Joint Relay Selection and Power Allocation for Cooperative Communication over Frequency Selective Fading Channels

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Abstract—In this paper, we consider the joint problem of relay selection and optimal power allocation for multi-relay amplify-and-forward (AF) cooperative communication system over frequency selective fading channels. An optimization model combined relay selection and power allocation under a total transmission power budget is formulated. Then, this combinatorial problem is solved in a distributed strategy. Relay selection with a new threshold-based multiple-relay selection (MRS) scheme is implemented at first, and then power is allocated between source and the selected relays in an optimized way to maximize channel capacity. Simulation result shows that the proposed joint scenario with relay selection and power allocation achieves better throughput performance than that of parallel-relay scenario (means that random relay is selected to forward data and allocated part of total power on average). Furthermore, the performances of the new MRS scheme and other relay selection strategies are also investigated.

Index Terms—relay selection, power allocation, amplify-and-forward, frequency selective fading, wireless networks, cooperative communication

I. INTRODUCTION

Cooperative communication via relays has been shown to be a promising technique to improve the performance of the wireless networks significantly by relaying signals for the user. It exploits spatial diversity in fading channels effectively, and transmits signal with lower energy consumption [1], [2]. A number of possible ways to improve the spectral efficiency are proposed in many references, while relay selection and power allocation are almost the two most considerable problems in this field.

The relay selection schemes in cooperative network are broadly be divided into two categories, single-relay selection (SRS) schemes [3] and MRS schemes [4], [5]. In the SRS schemes, the optimal relay node which can achieve the highest signal to noise ratio (SNR) at the destination is only chosen to participate in the transmission. And then, several MRS schemes are proposed by generalizing the idea of SRS to allow for multiple relays to cooperate. A novel relay selection algorithm for multi-user cooperative wireless networks with opportunistic relaying was proposed in [4]. Reference [5] analyzes the system where multiple relays that have the strongest signal strength at the destination are selected out of all potential relays and forward their received data from the source node to the destination node. These schemes achieve full diversity while maintaining high throughput.

Another efficient approach is to allocate the limited power among source and relay nodes optimally. Most of the optimal power allocation methods use total power constraint and outage probability [6], outage events [7] or the received SNR at the destination [8] as the optimization criterion. The above works demonstrate a significant performance improvement due to optimal power allocation over equal power allocation in cooperative networks.

Later, efforts are made to further improve the performance of cooperative diversity by jointing relay selection and optimal power allocation [9]-[11]. An scheme that performs relay selection while minimizing the total transmit power and satisfying certain quality-of-service requirement in multi-user decode-and-forward (DF) cooperative networks is present in [9] and [10]. Reference [11] presents an optimal joint RS and power allocation scheme for two-way relay networks to maximize the smaller SNR of the two transceivers under a total transmit power budget.

The joint scheme combines both the benefit of relay selection and power allocation to achieve better performance. However, all above the joint techniques are about the flat fading channels. In practical scenarios, these channels are likely to be frequency selective. Thus, in this paper, we consider the joint problem of relay selection and power allocation for multi-relay amplify-and-forward (AF) cooperative communication system over frequency selective fading channels.
selection and optimal power allocation for multi-relay AF cooperative communication system over frequency selective fading channels. In the beginning, the expression of optimization model combined relay selection and power allocation with a total transmission power limit is derived. Then, this combinatorial problem is solved in a distributed strategy. First, relay selection with a new threshold-based MRS scheme is implemented. Second, power allocation between source and the selected relays in an optimized way is executed to achieve the maximum channel capacity.

The remainder of the paper is organized as follows: Section II describes the system model, the signal transmission and channel capacity. In section III, the problem of joint relay selection and power allocation is formulated and solved. Section IV illustrates the simulation results. The conclusion of this paper is shown in Section V.

II. SYSTEM MODEL

A. System model

Consider a relay network with one source node $S$, one destination node $D$ and $N$ potential relay nodes $R_{ni}$, $i=0,1,2...K...N$, as depicted in Fig.1. All nodes are equipped with only one antenna. Both the source-to-relay $S-R_i$ and relay-to-destination $R_i-D$ channels are frequency selective with block-constant Rayleigh fading paths. There is no direct link between $S$ and $D$, but the results can be applied to the case with a direct link straightforwardly. The transmission of the signal from source to destination takes place in two phases with the cooperation of relays. We consider the AF protocol. In the first phase, the source broadcasts the signal to all the relays. RS based on our proposed strategies is applied in the second phase. Then, the selected relays amplify-and-forward the signal to the destination $D$, and the other relays remain silent. Similar to the techniques of [9], the destination is assumed to have a perfect $S-R_i$ and $R_i-D$ instantaneous channel state information to make the relay selection and power allocation. It then conveys the relay assignment and the optimized powers to source and relays through a low rate feedback channel.

