A Comprehensive Study of Resistor-Loaded Planar Dipole Antennas for Ground Penetrating Radar Applications

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Abstract

Ground penetrating radar (GPR) systems are increasingly being used for the detection and location of buried objects within the upper regions of the earth’s surface. The antenna is the most critical component of such a system. This thesis presents a comprehensive study of resistor-loaded planar dipole antennas for GPR applications using both theory and experiments. The theoretical analysis is performed using the finite difference time domain (FDTD) technique.

The analysis starts with the most popular planar dipole, the bow-tie. A parametric study is done to find out how the flare angle, length, and lumped resistors of the antenna should be selected to achieve broadband properties and good target detection with less clutter. The screening of the antenna and the position of transmitting and receiving antennas with respect to each other and ground surface are also studied. A number of other planar geometrical shapes are considered and compared with the bow-tie in order to find what geometrical shape gives the best performance. The FDTD simulations are carried out for both lossless and lossy, dispersive grounds. Also simulations are carried out including surface roughness and natural clutter like rocks and twigs to make the modeling more realistic.

Finally, a pair of resistor-loaded bow-tie antennas is constructed and both indoor and outdoor measurements are carried out to validate the simulation results.

Keywords: Ground penetrating radar, buried object detection, dipole antennas, FDTD methods, broadband properties, baluns, simulation, optimal design, lossy media, dispersive media, clutter.
Preface

This work has been carried out mainly at the Division of Electromagnetic Theory (TET, known as Electromagnetic Engineering at present), Royal Institute of Technology (KTH), Sweden. I express my deepest gratitude to my principal supervisor, Associate Prof. Martin Norgren for his excellent guidance and encouragement. Also I am very much grateful to Dr. Peter Fuks for his invaluable support and advice.

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List of papers


A comprehensive study of resistor-loaded planar dipole antennas …
Specification of my contributions to the included papers

Paper I. I did the main part. I modeled the antennas by using FDTD and carried out the numerical simulations for various antenna parameters.

Paper II. I did the main part. I carried out the modeling and simulation of the antennas and ground.

Paper III. I am the only author of this paper.

Paper IV. I did the main part. I carried out the modeling and simulation.

Paper V. I did the main part. I modeled and constructed the antennas. Carried out the measurements with Dr. Peter Fuks.
1. Introduction to Ground Penetrating Radar

1.1 What is ground penetrating radar?

Ground penetrating radar (GPR) is a radar which detects objects and interfaces buried beneath the earth’s surface. It is considered as a very effective tool for non-destructively sensing of the subsurface environment, since the radar can detect any object that has different electrical properties than the surrounding soil. Thus it senses both metallic and nonmetallic targets as opposed to metal detectors. A schematic diagram of a GPR is shown in Fig. 1.1.

![Schematic diagram of a GPR](image)

The history of GPR goes back to the beginning of the 20th century [1]. It was first used to detect interfaces like water and ore deposits under the ground. Since then the range of its applications has grown rapidly with the vast development in the field. GPR is used in military operations to detect land mines, unexploded ordnances (UXO), and tunnels. The civil engineering applications include water table and oil field detection, road and railbed quality assessment, pipe and cable detection etc. Another application is to locate archaeological remains and graveyards. In terms of environmental sensing, GPR has been applied to detect buried tanks, landfill debris, and contaminated fluids. Fig. 1.2 shows a photograph of a GPR survey in a graveyard.

The GPR system usually consists of transmitting and receiving antennas, a source connected to the transmitting antenna, and signal processing
equipment connected to the receiving antenna. Compared with designing an atmospheric radar, the design of a GPR is a more challenging task, especially the antennas [1], [2]. The type of antennas, choice of the transmitted signal, and method of signal processing are all dependent upon the particular application of the GPR system; that is, the object to be detected, depth at which the object is buried, and the electrical properties of the earth.

1.2 GPR antennas

The most critical component in a GPR system is the antenna, which couples the energy from the air into the ground, which is lossy and dispersive. Furthermore, since the antenna operates very close to the ground, the electrical properties of the ground will have a strong influence on the antenna’s input impedance and radiation characteristics. The total detection times for GPR systems are usually less than 100 ns. For shallow targets it can be even a few nanoseconds. Therefore, the transmitting signal should have a very short time duration and the antenna must be able to transmit the signal with little distortion and less ringing in order to avoid masking of the target echo. This demands an antenna with broadband properties and very short impulse response.

Usually separate transmit and receive antennas are used in GPR since it is difficult to isolate the receiver from transmission signal power due to the unavailability of fast switches [1]. The early time signals in the receiving signal have clutter like direct coupling and ground surface reflection. This
should be reduced as much as possible for better target detection. Screening of the two antennas by a conducting sheet is often used to reduce direct coupling. Orthogonal arrangement of the two antennas is another method. There are various types of broadband antennas in use for GPR today. They are described in the next chapter.

1.3 Operating frequency and depth of penetration

Many subsurface radar systems operate at frequencies below a few GHz. The higher the frequency the lower the depth of penetration since the attenuation of electromagnetic waves increases with the frequency. The depth of the detecting target can vary from a few centimeters to hundred meters depending on the type of application. When detecting interfaces, the required horizontal resolution of the radar is low. Therefore, the radar can operate at low frequencies like a few tens of MHz, but for localized targets, the resolution should be high in order to discriminate the objects, thus needing a larger bandwidth. A rough correlation between the horizontal resolution $\Delta x$, ground attenuation coefficient $\alpha$, and depth of penetration $d$ can be expressed by [1]

$$\Delta x = 4d \sqrt{\frac{\ln 2}{2 + \alpha d}}$$  \hspace{1cm} (0.1)

assuming a dependence of received signal power on $d$ in terms of $1/d^4$. Thus the horizontal resolution improves as the attenuation increases, which means as the frequency increases. Therefore, the operating frequency of the radar is determined based on the required depth of penetration, the type of detecting target, and the ground properties.

1.4 Ground properties

The electrical properties of the ground are quite complex. The soil can be lossy and dispersive. Therefore, its effective permittivity and effective conductivity increase with frequency. This results in high attenuations of electromagnetic waves at high frequencies. Wet materials have relatively high effective permittivities and conductivities. Therefore, signal attenuation increases with the moisture content. Wet clay soil, one of the most lossy
materials, can have an attenuation of 20-30 dB/m at 100 MHz and rising to over 100 dB/m at 1 GHz. Sea water has a loss of typically 200 dB/m at 100 MHz, rising to 300 dB/m at 1 GHz and forms an extreme case; fresh water is near the other end of the scale: 4 dB/m at 100 MHz and 40 dB/m at 1 GHz.

Ice, depending on whether it originated from fresh water or sea water, can have a loss between 1 and 50 dB/m at 1 GHz. Sandy soil tends to be among the lower loss material, having about 10 dB/m at 1 GHz. Loamy soils have losses of about 20-30 dB/m at 1 GHz [1].

The permittivity of soil depends mainly on the water content. At low microwave frequencies, water has a relative permittivity \( \varepsilon_r \) of approximately 80, while dry soils have \( \varepsilon_r \) in the range 2-6. Thus the measured values of \( \varepsilon_r \) for many soils lie mainly in the range 4-40. The permittivity of snow depends on its density and water content and \( \varepsilon_r \) varies between about 1.5 for new powder snow to 2.5 for old, compact snow.

Apart from the losses and dispersion, the ground can be inhomogeneous. Even in the same soil type, electrical properties can change with the soil density and moisture content. Moreover, the presence of ground surface roughness and localized inhomogeneities like rocks and twigs makes the target identification a challenging task.

1.5 Target detection

GPR systems operate either in time or frequency domain. In a time domain system a narrow fast rise time pulse is the incident signal. Often this pulse resembles a Gaussian pulse. Frequency modulated continuous wave (FMCW) and pulse synthesis systems operate in the frequency domain. In a FMCW system the frequency is continuously changed in a conventional chirped manner while in a pulse synthesis system a series of frequencies are radiated sequentially. Software is used to derive the time domain response in these systems.

Most practical GPR systems operate with a fairly consistent set of operating parameters [2]. Pulse widths can range from 1 to about 10 ns. The peak of the transmitted voltage can be from 30 V to several kV. Repetition rates range
from about 60 Hz to 1 MHz. The impulse generator, which is used for experiments in this research work, gives a pulse of 2 ns duration and 100 V peak voltage with a repetition rate up to 100 KHz.

The signal coming out from the receiving antenna contains both scattered signal from the target and other unwanted signals known as clutter. This clutter includes direct coupling, ground surface reflection, external interferences, and echoes from rocks and twigs in the ground. The signal strength of the target echo can be many times smaller than the clutter level mainly due to the attenuation caused by the ground. Therefore, in many cases, the signal coming from the receiving antenna has to pass through several processes before the target is identified. A block diagram of a typical GPR system is shown in Fig. 1.3.

As can be seen in Fig. 1.3 the received signal is first sent through a fast rise-time wideband amplifier to increase the power level. Next, the signal is sampled and digitized. Finally, the target signal is extracted by using signal processing techniques. The final outcome is often an image showing the buried objects.

In GPR terminology, the measurement taken by placing the GPR at a certain point on the ground is called an A-scan. Fig. 1.4 shows a typical A-scan time domain measurement where the time axis is proportional to the depth. The set of measurements obtained from repeated A-scan measurements at discrete points along a linear path is called a B-scan. This B-scan data are used to
construct images of the vertical sections through the ground under investigation.

![Graph showing received voltage over time](image)

Fig. 1.4 An A-scan measurement on a metal pipe buried in snow [ref. to paper V].

