 0.8 \end{pmatrix}, \end{aligned} \tag{27}$$

$$\begin{aligned} \left(e_{ij}\right)_{2\times 2} &= \begin{pmatrix} 0.4 & 0.6 \\ 0.5 & 0.3 \end{pmatrix}, \\ \left(f_{ij}\right)_{2\times 2} &= \begin{pmatrix} 0.3 & 0.2 \\ 0.5 & 0.4 \end{pmatrix}, \end{aligned} \tag{28}$$

$d_1 = 9, d_2 = 8$. Take $p_1 = p_2 = 0.5, \lambda = 1.1, \eta_1 = 0.5, \eta_2 = 0.5, \alpha_1 = 2.6, \alpha_2 = 1.6, \beta_1 = 1.3, \beta_2 = 1.7$, and from the definition of $\tau$, we obtain $\tau = 0.6931$. It is easy to check that Assumptions 1–3 are satisfied. Moreover, a simple computation yields

$$\begin{aligned} 2\eta_1 d_1 &= 9 \\ &> (1+\lambda)\eta_1 + \alpha_1 + \tau\beta_1 + \sum_{j=1}^{2}\eta_j\mu_{j1} \\ &\quad + \sum_{j=1}^{2}\eta_j\left|a_{j1}\right| L_1 + \eta_1\sum_{j=1}^{2}\left|a_{1j}\right| L_j \\ &\quad + \eta_1\sum_{j=1}^{2}\left|b_{1j}\right| M_j + \eta_1\sum_{j=1}^{2}\left|c_{1j}\right| N_j \\ &\quad + \sum_{j=1}^{2}\eta_j\left|d_{j1}\right| L_1 + \eta_1\sum_{j=1}^{2}\left|d_{1j}\right| L_j \\ &\quad + \eta_1\sum_{j=1}^{2}\left|e_{1j}\right| M_j + \eta_1\sum_{j=1}^{2}\left|f_{1j}\right| N_j = 5.14103, \end{aligned} \tag{29}$$

$$\begin{aligned} 2\eta_2 d_2 &= 8 \\ &> (1+\lambda)\eta_2 + \alpha_2 + \tau\beta_2 + \sum_{j=1}^{2}\eta_j\mu_{j2} \\ &\quad + \sum_{j=1}^{2}\eta_j\left|a_{j2}\right| L_2 + \eta_2\sum_{j=1}^{2}\left|a_{2j}\right| L_j \\ &\quad + \eta_2\sum_{j=1}^{2}\left|b_{2j}\right| M_j + \eta_2\sum_{j=1}^{2}\left|c_{2j}\right| N_j \\ &\quad + \sum_{j=1}^{2}\eta_j\left|d_{j2}\right| L_2 + \eta_2\sum_{j=1}^{2}\left|d_{2j}\right| L_j \\ &\quad + \eta_2\sum_{j=1}^{2}\left|e_{2j}\right| M_j + \eta_2\sum_{j=1}^{2}\left|f_{2j}\right| N_j = 4.39327, \end{aligned} \tag{30}$$

$$\begin{aligned} \alpha_1 &= 2.6 \\ &\ge \sum_{j=1}^{2} e^{\lambda\tau}\eta_j\nu_{j1} + \sum_{j=1}^{2} e^{\lambda\tau}\eta_j\left|b_{j1}\right| M_1 \\ &\quad + \sum_{j=1}^{2} e^{\lambda\tau}\eta_j\left|e_{j1}\right| M_1 = 0.889511, \end{aligned} \tag{31}$$

$$\begin{aligned} \alpha_2 &= 1.6 \\ &\ge \sum_{j=1}^{2} e^{\lambda\tau}\eta_j\nu_{j2} + \sum_{j=1}^{2} e^{\lambda\tau}\eta_j\left|b_{j2}\right| M_2 \\ &\quad + \sum_{j=1}^{2} e^{\lambda\tau}\eta_j\left|e_{j2}\right| M_2 = 0.471548, \end{aligned} \tag{32}$$

$$\begin{aligned} \beta_1 &= 1.3 \\ &\ge \frac{\tau}{(1-\delta)}\left(\sum_{j=1}^{2}\eta_j\left|c_{j1}\right| N_1 + \sum_{j=1}^{2}\eta_j\left|f_{j1}\right| N_1\right) \\ &= 0.1581, \end{aligned} \tag{33}$$

$$\begin{aligned} \beta_2 &= 1.7 \\ &\ge \frac{\tau}{(1-\delta)}\left(\sum_{j=1}^{2}\eta_j\left|c_{j2}\right| N_2 + \sum_{j=1}^{2}\eta_j\left|f_{j2}\right| N_2\right) \\ &= 0.1581. \end{aligned} \tag{34}$$

Hence, all the conditions of Theorem 4 are satisfied. System (22)-(23) is mean-square exponentially input-to-state stable. Obviously, system (22)-(23) is mean-square exponentially stable when $u(t) \equiv 0$.

