Worst-Case Weighted Sum-Rate Maximization for MISO Downlink Systems with Imperfect Channel Knowledge

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Abstract—We consider the worst-case weighted sum-rate maximization (WSRMax) problem under imperfect channel state information (CSI) in multicell downlink multiple-input single-output systems. The problem is known to be NP-hard even for the perfect CSI case. We propose a solution method, based on semi-definite relaxation (SDR) and branch and bound technique, which solves globally the nonconvex robust WSRMax problem with an optimality certificate. Novel bounding technique based on SDR is proposed.

Index Terms—Branch and bound, weighted sum-rate maximization, semi-definite programming, multicell networks, robust design.

I. INTRODUCTION

We consider the problem of weighted sum-rate maximization (WSRMax) for multicell downlink systems with linear precoding. The base stations (BSs) are assumed to have multiple antennas while all the receivers are equipped with single antenna. Furthermore, we assume all channels between the BSs and the receivers are known imperfectly.

The WSRMax problem under the perfect channel state information (CSI) has been addressed in literature, e.g., [1]–[4]. However, in practice the BSs can never have perfect CSI, due to the imperfect channel estimation and limited feedback bandwidth. Such CSI errors at BSs can be modeled either by assuming that the channel errors are random and follow a certain statistical distribution [5], or by assuming that the CSI errors lie in a bounded uncertainty region [6]–[9]. In this paper, we consider bounded uncertainty region to model the imperfect CSI errors, and then design the beamformers to optimize the worst-case WSRMax problem. Note that the WSRMax problem is an NP-hard problem [10], even for the perfect CSI case.

The main contribution of the paper is to propose a solution method, based on semi-definite programming (SDP) and branch and bound (BB) technique, which solves globally the nonconvex worst-case WSRMax problem with channel uncertainties within a pre-defined accuracy $\epsilon$. It is worth noting that this paper extends our recent work [11] to multi-cell multiple-input single-output (MISO) systems.

The proposed BB algorithm computes a sequence of asymptotically tight upper and lower bounds for the maximum worst-case weighted sum-rate, and it terminates when the difference between the upper and lower bound is smaller than $\epsilon$. The efficiency of BB algorithm depends on the formulation of computationally efficient lower and upper bound functions [12]. In [4] the perfect CSI is considered, thus the feasibility of the bounding functions can be easily checked by a second order cone program. In this paper, we consider imperfect CSI in all relevant channels for the WSRMax problem, which makes the problem much difficult that than in [4]. After the submission of the first manuscript we found out that, similar problem has been addressed using a different analysis technique in [13].

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we describe the network model and the channel uncertainty model used throughout the paper, and then formulate the worst-case WSRMax problem.

A. Network Model

A multicell MISO downlink system, with $N$ BSs each equipped with $T$ transmit antennas is considered. The set of all BSs is denoted by $\mathcal{N}$. Single data stream is transmitted for each user, and we denote the set of all data streams in the system by $\mathcal{L}$. The transmitter node (i.e., the BS) of $l$th data
stream is denoted by tran(l) and the receiver node of lth data stream is denoted by rec(l). We have \( \mathcal{L} = \cup_{n \in \mathbb{N}} \mathcal{L}(n) \), where \( \mathcal{L}(n) \) denotes the set of data streams transmitted by nth BS (see Figure 1).

The antenna signal vector transmitted by nth BS is given by
\[
x_n = \sum_{l \in \mathcal{L}(n)} d_l w_l,
\]
where \( d_l \in \mathbb{C} \) represent the information symbol and \( w_l \in \mathbb{C}^T \) denotes the transmit beamformer associated to lth data stream, respectively. We assume that \( d_l \) is normalized such that \( E[|d_l|^2] = 1 \). Moreover, we assume that data streams are independent, i.e., \( E \{ d_l d_j^* \} = 0 \) for all \( l, j \in \mathcal{L} \), where \( l \neq j \).

The signal received at \( \text{rec}(l) \) can be expressed as
\[
y_l = d_l h_{jl}^H w_l + \sum_{j \in \mathcal{L}(\text{tran}(l)), j \neq l} d_j h_{jl}^H w_j + \sum_{n \in \mathbb{N} \setminus \{ \text{tran}(l) \}, j \in \mathcal{L}(n)} d_j h_{jl}^H w_j + n_l,
\]
where \( h_{jl} \in \mathbb{C}^T \) is the channel vector from \( \text{tran}(j) \) to \( \text{rec}(l) \), and \( n_l \sim \mathcal{CN}(0, \sigma_l^2) \) represents the circularly symmetric complex gaussian noise received at \( \text{rec}(l) \). Note that the second right hand term in (2) represents the intra-cell interference and the third right hand term represents the out-of-cell interference.