### 1.6 Signal processing

GPR systems rely on signal processing, as the visual detection of the target echo is often not possible. The detection algorithms may range from simple energy detection procedures to sophisticated frequency domain target detection algorithms. Simple algorithms require little computer hardware while complicated ones require significant computer resources. Practically, potential raw data containing target information is filtered out from the rest using simple algorithms and then more complicated ones are applied to the filtered data to save both processing time and resources.

Among the detection algorithms, clutter reduction by subtraction is very common and simple. In this method, the radar is operated at several positions located little bit away from the target and by averaging, a rough estimate is obtained for clutter containing direct coupling and ground surface reflection. Then it is subtracted from the received signal when the radar is operating above the target. This method works well if the ground can be assumed to be homogeneous. Random noise in the received signal can be reduced by taking measurements repeatedly in conjunction with averaging.

More advanced methods need prior knowledge of the scattered signal, clutter, and noise. When the bandwidth of the scattered signal is known, filters can be used to pick out the appropriate signal and to modify its shape into a single peak to increase the detection probability. Target signatures are also used to identify targets. For small, localized, and regularly shaped objects the
spectrum of the scattered signal exhibits resonances, the location of which helps identify the geometry of the target. Long, thin targets like pipes and cables give special polarization properties to the scattered signal.

Once the target echo is extracted from the raw data obtained by placing the radar at different positions, a two or three dimensional image can be formed. The color or the intensity of a point in the image is selected to be proportional to the received signal amplitude at the point. Fig. 1.5 shows such a 2-D image showing a vertical section through the ground. The hyperbolic shapes represent localized objects.

![Fig. 1.5 Radar image across a pit containing pipes [1].](image)

This image can be translated into a more comprehensive form by applying signal processing techniques such as synthetic aperture radar (SAR), holographic method, and more advanced Kirchhoff methods [1]. Fig. 1.6 shows the converted image of Fig. 1.5.

![Fig. 1.6 Image showing the location of the pipes [1].](image)
2. GPR Antennas

As described in the previous chapter, the most critical part in a GPR system is the antenna. This chapter discusses various broadband antennas for GPR and the contribution to the field given by this thesis.

2.1 Types of antennas

2.1.1 Element antennas

Cylindrical monopoles and dipoles, Vee dipoles, biconical dipoles, bow-ties are widely used GPR antennas which fall into this category. These antennas have a limited bandwidth and must therefore be modified for subsurface radar applications. Broadband antennas can be made by resistively loading these antennas [3], [4]. The antenna can be fabricated by a resistive material or its ends can be loaded resistively. The resistive loading diminishes the wave gradually as it travels towards the antenna ends and thus diminishes the reflection of the wave at the ends. Hence, there will be no pronounced resonance associated with the dimensions of the antenna. As a result the antenna becomes broadband and the input signal can be transmitted with less distortion and reduced antenna ringing. These antennas have low directivities and produce linearly polarized radiation. Low efficiency is another feature due to loss loading. But the small physical volume is a major advantage.

Resistively loaded cylindrical dipoles or monopoles can be made by depositing a linearly tapered thin film of alloy on a glass rod [4]. Fig. 2.1 shows a photograph of a resistively loaded monopole.

Fig. 2.1 Photograph of a resistively loaded monopole [4].
Linear tapering changes the resistance per unit length along the dipole from a finite value at the feed point to a very large value at the end, according to

\[ r(z) \propto \frac{1}{1 - z/h} \]  

where \( z \) is the distance from the feed point, \( r(z) \) is the resistance per unit length at a distance \( z \) and \( h \) is the length of the arm (see Fig. 2.2).

Fig. 2.2 An arm of a dipole.

Due to linear polarization, the crossed dipole arrangement (see Fig. 2.3) can be used to reduce direct coupling in a GPR application. This arrangement is sensitive to targets other than planar interfaces parallel to the plane of antennas.

Fig. 2.3 Resistively loaded crossed-dipole GPR antenna.

A resistively loaded Vee dipole [5] is shown in Fig. 2.4. The resistance per unit length of each arm is increased towards the ends by tapering the width of each arm, \( w \) according to

\[ w(z) = w_0(1 - z/h) \]  

where \( z \) is the distance measured from the feed along the arm, \( h \) is the length of the arm, and \( w_0 \) is the width of the arm at the feed.
One advantage of this antenna is that its radiation characteristics are less affected by the ground properties since its feed is at a higher level above the ground. Reference [5] describes the successful application of this antenna for landmine detection.

Biconical dipole and its planar version, the bow-tie antenna (see Fig. 2.5) are another type used for subsurface radar. The latter is preferred for subsurface radar as it is easy to place above the ground. Brown and Woodward [6] have done the first comprehensive investigation of these antennas. Biconical dipoles and bow-ties have broader bandwidths than the cylindrical dipoles and the bandwidth can be further improved by resistive loading.

Resistive loading of bow-tie antennas can be done in several ways. The bowtie can be fabricated by a resistive sheet whose resistance increases toward the ends or a thin film of alloy of variable thickness can be deposited [7]. Due to limited availability of resistive sheets of varying resistivity and cost of thin film deposition, reference [7] suggests an easier method of constructing the antenna. Fig. 2.6 shows this antenna where it is fabricated with three different materials. The inner section is metal and the other
sections are of two different constant resistive materials. The end section is serrated to get the increasing resistance.

Fig. 2.6 Bow-tie with serrated ends.

Instead of a continuous loading, lumped resistors can be connected at the ends of the bow-tie as shown in Fig. 2.7. This thesis presents a thorough analysis on resistor-loaded bow-tie antennas and investigates other possible planar dipole geometries suitable for GPR applications.

Fig. 2.7 Resistor-loaded bow-tie antenna.

### 2.1.2 Horn antennas

Horn antenna is a commonly used aperture antenna for subsurface probing. Broadband properties are achieved either by ridge design [8] or loading the inside of the horn with a dielectric material [9]. Fig. 2.8 shows a photograph of a double-ridged horn. Quad-ridged horns are also common. The large physical volume of horn antennas is a drawback, which limits their use in low frequency applications.
2.1.3 Spiral and log-periodic antennas

These are very wideband antennas in which the upper and lower bound of the frequency range are determined by the size of the feed region and the overall size of the antenna respectively. Detailed descriptions of these antennas can be found in Rumsey [10] and Dyson [11]. The shortcoming with these types of antennas is their longer impulse response, due to nonlinear phase response. Therefore, in a short-range subsurface radar application, phase correction should be used to overcome this problem.

Spiral antennas which can be either planar or helical are of two types. i.e. Equiangular and Archimedian. Both produce circularly polarized radiation. But the former is preferred in GPR as the latter has a relatively longer impulse response. A diagram of an equiangular spiral antenna is shown in Fig. 2.9

![Four-arm equiangular spiral antenna](image)
Log-periodic antennas are not as broadband as spiral antennas. The main difference between this antenna and the spiral is that the log-periodic antenna is linearly polarized. Fig. 2.10 shows a log-periodic antenna.

![Log-periodic antenna](image)

Fig. 2.10 Log-periodic antenna [1].

### 2.2 Motivation and contribution to the field

As described in the previous section, the resistor-loaded bow-tie is a popular broadband antenna for GPR applications. It is useful in the upper MHz and lower GHz frequency range to achieve depths of penetrations up to several meters in many different applications.

The first comprehensive study on bow-tie antennas was performed by Brown and Woodward in 1952 [6]. They experimentally studied the antenna’s input impedance and radiation characteristics in free space. The bow-tie antenna has been optimized for pulse radiation by continuous resistive loading in [7]. But a comprehensive study is not available for resistor-loaded bow-tie antennas, specially when it comes for GPR applications where the antennas are enclosed in conducting cavities and operating in the vicinity of the ground.

This thesis thoroughly analyses the resistor-loaded bow-tie antennas and provides a guideline on the selection of the antenna dimensions, lumped resistor values and their position, and shielding to suit GPR applications. Also the study is extended to other planar dipole geometries. The presented simulations of the GPR with real ground conditions and results of experiments carried out using a bow-tie antenna setup provide a good understanding of the antenna’s performance.
3. Baluns for GPR antennas

Many of the antennas are fed by a balun, in order to match the unbalanced transmission line to the balanced radiating structure. This prevents an unwanted current flowing in the outer surface of the transmission line, which leads to distortion of the radiation characteristics of the antenna. It is often called a balun transformer as an impedance transformation is also accompanied with a balance-unbalance conversion.

Bazuka balun (1:1) and $\lambda/2$ coaxial balun (4:1) are two commonly used narrowband balun transformers. But these baluns cannot be used with GPR antennas, which operate in a wide frequency range. Thus broadband baluns such as ferrite-core transmission line transformers are required to feed GPR antennas. The bow-tie antenna used for the measurements in this study is fed by a 4:1, 50-1000 MHz, ferrite-core balun from Mini-Circuits. In the following section, a brief description of ferrite-core transmission line transformers is given in order to understand its operation and effect on the antenna’s performance.

