Testing the positive theory of government finance*

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Received October 1988, final version received June 1990

Researchers characterizing optimal tax policies for dynamic economies have reasoned that tax rates should follow a random walk. We conduct a frequency-domain examination of U.S. tax rates and reject the hypothesis that the first difference in the series is white noise. The source of the rejection is pronounced activity of tax changes at an eight-year cycle. Regression analysis confirms the finding that there is a cyclical component to tax changes corresponding to changes in political party administration. The results suggest that the positive theory of government finance needs to be refined to incorporate features of political equilibrium.

1. Introduction

Principally due to Barro (1979), the life-cycle model of consumption has been reinterpreted as a positive theory of government finance. Whereas the life-cycle model of consumption considers a representative agent who chooses a sequence of consumption to maximize utility subject to exogenous income flows and a law of motion for individual wealth, a descriptive theory of government finance considers an infinitely-lived government that chooses a sequence of tax rates to minimize deadweight loss subject to exogenous...
government spending flows and a law of motion for government wealth (debt).

According to this theory of government finance, the tax rate at any point in time solves a dynamic programming problem. In particular, minimization of the distortions imposed by a government's revenue requirements requires that marginal inefficiencies be equated across time. Furthermore, a quadratic loss function for the government implies that at the global optimum the tax rate approximately equals the marginal inefficiency and therefore possesses a martingale sample path.

This paper tests the positive theory of government finance by studying the testable implications of the martingale hypothesis. The guiding principle is that the martingale hypothesis imposes restrictions on the entire range of autocorrelations of a differenced tax-rate time series. These restrictions lead us to perform a set of frequency-domain econometric exercises that capture all implications of the null model. Our belief is that these investigations shed light on the empirical significance of particular elaborations of the standard model, and thus serve as a useful guide for further research on tax-rate determination.

The test statistics we employ allow us to discriminate among potential reasons why the basic martingale tax model may fail. This is important because the standard model that yields the martingale hypothesis for tax rates should be regarded as an approximation of a more sophisticated underlying model of dynamic taxation. A more sophisticated underlying model might not have exactly the martingale property but would still be consistent with tax smoothing, and thus should still conclude that optimal tax rates are well-predicted by current levels. Thus, a finding that taxes are not exactly a martingale need not imply that the data are inconsistent with the essential elements of the positive theory of government finance.

For example, a critical assumption used to derive the martingale property is that either a single infinitely-lived government is able to tie its own hands and enforce a fully state-contingent tax-policy rule set at the beginning of time, or that sequences of governments are able to bind their successors by means of an unspecified commitment technology. Time-consistent solutions typically do not imply an exact martingale in tax rates. Interestingly, Poterba and Rotemberg (1988) examine international aggregate tax data and find qualified evidence against the commitment solution and in favor of the time-consistent solution. However, Judd (1989) emphasizes that capital and labor should be taxed differently in the optimum. Upon disaggregating taxes appropriately, he finds support for optimality (and hence, commitment).

In a significant enhancement of the basic Barro (1979) model, Lucas and Stokey (1983) demonstrate that the optimal tax policy can be time-consistent. Although their model shares central components with the simpler martingale model, the Lucas–Stokey environment is more general than Barro's in two
ways. First, there is no linear-quadratic structure. Second, Lucas-Stokey allows for fully state-contingent government debt. If bonds are permitted to play an insurance role, then in an optimum there is a single shadow price of government revenue across states of nature. Tax rates in any period are consequently governed by conventional optimal-tax rules concerning the current elasticity of the supply of labor. If this elasticity were constant across all states, then the optimal tax rate would also be constant. Barro's martingale theory of the evolution of taxes, however, is derived in the absence of state-contingent debt. With perfectly safe bonds, the government cannot shift revenue across states without changing taxes, and so the shadow price of revenue is not constant. For this reason, according to the martingale hypothesis, tax rates might change even though the current elasticity of the supply of labor has not changed. Thus, while the Lucas-Stokey model does not imply that tax movements exactly obey a martingale, it does imply some smoothing of taxes.

Given that these extensions of the Barro model entail substantial smoothing of taxes, it is not surprising that regression tests of the tax-rate martingale hypothesis by Barro (1981), Kingston (1987), and Mankiw (1987) have found that the evolution of United States tax rates over time is roughly consistent with the martingale property. Important evidence contrary to the martingale hypothesis was documented by Sahasakul (1986), who found that movements in tax rates were predictably related to wars and recessions. However, it is not clear that Sahasakul's results constitute significant evidence against the idea that tax rates are approximately dynamically optimal, since wars and large business-cycle movements are precisely the circumstances under which the linear-quadratic specification of the martingale approximation is likely to be the poorest. In this vein, the labor-supply elasticity might not be constant in wartime.