The received signal-to-interference-plus-noise ratio (SINR) at \( \text{rec}(l) \) is given by
\[
\Gamma_l = \frac{|h_{jl}^H w_l|^2}{\sigma_l^2 + \sum_{j \in \mathcal{L}(\text{tran}(l)), j \neq l} |h_{jl}^H w_j|^2 + \sum_{n \in \mathbb{N} \setminus \{ \text{tran}(l) \}, j \in \mathcal{L}(n)} |h_{jl}^H w_j|^2},
\]
(3)

**B. Channel Uncertainty Model**

We assume that all the channels are imperfectly known at the network controller, but they belong to a known compact sets of possible values. Specifically, we assume that the channel vectors belong to a known ellipsoidal uncertainty set.

We model the channel vector, \( h_{jl} \), from \( \text{tran}(j) \) to \( \text{rec}(l) \) as the sum of two components, i.e.,
\[
h_{jl} = \hat{h}_{jl} + e_{jl}, \quad j, l \in \mathcal{L},
\]
where \( \hat{h}_{jl} \in \mathbb{C}^T \) denotes the estimated value of the channel at the network controller and \( e_{jl} \in \mathbb{C}^T \) represents the corresponding channel estimation error. It is assumed that \( e_{jl} \) can take any value inside a \( T \)-dimensional complex ellipsoid, which is defined as
\[
\mathcal{E}_{jl} = \{ e_{jl} : |h_{jl}^H e_{jl}| \leq 1 \},
\]
where \( Q_{jl} \) is a complex Hermitian positive definite matrix, assumed to be known, which specifies the size and shape of the ellipsoid. For example, when \( Q_{jl} = (1/\xi_{jl}) I \), the ellipsoidal channel error model (5) reduces to a ball uncertainty region with uncertainty radius \( \xi_{jl} \) [14].

Now, with the channel uncertainty model (4), the received SINR of lth data stream (3) can be expressed as in (6).

**C. Problem Formulation**

Let \( \beta_l \) be an arbitrary nonnegative weight associated with data stream \( l \). We consider the case where all receivers are using single-user detection. Assuming that the power allocation is subject to a maximum power constraint \( \sum_{l \in \mathcal{L}(n)} |w_l|^2 \leq p_n^{\text{max}} \) for each BS \( n \in \mathcal{N} \), the problem of worst-case WSRMax can be expressed as
\[
\begin{align*}
\text{maximize} & \quad \min_{e_{jl} \in \mathcal{E}_{jl}, j \in \mathcal{L}} \sum_{l \in \mathcal{L}(n)} \beta_l \log(1 + \Gamma_l) \\
\text{subject to} & \quad \sum_{l \in \mathcal{L}(n)} |w_l|^2 \leq p_n^{\text{max}}, \quad n \in \mathcal{N},
\end{align*}
\]
(7)
with variables \( e_{jl} \) and \( w_l \) for all \( j, l \in \mathcal{L} \).

By noting \( \log(\cdot) \) is a monotonically increasing function and changing the sign of the objective function, we can equivalently reformulate problem (7) as
\[
\begin{align*}
\text{minimize} & \quad -\sum_{l \in \mathcal{L}} \beta_l \log(1 + \gamma_l) \\
\text{subject to} & \quad \gamma_l \leq \inf_{e_{jl} \in \mathcal{E}_{jl}, j \in \mathcal{L}} \Gamma_l, \quad l \in \mathcal{L}, \\
& \quad \sum_{l \in \mathcal{L}(n)} |w_l|^2 \leq p_n^{\text{max}}, \quad n \in \mathcal{N},
\end{align*}
\]
(8)
with variables \( \gamma_l \) and \( w_l \) for all \( j, l \in \mathcal{L} \).

**III. Branch and Bound Algorithm**

We start by equivalently reformulating problem (8) as minimization of a nonconvex function over an \( L \)-dimensional rectangle. Then we apply BB techniques [12] to minimize the nonconvex function over the \( L \)-dimensional rectangle.