3.1 Transmission line transformers (TLTs)

The TLT transmits the energy from input to output by a transmission line mode and not by flux-linkages as in the conventional transformer. As a result the TLT has a much wider bandwidth and a higher efficiency than its conventional counterpart. Its larger bandwidth is achieved by employing ferrite cores, which can withstand high frequencies. TLTs of 1:1 and 4:1 impedance ratios (balun ratios) are shown in Fig. 3.1 and 3.2, respectively.
The operation of TLTs can be described by the transmission line theory as its name implies. The parallel coils wound in the core act as a parallel transmission line. What follows next is an analysis of the 4:1 balun using this theory.
Fig. 3.3 shows the high frequency model of the 4:1 balun. Voltages $V_1$, $V_2$ and currents $I_1$, $I_2$ at either end of the transmission line can be related by

$$V_1 = V_2 \cos kl + jZ_0 I_2 \sin kl$$  \hspace{1cm} (2.1)$$

$$I_1 = I_2 \cos kl + j\frac{V_2}{Z_0} \sin kl$$  \hspace{1cm} (2.2)$$

where $k$ is the wave number, $l$ is the length of the transmission line (same as the length of a coil), and $Z_0$ is the characteristic impedance of the line.

The impedances at low impedance side $Z_1$ and high impedance side $Z_2$ of the balun can be written as

$$Z_1 = \frac{V_1}{I_1 + I_2}$$  \hspace{1cm} (2.3)$$

$$Z_2 = \frac{V_1 + V_2}{I_2}.$$  \hspace{1cm} (2.4)$$

Using equations (2.1)- (2.4), $V_1$ and $V_2$ can be removed to get

$$Z_1(1 + \cos kl) I_1 = I_2 \left( (Z_2 - Z_1) \cos kl - Z_1 + jZ_0 \sin kl \right)$$  \hspace{1cm} (2.5)$$

$$(Z_0 + jZ_1 \sin kl) I_1 = I_2 \left( Z_0 \cos kl + j(\frac{Z_2 - Z_1}{Z_1}) \sin kl \right).$$  \hspace{1cm} (2.6)$$
Dividing (2.5) by (2.6) and some simplification deduces

\[ Z_1 = \frac{Z_0 (Z_2 \cos kl + jZ_0 \sin kl)}{2Z_0 (1 + \cos kl) + jZ_2 \sin kl}. \]  

(2.7)

Equation (2.7) clearly shows the frequency dependence of the relation between the input and output impedances of the balun transformer. As \( k \to 0 \), equation (2.7) simplifies to \( Z_1 = Z_2 / 4 \), which is the designed relation. In order to get the optimum performance (i.e. to make the impedance ratio \( |Z_2 / Z_1| \) closer to 4:1 over the designed frequency band), the characteristic impedance \( Z_0 \) should be equal to the geometric mean of the input and output impedances of the TLT [12]. E.g. if a 50\( \Omega \) coaxial transmission line is to be connected to a 200\( \Omega \) resistor through a 4:1 TLT, its \( Z_0 \) should be \( \sqrt{50 \times 200} = 100 \Omega \). But this optimum performance cannot be achieved if the balun is connected to a load of variable impedance such as an antenna.

The characteristic impedance of the TLT depends on the ferrite material, spacing between the wires, and wire diameter. Fig. 3.4 shows the variation of the TLT’s impedance ratio, \( |Z_2 / Z_1| \) with frequency for two different fixed impedances at the high impedance side (\( Z_2 = 200 \Omega \) and \( Z_2 = 300 \Omega \)) as given by equation (2.7). The other parameter values are \( k = 5k_0 \) (\( k_0 \) is the free space wave number), \( Z_0 = 100 \Omega \) (the optimum value for \( Z_2 = 200 \Omega \)), and \( l = 2 \) cm. Fig. 3.4 clearly shows the better performance of the optimum design.

Fig. 3.4 Variation of the impedance ratio of the 4:1 TLT with frequency.
In commercially available TLT’s, the performance is expressed in terms of the return loss, insertion loss, and frequency bandwidth. These are evaluated when the input and output of the TLT is connected with its designed impedances. The return loss is the fraction of power reflected back from the input side and the insertion loss is the ratio of output power to the input power. Normally the frequency range in which the insertion loss is less than 1,2 or 3 dB is expressed as the bandwidth of the TLT.
4. Modeling of GPR Systems Using FDTD

Simulation of ground penetrating radar systems assists the subsequent design of high performance radar hardware and software. Modeling of actual GPR environments has become feasible with the rapid growth in the computer field in terms of both memory and speed.

Novel GPR configurations and optimized antennas can be designed by using the simulation results. As mentioned in section 1.6, a prior knowledge of the scattered signal, clutter, and noise is required for the development and application of advanced detection algorithms. Accurate modeling of a realistic GPR is one of the methods to get this information. The finite difference time domain (FDTD) method [13], [14] has proven to be an efficient tool when analyzing GPR problems, and it is also the method that is used for the simulation studies in this thesis. This chapter describes some FDTD techniques used to model planar dipoles and the ground in GPR scenarios.

4.1 Finite difference time domain method

Finite difference time domain (FDTD) technique is a second order accurate direct solution method for Maxwell’s time dependent curl equations:

\[ \nabla \times \mathbf{H} = \mathbf{J}_e + \frac{\partial \mathbf{D}}{\partial t} \]  
(3.1)

\[ \nabla \times \mathbf{E} = -\mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t} \]  
(3.2)

where \( \mathbf{E} \) is the electric field, \( \mathbf{H} \) is the magnetic field, \( \mathbf{D} \) is the electric flux density, and \( \mathbf{B} \) is the magnetic flux density. \( \mathbf{J}_e \) and \( \mathbf{J}_m \) are electric and magnetic current densities respectively.

For implementing the FDTD algorithm, the computational domain is divided into cubic cells as shown in Fig. 4.1. The \( \mathbf{E} \)- & \( \mathbf{H} \)-field components at each cell are calculated from the finite difference expressions based on Yee’s algorithm [15]. Each \( \mathbf{E} \) component is surrounded by four \( \mathbf{H} \) components and each \( \mathbf{H} \) component is surrounded by four \( \mathbf{E} \) components in the grid. The algorithm calculates field components separated by \( \Delta t \) in time according to what is termed as the leapfrog arrangement: \( \mathbf{H} \) components are in front of...
and behind $E$ components by a time $\Delta t/2$. Thus the $H$ -field values at a
time are found from the previously found values of $E$. Then this newly
calculated $H$ -field values are used to find the next set of $E$ -field values.

![FDTD cell](image)

Fig. 4.1 FDTD cell.

Deriving finite difference expressions for a homogeneous, lossy, and
nondispersive medium is described below. For this medium, the equation
(3.2) can be expressed in rectangular coordinates as follows:

$$
\begin{align*}
\left( \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) & = \left( \frac{\partial E_x}{\partial x} - \frac{\partial E_z}{\partial z} \right) \\
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} & = \left( \mu \frac{\partial}{\partial t} + \sigma^* \right) H_y \\
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} & = \left( \frac{\partial}{\partial t} \right) H_z
\end{align*}
$$

(3.3)

where $\mu$ is the permeability and $\sigma^*$ is the magnetic conductivity in the
medium. Considering only the $x$ component of $H$ yields

$$
\frac{\partial E_z}{\partial z} - \frac{\partial E_x}{\partial y} = \mu \frac{\partial}{\partial t} H_x + \sigma^* H_y.
$$

(3.4)

Next this equation is applied to a point with coordinates
$(x = i\Delta, y = (j + 1/2)\Delta, z = (k + 1/2)\Delta)$ where $\Delta$ is the cell size. The selected
time is $t = n\Delta t$ at which $H$ -field values do not exist but the $E$ -field values,
according to the leapfrog arrangement. Now applying second order accurate
central difference scheme for space & time derivatives and semi implicit
scheme for $H_x$ at this time instance gives
\[
\frac{E^n_x(i, j+1/2, k+1) - E^n_x(i, j+1/2, k)}{\Delta} - \frac{E^n_x(i, j+1, k+1/2) - E^n_x(i, j, k+1/2)}{\Delta} = \mu_0 \frac{H^{n+1/2}_x(i, j+1/2, k+1/2) - H^{n+1/2}_x(i, j+1, k+1/2)}{\Delta t} + \sigma \frac{H^{n+1/2}_x(i, j+1/2, k+1/2) + H^{n+1/2}_x(i, j+1/2, k+1/2)}{2}
\]

where the conventional notation \( H^n_x(i, j, k) = H_x(x=i\Delta, y=j\Delta, z=k\Delta, \ t=n\Delta) \) is used. Now rearranging the terms gives the finite difference expression for \( H_x \) as follows:

\[
H^{n+1/2}_x(i, j+1/2, k+1/2) = \left( \frac{1 - \sigma^* \Delta t / 2\mu}{1 + \sigma^* \Delta t / 2\mu} \right) H^{n+1/2}_x(i, j+1/2, k+1/2) + \left( \frac{\Delta t / \Delta \mu}{1 + \sigma^* \Delta t / 2\mu} \right) \left[ \frac{E^n_x(i, j+1/2, k+1) - E^n_x(i, j+1/2, k)}{\Delta} - \frac{E^n_x(i, j+1, k+1/2) - E^n_x(i, j, k+1/2)}{\Delta} \right]
\]

Similar expressions can be derived for the other components of \( E \) and \( H \).

Equation (3.7) gives the finite difference expression for \( E_x \).

\[
E^{n+1}_x(i+1/2, j, k) = \left( \frac{1 - \sigma^* \Delta t / 2\epsilon}{1 + \sigma^* \Delta t / 2\epsilon} \right) E^n_x(i+1/2, j, k) + \left( \frac{\Delta t / \Delta \epsilon}{1 + \sigma^* \Delta t / 2\epsilon} \right) \left[ \frac{H^{n+1/2}_x(i+1/2, j+1/2, k) - H^{n+1/2}_x(i+1/2, j-1/2, k)}{\Delta} - \frac{H^{n+1/2}_x(i+1/2, j, k+1/2) - H^{n+1/2}_x(i+1/2, j, k-1/2)}{\Delta} \right]
\]
where $\varepsilon$ is the permittivity of the medium and $\sigma$ is the electric conductivity in the medium.