The results we present may be given an interpretation that distinguishes this paper from previous work. The empirical evidence clearly rejects the martingale hypothesis and instead points to a political business cycle: deviations from the null hypothesis of a martingale occur along certain electoral frequencies. The behavior of the sample spectral density of first-differences at the \( \pi/4 \) frequency (an eight-year cycle) significantly departs from the hypothesized spectral density of white noise at that frequency. The calendar dates of these deviations correspond to the two years prior to a successful

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1Elsewhere, we question the strict logical correctness of the martingale property in the linear-quadratic framework. In particular, the martingale convergence theorem implies that a martingale in tax rates requires either that tax rates are asymptotically constant or that the asymptotic variance of a tax-rate series is infinite. Since the latter possibility entails a positive probability that either the private sector's or public sector's budget constraints are violated, a researcher would reject the presence of a nondegenerate martingale almost surely with a sufficiently large sample. See Bizer and Durlauf (1990).
re-election bid for the presidency, and regression analysis reveals that taxes are regularly lowered at these points in time. This departure from the martingale null appears to be inconsistent with the more sophisticated versions of the basic model. In particular, because the Lucas–Stokey model suggests that tax changes only occur when government spending requirements alter the current elasticity of the supply of labor, and because there seems to be little reason to believe that there is pronounced movement in the labor-supply elasticity at a $\pi/4$ frequency, our results are difficult to reconcile fully with their theory. Instead, we interpret the findings as evidence that the positive theory of government finance neglects an important determinant of tax-rate changes; namely, it abstracts from the political process that results in changes in government. Recent work by Cukierman and Meltzer (1986), Rogoff and Sibert (1988), and Rogoff (1990) suggesting that tax reductions occur prior to incumbent re-elections is correspondingly given substantial support by the evidence. Our conclusion is that attempts to model government policy as arising out of the decisions of some unchanging, infinitely-lived social planner inherently neglect fundamental determinants of tax-rate decisions. The evidence indicates a need to refine the positive theory of government finance to incorporate features of political equilibrium into the analysis.

The paper is organized as follows. Section 2 briefly describes the optimal tax model and discusses frequency-domain tests of the martingale theory of tax fluctuations. These tests largely reject the martingale hypothesis. In order to more closely pursue an interpretation for these rejections, section 3 performs regression analysis on the tax-rate time series. These tests again reject the model and strongly indicate that political factors have significant explanatory power for tax changes. Section 4 summarizes and concludes.

2. Testable implications of the martingale hypothesis

The martingale property of optimal tax rates typically derives from an optimal-control problem comparable to solving at each $t$:

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[ -a_0 \tau_{t+j} - \frac{a_1}{2} \tau_{t+j}^2 \right], \quad 0 < \beta < 1,$$

by choosing $\{\tau_t, B_{t+1} \}_{t=0}^{\infty}$. Maximization is performed subject to the budget constraint:

$$B_{t+1} = R[B_t + g_t - \tau_t], \quad R > 1,$$

$$B_t \text{ bounded, for all } t.$$

The expectations operator is present because the sequence $\{g_t\}_{t=0}^{\infty}$ is an
exogenous stochastic process describing the ratio of government spending to GNP. \( \tau_t \) is the tax collected as a percentage of GNP, and may be interpreted as an average tax rate. \( B_t \) is the value of a one-period bond to be repaid in period \( t \), also as a percentage of GNP, and \( R \) is the relative price of tomorrow's output. Notice that \( B_t \) is not indexed by states of nature, so that all debt is risk-free. This assumption seems reasonable in the context of a study of U.S. tax rates, because U.S. government bonds are similarly not fully state-contingent. The constraint that \( B_t \) is bounded is necessary to impose a requirement that the government cannot increase the debt to GNP ratio without limit into the infinite future. The objective function is identical to that employed by Barro (1981) and others, and may be justified by models in public finance where deadweight loss is proportional to the square of the tax rate. The Euler equation that follows from this maximization problem implies:

\[
E_t(\tau_{t+1}) = \alpha(a_0, a_1, [\beta R]^{-1}) + [\beta R]^{-1} \tau_t. \tag{3}
\]

In this formulation, when \([\beta R]^{-1} = 1\), it can be shown that \(\alpha(a_0, a_1, [\beta R]^{-1}) = 0\), and hence that tax rates obey a martingale. Condition (3) can be regarded as the null hypothesis of the dynamic optimal taxation problem as it has been traditionally treated.

We now examine the properties of the tax-rate series to see whether the martingale hypothesis is consistent with the data. An implication of our empirical exercises is that evidence in the empirical literature in favor of the hypothesis may have been generated by examining a narrow subset of the potential alternative hypotheses.