Let us first define the objective function of problem (8) as
\[
\begin{align*}
f_0(\gamma) = & \sum_{l \in \mathcal{L}} -\beta_l \log(1 + \gamma_l) \\
\text{subject to} & \quad \gamma_l \leq \inf_{e_{jl} \in \mathcal{E}_{jl}, j \in \mathcal{L}} \Gamma_l, \quad l \in \mathcal{L}, \\
& \quad \sum_{l \in \mathcal{L}(n)} |w_l|^2 \leq p_n^{\text{max}}, \quad n \in \mathcal{N}
\end{align*}
\]
Then problem (8) can be expressed equivalently as
\[
\begin{align*}
\text{minimize} & \quad f_0(\gamma) \\
\text{subject to} & \quad \gamma \in \mathcal{G},
\end{align*}
\]
(9)
with variable \( \gamma \).

For clarity, let us define a new function \( \hat{f} : \mathbb{R}_+^L \to \mathbb{R} \) as
\[
\hat{f}(\gamma) = \begin{cases} f_0(\gamma) & \text{if } \gamma \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases}
\]
(11)
and note that for any $S \subseteq \mathbb{R}^2_+$ such that $G \subseteq S$, we have
\[
\inf_{\gamma \in S} \tilde{f}(\gamma) = \inf_{\gamma \in G} f_0(\gamma) = p^*,
\]
where $p^*$ is the optimal value of problem (10). Note that the first equality follows from the fact that for any $\gamma \in \mathbb{R}^2_+$ we have $f_0(\gamma) \leq 0$. It is also worth noting that the function $\tilde{f}$ is nonconvex over $S$ and $f_0$ is a global lower bound on $\tilde{f}$, i.e., $f_0(\gamma) \leq \tilde{f}(\gamma)$ for all $\gamma \in S$.

Let us define the $L$-dimensional rectangle $Q_{\text{init}}$ as
\[
Q_{\text{init}} = \left\{ \gamma \mid 0 \leq \gamma_l \leq \inf_{e_l \in \ell_l} \frac{\|\hat{h}_{ll} + e_{ll}\|^2_{P_{\text{trans}}(l)}}{\sigma^2_l}, \ l \in L \right\}.
\]
(13)

It is easy to check that $G \subseteq Q_{\text{init}}$. Furthermore, we can choose any $e_l$ inside the uncertainty region to obtain an upper bound for $\gamma_l$ in (13). Hence, without loss of generality we choose $e_l = 0$. Then the $L$-dimensional rectangle can be defined as
\[
Q_{\text{init}} = \left\{ \gamma \mid 0 \leq \gamma_l \leq \frac{\|\hat{h}_{ll}\|^2_{P_{\text{trans}}(l)}}{\sigma^2_l}, \ l \in L \right\}.
\]
(14)

Therefore, from (12), it follows that $\inf_{\gamma \in Q_{\text{init}}} \tilde{f}(\gamma) = p^*$. Thus, we have reformulated problem (10) equivalently as a minimization of the nonconvex function $\tilde{f}$ over the rectangle $Q_{\text{init}}$. To maintain a cohesive presentation, in the sequel, we review briefly the BB method introduced in [4] to minimize $\tilde{f}$ over $Q_{\text{init}}$.

For any $L$-dimensional rectangle $Q = \{\gamma | \gamma_{l,\min} \leq \gamma_l \leq \gamma_{l,\max}, l \in L\}$ such that $Q \subseteq Q_{\text{init}}$, let us define a function $\phi_{\text{min}}(Q)$ as
\[
\phi_{\text{min}}(Q) = \inf_{\gamma \in Q} \tilde{f}(\gamma).
\]
(15)

By using (12) and (15), it can be easily verified that $\phi_{\text{min}}(Q_{\text{init}}) = \inf_{\gamma \in Q_{\text{init}}} \tilde{f}(\gamma) = p^*$.

The key idea of the BB algorithm is to generate a sequence of asymptotically tight upper and lower bounds for $\phi_{\text{min}}(Q_{\text{init}})$. At each iteration $k$, the lower bound $L_k$ and the upper bound $U_k$ are updated by partitioning $Q_{\text{init}}$ into smaller rectangles. To ensure the convergence, the bounds should become tight as the number of rectangles in the partition of $Q_{\text{init}}$ grows. To do this, the BB uses two functions $\phi_{\text{ub}}(Q)$ and $\phi_{\text{lb}}(Q)$, defined for any rectangle $Q \subseteq Q_{\text{init}}$ such that 1) $\phi_{\text{ub}}(Q) \leq \phi_{\text{min}}(Q) \leq \phi_{\text{lb}}(Q)$ and 2) $\phi_{\text{ub}}(Q) - \phi_{\text{lb}}(Q) \rightarrow 0$ as the maximum half length of the sides of $Q$ (i.e., $\max_{l \in L} (\gamma_{l,\max} - \gamma_{l,\min})$) goes to zero [15, 16].