Before transmission, a cyclic prefix is inserted to the signal at $S$ to combat the inter-symbol interference, and the received signal at relay $R_i$ can be expressed in vector form [12], [13] after removing the cyclic prefix as

$$r_i = H_i s + n_i$$  (1)

where $s$ is a length-$M$ input signal vector $s \sim (0, Q_s)$, $H_i$ is a $M \times M$ circulant channel matrix with entries of $h_{ij}$, $j=1,2...J$, $J$, is the number of the paths $S-R_i$. $H_i$ represents the frequency selective fading. And $n_i$ denotes the additive white Gaussian noise (AWGN) vector at the relay $R_i$ with zero mean and covariance matrix $\sigma_n^2 I$. Without any loss of generality, we consider the AWGN of all links having zero mean and equal variance. In AF systems, relays only linear amplify the received signal and forward it to destination. Thus, the transmit signal of $R_i$ can be expressible as

$$x_i = A_i (H_i s + n_i)$$  (2)

where the $A_i$ denotes the linear amplification. The same as source node, a cyclic prefix is also inserted to the signal at relay $R_i$. Thus, the signal received by destination from the relay $R_i$ is

$$y_i = G_i A_i (H_i s + n_i) + n$$  (3)

where the $G_i$ denotes the $M \times M$ circulant channel matrix of the paths $R_i-D$ as the $H_i$. We consider a special case that there are no relative transmission delays among different $S-R_i-D$ links. The destination $D$ combines all received signals from the $N$ links. Then, we can express the received signal at destination as

$$y = \sum_{i=1}^{N} G_i A_i (H_i s + n_i) + n$$  (4)

B. Channel capacity

As shown by [13], understand the multi-relay AF systems in multipath fading through the concept of virtual sub-channels. Matrix $H_i$, $G_i$, $Q_i$ and $A_i$ has the following structure

$$H_i = F \text{diag} [\alpha_1,\ldots,\alpha_{M}] F$$
$$G_i = F \text{diag} [\beta_1,\ldots,\beta_{M}] F$$
$$Q_i = F \text{diag} [\rho_1,\ldots,\rho_{M}] F$$
$$A_i = F \text{diag} [\sigma_1,\ldots,\sigma_{M}] F$$  (5)

where $*$ denotes conjugate transposition. $F$ is the unitary DFT (discrete Fourier transform) matrix and $\{\alpha_j\}, \{\beta_j\}$ denote the eigenvalues of $H_i$ and $G_i$. The $P_m$ is transmission power at $S$ and the $a_m$ is amplification coefficient of relay $R_i$ for the $m$ th sub-channel. Accordingly, the equation (4) can be written as
\[ y = \sum_{i=1}^{N} G_i A_i H_i s + \sum_{i=1}^{N} G_i A_i n_i + n \]  

Then the covariance matrix of the received signal \( y \) can be expressed as

\[
E[yy^*] = \sum_{i=1}^{N} G_i A_i H_i Q_i H_i^* A_i^* G_i^* + \sigma_n^2 \left( \sum_{i=1}^{N} G_i A_i A_i^* G_i^* + I \right)
\]

According to the definition of channel capacity, we have the expression

\[
C = \frac{1}{N} \log \det \left( \frac{1}{\sigma_n^2} E[yy^*] \right)
\]

Inserting (5) into (11), we obtain

\[
C = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{P_m \sum_{m=1}^{M} \alpha_m^2 \beta_m^2}{\sigma_n^2 \left( \sum_{i=1}^{N} |\beta_m|^2 + |\alpha_m|^2 \right) + 1} \right)
\]

The amplification coefficient \( a_m \) can be expressed as in [13]

\[
a_m = \frac{f \alpha_m^2 \beta_m^2}{P_m |\alpha_m|^2 + P_m |\beta_m|^2 + \sigma_n^2}
\]

where

\[
f = \sqrt{p_m \sum_{m=1}^{M} \left( P_m |\alpha_m|^2 + \sigma_n^2 \right)} \frac{1}{\left( P_m |\beta_m|^2 + \sigma_n^2 \right)}
\]

\[ p_m \] is the transmission power of all the work relays for the \( m \) th sub-channel and it is expressed in the next section. From (9), (10) and (11), we can obtain the channel capacity finally

\[
C = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{P_m \sum_{m=1}^{M} \alpha_m^2 \beta_m^2}{\sigma_n^2 \left( \sum_{i=1}^{N} |\beta_m|^2 + |\alpha_m|^2 \right) + 1} \right)
\]

