Performance Analysis of Wireless Link Operating in $\alpha$-$\mu$ Fading Channel

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Abstract—The $\alpha$-$\mu$ distribution is a generalized fading distribution which explores nonlinearities of the wireless propagation medium. This distribution includes the other distributions such as Gamma, Nakagami, exponential, Gaussian, Rayleigh etc. This makes $\alpha$-$\mu$ distribution very interesting. In this paper probability density function and performance metrics such as outage and bit error rate of $\alpha$-$\mu$ distribution is discussed with Monte-Carlo simulation results.

Keywords—$\alpha$-$\mu$ Distribution, Bit Error Rate (BER), Generalized Fading Channels, Outage Probability, Probability Density Function.

I. INTRODUCTION

There are many type of distributions which describe the mobile radio signal. In the recent past alpha-mu ($\alpha$-$\mu$) fading model [1] has been proposed considering two important phenomenon of radio propagation non-linearity and clustering. The $\alpha$-$\mu$ represents a generalized fading distribution for small-scale variation of the fading signal in a non-line-of-sight fading condition. The $\alpha$-$\mu$ distribution is flexible, general and has easy mathematical tractability. In fact, the Generalized Gamma Distribution (GGD) also known as Stacy distribution [2, 24] has been renamed as $\alpha$-$\mu$ distribution, indicating the physical parameters involved. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely $\alpha$ and $\mu$. The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters.

In earlier works by indoor and outdoor field trial measurements [3], the autocorrelation and power spectrum functions of $\alpha$-$\mu$ distribution have been derived and validated whereas in [5] the probability density function has been obtained. The accurate approximations for the outage probability of equal gain receivers subject to arbitrary independent co-channel interferers are proposed in [4].

The exact expressions for the level crossing rate (LCR) and average fade duration (AFD) has been derived in [6] for multi-branch selection, equal-gain, and maximal-ratio combiners operating over independent non-identical $\alpha$-$\mu$ fading channels. Expressions for probability density function (PDF), cumulative distribution function (CDF) and moments for the product of two independent and non-identically distributed $\alpha$-$\mu$ variates have been derived in [7]. The joint probability density function and joint cumulative distribution function for multivariate $\alpha$-$\mu$ distribution has been derived in [8]-[11]. Based on moment estimators approach closed form approximations for the LCR of multi-branch equal-gain and maximal-ratio combiners operating on independent non-identically distributed Nakagami-$\mu$ fading channels has been derived in [12]. The exact expressions for LCR and AFD of equal-gain and maximal-ratio combiners have been derived in [13].

The average channel capacity for generalized fading scenarios are given in [14, 17]. Highly accurate closed-form approximations to PDF and CDF of the sum of i.i.d. $\alpha$-$\mu$ variates have been provided in [15]. Moment generating function (MGF) for the PDF characterizing of $\alpha$-$\mu$ fading channel has been derived in [16]. MGF is further used for evaluating BER. Reference [18, 19] derive the switching rate of a dual branch selection diversity combiner for generalized fading. The Shannon capacity of the $\alpha$-$\mu$ fading channel is derived in [21]. Performance analysis of dual selection combing diversity receiver over correlated $\alpha$-$\mu$ fading channels is presented in [22]. Reference [23] provides performance analysis of signal-to-interference ratio based selection combining diversity system over $\alpha$-$\mu$ fading distributed and correlated channels. Fading models for $\kappa$-$\mu$ distribution and $\eta$-$\mu$ distributions are presented in [25]. The performance of a dual-branch switched-and-stay combining diversity receiver, operating over correlated $\alpha$-$\mu$ fading channel is discussed in [26]. The performance of the system with dual selection combing over correlated Weibull channel in the presence of $\alpha$-$\mu$ distributed co-channel interference is studied in [27].
Capacity analysis of dual-hop wireless communication systems over $\alpha\mu$ fading channels is carried out in [28]. BER for i.i.d. $\alpha\mu$ fading channel with a maximal ratio combining receiver is carried out in [29]. Performance analysis of wireless communication over $\alpha\eta\mu$ fading channel has been investigated in [31], and outage, PDF and CDF of received signal to interference ratio has been derived. In reference [32] two new generalized fading distribution namely $\alpha\eta\mu$ and $\alpha\kappa\mu$ distributions have been discussed. The [33] explains how cognitive wireless networks fulfills the need of additional wireless spectrum. Deployment of multi-input multi-output antenna systems with orthogonal frequency division multiplexing over wireless channels has been identified in [34] as one of the most promising techniques for future wireless services.