For clarity, we first summarize the generic BB algorithm, and the bounding functions are defined in Section IV.

\textbf{Algorithm 1: Branch and bound algorithm}

1) Initialization: given tolerance $\epsilon > 0$. Set $k = 1$, $B_1 = \{Q_{\text{init}}\}$, $U_1 = \phi_{\text{ub}}(Q_{\text{init}})$, and $L_1 = \phi_{\text{lb}}(Q_{\text{init}})$.

2) Stopping criterion: if $U_k - L_k > \epsilon$ go to Step 3, otherwise STOP.

3) Branching: a) pick $Q \in B_k$ for which $\phi_{\text{ub}}(Q) = L_k$ and set $Q_k = Q$.

b) split $Q_k$ along one of its longest edge into $Q_l$ and $Q_{ll}$.

c) Let $B_{k+1} = \{B_k \setminus \{Q_k\}\} \cup \{Q_l, Q_{ll}\}$. 

4) Bounding: 

a) set $U_{k+1} = \min_{Q \in B_{k+1}} \{\phi_{\text{ub}}(Q)\}$.

b) set $L_{k+1} = \min_{Q \in B_{k+1}} \{\phi_{\text{lb}}(Q)\}$.

5) Pruning: 

a) pick all $Q \in B_{k+1}$ for which $\phi_{\text{ub}}(Q) > U_{k+1}$.

b) update $B_{k+1}$ by removing all $Q$ obtained in the above step (5-a).

6) Set $k = k + 1$ and go to step 2.

\section{IV. UPPER AND LOWER BOUND FUNCTIONS}

\subsection{A. Bounding Functions}

We start by expressing the bounding functions $\phi_{\text{ub}}$ and $\phi_{\text{lb}}$ proposed in [4] for the case of perfect CSI in MISO systems as
\[
\phi_{\text{ub}}(Q) = \left\{ \begin{array}{ll}
f_0(\gamma_{\text{max}}) & \gamma_{\text{min}} \in G \\
0 & \text{otherwise}
\end{array} \right. \quad (16)
\]
and
\[
\phi_{\text{lb}}(Q) = \tilde{f}(\gamma_{\text{min}}) = \left\{ \begin{array}{ll}
f_0(\gamma_{\text{min}}) & \gamma_{\text{min}} \in G \\
0 & \text{otherwise}
\end{array} \right. \quad (17)
\]
where $\gamma_{\text{max}} = [\gamma_{l,\max}, \ldots, \gamma_{l,\max}]^T$, $\gamma_{\text{min}} = [\gamma_{l,\min}, \ldots, \gamma_{l,\min}]^T$, and $G$ is defined in (9). These general expressions hold true for the case of robust MISO systems as well. However, checking the condition $\gamma_{\text{min}} \in G$, which is central to calculating $\phi_{\text{ub}}$ and $\phi_{\text{lb}}$, is much more difficult in the case of uncertain CSI. An efficient method based on semi-definite programming is presented in the sequel.

Let $\{\gamma_l\}_{l \in L}$ be a specified set of SINR values. Checking the condition that these values are achievable (i.e., testing if $\{\gamma_l\}_{l \in L} \in G$) is equivalent to solving the following feasibility problem
\[
\begin{align*}
\text{find} & \quad \{w_l, e_{jl}\}_{j,l \in L} \\
\text{subject to} & \quad \gamma_l \leq \Gamma_l, \quad l \in L \\
& \quad e_{jl}^H Q_{jl} e_{jl} \leq 1, \ j, l \in L \\
& \quad \sum_{j \in L(n)} \|w_l\|^2_{P_{\text{trans}}(l)} \leq \mu_n^2, \ n \in N.
\end{align*}
\]
(18)

with variables $w_l$ and $e_{jl}$, $j, l \in L$.