When the dispersive properties are included into the medium, the finite difference expressions get more complicated. Also it requires more computer memory to run the FDTD code. There are three methods in use today to include dispersion into the FDTD algorithm: Recursive convolution method [16, 17], auxiliary differential equation method (ADE), [18-20], and $z$-transform method [21]. The last two methods are more accurate but not suitable for higher order dispersion.

### 4.2 Absorbing boundary conditions

When using FDTD, in order to simulate the computational domain extending to infinity, absorbing boundary conditions (ABC) should be used at the boundary.

![Fig. 4.2 Computational domain with ABC layer.](image)

Perfectly matched layer (PML) is the most accurate one. Berenger PML [22] is the simplest and is used in the thesis, when simulating lossless media. Other PMLs like uniaxial PML (UPML) [23] are developed to match lossy and dispersive media.
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4.2.1 Berenger PML

This PML applies for a lossless nondispersive medium. The following set of equations gives Berenger’s modified Maxwell equations for a perfectly matched layer in rectangular coordinates.

\[\mu \frac{\partial H_{xy}}{\partial t} + \sigma' H_{xy} = -\frac{\partial E_x}{\partial y}\]  
(3.8)

\[\mu \frac{\partial H_{xt}}{\partial t} + \sigma' H_{xt} = \frac{\partial E_y}{\partial z}\]  
(3.9)

\[\mu \frac{\partial H_{xz}}{\partial t} + \sigma' H_{xz} = -\frac{\partial E_x}{\partial z}\]  
(3.10)

\[\mu \frac{\partial H_{yx}}{\partial t} + \sigma' H_{yx} = \frac{\partial E_x}{\partial x}\]  
(3.11)

\[\mu \frac{\partial H_{yx}}{\partial t} + \sigma' H_{yx} = -\frac{\partial E_y}{\partial x}\]  
(3.12)

\[\varepsilon \frac{\partial E_{xy}}{\partial t} + \sigma' E_{xy} = \frac{\partial H_x}{\partial y}\]  
(3.13)

\[\varepsilon \frac{\partial E_{zx}}{\partial t} + \sigma' E_{zx} = -\frac{\partial H_y}{\partial z}\]  
(3.14)

\[\varepsilon \frac{\partial E_{zy}}{\partial t} + \sigma' E_{zy} = \frac{\partial H_x}{\partial z}\]  
(3.15)

\[\varepsilon \frac{\partial E_{yx}}{\partial t} + \sigma' E_{yx} = \frac{\partial H_z}{\partial z}\]  
(3.16)

\[\varepsilon \frac{\partial E_{xx}}{\partial t} + \sigma' E_{xx} = -\frac{\partial H_y}{\partial x}\]  
(3.17)
\[
\varepsilon \frac{\partial E_{x}}{\partial t} + \sigma_{x} E_{x} = \frac{\partial H_{y}}{\partial x} \quad (3.18)
\]

\[
\varepsilon \frac{\partial E_{y}}{\partial t} + \sigma_{y} E_{y} = -\frac{\partial H_{x}}{\partial y} \quad (3.19)
\]

where \( \varepsilon \) and \( \mu \), respectively, are the permittivity and the permeability of the medium to which the PML is matched. Parameters \( \sigma_{x}, \sigma_{y}, \) and \( \sigma_{z} \) denote electric conductivities and \( \sigma_{x}^{*}, \sigma_{y}^{*}, \) and \( \sigma_{z}^{*} \) denote magnetic conductivities in the PML. Also the split fields satisfy the following relations:

\[
H_{xy} + H_{xz} = H_{x} \quad (3.20)
\]

\[
H_{yx} + H_{yz} = H_{y} \quad (3.21)
\]

\[
H_{zx} + H_{zy} = H_{z} \quad (3.22)
\]

\[
E_{xy} + E_{xz} = E_{x} \quad (3.23)
\]

\[
E_{yx} + E_{yz} = E_{y} \quad (3.24)
\]

\[
E_{zx} + E_{zy} = E_{z} \quad (3.25)
\]

\( \sigma_{\Omega} \) and \( \sigma_{\Omega}^{*} \) are nonzero only in the PML layers perpendicular to \( \Omega \) axis, \( (\Omega \equiv x, y, z) \) in the hexahedral computational domain (see Fig. 4.2). For perfect matching, the electric conductivity \( \sigma \) and the magnetic conductivity \( \sigma^{*} \) of the PML should satisfy the following equation:

\[
\frac{\sigma}{\varepsilon} = \frac{\sigma^{*}}{\mu}. \quad (3.26)
\]
The conductivity at any point in the PML is calculated by

$$\sigma(\rho) = \sigma_{\text{max}} \left( \frac{\rho}{\delta} \right)^n$$  \hspace{1cm} (3.27)

where $\rho$ is the distance from the inner surface of the PML region (see Fig. 4.3), $\sigma_{\text{max}}$ is the maximum conductivity inside the layer, $\delta$ is the thickness of the PML and $n$ is the order of PML loss grading. The maximum conductivity inside the layer is calculated by the expression for the reflection coefficient for normal incidence $R(0)$ at the outer walls of PML. This expression is shown below:

$$R(0) = \exp(-2\sigma_{\text{max}} \delta/(n+1)\varepsilon u)$$  \hspace{1cm} (3.28)

where $u$ is the velocity of light in the medium to which the PML is matched. A suitable value for $R(0)$ is assumed to find $\sigma_{\text{max}}$ in equation (3.28). The magnetic conductivity $\sigma^*$ is calculated by the equation (3.26). Typical finite difference expressions for $H_{xy}$ and $E_{xy}$ (based on equations (3.8) and (3.14)) are shown below:

$$H_{xy}^{n+1/2}(i, j, k) = Da_{ij,k} H_{xy}^{n+1/2}(i, j, k) + Db_{ij,k} \left\{ -[E_z^n(i, j+1/2, k) - E_z^n(i, j-1/2, k)] \right\}$$  \hspace{1cm} (3.29)
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\[ E_{xy}^{n+1}(i, j, k) = C_{a_{ij},k} E_{xy}^{n}(i, j, k) + C_{b_{ij},k} \left( H_{z}^{n+1/2}(i, j+1/2, k) - H_{z}^{n+1/2}(i, j-1/2, k) \right) \]  

(3.30)

where \( C_{a_{ij},k} = \left( \frac{1 - \sigma_y(i, j, k) \Delta t / 2 \epsilon}{1 + \sigma_y(i, j, k) \Delta t / 2 \epsilon} \right) \); \( C_{b_{ij},k} = \left( \frac{\Delta t / \epsilon \Delta}{1 + \sigma_y(i, j, k) \Delta t / 2 \epsilon} \right) \);

\( D_{a_{ij},k} = \left( \frac{1 - \sigma_x(i, j, k) \Delta t / 2 \mu}{1 + \sigma_x(i, j, k) \Delta t / 2 \mu} \right) \); \( D_{b_{ij},k} = \left( \frac{\Delta t / \mu \Delta}{1 + \sigma_x(i, j, k) \Delta t / 2 \mu} \right) \).

Here the conventional notation \( H_{xy}^{n}(i, j, k) = H_{xy}(x = i \Delta, y = j \Delta, z = k \Delta, t = n \Delta t) \) is used where \( \Delta t \) is the time step and \( \Delta \) is the spatial cell size.

### 4.2.2 Uniaxial PML

This was first introduced by Sacks et al. [23] and developed for 3-D and dispersive media by Gedney [24], [25]. The medium is anisotropic and when it is matched to a 3-D medium of permittivity \( \epsilon \) and permeability \( \mu \), the Maxwell curl equations take the following form in rectangular coordinates:

\[
\begin{pmatrix}
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} \\
\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial x} \\
\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y}
\end{pmatrix} = j \omega \epsilon_i \begin{pmatrix}
\frac{s_x s_y}{s} & 0 & 0 \\
0 & \frac{s_x s_z}{s} & 0 \\
0 & 0 & \frac{s_y s_z}{s}
\end{pmatrix} \begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
\]  

(3.31)
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\[
\begin{pmatrix}
\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \\
\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{pmatrix}
= j \omega \mu \begin{pmatrix}
\frac{s_x s_z}{s_x} & 0 & 0 \\
0 & \frac{s_y s_z}{s_y} & 0 \\
0 & 0 & \frac{s_z s_x}{s_z}
\end{pmatrix} \begin{pmatrix}
H_x \\
H_y \\
H_z
\end{pmatrix}
\] (3.32)

where \( \omega \) is the angular frequency and \( s_i \) \((i = x, y, z)\) is given by

\[ s_i = \kappa_i + \sigma_i / j \omega \varepsilon_0. \] (3.33)

\( s_i = 1 \) when the PML is not perpendicular to axis \( i \) (see Fig. 4.2). When the PML is perpendicular to axis \( i \), \( \sigma_i \) (normalized with respect to the relative permittivity) is increased from the inner boundary to outer boundary of the computational domain as given by the equation (3.27) (as in Berenger PML). Also the maximum conductivity in this equation is calculated by the equation (3.28). Using a value not equal to unity for \( \kappa_i \) is optional. The permittivity and permeability tensors are defined as \( \varepsilon = \varepsilon_0 \varepsilon \) and \( \mu = \mu_0 \mu \) respectively where

\[
\varepsilon = \begin{pmatrix}
\frac{s_x s_z}{s_x} & 0 & 0 \\
0 & \frac{s_y s_z}{s_y} & 0 \\
0 & 0 & \frac{s_z s_x}{s_z}
\end{pmatrix}
\] (3.34)

The electric flux density \( D \) and the magnetic flux density \( B \) are related to the \( E \) - and \( H \) - fields respectively as follows:

\[
D = \varepsilon E
\] (3.35)

\[
B = \mu H.
\] (3.36)
UPML can be easily applied for lossy and dispersive media by replacing \( \varepsilon \) and \( \mu \) by their corresponding complex values. The derivation of the FDTD scheme for UPML is described in section 4.6.