*Example 2* (3-dimension case). Consider the case of 3-dimension stochastic recurrent neural networks with multi-proportional delays

$$dx_i(t) = \left[ -d_i x_i(t) + \bigwedge_{j=1}^{3} a_{ij} f_j(x_j(t)) + \bigwedge_{j=1}^{3} b_{ij} g_j(x_j(p_j t)) + \bigwedge_{j=1}^{3} c_{ij} \int_{t-\tau(t)}^{t} h_j(x_j(s))\, ds + \bigvee_{j=1}^{3} d_{ij} f_j(x_j(t)) + \bigvee_{j=1}^{3} e_{ij} g_j(x_j(p_j t)) + \bigvee_{j=1}^{3} f_{ij} \int_{t-\tau(t)}^{t} h_j(x_j(s))\, ds + u_i(t) \right] dt + \sum_{j=1}^{3} \sigma_{ij}(x_j(t), x_j(p_j t))\, dw_j(t), \tag{35}$$

$$x_i(t) = \varphi_i(t), \quad t \in [p, 1], \ i = 1, 2, 3, \tag{36}$$

where

$$\begin{aligned} f(x) = g(x) = h(x) &= 0.2 \tanh(x), \\ u(t) &= 0.1 \sin(t), \end{aligned} \tag{37}$$

$$(\sigma_{ij})_{3\times 3} = \begin{pmatrix} 0.2x_1(t) & 0.2(x_2(p_2 t)) & 0.3x_3(t) \\ 0.4x_1(p_1 t) & 0.4(x_2(t) + x_2(p_2 t)) & 0.2x_3(p_3(t)) \\ 0.2x_1(t) & 0.1x_2(t) & 0.4(x_3(t) + x_3(p_3(t))) \end{pmatrix}, \tag{38}$$

Other parameters of system (22)-(23) are given as follows:

$$(a_{ij})_{3\times 3} = \begin{pmatrix} -0.8 & 0.5 & 0.5 \\ 0.9 & 0.3 & 0.5 \\ 0.2 & 0.6 & -0.3 \end{pmatrix}, \quad (b_{ij})_{3\times 3} = \begin{pmatrix} 0.4 & 0.3 & -0.2 \\ 0.5 & 0.8 & 0.4 \\ 0.9 & -0.3 & 0.6 \end{pmatrix}, \tag{39}$$

$$(c_{ij})_{3\times 3} = \begin{pmatrix} 0.6 & -0.4 & 0.5 \\ 0.3 & 0.2 & 0.3 \\ -0.6 & 0.2 & 0.5 \end{pmatrix}, \quad (d_{ij})_{3\times 3} = \begin{pmatrix} 0.7 & 0.4 & 0.5 \\ 0.6 & 0.2 & 0.3 \\ 0.3 & 0.2 & 0.4 \end{pmatrix}, \tag{40}$$

$$(e_{ij})_{3\times 3} = \begin{pmatrix} 0.5 & 0.2 & 0.2 \\ 0.4 & 0.3 & 0.5 \\ 0.7 & 0.1 & 0.3 \end{pmatrix}, \quad (f_{ij})_{3\times 3} = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}, \tag{41}$$

$d_1 = 8, d_2 = 9, d_3 = 10$. Take $p_1 = p_2 = p_3 = 0.5, \lambda = 1.3, \eta_1 = 0.4, \eta_2 = 0.4, \eta_3 = 0.4, \alpha_1 = 1.4, \alpha_2 = 1.3, \alpha_3 = 1.4, \beta_1 = 1.3, \beta_2 = 1.2, \beta_3 = 1.1$, and it follows from the definition of $\tau$ that $\tau = 0.6931$. It is easy to check that Assumptions 1–3 are satisfied. A direct computation gives

$$\begin{aligned} 2\eta_1 d_1 &= 6.4 \\ &> (1+\lambda)\eta_1 + \alpha_1 + \tau\beta_1 + \sum_{j=1}^{3} \eta_j \mu_{j1} \\ &\quad + \sum_{j=1}^{3} \eta_j |a_{j1}| L_1 + \eta_1 \sum_{j=1}^{3} |a_{1j}| L_j \\ &\quad + \eta_1 \sum_{j=1}^{3} |b_{1j}| M_j + \eta_1 \sum_{j=1}^{3} |c_{1j}| N_j \\ &\quad + \sum_{j=1}^{3} \eta_j |d_{j1}| L_1 + \eta_1 \sum_{j=1}^{3} |d_{1j}| L_j \\ &\quad + \eta_1 \sum_{j=1}^{3} |e_{1j}| M_j + \eta_1 \sum_{j=1}^{3} |f_{1j}| N_j = 5.60903, \end{aligned} \tag{42}$$

$$\begin{aligned} 2\eta_2 d_2 &= 7.2 \\ &> (1+\lambda)\eta_2 + \alpha_2 + \tau\beta_2 + \sum_{j=1}^{3} \eta_j \mu_{j2} \\ &\quad + \sum_{j=1}^{3} \eta_j |a_{j2}| L_2 + \eta_2 \sum_{j=1}^{3} |a_{2j}| L_j \\ &\quad + \eta_2 \sum_{j=1}^{3} |b_{2j}| M_j + \eta_2 \sum_{j=1}^{3} |c_{2j}| N_j \end{aligned}$$