Problem (18) is not convex in this sequel, so we propose an efficient method based on SDP to solve problem (18). By following the approach of [11, Sec.III-B] and introducing variables $I_{jl} = \|\hat{h}_{jl} + e_{jl}\|^2_{P_{\text{trans}}(l)}$ for all $j \in L(n), n \in \ldots$
\( \mathcal{N}\{\text{tran}(l)\} \), \( l \in \mathcal{L} \), problem (18) can be equivalently reformulate as

\[
\begin{align}
\text{find} & \quad \{w_l, e_{jl}, I_{jl}\}, j, l \in \mathcal{L} \\
\text{subject to} & \quad (\hat{h}_{jl} + e_{jl})^H \mathbf{V}_l (\hat{h}_{jl} + e_{jl}) \geq \sum_{n \in \mathcal{N}\{\text{tran}(l)\}} \sum_{j' \in \mathcal{L}(n)} I_{jl} + \sigma_l^2, l \in \mathcal{L} \\
& \quad e_{jl}^H Q_{jl} e_{jl} \leq 1, \quad l \in \mathcal{L} \quad \text{(19a)} \\
& \quad (\hat{h}_{jl} + e_{jl})^H w_l (\hat{h}_{jl} + e_{jl}) \leq I_{jl}, \quad j \in \mathcal{L}(n), n \in \mathcal{N}\{\text{tran}(l)\}, l \in \mathcal{L} \quad \text{(19b)} \\
& \quad e_{jl}^H Q_{jl} e_{jl} \leq 1, \quad j \in \mathcal{L}(n), n \in \mathcal{N}\{\text{tran}(l)\}, l \in \mathcal{L} \quad \text{(19c)} \\
& \quad \sum_{l \in \mathcal{L}(n)} w_l^H w_l \leq p_n^{\text{max}}, \quad n \in \mathcal{N} \quad \text{(19d)}
\end{align}
\]

where \( \mathbf{V}_l = \left[ \frac{\mathbf{w}_l^H}{\eta} - \sum_{j' \in \mathcal{L}(n)} \mathbf{w}_{j'} \right] \). Note that we have re-written second constraint of problem (18) as two separate ones, i.e., constraints (19c) and (19e). Furthermore, it is easy to show (e.g., by contradiction) that constraints (19d) are tight (i.e., they hold with equality at optimality). Hence, problem (19) is an equivalent reformulation of problem (18).

The outer product \( w_l^H w_l^H \) in problem (19) is a rank one positive semidefinite matrix. We introduce a new set of variables \( \mathbf{W}_l = w_l^H w_l^H \) for all \( l \in \mathcal{L} \). Then, by following the approach of [11, Sec. IV] and using S-procedure [17, 18], problem (19) can be equivalently reformulated as

\[
\begin{align}
\text{find} & \quad \{\mathbf{W}_l, \mu_{jl}, I_{jl}\}, j, l \in \mathcal{L} \\
\text{subject to} & \quad \mathbf{\Delta}_{jl} \geq 0, \quad l \in \mathcal{L} \\
& \quad \Phi_{jl} \geq 0, \quad j \in \mathcal{L}(n), n \in \mathcal{N}\{\text{tran}(l)\}, l \in \mathcal{L} \\
& \quad \mu_{jl} \geq 0, \quad j, l \in \mathcal{L} \\
& \quad \mathbf{W}_l \geq 0, \quad l \in \mathcal{L} \\
& \quad \sum_{l \in \mathcal{L}(n)} \text{Trace}(\mathbf{W}_l) \leq p_n^{\text{max}}, \quad n \in \mathcal{N} \\
& \quad \text{Rank}(\mathbf{W}_l) = 1, \quad l \in \mathcal{L}, \quad \text{(20)}
\end{align}
\]

with variables \( \mathbf{W}_l, \mu_{jl} \) and \( I_{jl} \) for \( j, l \in \mathcal{L} \), where \( \mathbf{\Delta}_{jl} \) given by (21) and \( \Phi_{jl} \) given by (22) respectively. Note that if the rank constraints of problem (20) are neglected, then problem (20) can be solved by using a standard SDP solver [19].

B. Optimality Conditions

It turns out that we can employ a simple trick to handle also the rank constraint of problem (20). The procedure is based on replacing the dummy objective function in (20) with the sum power minimization of the network. Then the feasibility problem (20) can be written as

\[
\begin{align}
\text{minimize} & \quad \sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}(n)} \text{Trace}(\mathbf{W}_l) \\
\text{subject to} & \quad \mathbf{\Delta}_{jl} \geq 0, \quad l \in \mathcal{L} \\
& \quad \Phi_{jl} \geq 0, \quad j \in \mathcal{L}(n), n \in \mathcal{N}\{\text{tran}(l)\}, l \in \mathcal{L} \\
& \quad \mu_{jl} \geq 0, \quad j, l \in \mathcal{L} \\
& \quad \mathbf{W}_l \geq 0, \quad l \in \mathcal{L} \\
& \quad \sum_{l \in \mathcal{L}(n)} \text{Trace}(\mathbf{W}_l) \leq p_n^{\text{max}}, \quad n \in \mathcal{N}, \quad \text{(23)}
\end{align}
\]