### 4.3 Subcell models for planar dipole antennas

When modeling planar dipole antennas, it is assumed that the antenna sheet is a very thin perfect electric conductor (PEC). Therefore tangential electric components along the PEC sheet are equaled to zero. Some boundary surfaces of the antenna do not fit into FDTD square cells. Therefore, a subcell method should be used to model its boundaries. The simplest subcell model is the staircase model. In this method, the edges of the antenna are approximated by staircase type boundaries as shown in Fig. 4.4.

![Staircase model](image)

**Fig. 4.4 Staircase model.**

The errors in the antenna impedance and the near field, caused by this subcell method, are quite large. The target in GPR applications is in the near field and therefore, this staircase method is not good enough to accurately model the antenna.

In this thesis, the antennas are modeled using the more accurate conformal model [26], where the FDTD cells near the boundary are deformed so that they are in a manner conforming with the edges of the antenna. The finite difference expressions of the FDTD scheme has to be modified at these cells. Applying this model to the bow-tie is explained next.
Fig. 4.5 shows how the electric field contours surrounding $H_z$ components at the boundary of the bow-tie are deformed. In the Figure, $a_i, b_i \ (i = 1, 2, 3)$ are the lengths of the segments carrying $E_z$-field components and $A_i \ (i = 1, 2, 3)$ are the areas of the contours.

There are three different cases to be considered here. In contour (i), all the $E$-field components are calculable and modified time stepping for $H_z^1$ is given by
where $\mu_0$ is the permeability of free space. The usual notation $H_z(t) = H_z(t = n\Delta t)$ is used here. In contour (ii), one of the two $E_y$ components needed to calculate $H_z^2$ is inside the plate. Therefore it is borrowed from the nearby cell, which is one space cell away from the plate in the collinear direction (collinear borrow). The modified time stepping for $H_z^2$ is shown below:

$$H_z^2(n+1/2) = H_z^2(n-1/2) + \frac{\Delta t}{\mu_0 A_z} \left[ \left[ E_y^{b1}(n) \times a_z - E_y^{b2}(n) \times b_z \right] + E_z^2(n) \times \Delta \right]. \quad (3.37)$$

In contour (iii), both the $E_y$ components are outside the plate. But an adjacent $H_z$ component needed to calculate one of these $E_y$ components is inside the plate. Therefore $E_y$ is borrowed in the similar manner for the affected segment of the contour as shown below:

$$H_z^2(n+1/2) = H_z^2(n-1/2) + \frac{\Delta t}{\mu_0 A_z} \left[ \left[ E_y^{b3}(n) \times a_z - E_y^{b2}(n) \times b_z \right] + E_y^2(n) \times \Delta \right]. \quad (3.38)$$

To calculate $E_z$ or $E_y$ if one of two adjacent $H_z$ is inside the plate, it ($E_z$ or $E_y$) is borrowed from the nearby cell as described above. Modified time stepping is only necessary for $H_z$ components adjacent to the boundary.

When the antenna has a curved surface, the deformed cells can be oriented in either $x$ or $y$ direction as shown in Fig. 4.6. Therefore, for $H_z$ calculations,
collinear borrow will be done on either $E_x$ or $E_y$. Also there are special deformed cells where collinear borrow is necessary on both $E_x$ and $E_y$.

![Diagram of conformal subcell model for the curved surface of a planar dipole.]

**Fig. 4.6** Conformal subcell model for the curved surface of a planar dipole.

### 4.4 Antenna feed model

Accurate results can be obtained if the antenna feed region is modeled precisely. A 1-D FDTD scheme known as one-way injector model [27] is used for this purpose. In this model, the feeding transmission line is modeled under the assumption that electromagnetic waves in the line are transverse electromagnetic (TEM).

This FDTD scheme can be built by starting from the famous transmission line equations:

$$L \frac{dl}{dt} = -\frac{dV}{dz}$$  \hspace{1cm} (3.40)
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\[ \frac{dV}{dt} = -\frac{dI}{dz} \]  

(3.41)

where \( V \) and \( I \) are line voltage and current respectively. \( L \) and \( C \) denote the inductance and capacitance per unit length respectively. The corresponding finite difference expressions for \( V \) and \( I \) are given by

\[ V^{n+1}(l) = V^n(l) - \frac{Z_0 v \Delta t}{\Delta l} \left[ I^{n+1/2}(l + 1/2) - I^{n+1/2}(l - 1/2) \right] \]  

(3.42)

\[ I^{n+1/2}(l + 1/2) = I^{n-1/2}(l + 1/2) - \frac{V^n(l + 1)}{Z_0 \Delta l} \left[ V^n(l + 1) - V^n(l) \right] \]  

(3.43)

where \( v = 1/\sqrt{LC} \) is the velocity of propagation of the signal in the line and \( Z_0 \) is the characteristic impedance of the line. For an open-air transmission line, \( v = c \) where \( c \) is the velocity of light in free space. The conventional notation \( V^n(l) = V(z = l \Delta', t = n \Delta t) \) is used, where \( \Delta' \) is the 1-D cell size along the line and \( \Delta t \) is the time step. If the input signal is inserted as a hard source at a point in the line, it gives furious reflections, when the reflected
signal from the antenna feed point comes back to the source point. As a remedy, in the one-way injector model, the transmission line is divided into two regions. The region near the antenna feed point records the total line voltage and current while the other region records only the reflected voltage and current. (See Fig. 4.7). Input signal is introduced at the total/reflection interface (at $l_s$), where the finite difference expressions for line voltage and current take the following form:

\begin{align}
V^{n+1}(l_s) &= V^n(l_s) - \frac{Z_0 \nu \Delta t}{\Delta'} \left[ I^{n+1/2}(l_s + 1/2) - I^{n+1/2}(l_s - 1/2) \right] + \\
&\quad \frac{Z_0 \nu \Delta t}{\Delta'} I_{inc}^{n+1/2}(l_s + 1/2) \\
I^{n+1/2}(l_s + 1/2) &= I^{n-1/2}(l_s + 1/2) - \frac{\nu \Delta t}{Z_0 \Delta'} \left[ V^n(l_s + 1) - V^n(l_s) \right] + \\
&\quad \frac{\nu \Delta t}{Z_0 \Delta'} V_{inc}^n(l_s)
\end{align}

where $I_{inc}^{n+1/2}(l_s + 1/2)$ is the incident current at $(l_s + 1/2)$ and $V_{inc}^n(l_s)$ is the incident voltage at $l_s$. The current on top of the line (at $l_f + 1/2$) is calculated by the $H$-field components surrounding $E_x$ at the feed point, $\left( E_x(i_f, j_f, k_f) \right)$, using the Ampere’s law as follows:

\begin{align}
I^{n+1/2}(l_f + 1/2) &= \Delta \left[ H_x^{n+1/2}(i_f, j_f + 1/2, k_f) - H_x^{n+1/2}(i_f, j_f - 1/2, k_f) \right] + \\
&\quad \Delta \left[ H_y^{n+1/2}(i_f, j_f, k_f - 1/2) - H_y^{n+1/2}(i_f, j_f, k_f + 1/2) \right].
\end{align}

Again the notation $H^x(i, j, k) = H_x(x = i \Delta x, y = j \Delta y, z = k \Delta z, t = n \Delta t)$ is used where $\Delta x = \Delta y = \Delta z = \Delta$ and $\Delta$ is the spatial cell size in the main grid. The bottom of the transmission line ($l = 0$) is terminated by a first order accurate Mur ABC:
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\[ V^{n+1}(0) = -V^{n+1}(1) + \frac{v\Delta t - \Delta'}{v\Delta t + \Delta'} \left[ V^{n+1}(1) + V^{n+1}(0) \right] + \frac{2\Delta'}{v\Delta t + \Delta'} \left[ V^x(0) + V^x(1) \right]. \]  

\[ (3.47) \]

\[ E_x \text{ at the feed point } (i_f, j_f, k_f) \text{ is calculated by simply dividing the gap voltage by the gap. That is} \]

\[ E_x^{n+1}(i_f, j_f, k_f) = \frac{-V_{ij_k}^{n+1}}{\Delta_f} \]

\[ (3.48) \]

where \( \Delta_f \) is the gap at the feed point. When the antenna is operating in the receiving mode, the received voltage can be calculated using the same transmission line model making the input voltage and current zero.

### 4.5 Modeling the lumped resistors in the antenna

The planar dipole antennas under investigation in this thesis are resistor-loaded as described in section 2.1.1. The finite difference expressions of the FDTD scheme should be modified at these lumped resistors. In the simulations, the resistors are assumed to be frequency independent. Consider Fig. 4.8 in which lumped resistors are connected to the dipole arm along the \( x \) axis.

[Fig. 4.8 Location of lumped resistors with respect to the FDTD grid.]

34
The resistance of each resistor is assumed to be uniformly distributed over a few FDTD cells. The resistors are placed along \( E_x \) components of these cells. Therefore it is necessary to modify the finite difference expression for these \( E_x \) components in order to account for the current flow in the resistors [28].