The evolution of $\alpha\mu$ distribution can be traced in [35] where two-variate PDF, for correlated variates, each of which has marginal Gaussian distribution is explained. The physical basis for the GGD is discussed in [36]. In [37] authors have brought out that GGD is a flexible distribution and has exponential, gamma, and Weibull as subfamilies, and lognormal as limiting distribution. In flat fading environment channel estimation has been done in [38] using phase estimation of the transmitted signal. A framework based on Mellin-transform for deriving closed form expression for symbol error rate of $\alpha\mu$ fading channel for single branch and maximal ratio combining receivers have been presented in [39]. In [40] the problem of energy detection of an unknown deterministic signal over fading channel is revisited. Reference [41] presents the $\kappa\mu$ fading distribution, which is used for characterising the mobile radio propagation under severe fading conditions. Whereas in [42] natural generalization of the $\kappa\mu$ fading channel in which the line of sight component is subject to shadowing is investigated. A novel characterisation of fading experienced in body to body communication channels is carried out in [43] for fire and rescue personnel using the $\kappa\mu$ distribution.

Further, exact closed-form expression is derived for outage probability in $\eta\mu$ fading channels in [44]. Closed-form expressions for the averages of the Gaussian Q-function and product of two Gaussian Q-functions over the generalised $\eta\mu$ and $\kappa\mu$ distributions have been obtained in [45]. BER performance of switched diversity receivers is analyzed in [46] over $\kappa\mu$ and $\eta\mu$ fading channels using moment generation function based approach. Performance analysis of $\alpha\eta\mu$ fading channel is carried out in [47], when the communication is subjected to influence of co-channel interference. In [48], MATLAB based approach for mobile radio channels modelling for flat fading is presented. End-to-end performance of two-hops wireless communication systems with non regenerative relays over flat Rayleigh-fading channels is presented in [49]. An overview of the physical insight and the various performance metrics of fading channels is discussed in [50]. Unified analytical framework is presented in [51] to determine the exact average symbol-error rate of linearly modulated signals over generalized fading channels. Unified approach for evaluating the error rate performance of digital communication systems operating over a generalized fading channel is given in [52]. Reference [53] develops a novel generic framework for the capacity analysis of L-branch equal gain combining/maximal ratio combining over generalized fading channels.

II. THE ALPHA-MU FADING MODEL REVISITED

In the $\alpha\mu$ distribution, it is considered that a signal is composed of clusters of multipath waves propagating in a non-homogenous environment. In any one cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. It is assumed that the clusters of multipath waves have the scattered waves with identical powers. Thus, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components.
Fig. 1. Matching of analytical and simulated results of PDF of $\alpha$ for various $\alpha$ and $\mu$ (a) $\alpha=2$ and $\mu=2$, (b) $\alpha=2$ and $\mu=3$, (c) $\alpha=3$ and $\mu=2$, (d) $\alpha=3$ and $\mu=3$, (e) $\alpha=4$ and $\mu=2$, (f) $\alpha=4$ and $\mu=3$.
Assuming that the received signal at the $i^{th}$ branch ($i = 1, ..., M$) includes a certain number $n_i$ of multipath clusters, the resulting $\alpha$-$\mu$ envelope $R_i$ at the $i^{th}$ branch is written as

$$R_i^\alpha = \sum_{i=1}^{n} (X_{il}^2 + Y_{il}^2)^{\alpha/2}$$

(1)

Where $\alpha > 0$ is power parameter, $X_{il}$ and $Y_{il}$ are zero mean mutually independent Gaussian processes with identical variances i.e.

$$V(X_{il}) = V(Y_{il}) = \sigma_i^2 = \frac{\hat{r}^\alpha}{2n_i}$$

\[ \hat{r} = \alpha - \text{root mean value of } R_i^\alpha = \sqrt{\frac{\alpha}{\Gamma(\alpha)}} \]

With $E(\cdot)$ and $V(\cdot)$ are mean & variance operators respectively.