Our empirical work concentrates on frequency-domain-based hypothesis tests applied to the history of tax changes. While there is a mapping from the time-domain autocorrelations to the spectral density, frequency-domain techniques permit a more straightforward decomposition of a time series into long- versus short-run components. We find this decomposition particularly instructive because there are alternative hypotheses to the underlying model for which one-step-ahead tax rates are likely to be well-predicted by current levels. Indeed, any model that implies substantial smoothing of taxes will have this result. The likely violations of the martingale properties might then be associated with longer-run mean reversion, and regression analysis that presents evidence that short-run autocorrelations are insignificantly different from zero will not be a powerful test of the null hypothesis. Moreover, the frequency-domain analysis permits us to search for this reversion without asserting any prior knowledge of which autocorrelations are important, as is necessary in standard regression analyses. In searching for departures from
the martingale specification over the entire range of frequencies, we are able to locate deviations without presuming to know the specific frequencies at which these deviations occur. While our conclusions do not uniformly indicate rejection, they do suggest that previous time-domain empirical tests have overlooked violations because of an exclusive focus on low-order autocorrelations.

We take as the null hypothesis that the series is a nondegenerate martingale, and then attempt to identify where the null hypothesis fails. It is straightforward to interpret this null as placing restrictions on values of the spectral density of tax changes. For example, an obvious location to search for a violation is the value of the spectral density of the differenced series at the zero frequency. Under the null hypothesis of a martingale, the value of the spectral density of the differenced series at the zero frequency should be one (appropriately normalized). Thus, a straightforward first check of the martingale hypothesis comes from an examination of the value of the spectral density at the zero frequency. Under the alternative hypothesis that the tax series is stationary, the spectral density of the differenced series should be zero at the zero frequency.

More generally, the martingale theory imposes restrictions at every frequency. Consequently, we develop tests that capture deviations of the entire spectral density from its hypothesized functional form. If the tax-rate series is a martingale, then the value of the spectral density of first-differences should be a constant across all frequencies. The cumulated periodogram tests we perform are well-suited for this question because they explore the implications of deviations from the null hypothesis by implicitly averaging over the deviations at all frequencies. These tests are general goodness-of-fit procedures which are consistent against all deviations from the null. Periodogram tests are thus more appropriate as a measure of the utility of the martingale approximation than a low-order autocorrelation test since all implications of the null are simultaneously explored. The periodogram tests we perform predominantly reject the martingale null.

The first set of results in this section are complementary to those of Sahasakul (1986) who found that there is some predictability of tax changes from wars and recessions. Our nonparametric approach shows that the deviations from the null hypothesis found by Sahasakul are also embedded in the history of tax changes. However, the violations that we discover are different in kind than those located by Sahasakul, and are attributable to political factors that recur with regular, long periodicity.

The decomposition of a time series into long- and short-run components is best expressed in the frequency domain. By the Cramér Representation Theorem, any stationary time series can be expressed as the sum of randomly weighted sinusoidal terms $\cos(\omega t)$ and $\sin(\omega t)$, $\omega \in [-\pi, \pi]$, where the variances of the different weights determine the contributions of the different
sinusoids to the variance of the entire time series. Values of $\omega$ near zero identify terms which contribute to the long-run characteristics of the series whereas values near $-\pi$ or $\pi$ represent the short-run components of the time series. To see this, notice that at low frequencies, the cosine wave $\cos(\omega t)$ varies slowly with $t$. Thus, at low values of $\omega$, only large differences in $t$ will yield changes in the contribution of the term to the time series. To say that a time series is white noise is to say that the long- and short-run components of the time series contribute equally to the variance, i.e., that the spectral density is a rectangle over the interval $[-\pi, \pi]$. This interpretation is consistent with the representation of the spectral density as the Fourier transform of the autocovariance function, $\sigma_{\Delta \tau}(j)$:

$$f_{\Delta \tau}(\omega) = (2\pi)^{-1} \sum_{j=-\infty}^{\infty} \sigma_{\Delta \tau}(j)e^{-ij\omega},$$

$$= \frac{\sigma^2_{\Delta \tau}}{2\pi} \quad \text{for white noise \(\Delta \tau_j\).} \quad \text{(4)}$$

Notice that long-run mean reversion will manifest itself as a deficiency of spectral power, relative to a rectangle, in the lower frequencies.