Assuming the current is uniformly distributed over the cross section of a cell, the following expression gives the current density \( J \) at the point \((i, j, k)\) (see Fig. 4.8):

\[
J(i, j, k) = \frac{E_x(i, j, k)}{r\Delta}
\]

(3.49)

where \( r \) is the resistance per cell. This current density comes in the Ampere’s law of Maxwell’s equation as follows:

\[
\frac{E_x}{r\Delta} + \varepsilon_0 \frac{\partial}{\partial t} E_x = \frac{\partial}{\partial y} H_z - \frac{\partial}{\partial z} H_y
\]

(3.50)

where \( \varepsilon_0 \) is the permittivity of free space. The modified finite difference expression for \( E_x \) is as follows:

\[
E_x^{n+1}(i, j, k) = \left\{ \begin{array}{l}
\frac{1 - \Delta t/2r\Delta\varepsilon_0}{1 + \Delta t/2r\Delta\varepsilon_0} E_x^n(i, j, k) + \frac{\Delta t/\varepsilon_0\Delta}{1 + \Delta t/2r\Delta\varepsilon_0} \\
\left[ H_z^{n+1/2}(i, j + 1/2, k) - H_z^{n+1/2}(i, j - 1/2, k) \right] - \\
\left[ H_y^{n+1/2}(i, j, k + 1/2) - H_y^{n+1/2}(i, j, k - 1/2) \right]
\end{array} \right. (3.51)
\]

4.6 Modeling of lossy and dispersive soil

In a lossy, dispersive medium, the permittivity and permeability can be functions of frequency. As mentioned in section 4.1, there are three different methods to make FDTD modeling of lossy, dispersive soil. The type of method suitable for a particular application can depend on the type of mathematical model used to represent the soil. In this thesis a two term Debye model is used together with a static conductivity to model the soil.
The FDTD scheme is obtained for this soil model by using the ADE method with UPML. This is explained in the rest of this section.

In the two term Debye model with static conductivity, the relative permittivity $\varepsilon_r$ takes the following form:

$$
\varepsilon_r(\omega) = \varepsilon_\infty + \sum_{k=1}^{2} G_k(\varepsilon_s - \varepsilon_\infty) + \frac{\sigma_s}{j\omega\varepsilon_0}
$$

(3.52)

where $\omega$ is the frequency, $\varepsilon_s$ is the relative permittivity at DC, $\varepsilon_\infty$ is the permittivity of free space, and $\sigma_s$ is the static electric conductivity, and $\sum_{k=1}^{2} G_k = 1$. The medium is assumed to be non magnetic so that its permeability is equal to that of free space, $\mu_0$. The Maxwell curl equations can thus be expressed in the following form for this model in rectangular coordinates:

$$
\begin{pmatrix}
\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y} \\
\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \\
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x}
\end{pmatrix} = j\omega\varepsilon_0 \left( \varepsilon_\infty + \sum_{k=1}^{2} \frac{A_k}{1 + j\omega t_k} + \frac{\sigma_s}{j\omega\varepsilon_0} \right) \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}
$$

(3.53)

$$
\begin{pmatrix}
\frac{\partial E_z}{\partial z} - \frac{\partial E_y}{\partial y} \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial z} \\
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{pmatrix} = j\omega\mu_0 \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}
$$

(3.54)
where $s$ is given by the equation (3.34). Its elements $s_i (i = x, y, z)$ are defined in (3.33). When the above two curl equations apply to the outside of the PML, $\widetilde{s}$ becomes an identity matrix and the equations (3.55) and (3.56) reduce to (3.53) and (3.54) respectively.

The time domain finite difference expressions for the above curl equations can be obtained by using the ADE method. The order of the differential equations is kept at two by suitably selecting the constitutive relationships to decouple the frequency-dependent terms in the equations.

First, the following relationships are used:

$$P_x = \frac{s_x}{s} E_x; \quad P_y = \frac{s_y}{s} E_y; \quad P_z = \frac{s_z}{s} E_z$$

$$P'_x = s_x P_x; \quad P'_y = s_y P_y; \quad P'_z = s_z P_z.$$
Then the equation (3.55) reduces to

\[
\begin{pmatrix}
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} \\
\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial x} \\
\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y}
\end{pmatrix} = j\omega\varepsilon', (\omega) \begin{pmatrix} P'_x \\ P'_y \\ P'_z \end{pmatrix} + \sigma_i \begin{pmatrix} P'_x \\ P'_y \\ P'_z \end{pmatrix}
\]

(3.59)

where

\[
\varepsilon', (\omega) = \varepsilon, + \sum_{r=1}^{2} \frac{A_r}{1 + j\omega t_r}
\]

(3.60)

Then it is assumed that

\[
\mathbf{D}_i = \varepsilon_i \varepsilon', (\omega) P'_i
\]

(3.61)

where \( i \equiv (x, y, z) \). Now (3.59) takes the following form in time domain:

\[
\begin{pmatrix}
\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} \\
\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial x} \\
\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y}
\end{pmatrix} = \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{D}_x \\ \mathbf{D}_y \\ \mathbf{D}_z \end{pmatrix} + \sigma_i \begin{pmatrix} P'_x \\ P'_y \\ P'_z \end{pmatrix}
\]

(3.62)

The equation (3.62) gives the finite difference expression for \( \mathbf{D}_x \) as
4. Modeling of GPR systems using FDTD

\[
D_{x}^{n+1}(i, j, k) = D_{x}^{n}(i, j, k) - \frac{\Delta t \sigma_{x}}{2} \left( P_{x}^{n+1}(i, j, k) + P_{x}^{n}(i, j, k) \right) \\
+ \frac{\Delta t}{2} \left( H_{x}^{n+1/2}(i, j + 1/2, k) - H_{x}^{n+1/2}(i, j - 1/2, k) \right) - \left( H_{y}^{n+1/2}(i, j, k + 1/2) - H_{y}^{n+1/2}(i, j, k - 1/2) \right)
\]

(3.63)

Substituting (3.60) in (3.61) gives

\[
D_{t} = \varepsilon_{0} \left( \varepsilon_{\infty} + \sum_{k=1}^{3} A_{k} \right) P_{t}'.
\]

(3.64)

Finite difference expressions can be obtained for this by using the ADE method. Considering the \(x\) component in (3.64) and multiplying it by the common denominator gives

\[
D_{x} \left( 1 + j \omega t_{1} \right) \left( 1 + j \omega t_{2} \right) = \varepsilon_{0} \varepsilon_{\infty} \left( 1 + j \omega t_{1} \right) \left( 1 + j \omega t_{2} \right) P_{x}' \\
+ \varepsilon_{0} A_{1} \left( 1 + j \omega t_{2} \right) P_{x}' + \varepsilon_{0} A_{2} \left( 1 + j \omega t_{1} \right) P_{x}'.
\]

(3.65)

This can be converted into time domain as follows:

\[
t_{1} t_{2} \frac{\partial^{2}}{\partial t^{2}} D_{x} + D_{x} + \left( t_{1} + t_{2} \right) \frac{\partial}{\partial t} D_{x} = \left( \varepsilon_{0} \varepsilon_{\infty} + \varepsilon_{0} A_{1} + \varepsilon_{0} A_{2} \right) P_{x}' \\
+ \left[ \varepsilon_{0} \varepsilon_{\infty} \left( t_{1} + t_{2} \right) + \varepsilon_{0} A_{2} t_{1} + \varepsilon_{0} A_{1} t_{2} \right] \frac{\partial}{\partial t} P_{x}' + t_{1} t_{2} \varepsilon_{0} \varepsilon_{\infty} \frac{\partial^{2}}{\partial t^{2}} P_{x}'.
\]

(3.66)

Applying a second order accurate central difference scheme centered at time step \(n\) (with a semi implicit specification of the fields located at that time point), (3.66) yields
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\[ t_1 t_2 \left( \frac{D_x^{n+1}(i,j,k) - 2D_x^n(i,j,k) + D_x^{n-1}(i,j,k)}{(\Delta t)^2} \right) + \frac{D_x^{n+1}(i,j,k) + D_x^{n-1}(i,j,k)}{2} + (t_1 + t_2) \left( \frac{D_x^{n+1}(i,j,k) - D_x^{n-1}(i,j,k)}{2\Delta t} \right) \]

\[ = (\varepsilon_0\varepsilon_\infty + \varepsilon_0A_1 + \varepsilon_0A_2) \left( \frac{P_x^{n+1}(i,j,k) + P_x^{n-1}(i,j,k)}{2} \right) \]

\[ + \left[ \varepsilon_0\varepsilon_\infty (t_1 + t_2) + \varepsilon_0A_1 + \varepsilon_0A_2 \right] \left( \frac{P_x^{n+1}(i,j,k) - P_x^{n-1}(i,j,k)}{2\Delta t} \right) \]

\[ + t_1 t_2 \varepsilon_0\varepsilon_\infty \left( \frac{P_x^{n+1}(i,j,k) - 2P_x^n(i,j,k) + P_x^{n-1}(i,j,k)}{(\Delta t)^2} \right). \]  

Solving (3.67) for \( P_x^{n+1} \) gives

\[ P_x^{n+1}(i,j,k) = \frac{\alpha_2P_x^n(i,j,k) + \alpha_3P_x^{n-1}(i,j,k) + \beta_1D_x^{n+1}(i,j,k)}{\alpha_1} \]  

(3.68)

where

\[ \alpha_1 = \varepsilon_0 \left[ \varepsilon_\infty \left( \frac{1}{2} + \frac{t_1 + t_2}{2\Delta t} \right) + A_1 \left( \frac{1}{2} + \frac{t_2}{2\Delta t} \right) + A_2 \left( \frac{1}{2} + \frac{t_1}{2\Delta t} \right) \right] \]  