### III. JOINT RELAY SELECTION AND POWER ALLOCATION

In this section, the optimization problem which combines relay selection and power allocation under a total transmission power budget is formulated at first. And then, we solve this combinatorial problem in a distributed strategy.

#### A. Problem formulation

The joint design problem is equivalent to the relay selection and achieving the optimal power allocation such that the channel capacity is maximized under a total transmission power constraint \( P \). It can be written as

\[
\max \sum_{n=1}^{M} \log_2 \left( 1 + \frac{P_m \sum_{m=1}^{M} \alpha_m^2 \beta_m^2}{\sigma_n^2 \left( P_m |\alpha_m|^2 + P_m |\beta_m|^2 + \sigma_n^2 \right)} \right)
\]

subject to: \( P_m + P_r = P \)

\[
p_m \geq 0, \quad P_m \geq 0
\]

\[ \xi \in \{0, 1\} \]

where \( P_m \) and \( P_r \) denote the power consumed at the source and relays, respectively. \( \xi \) is the relay selection variable. \( \xi = 1 \) if the relay \( R_i \) is selected, otherwise \( \xi = 0 \). The specific selection strategies are discussed in the next section. We can rewrite the power as

\[
P_m = \sum_{n=1}^{M} p_m, \quad P_r = \sum_{n=1}^{M} p_m
\]

\[
P = \sum_{m=1}^{M} p_m
\]

\[
= \sum_{m=1}^{M} \left( p_m + p_m + \xi p_m \right)
\]

where

\[
p_m = |\alpha_m|^2 \left( P_m |\alpha_m|^2 + \sigma_n^2 \right)
\]

For the \( m \) th sub-channel, assume the transmission power constraint \( p_m \) is given. Then we can write the optimization problem

\[
\max \log_2 \left( 1 + \frac{p_m \sum_{m=1}^{M} \alpha_m^2 \beta_m^2}{\sigma_n^2 \left( p_m |\alpha_m|^2 + p_m |\beta_m|^2 + \sigma_n^2 \right)} \right)
\]

subject to: \( p_m + p_m = P_m \)

\[
p_m \geq 0, \quad p_m \geq 0
\]

\[ \xi \in \{0, 1\} \]

Next, we solve above the combinatorial problem in a distributed strategy. First, the value of \( \xi \) is determined,
namely, relay selection is implemented, and then power is allocated between source and the selected relays in an optimized way to maximize the objective function (17).

**B. Distributed implementation**

1) Relay selection: In order to achieve a better compromise between the error rate and spectral efficiency in cooperative communication network, an adaptive MRS scheme that can adjust the number of relays according to the threshold is proposed in this subsection. The $K$ relays that have the highest SNR are sequentially selected out from the $N$ relays such that the SNR of the combined $K$ relayed paths exceeds the preset threshold. The value of SNR threshold depends on the system situation and can be chosen to be the minimum symbol decoding required for a given modulation scheme [15], but it is beyond our scope in this paper. According to (9), for each relay $R_i$, namely $i = 1$ to $N$ in (12), the SNR expression for the $m$th sub-channel can be expressed as

$$\gamma^m = \frac{p_m |\beta_m a_m|^2}{\sigma_n^2 (1 + \beta_m a_m^2)}$$

(18)

Then, the average SNR of the received packet for each relayed path can be represented as in [14]

$$\overline{\gamma^m} = E[\gamma^m]$$

(19)

Fig.2 is the flow chart which illustrates the relay selection process in detail. First, the end-to-end SNR of each relay $R_i$ at destination is arranged to be the order statistics $\overline{\gamma_1} \geq \overline{\gamma_2} \geq ... \geq \overline{\gamma_N}$. Next, threshold checking of the combined SNR is implemented. If the combined SNR has exceeded the preset threshold $\gamma_0$, no more relays are selected. Otherwise, the relays $R_2, R_3, ..., R_k$ are sequentially selected from the $N$ relays until the combined SNR exceeds $\gamma_0$ or all the $N$ relays have been selected out. For the selected relays, the $\xi_i$ is set to be $1$ in (17).