Testing for the martingale property of taxes is equivalent to testing for the whiteness of the tax rate changes, $\Delta \tau_j$. We will examine the sample spectral density of the time series of tax rate changes. Sample spectral density estimates are normally based upon the periodogram, $I_\tau(\omega)$:

$$I_\tau(\omega) = (2\pi)^{-1} \sum_{j=-T+1}^{T-1} \hat{\sigma}_{\Delta \tau}(j)e^{-ij\omega},$$

where

$$\hat{\sigma}_{\Delta \tau}(j) = T^{-1} \sum_{t=1}^{T-j} \Delta \tau_t \Delta \tau_{t+j}. \quad \text{(6)}$$

Our empirical analysis is based upon an annual time series extending from 1879 to 1986 on total nominal tax receipts collected by the Federal government divided by nominal GNP. Tax receipts include all revenues collected by the Federal government minus transfers from the Federal Reserve to the U.S. Treasury. This series is an updated version of that used in Barro (1981).

Fig. 1 presents the periodogram of the tax rate changes. Superimposed on the periodogram is the theoretical spectral density under the null hypothesis. The periodograms reflect evidence of departures from the rectangular null, especially around the frequency $\pi/4$. In addition, there appears to be some

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concentration of power around the lower half of the frequencies. It is difficult to see any other significant departures from white noise.

Fig. 2 contrasts the periodograms for tax changes pre- and post-World War II. The pre-war periodogram exhibits substantially more pronounced peaks and troughs than the post-war periodogram. In addition, the pre-war periodogram is relatively more concentrated in the low frequencies. The casual evidence indicates that the pre-war data are primarily responsible for violations of the null.

Fig. 2
In order to formalize the idea that the sample spectral density should be shaped like a rectangle, we need to develop some properties of the periodogram when treated as a random function. A standard result in spectral analysis shows that the periodogram is an inconsistent estimate of the spectral density at individual frequencies. However, our present concern is not with individual frequencies but rather with the entire spectral density. Under the null hypothesis that the time series is white noise, the properties of the periodogram as a whole may be exploited even when the individual frequencies cannot be estimated consistently. This leads us to consider behavior of the sample spectral distribution function. We consider the periodogram-based estimate of the spectral distribution function, when normalized so as to define a function on the interval \([0, 1]\):

\[
F_T(t) = \int_0^t I_T(\mu) \, d\mu, \quad t \in [0, 1].
\]  

(7)

As the integral of a rectangle is a diagonal line, all testable implications of the null hypothesis may be re-expressed by the requirement that the sample spectral distribution function is approximately shaped as a diagonal.

Our analysis tests the shape of the complete spectral distribution function. A justification for our hypothesis tests of spectral shape is in Durlauf (1990a). These tests examine the behavior of the deviations of the sample spectral distribution from the null hypothesis diagonal. To do this, we need to employ a Brownian Bridge, denoted as \(U(t)\) for \(t \in [0, 1]\). The Brownian Bridge is a tied-down Brownian Motion process (i.e., its endpoints are zero). Its widest use comes in the analysis of empirical distribution functions. We employ the following Theorem which is valid for any process fulfilling the Hannan and Heyde (1972) conditions for asymptotic normality of sample autocorrelations.

**Theorem.** If \(\Delta_t\) is a uniformly bounded martingale difference sequence, then

(\(a\)) \(U_T(t) = \sqrt{2} T^{1/2} \int_0^\mu \left( \frac{I_T(\mu)}{\widehat{\sigma}^2_{\Delta_t}} - \frac{1}{2\pi} \right) d\mu \Rightarrow U(t)\) on \(t \in [0, 1]\);

(\(b\)) \(CVM_T = \int_0^1 U_T(t) \, dt \Rightarrow \text{Cramér-von Mises statistic},\)

\(KS_T = \sup_{[0, 1]} |U_T(t)| \Rightarrow \text{Kolmogorov-Smirnov statistic},\)

\(3\)See Shorack and Wellner (1987) for discussion of the Brownian Bridge.

\(4\)The specific conditions developed by Hannan and Heyde are technical and are omitted. The conditions permit a great deal of heterogeneity in the data-generating process, and impose weak restrictions on the tax-rate series. For example, conditional heteroskedasticity is allowed.
The positive theory of government finance

\[ K_T = \sup_{0 \leq s, t \leq 1} |U_T(t) - U_T(s)| \Rightarrow \text{Kuiper statistic,} \]

\[ R_T = \sup_{0 < \alpha < 1} \left| \frac{U_T(t)}{t} \right| \Rightarrow \text{Renyi statistic;} \]

(c) \( CVM_T, KS_T, K_T, \) and \( R_T \) all diverge if \( \Lambda_{\tau_i} \) is any other MA process satisfying the conditions of the Theorem.