(3.69)

\[ \alpha_2 = \frac{2\varepsilon_0\varepsilon_\infty t_1 t_2}{(\Delta t)^2} \]  

(3.70)

\[ \alpha_3 = \varepsilon_0 \left[ \varepsilon_\infty \left( \frac{1}{2} + \frac{t_1 + t_2}{2\Delta t} \right) - \frac{t_1 t_2}{(\Delta t)^2} \right] + A_1 \left( \frac{1}{2} + \frac{t_2}{2\Delta t} \right) + A_2 \left( \frac{1}{2} + \frac{t_1}{2\Delta t} \right) \]  

(3.71)
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\[ \beta_1 = \frac{t_1 t_2}{(\Delta t)^2} + \frac{1}{2} \frac{t_1 + t_2}{2\Delta t} \]  
(3.72)

\[ \beta_2 = \frac{2t_1 t_2}{(\Delta t)^2} \]  
(3.73)

\[ \beta_3 = \frac{t_1 t_2}{(\Delta t)^2} + \frac{1}{2} \frac{t_1 + t_2}{2\Delta t} \]  
(3.74)

The value of \( P_{x_{n+1}} \) in (3.68) can be substituted in (3.63) to find \( D_{x_{n+1}} \) in terms of previous values of \( D_x \) & \( P_x \) and present \( H \)-field components. Now (3.63) yields

\[ D_{x_{n+1}}(i, j, k) = \frac{\gamma_2}{\gamma_1} D_x^n(i, j, k) - \frac{\gamma_1}{\gamma_1} D_x^{n-1}(i, j, k) \]

\[ -\frac{\gamma_2}{\gamma_1} \left[ 1 + \frac{\alpha_2}{\alpha_1} \right] P_x^n(i, j, k) + \frac{\alpha_2}{\alpha_1} P_x^{n-1}(i, j, k) \]

\[ + \frac{\gamma_2}{\gamma_1} \left[ \left( H_x^{n+1/2}(i, j + 1/2, k) - H_x^{n+1/2}(i, j - 1/2, k) \right) \right] \]

\[ - \frac{\gamma_2}{\gamma_1} \left[ \left( H_y^{n+1/2}(i, j, k + 1/2) - H_y^{n+1/2}(i, j, k - 1/2) \right) \right] \]

where

\[ \gamma_1 = 1 + \frac{\Delta t \sigma \beta_1}{2 \alpha_1} \]  
(3.76)

\[ \gamma_2 = 1 + \frac{\Delta t \sigma \beta_2}{2 \alpha_1} \]  
(3.77)

\[ \gamma_3 = \frac{\Delta t \sigma \beta_3}{2 \alpha_1} \]  
(3.78)
\[ \gamma_s = \frac{\Delta t \sigma_s}{2} \] 
\[ \gamma_s = \frac{\Delta t}{\Delta} \] 

Now consider (3.58). \( P'_x \) is related to \( P_x \) by
\[ P'_x = s_y P_x. \] 

Substituting (3.33) in this equation gives
\[ P'_x = \left( \kappa_y + \frac{\sigma_y}{j \omega \varepsilon_0} \right) P_x. \] 

Now the ADE method can be used to get the finite difference expressions for \( P_x \). Multiplying (3.81) by \( j \omega \varepsilon_0 \) yields
\[ j \omega \varepsilon_0 P'_x = j \omega \varepsilon_0 \kappa_y P_x + \sigma_y P_x. \] 

Converting this equation into time domain gives
\[ \frac{\partial}{\partial t} P'_x = \kappa_y \frac{\partial}{\partial t} P_x + \sigma_y \frac{P_x}{\varepsilon_0}. \] 

Then the finite difference expression for \( P^{n+1}_x \) is given by
\[ P^{n+1}_x(i, j, k) = P^{n+1}_x(i, j, k) - P''_x(i, j, k) + P''_x(i, j, k) \left( \kappa_y - \frac{\sigma_y \Delta t}{2 \varepsilon_0} \right) \] 
\[ \left( \kappa_y + \frac{\sigma_y \Delta t}{2 \varepsilon_0} \right). \] 

Next consider (3.57) where \( E_x \) is related to \( P_x \) by
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\[ P_x = \frac{s_x}{s_x} E_x. \]  

(3.57)

Substituting (3.33) in this equation yields

\[ P_x = \left( \kappa_z + \frac{\sigma_z}{j \omega \varepsilon_0} \right) E_x. \]  

(3.85)

Rearranging the terms gives

\[ j \omega \varepsilon_k \kappa_x P_x + \sigma_x P_x = j \omega \varepsilon_k \kappa_x E_x + \sigma_x E_x. \]  

(3.86)

Now converting the equation into time domain yields

\[ \varepsilon_k \kappa_x \frac{\partial}{\partial t} P_x + \sigma_x P_x = \varepsilon_k \kappa_x \frac{\partial}{\partial t} E_x + \sigma_x E_x. \]  

(3.87)

This gives the finite difference expression for \( E_x^{n+1} \) as follows:

\[ E_x^{n+1}(i,j,k) = \frac{\kappa_z + \frac{\sigma_z \Delta t}{2 \varepsilon_0}}{\kappa_z + \frac{\sigma \Delta t}{2 \varepsilon_0}} P_x^{n+1}(i,j,k) - \frac{\kappa_z - \frac{\sigma \Delta t}{2 \varepsilon_0}}{\kappa_z + \frac{\sigma \Delta t}{2 \varepsilon_0}} P_x^n(i,j,k) \]

\[ + \frac{\kappa_z - \frac{\sigma \Delta t}{2 \varepsilon_0}}{\kappa_z + \frac{\sigma \Delta t}{2 \varepsilon_0}} E_x^n(i,j,k). \]  

(3.88)

When the present values of \( H \)-field components are known, the calculation of \( E_x^{n+1} \) has to come through several steps. First from (3.75) \( D_x^{n+1} \) should be found. Then \( P_x^{n+1} \) should be found from (3.68). This parameter can be used to find \( P_x^{n+1} \) from (3.84) next. Finally the equation (3.88) gives \( E_x^{n+1} \).
Similar expressions can be derived for $y$ and $z$ components. The finite difference expressions for $H$-field components can be obtained from (3.56) in the same way as for the $E$-field components. In order to use the ADE method, the following intermediate relations are used:

$$B_x = \mu_0 \frac{s_x}{s_z} H_x$$

(3.89)

$$B_y = \mu_0 \frac{s_y}{s_z} H_y$$

(3.90)

$$B_z = \mu_0 \frac{s_z}{s_z} H_z.$$  

(3.91)

Substituting these relations in (3.56) yields

$$\begin{pmatrix}
\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \\
\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{pmatrix}
= j \omega \begin{pmatrix}
s_y & 0 & 0 \\
0 & s_z & 0 \\
0 & 0 & s_z
\end{pmatrix}
\begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix}. 

(3.92)

Substituting (3.33) in this equation and converting it into time domain gives

$$\begin{pmatrix}
\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \\
\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \\
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}
\end{pmatrix}
= \frac{\partial}{\partial t} \begin{pmatrix}
\kappa_y & 0 & 0 \\
0 & \kappa_z & 0 \\
0 & 0 & \kappa_z
\end{pmatrix}
\begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix} + \frac{1}{\epsilon_0} \begin{pmatrix}
\sigma_y & 0 & 0 \\
0 & \sigma_z & 0 \\
0 & 0 & \sigma_z
\end{pmatrix}
\begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix}.

(3.93)
Now the finite difference expressions for $B_x$, $B_y$ and $B_z$ can be derived from (3.93). The Equation (3.94) shows the expression for $B_x$:

$$B_x^{n+1/2}(i, j, k) = \frac{\kappa_x - \sigma_x \Delta t}{2\varepsilon_0} B_x^{n-1/2}(i, j, k) - \frac{\Delta t}{\Delta \left(\kappa_x + \sigma_x \Delta t \frac{1}{2\varepsilon_0}\right)} \begin{bmatrix} E_x^*(i, j, k+1/2) - E_x^*(i, j, k-1/2) \\ -E_y^*(i, j, k+1/2) - E_y^*(i, j, k-1/2) \end{bmatrix}.$$  (3.94)

Using the value of $B_x^{n+1/2}$, $H_x^{n+1/2}$ can be found from the finite difference expression for equation (3.89). This is shown next:

$$H_x^{n+1/2} = \frac{\kappa_x - \sigma_x \Delta t}{2\varepsilon_0} B_x^{n+1/2} - \frac{\kappa_x - \sigma_x \Delta t}{2\varepsilon_0} B_x^{n-1/2} B_x^{n+1/2} = \mu_0 \left(\kappa_x + \sigma_x \Delta t \frac{1}{2\varepsilon_0}\right) \begin{bmatrix} \kappa_x - \sigma_x \Delta t \\ \kappa_x + \sigma_x \Delta t \frac{1}{2\varepsilon_0} \end{bmatrix} B_x^{n+1/2}.$$  (3.95)

Similar expressions can be derived for $y$ and $z$ components.

As mentioned in the section 4.2.2, using a value not equal to unity for $\kappa_i$ ($i = x, y, z$) is optional. Therefore it is taken $\kappa_i = 1$ in the simulations.
5. Summary of Papers

5.1 Paper I

This paper describes how the parameters of resistor-loaded bow-tie antennas affect the target detection using the FDTD simulations. The antennas are enclosed in rectangular cavities to suit the GPR applications. The parametric study includes the antenna’s flare angle, length, lumped resistors, and position of the transmitting and receiving antennas with respect to each other and ground surface. The design guidelines of getting a better target discrimination with less clutter from the bow-tie antenna have been presented. In the simulations, it is considered that the ground properties would not affect the optimization of antenna parameters. Therefore the ground is assumed to be lossless and nondispersive with perfectly smooth surface.