2) Power allocation: In this subsection, the optimal power allocation is to be achieved. For given $p_m$, we utilize $p_{m,s} = \psi_n p_m$ to denote the power allocated to the source, and $p_{m,r} = (1-\psi_n) p_m$ to denote the power allocated to the selected relays where $0 < \psi_n < 1$. Inserting the above relations into (17), the optimization problem can be written as

$$\max \log \left(1 + \sum_{i=1}^K \psi_n (1-\psi_n) p_m |\alpha_m a_m|^2 + \frac{\sigma_n^2}{p_m} \right)$$

(20)

We need derive the optimal $\psi_n$ to make (20) to be maximal, which is equivalent to maximizing threshold-based relay selection process

$$z_n = \sum_{i=1}^K \psi_n (1-\psi_n) p_m |\alpha_m a_m|^2 + \frac{\sigma_n^2}{p_m}$$

(21)

We can derived that

$$\frac{\partial^2 z_n}{\partial \psi_n^2} < 0$$

(22)

Thus the optimal power allocation coefficient $\psi_n$ can be calculated finally as the solution of

$$\frac{\partial z_n}{\partial \psi_n} = 0$$

(23)

Then, the optimal transmission power of source and relays can be derived by using $p_{m,s} = \psi_n p_m$, $p_{m,r} = (1-\psi_n) p_m$. Next, the amplification coefficient $a_m$ can be calculated by (10) and the transmission power of relay $R_i$ for the $m$th sub-channel $p_{m,s}$ is achieved by using (16).
IV. SIMULATION RESULTS

This section, in order to illustrate the performance of the proposed joint relay selection and power allocation algorithms for cooperative communication network over frequency-selective fading channels, some numerical results are presented. We assume that a length $M=16$ symbol modulated by BPSK is transmitted through a system with $N=10$ potential relays. Both the $S-R_i$ and $R_i-D$ links suffer from frequency selective fading and each link consists of 4 independent identically distributed (i.i.d.) Rayleigh paths. The noises at the relays and receiver have zero mean and equal variance.

In Fig.3 and Fig.4, the performance of the proposed joint scenario (threshold-based selection with optimal power) is compared with parallel-relay scenario (random four relays selection with average power). Fig.3 shows that the bit error rate performance of joint scenario is much better than that of the later. The difference is growing as the increase of input SNR. As expected, the threshold-based selection with optimal power achieves the larger channel capacity in Fig.4. The results show that the joint scheme can achieve better performance which combines both the benefit of threshold-based relay selection and optimal power allocation.

In order to further investigate the performance of threshold-based selection scheme with optimal power, Fig.5 makes the compare of the proposed scheme with best one relay, best four relays and all relays scheme. $K$ is the number of selected relays in the figure. For the proposed scheme, we can find that the number of the selected relays $K$ depends on both the value of threshold $\gamma_0$ and the SNR. More relays need to be selected when the threshold is higher. And as the input SNR increases, $K$ decreases on the whole. It shows that the scheme could select the relays adaptively under different situations. The bit error rate performance of the threshold-based selection scheme is better than the best one relay and best four relays schemes. And this is not surprising that the all relays selection achieve the lowest bit error rate since it select all the potential relays. It shows that the bit error rate goes down with an increase of number of selected relays. However, this performance gain of all relays selection scheme is achieved at the cost of additional bandwidth and power. Relatively, the threshold-based selection scheme achieves a better compromise between bit error rate and spectral efficiency in cooperative communication network.

V. CONCLUSION

In this paper, we consider the joint problem of relay selection and optimal power allocation for multi-relay AF cooperative communication system over frequency selective fading channels. An optimization model combined relay selection and power allocation under a total transmission power budget is formulated. Then, this combinatorial problem is solved in a distributed strategy. Relay selection with a new adaptive threshold-based MRS scheme is implemented at first, and then power is allocated between source and the selected relays in an optimized way to maximize channel capacity. Simulation result shows that the proposed joint scheme achieves better throughput performance than that of parallel-relay scenario. Besides, we find that the threshold-based selection scheme can achieve a better compromise...
between bit error rate and spectral efficiency in cooperative communication network by the comparison.

ACKNOWLEDGMENT

This research was supported by the Scientific Research Foundation of Harbin Institute of Technology at Weihai (HIT(WH)X201101), Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT.NSRIF.201116) and National Natural Science Foundation of China (61001093).

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