**Proof.** Durlauf (1990a).

The idea of the Theorem is as follows. Under the null hypothesis, the cumulated periodogram of the time series should approximate a diagonal. If one normalizes the periodogram by the estimated variance, the cumulated deviations must sum to zero since the integral of the periodogram must equal the sample variance. A particular normalization of the deviations, under the martingale null, converges to a Brownian Bridge. The consistency of the test occurs because under the null, these deviations will be bounded almost surely, whereas under the alternative they will explode, because of the \( T^{1/2} \) term which blows up the deviations of the cumulated periodogram from the null hypothesis.

Testing the shape of a random function is less straightforward than testing the value of a scalar random variable. The four different statistics in the Theorem provide various metrics for this shape relative to the martingale null. One advantage of these tests is that asymptotically they possess power 1 against all fixed MA alternatives, unlike time-domain tests which examine a subset of the autocorrelations and correspondingly may be inconsistent.

Table 1 gives the results of calculating the four statistics on the annual tax data from 1879 to 1986. We focus first on the entire sample period, and then include values for the test statistics applied to pre- and post-war subsamples.

By these metrics, one overall rejects the white-noise null hypothesis. Three of the four statistics consistently reject for the entire sample period. For the two subsamples, the evidence is apparently stronger against the null for the pre-war rather than the post-war period. For the post-war data only one of the four statistics, the Cramér–von Mises statistic, is inconsistent with the null hypothesis at the asymptotic significance level. From inspection of the periodogram in fig. 1, the source of the rejections for the entire sample appears to be the concentration of large values of the periodogram in the low

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5 The finite-sample properties of the periodogram-shape tests are relatively unknown, but simulation evidence in Bernard (1989) and Durlauf (1990a,b) suggests that the statistics will not lead to excessive (relative to 5%) rejections of the null.
Table 1
Periodogram-based tests of model noise.\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>(CVM_T)</th>
<th>(KS_T)</th>
<th>(K_T)</th>
<th>(R_T)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1879–1986)</td>
<td>0.81(^c)</td>
<td>1.59(^c)</td>
<td>1.65</td>
<td>11.2(^c)</td>
<td></td>
</tr>
<tr>
<td>Pre-war</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1879–1945)</td>
<td>0.95(^c)</td>
<td>1.63(^c)</td>
<td>1.65</td>
<td>11.3(^c)</td>
<td></td>
</tr>
<tr>
<td>Post-war</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1946–1986)</td>
<td>0.65(^c)</td>
<td>1.33</td>
<td>1.34</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>5% critical value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.46</td>
<td>1.36</td>
<td>1.75</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>0.74</td>
<td>1.64</td>
<td>2.01</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>0.1%</td>
<td>1.16</td>
<td>1.96</td>
<td>2.32</td>
<td>10.5</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Sources: Shorack and Wellner (1987) and Owen (1962). \(CVM_T\) denotes values of the Cramér-von Mises statistic, \(KS_T\) denotes values of the Kolmogorov-Smirnov statistic, \(K_T\) denotes values of the Kuiper statistic, and \(R_T\) denotes values of the Renyi statistic.

\(^b\)Computed for \(\alpha = 0.1\).

\(^c\)Significant at asymptotic 5% level.

If we think of the cumulated periodogram deviations as the realization of a Brownian Bridge, then the volatility and height of the periodogram should not differ between the intervals \([0, \pi/2]\) and \([\pi/2, \pi]\). The estimates of \(CVM_T\) are especially sensitive to this imbalance.

The acceptance of the null hypothesis by the Kuiper statistic is not surprising, given the nature of deviations of the cumulated periodogram from a diagonal line, as illustrated in fig. 3. The deviations are almost never negative, which means that the deviation between the maximum and minimum of the series is essentially equal to the maximum. Therefore, \(KS_T\) and \(K_T\) take on similar values with different significance levels. The Kuiper statistic has lower power in relation to the other statistics against the alternative of spectral density concentration in the low frequencies. As noted above, the periodogram reveals some concentration of power around the frequency \(\pi/4\). Interestingly, this is the frequency which corresponds to a cyclical component to the time series with period of eight years. Eight years is