The study on the lumped resistors shows that the antenna’s input impedance depends on the total parallel end resistance and not on the number of lumped resistors used. The lumped resistors effectively suppress the currents towards the ends of the antenna when they are placed at the corners. The study reveals that the lumped resistors mostly affect the antenna’s input impedance at low frequencies and a total parallel end resistance of around 100-150 $\Omega$ is suitable to obtain a good GPR response with low clutter level.

The antenna’s flare angle is varied in the range 40°-80° by keeping the gap between the antenna cavities the same. A larger peak-to-peak scattered signal amplitude with less clutter could be observed with increasing flare angle. For a given frequency band of operation, the bandwidth of the antenna can be improved by reducing its length. Also this reduces the radiated signal ringing period. Therefore, shorter antennas are useful for the detection of shallowly buried targets. A larger peak-to-peak scattered signal amplitude can be obtained by increasing the antenna length. But this has an upper limit as the resulting increase in the distance between antenna feed points has a negative effect.

Simulation results show that when the antennas are raised from the ground, the scattered signal strength starts to decrease and the antenna becomes less
broadband. Moreover, it shortens the clutter signal ringing period. For this antenna setup, a height of around 1-2 cm from the ground seems to be ideal. When the separation between the antennas is increased, the received signal strength starts to drop and clutter signal ringing period gets longer. Placing the antennas very close to each other can mask the scattered signal for a real GPR application since the clutter is larger and the optimum separation will depend on the depth of the buried target as well. The change in the input impedance of the transmitting antenna for different gaps is minor.

5.2 Paper II

In this paper, a realistic FDTD simulation has been done on a GPR system consisting of a pair of resistor-loaded bow-tie antennas enclosed in rectangular conducting cavities. Aiming at a good target detection with low clutter, suitable antenna parameters are selected based on the results of paper I. The simulation includes the features in a practical GPR environment such as lossy and dispersive media, ground surface roughness and inhomogeneities in soil.

The paper investigates the performance of the antenna in two different soil types: Puerto Rico and San Antonio clay loams, with different moisture contents of 2.5%, 5%, and 10%. The electrical properties of these soils are modeled by using a two-term Debye model with a static conductivity and model parameters are calculated by curve fitting the experimental data found in [29]. The detecting targets are buried metal and plastic pipes. First, simulations are carried out for perfectly smooth ground surface with no soil inhomogeneities. The time domain GPR responses show the unique shapes of scattered signals of metal and plastic pipes, which are useful in target identification. The input impedance variation of the antenna is also found and the results show that the soil properties do not considerably affect the antenna’s input impedance, an important feature for a GPR antenna. The target polarization characteristics associated with thin long targets are also analyzed with the metal and plastic pipes and simulation results are in good agreement with the theory.

The surface roughness at the air-ground interface is simulated by randomly placing 100 rectangular holes of different sizes on the ground surface. The results show that adding surface roughness has distorted the early time
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signals of the GPR response which mainly consists of the ground surface reflection. But the scattered signals of the pipes are almost unaffected. Therefore, the detectibility of the target by the antenna is not harmed by the addition of the surface roughness.

Next, soil inhomogeneities with different sizes and material properties are added to the soil randomly. These scatterers are assumed to be lossy and nondispersive and their relative permittivity and conductivity values are chosen in the typical range for soil inhomogeneities like rocks and twigs. Simulation results show that small scatterers, spread all over the soil, have negligible effect on the GPR response when compared to the surface roughness. If the target is as small as the scatterers, visual detection would be almost impossible and more advanced detection algorithms would have to be used to pick the target echo.

5.3 Paper III

This paper describes the performance of planar parabolic dipoles for GPR applications using FDTD. The antenna is resistor-loaded and shielded to suit the GPR applications. The clutter level and the GPR response of the antenna when it is operating above both lossless and lossy media are analyzed. The input impedance and radiation characteristics are also described. Simulations are carried out for different antenna dimensions and results are compared with the bow-tie to assess the antenna’s performance.

The simulation results in time and frequency domain show that this antenna has broadband properties and it performs well as a GPR antenna. It has a lower clutter level and increased scattered signal strength, although the improvement is marginal over the corresponding bow-tie having the same length and width. This same result could be observed irrespective of the antenna dimensions and ground properties.

5.4 Paper IV

The first part of this paper investigates several different geometries of planar dipoles in order to find what shape gives the best performance in GPR applications using FDTD. Both lossless and lossy, dispersive media are
considered and a rectangular conducting sheet is used as the target for the simulations. The peak-to-peak scattered signal amplitudes given by different antennas are compared. It could be observed that the bow-tie antenna outperforms the other antennas considered except the planar parabolic dipole, which gives a marginal improvement over the bow-tie. In order to better assess the performance of the antennas, integrated power of the scattered signals is also considered. It clearly differentiates the target detectability of the antennas considered. Similar results could be observed for different antenna lengths and widths and for different targets and ground properties.

Next, the effect of adding a wave absorbing material to the inner walls of conducting cavities, which screens the planar dipoles, is studied. Generally, wave absorbers are employed to suppress signal reflections inside the screening cavities of GPR antennas. But its usefulness depends on the antenna type and this paper studies its applicability to planar dipoles. This is accomplished by using perfectly matched layer (PML) absorbing boundary conditions to simulate the absorbing layer. It is evident from the simulation results that an absorbing layer increases the direct coupling between the transmitting and receiving antennas and degrades the antenna’s ability to detect the target echo. Therefore, the placement of a wave-absorbing layer inside the cavity is not suitable for planar dipoles. This could be observed for both lossless and lossy grounds considered.

Finally, a genetic algorithm is used to optimize the screening cavity height and lumped resistor values to improve the received signal strength with a low clutter level. The optimum values are searched in the range of 50-750Ω for the lumped resistors and 2.25-13.5 cm for the cavity height. The optimization gives a total parallel end resistance of 144.05 Ω and a cavity height of 11.25 cm for the bow-tie. The optimized values for the planar parabolic dipole are 127.4 Ω and 11.25 cm. Simulations performed with these optimized values clearly show the improved performance for both lossless and lossy ground media.

5.5 Paper V

This paper presents practical aspects of using a pair of resistor-loaded bow-tie antennas for GPR applications and provides a validation for the FDTD simulations performed in the previous four papers. Two antennas, enclosed in
rectangular conducting cavities, are constructed and equipped with suitable ferrite core transmission line balun transformers (TLTs) for impedance matching. The GPR system is driven by a narrow pulse generated by an impulse generator and the signal in the receiving antenna is measured by a high speed oscilloscope. The incident pulse is an approximate Gaussian pulse with a time duration of about 2 ns and a peak voltage of 30 V.

In order to compare the experimental results with theory, some indoor measurements are carried out. The antenna’s $S_{11}$ parameter is measured by a network analyser when the antenna is radiating into free space. Also the time domain response is measured by the oscilloscope when the antenna set up is placed symmetrically 87.5 cm away from a rectangular conducting sheet (40 cm × 40 cm) in free space. These measurements are compared with the FDTD simulation results. The results are in agreement and the deviations are mainly due to the inherent problems of the balun transformer as explained in section 3.1.

The outdoor measurements are taken on a rectangular conducting sheet (75 cm × 65 cm) and two kinds of pipes (metal and plastic) buried at a depth of 50 cm in snow. The metal pipe has an outer diameter of 15 cm and the plastic pipe has an outer diameter of 11 cm with 5 mm thickness. The A-scan and B-scan measurements are obtained with these targets buried one at a time. The sheet and the metal pipe could be visually detected. But the plastic pipe could not be identified visually. The images of snow in the presence of the targets are formed using B-scan data and presented in the paper.

The results show that this GPR system looks promising for detecting localized targets buried within a depth of around 1 m. Deeper penetrations up to several meters would also be achieved by suitably selecting the operating frequency band and the antenna dimensions.
6. Conclusion

This thesis presents a detailed study on the use of resistor-loaded planar dipole antennas for ground penetrating radar applications. The analysis has been done using FDTD simulations and measurements have been carried out to validate the simulations by constructing a bow-tie antenna system. Losses and dispersive properties of the ground, surface roughness, and soil inhomogeneities are all included in the simulations to obtain more realistic results.

Among different planar dipoles considered, the bow-tie and planar parabolic dipole are shown to be the best antenna geometries for the GPR applications. These antennas, when loaded with lumped resistors, have broadband properties and have the ability to radiate narrow pulses with less distortion and ringing. Among the two, the planar parabolic dipole shows a marginally improved performance. A design guideline for GPR applications has been presented for the resistor-loaded bow-tie antenna, which includes details on the selection of flare angle, lumped resistors, length, shielding, etc of the antenna. The same guideline can be used for the planar parabolic dipoles.

The planer dipole antennas are useful in the upper MHz and lower GHz frequency range to achieve depths of penetrations up to several meters. Their small physical volume and ability to operate closer to the ground surface are quite useful in GPR applications. But their symmetrical structure demands the use of a balun transformer to connect the antenna feed point to the unbalanced transmission line, which feeds the antenna. The lack of availability of very broadband balun transformers with a constant impedance ratio can degrade the performance of these antennas.
7. References


[22] J. P. Berenger, "A perfectly matched layer for the absorption of
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