$$+\sum_{j=1}^{3}\eta_j\left|d_{j2}\right|L_2+\eta_2\sum_{j=1}^{3}\left|d_{2j}\right|L_j$$
$$+\eta_2\sum_{j=1}^{3}\left|e_{2j}\right|M_j+\eta_2\sum_{j=1}^{3}\left|f_{2j}\right|N_j=5.46372, \tag{43}$$

$$\begin{aligned}2\eta_3 d_3 &= 8\\ &> (1+\lambda)\,\eta_3+\alpha_3+\tau\beta_3+\sum_{j=1}^{3}\eta_j\mu_{j3}\\ &\quad+\sum_{j=1}^{3}\eta_j\left|a_{j3}\right|L_3+\eta_3\sum_{j=1}^{3}\left|a_{3j}\right|L_j\\ &\quad+\eta_3\sum_{j=1}^{3}\left|b_{3j}\right|M_j+\eta_3\sum_{j=1}^{3}\left|c_{3j}\right|N_j\\ &\quad+\sum_{j=1}^{3}\eta_j\left|d_{j3}\right|L_3+\eta_3\sum_{j=1}^{3}\left|d_{3j}\right|L_j\\ &\quad+\eta_3\sum_{j=1}^{3}\left|e_{3j}\right|M_j+\eta_3\sum_{j=1}^{3}\left|f_{3j}\right|N_j=5.59841,\end{aligned} \tag{44}$$

$$\begin{aligned}\alpha_1 &= 1.4\\ &\ge \sum_{j=1}^{3}e^{\lambda\tau}\eta_j\nu_{j1}+\sum_{j=1}^{3}e^{\lambda\tau}\eta_j\left|b_{j1}\right|M_1\\ &\quad+\sum_{j=1}^{3}e^{\lambda\tau}\eta_j\left|e_{j1}\right|M_1=1.29802,\end{aligned} \tag{45}$$

$$\begin{aligned}\alpha_2 &= 1.3\\ &\ge \sum_{j=1}^{3}e^{\lambda\tau}\eta_j\nu_{j2}+\sum_{j=1}^{3}e^{\lambda\tau}\eta_j\left|b_{j2}\right|M_2\\ &\quad+\sum_{j=1}^{3}e^{\lambda\tau}\eta_j\left|e_{j2}\right|M_2=1.08332,\end{aligned} \tag{46}$$

$$\begin{aligned}\alpha_3 &= 1.4\\ &\ge \sum_{j=1}^{3}e^{\lambda\tau}\eta_j\nu_{j3}+\sum_{j=1}^{3}e^{\lambda\tau}\eta_j\left|b_{j3}\right|M_3\\ &\quad+\sum_{j=1}^{3}e^{\lambda\tau}\eta_j\left|e_{j3}\right|M_3=1.31969,\end{aligned} \tag{47}$$

$$\begin{aligned}\beta_1 &= 1.3\\ &\ge \frac{\tau}{(1-\delta)}\left(\sum_{j=1}^{3}\eta_j\left|c_{j1}\right|N_1+\sum_{j=1}^{3}\eta_j\left|f_{j1}\right|N_1\right)\\ &= 0.48781,\end{aligned} \tag{48}$$

$$\begin{aligned}\beta_2 &= 1.2\\ &\ge \frac{\tau}{(1-\delta)}\left(\sum_{j=1}^{3}\eta_j\left|c_{j2}\right|N_2+\sum_{j=1}^{3}\eta_j\left|f_{j2}\right|N_2\right)\\ &= 0.28908,\end{aligned} \tag{49}$$

$$\begin{aligned}\beta_3 &= 1.1\\ &\ge \frac{\tau}{(1-\delta)}\left(\sum_{j=1}^{3}\eta_j\left|c_{j3}\right|N_3+\sum_{j=1}^{3}\eta_j\left|f_{j3}\right|N_3\right)\\ &= 0.36134.\end{aligned} \tag{50}$$

Hence, all the conditions of Theorem 4 are satisfied. System (35)-(36) is mean-square exponentially input-to-state stable. Obviously, system (22)-(23) is mean-square exponentially stable when $u(t) \equiv 0$.

*Remark 3.* Obviously, the obtained results in [44, 46] do not apply to Examples 1 and 2 since many factors such as fuzzy logic, multiproportional delays, and distributed delays are considered in Examples 1 and 2.

## 5. Concluding Remarks

In this paper, we have studied mean-square exponential input-to-state stability of a class of stochastic fuzzy recurrent neural networks with multiproportional delays and distributed delays. A key characteristics of this paper is that the nonlinear transformation $y(t) = x(e^t)$ is employed to transform the considered system into stochastic recurrent neural networks with constant delays and variable coefficient, which overcomes the difficulty from multiproportional delays. Moreover, we also consider the effects of distributed delays and fuzzy. In our future works, we will apply the method developed in this paper to study some other important problems such as the stability of multiagent systems.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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