\(^b\)Following Working (1960), if taxes follow a continuous-time random walk, then the first-difference of a time-averaged tax series is not white noise; rather, it obeys an MA(1) with coefficient 0.25. To correct for any influence time aggregation might have on our results, we recalculated the test statistics in table 1 using an appropriately filtered periodogram. The test statistics computed in table 1 using the filtered periodogram become insignificantly different from zero. However, this may be explained by the tendency of the filter to eradicate spectral power in the low frequencies. The filter consequently tends to reduce testing power in the spectral shape tests against alternatives concentrated in lower frequencies, which is precisely where violations of the null are likely to be important. Other filters that reduce spectral power in the low frequencies also exhibit this phenomenon [e.g., if one filters the data assuming the process is an AR(1) with coefficient 0.25, periodogram tests will again accept the null]. Further testing rejects a time-aggregated null hypothesis. First, 7 of the autocorrelations between 2 and 30 are statistically significant, which is inconsistent with a time-aggregated null. Second, spectral density estimates derived from the filtered periodogram verify that there is too much spectral power in frequencies around \(\pi/4\) to be consistent with the time-aggregated null.
the most common length of time for continuous political-party control of the presidency, and therefore is a natural frequency to use in examining the alternative hypothesis that political parties exert influence on the intertemporal tax-smoothing calculus. Similarly, \( \pi/2 \) corresponds to the frequency of presidential elections, and thus is also of interest. Of course, we are also interested in mean reversion, so that the zero frequency is of interest. Our second set of tests therefore examines these 'electoral' frequencies and the zero frequency.

In order to construct test statistics based upon the spectral density at particular values, it is customary to use a window estimator in place of the (inconsistent) periodogram estimate. For this purpose, we employ a Bartlett window, which modifies the periodogram by weighting high-order autocovariances towards zero. The procedure, described in Priestley (1981), is to define weights of the form:

\[
\lambda(j) = 1 - |j|/M, \quad |j| \leq M, \\
= 0, \quad \text{otherwise},
\]

where \( M \) is an increasing function of \( T \). Multiplying the autocovariances by \( \lambda(j) \) produces a spectral density estimate \( B_T(M, \omega) \):

\[
B_T(M, \omega) = (2\pi)^{-1} \sum_{j = -M}^{M} \lambda(j) \hat{\sigma}_3(j)e^{-i\omega j}.
\]

If \( M/T \to 0 \), these modified estimates will be consistent under the null
hypothesis, and when normalized are asymptotically normally distributed. Fig. 4 illustrates various Bartlett estimates of the spectral density.

If taxes are stationary about some mean, then the spectral density of the first-difference equals zero at the zero frequency. Under the martingale null, $2\pi$ times the value of the spectral density at the zero frequency will equal one. Cochrane (1988) and Lo and MacKinlay (1988) among others have interpreted zero-frequency values below one as evidence of long-run mean reversion. From the perspective of the utility of the martingale approximation, one would expect this test to reject the null.

Table 2 surprisingly reveals little evidence of mean reversion. Even at 60 lags, there is no evidence against the null hypothesis of a martingale. The basic difficulty apparently is that the data series is too short to yield useful information on long-run mean reversion. This problem is endemic to many empirical questions, as discussed in Cochrane (1988). Unfortunately, the confidence intervals of the test statistic at the zero frequency are so large that we cannot reject the martingale null at this frequency.

Our next set of tests explores the behavior of the spectral density at electoral cycles. Table 3 reports the estimates of the spectral density at the $\pi/2$ and $\pi/4$ frequencies. The table has two noteworthy features. First, we find no evidence that the spectral density at the $\pi/2$ frequency deviates from a value of one. In the language of political business cycles, we do not find a

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7In reporting estimates of the spectral density at selected frequencies, we use the normalized Bartlett estimates [i.e., $B_r(M, \omega)$ is calculated using sample autocorrelations]. We have also multiplied the estimates by $2\pi$ for ease of interpretation.
D.S. Bizer and S.N. Durlauf, The positive theory of government finance

Table 2
Bartlett estimates at zero frequency.\(^a\)

<table>
<thead>
<tr>
<th>Window size ((M))</th>
<th>(2\pi B_T(M, 0))</th>
<th>(H_0: 2\pi f(0) = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.38</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.47</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1.57</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1.72</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.83</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>(0.79)</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.87</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>(0.86)</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Standard errors are in parentheses. Column 2 reports the Bartlett Window estimates at the zero frequency and column 3 reports the \(t\) statistic associated with \(H_0\).

Table 3
Bartlett estimates at 'electoral' frequencies.\(^a\)

<table>
<thead>
<tr>
<th>Window size ((M))</th>
<th>(2\pi B_T(M, \pi/4))</th>
<th>(H_0: 2\pi f(\pi/4) = 1)</th>
<th>(2\pi B_T(M, \pi/2))</th>
<th>(H_0: 2\pi f(\pi/2) = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.66</td>
<td>2.64(^b)</td>
<td>0.86</td>
<td>-0.56</td>
</tr>
<tr>
<td></td>
<td>(0.25)</td>
<td>(0.25)</td>
<td>(0.25)</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2.25</td>
<td>3.57(^b)</td>
<td>0.73</td>
<td>-0.77</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.35)</td>
<td>(0.35)</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2.85</td>
<td>4.30(^b)</td>
<td>0.50</td>
<td>-1.16</td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td>(0.43)</td>
<td>(0.43)</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>3.34</td>
<td>4.68(^b)</td>
<td>0.42</td>
<td>-1.16</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.50)</td>
<td>(0.50)</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>3.60</td>
<td>4.64(^b)</td>
<td>0.39</td>
<td>-1.09</td>
</tr>
<tr>
<td></td>
<td>(0.56)</td>
<td>(0.56)</td>
<td>(0.56)</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>3.78</td>
<td>4.56(^b)</td>
<td>0.37</td>
<td>-1.03</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.61)</td>
<td>(0.61)</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Standard errors are in parentheses. Columns 2 and 4 report the Bartlett Window estimates at the \(\pi/4\) and \(\pi/2\) frequencies, respectively. Columns 3 and 5 report the \(t\)-statistics associated with \(H_0\).