Thus, for a ($\alpha$-$\mu$) fading signal with envelope $R$, an arbitrary parameter ($\alpha > 0$), a $\alpha$-root mean value of $R^\alpha$ is given as

$$\hat{r} = \sqrt{2\mu\sigma^2}$$

The $\alpha$-$\mu$ probability density function (PDF), $f_R(r)$ of $R$ is given [1] as

$$f_R(r) = \frac{\alpha}{\Gamma(\alpha)\mu^{\alpha\mu}} r^{\alpha\mu-1} \exp\left[-\mu r^\alpha\right]$$

(2)

Where $\mu > 0$, is the inverse of the normalized variance of $R^\alpha$

The outage probability ($P_{out}$) of $\alpha$-$\mu$ is defined in [4] as the probability that the error rate exceeds a pre-defined value or equivalently, the received SNR drops below a pre-defined threshold ($\gamma_{thr}$).

$$P_{out} = \frac{\Gamma\left(\mu, \frac{\gamma_{thr}^{\alpha/2}}{\gamma}\right)}{\Gamma(\mu)}$$

(3)
The BER of $\alpha$-$\mu$ fading channel is derived in [16, (7)] using MGF approach.

III. SIMULATION RESULTS AND DISCUSSIONS

The PDF of fading envelope of $\alpha$-$\mu$ fading channel is simulated using MATLAB environment by choosing 5000 samples. The result presented in Fig.1 and Fig.2. are simulated and analytical results obtained by varying the values of $\alpha$ and $\mu$ as follows:

Case –I: Fig.1.(a), (c), (e); varying $\alpha$ from 2 to 4, and $\mu$ = 2.
Case –II: Fig.1.(b), (d), (f); varying $\alpha$ from 2 to 4, and $\mu$ = 3.
Case –III: Fig.2.(a), (c), (e); varying $\alpha$ from 2 to 4, and $\mu$ = 4.
Case –IV: Fig.2.(b), (d), (f); varying $\alpha$ and $\mu$ for special cases of Rayleigh, Exponential and Nakagami-$m$.

The simulated results matches with analytical curve. It is observed that by increase in $\alpha$ the base of the PDF curve shrinks which means variance decreases and vice versa. Whereas by increase in $\mu$ the curve becomes more peaky which means probability of mean increases.

Outage performance and BER for BPSK modulated $\alpha$-$\mu$ fading channel is shown in Fig. 3 and Fig.4 respectively. In these simulation 500000 samples have been considered for each combination of $\alpha$ and $\mu$. It is observed that for a given value of $\mu$, at zero dB SNR outage is same irrespective of $\alpha$ but outage decreases with increase in $\alpha$ at higher value of SNR. BER for $\alpha$-$\mu$ fading channel is shown in Fig.4. Here following cases of BER curves are illustrated:

Case –I: Varying $\alpha$ from 2 to 4, and keeping $\mu$ = 2. It is seen that at 7 dB SNR the curves crosses each other.
Case –II: Varying $\alpha$ from 2 to 4, and keeping $\mu$ = 3. It is seen that at 8 dB SNR the curves crosses each other.
Case –III: Varying $\alpha$ from 2 to 4, and keeping $\mu$ = 4. It is seen that at 9 dB SNR the curves crosses each other.

Let us call the SNR on these crossing points as critical SNR (i.e. $SNR_c$) for a given value of $\mu$. It is observed that if SNR of the channel is below $SNR_c$ then BER is proportional to $\alpha$ but for SNR greater than $SNR_c$ the BER is inversely proportional to $\alpha$. 

Fig. 4. Bit Error Rate of $\alpha$-$\mu$ fading channel

Fig. 5. BER of $\alpha$-$\mu$ fading channel $\alpha$ varied from 1 to 7, keeping $\mu$=1
In this correspondence, PDF, Outage and BER of $\alpha$-$\mu$ fading model have been discussed. The simulated and analytical results of performance metrics have been illustrated. The effect of $\alpha$ and $\mu$ parameters variation on BER and outage is discussed. The result obtained in this letter motivates researcher to explore more the $\alpha$-$\mu$ generalized fading model.

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REFERENCES


[32] Gustavo Fraidenraich, Michel Daoud Yacoub, “The $\alpha$-$\eta$-$\mu$ and $\alpha$-$\kappa$-$\mu$ Fading Distributions” IEEE 9th International Symposium on Spread Spectrum Techniques and Applications 2006.