\(^b\)Significant at asymptotic 5\% level.
systematic relationship between tax changes and elections. However, at the \( \pi/4 \) frequency, we find strong evidence that the null hypothesis of a martingale is uniformly rejected.\(^8\) This result may be tentatively interpreted as saying that there is a tendency towards mean reversion across political-party administrations. This result makes intuitive sense since, historically, presidential administrations are most active in the early years of the first term and these parties typically remain in office for an eight-year tenure. Since the beginning of the sample, there have been twenty-eight elections, but only twelve changes in party control, implying an average political-party change roughly every other election. Indeed, with the exception of the Carter presidency, since World War II every administration has been a two-term presidency.

3. Regression analysis

The higher concentration of tax changes at a frequency of eight years provocatively suggests that political events account for significant variability in tax rates. To investigate this possibility more closely, we attempt to identify the source of these tax changes. Unfortunately, spectral analysis is not capable of identifying these linkages because while it reveals that there is pronounced activity every eight years, it does not reveal which eight-year intervals are critical. Regression analysis is more properly suited for this task. Thus, our final examination of the behavior of tax rates consists of identifying in calendar time the eight-year cycle responsible for the spike in the spectral density at \( \pi/4 \), and then ascertaining whether these dates have a political interpretation.

Table 4 presents the results of a regression of tax changes on different eight-year cyclical dummy variables. Each dummy variable is given a value of one every eight years: the variables are distinguished by the consecutive calendar dates in which they turn ‘on’. Thus, \( D1 \) attains a value of one beginning at the start of the sample, 1879, 1887,\ldots, 1983. \( D2 \) attains a value of one in 1880, 1888,\ldots, and \( D8 \) attains a value of one in 1886, 1894,\ldots, 1982. Entering each of the eight dummy variables separately, we run regressions of the form:

\[
\Delta t_i = C + \beta_1 GF_{t-1} + \beta_2 R_{t-1} + \beta_3 D_{i} + \varepsilon_t, \quad i = 1, \ldots, 8. \quad (10)
\]

The regressors include a constant, a government-spending variable \([GF\) is total federal government expenditures less transfers from the Federal Reserve divided by nominal GNP, as used in Barro (1981)], and a recession

\(^8\)The spectral density at \( \pi/4 \) is also statistically significant against the temporally aggregated null that tax changes obey \( \Delta t_i = \varepsilon_i + 0.25 \varepsilon_{i-1} \) for lag lengths 20–60.
Table 4
Eq. (10) regression estimates of tax cycles.a

<table>
<thead>
<tr>
<th>Dummy variable</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D1$</td>
<td>0.007</td>
<td>-0.004</td>
<td>0.006</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$D2$</td>
<td>0.009</td>
<td>-0.004</td>
<td>-0.002</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$D3$</td>
<td>0.008</td>
<td>-0.004</td>
<td>0.001</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>$D4$</td>
<td>0.009</td>
<td>-0.004</td>
<td>-0.008</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$D5$</td>
<td>0.008</td>
<td>-0.004</td>
<td>-0.002</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$D6$</td>
<td>0.008</td>
<td>-0.004</td>
<td>-0.003</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$D7$</td>
<td>0.009</td>
<td>-0.004</td>
<td>0.003</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>$D8$</td>
<td>0.009</td>
<td>-0.003</td>
<td>0.005</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
</tr>
</tbody>
</table>

= 1 two years prior to any election 0.008 -0.004 -0.007 0.55
= 0 otherwise (0.011) (0.002) (0.003)

= 1 two years prior to a successful party re-election 0.009 -0.004 -0.007 0.02
= 0 otherwise (0.011) (0.002) (0.003)

= 1 two years prior to any change in presidential party 0.009 -0.004 -0.004 0.23
= 0 otherwise (0.011) (0.002) (0.003)

aStandard errors in parentheses. $\beta_1$, $\beta_2$, and $\beta_3$ are the estimated coefficients on lagged government spending, the lagged recession indicator, and the cyclical dummy variable, respectively. The last column is the marginal significance level of the t-statistic for exclusion of the dummy variable. $D1$–$D8$ are given a value of one every eight years: the variables are distinguished by the consecutive calendar dates in which they turn "on". Thus, $D1$ attains a value of one beginning at the start of the sample, 1879,1887, ..., 1983; $D2$ attains a value of one in 1880,1888, ... ,1984, ... ; and $D8$ attains a value of one in 1886,1894, ... ,1982.

dummy (R is one if there was a recession that year according to NBER business-cycle dates). These variables are designed to control for violations of the null hypothesis associated with the eight-year cycle. According to the null hypothesis, the error term of this regression is a function of the contemporaneous shock to the government-spending ratio variable. Thus, the error term can deviate from its mean from either a contemporaneous shock to the level

Sahasakul (1986) found a temporary-government-spending variable and business-cycle movements to be significant in a test of the optimal-tax hypothesis.
of government spending or from a contemporaneous shock to nominal GNP, so that both \( GF \) and \( R \) must be lagged in the regression.\(^{10}\)

Table 4 reveals that only \( D1 \) and \( D4 \) are significant at values near the five percent level. These variables are prominent in two respects: they coincide with wartime build-ups and they often coincide with the year or two preceding a presidential re-election bid. The second possibility is consistent with models of electoral competition that conclude that taxes should be manipulated prior to re-election by incumbents in order to signal competence [Cukierman and Meltzer (1986), Rogoff and Sibert (1988), and Rogoff (1990)]. In order to investigate this link more directly, we run regressions on three dummy variables created to reflect explicitly electorally significant dates. The first variable is defined as one for dates two years prior to any election, and zero otherwise. The second is defined as one for dates two years prior to a successful re-election bid (where success is defined as maintaining party control of the presidency), and is zero otherwise. The third variable is defined as one for dates two years prior to any change in political-party control of the presidency (i.e., an unsuccessful re-election bid). One's prior expectations for the political dummies must be low since these variables restrict attention to the two-year lead on a re-election bid. In doing so, the variables impose greater temporal structure on the political influence on tax rates than models of electoral competition generally assert. Put another way, models of electoral competition could be an important driving force in tax-rate changes even if tax-rate changes occur sometimes at one-year leads prior to a re-election bid, and other times at two-year leads. Nevertheless, the result for the indicator on successful re-election bids is striking. The coefficient is significant at the 2 percent level and has the negative sign predicted by the electoral-competition literature: tax burdens are reduced prior to successful re-election bids. The other two political dummy variables are statistically insignificant.\(^{11}\) The evidence points to a significant relationship between tax reduction and re-election success. This suggests that politicians that lower taxes are better able to retain office, but at the same time leads to the question of why all politicians do not try to reduce taxes. One interpretation is that politicians are not solely interested in being re-elected but rather have broader objectives that sometimes conflict with re-election success.

4. Conclusions

This paper has been designed to study the claim in the public-finance literature that tax rates follow a random walk. We have attempted to

\(^{10}\)See Sargent (1987).

\(^{11}\)We ran regressions that included our three political-indicator variables with different leads and lags. No other regressions yielded significant coefficients. Results are available upon request.
reconcile our evidence against this theory of government finance with the mixed empirical support that the martingale hypothesis has received in the literature. A reconciliation can be achieved by recognizing that previous tests restricted attention to short-run autocorrelations, whereas the null hypothesis imposes restrictions on all autocorrelations. Tests robust to violations at all frequencies reveal that the martingale approximation is not strong for the overall shape of the spectral distribution function (the presence of long-run mean reversion cannot, however, be confirmed). The approximation is especially poor at one of the electoral seasonals. This evidence points compellingly to the hypothesis that tax rates contain a predictable component corresponding to electoral events. Specifically, there appears to be a cyclical component to tax changes with a period of eight years, which roughly corresponds to changes in political-party administration. Regression analysis confirms that taxes are reduced two years prior to successful presidential re-election attempts.

In terms of future research, our evidence on behalf of an electoral seasonal in taxes suggests that the interaction of taxes with political considerations should receive greater scrutiny, and that the positive theory of tax-rate determination should be modified to incorporate important elements of political equilibrium. Government financial decisions are made by political regimes, and these regimes change over time. The regularity of this phenomenon induces a distinctive regularity in the time-series behavior of tax rates, despite the fact that tax rates appear otherwise quite smooth over time. A more careful theoretical delineation of the relationship between political competition, political change, and tax-rate determination would therefore seem valuable, and would lead to a more accurate description of the actual evolution of tax rates.

References