with variables \( \mathbf{W}_l, \mu_{jl} \) and \( I_{jl} \) for all \( j, l \in \mathcal{L} \). Clearly, problem (20) is feasible (when the Rank constraint is relaxed) if and only if problem (23) is feasible (because they have the same set of constraints). The proposition [9, Proposition 1] ensures that for the given conditions\(^2\) problem (23) returns a set of rank one matrices \( \mathbf{W}_l \) for all \( l \in \mathcal{L} \).

V. SIMULATION RESULTS

We illustrate the convergence of the proposed algorithm by a numerical example with \( N = 2 \) BSs with \( T = 4 \) transmit antennas at each one (see Figure 1). The distance between the BSs denoted by \( D_{BS} \). We assume circular sectors, where the radius of each one is denoted by \( R_{BS} \). For simplicity, we assume that the network is transmitting \( L = 4 \) data streams, 2 streams per each BS. The locations of the users associated with each data stream are arbitrarily chosen as shown in Figure 1.

We assume an exponential path loss model for the estimated values of the channels, where the channel vector from the transmitter of data stream \( j \) (i.e., BS \( \text{tran}(j) \)) to the receiver of data stream \( l \) (i.e., user \( \text{rec}(l) \)) is modeled as

\[
\hat{h}_{jl} = \left( \frac{d_{jl}}{d_0} \right)^{-\eta/2} e_{jl}
\]

where \( d_{jl} \) is the distance from \( \text{tran}(j) \) to \( \text{rec}(l) \), \( d_0 \) is the far field reference distance [20], \( \eta \) is the path loss exponent, and \( e_{jl} \in \mathbb{C}^T \) is arbitrarily chosen from the distribution \( \mathcal{CN}(0, I) \) (i.e., frequency-flat fading channel with uncorrelated antennas). Further, we assume that \( \mathbf{Q}_{jl} = (1/\xi^2)I \) and set \( \xi = 0.1 \) for all \( j, l \in \mathcal{L} \). Thus, an estimation error can take any value inside a ball with radius \( \xi \).

We assume \( p_n^{\text{max}} = p_0^{\text{max}} \) for all \( n \in \mathcal{N} \), and \( \sigma_1 = \sigma = \sigma_l^{\text{max}} \) for all \( l \in \mathcal{L} \). We define the signal-to-noise ratio (SNR) operating point at a distance \( R \) as \( \text{SNR}(R) = (R/d_0)^{-\eta} p_0^{\text{max}}/\sigma^2 \). In the simulations we set \( \text{SNR}(R_{BS})=10\text{dB} \), \( d_0 = 1 \), \( \eta = 4 \), \( p_0^{\text{max}}/\sigma^2 = 40\text{dB} \), \( \sigma^2 = 1 \) and we let \( D_{BS}/R = 1.6 \).

\(^2\)The conditions of proposition [9, Proposition 1] is not reproduced here due to space limitation.

\[
\begin{align}
\mathbf{\Delta}_{jl} & \triangleq \left[ \begin{array}{c}
\mathbf{V}_l \\
\hat{h}_{jl}^H \mathbf{V}_l \\
\hat{h}_{jl}^H \mathbf{V}_l - \sum_{n \in \mathcal{N}\{\text{tran}(l)\}} \sum_{j' \in \mathcal{L}(n)} I_{jl} - \sigma_n^2 \\
\end{array} \right] + \mu_{jl} \left[ \begin{array}{c}
\mathbf{Q}_{jl} \\
0 \\
-1 \\
\end{array} \right] \geq 0, \quad l \in \mathcal{L} \\
\Phi_{jl} & \triangleq \left[ \begin{array}{c}
-\mathbf{W}_l \\
-\hat{h}_{jl}^H \mathbf{W}_l \\
I_{jl} - \hat{h}_{jl}^H \mathbf{W}_l \hat{h}_{jl} \\
\end{array} \right] + \mu_{jl} \left[ \begin{array}{c}
\mathbf{Q}_{jl} \\
0 \\
-1 \\
\end{array} \right] \geq 0, \quad j \in \mathcal{L}(n), n \in \mathcal{N}\{\text{tran}(l)\}, l \in \mathcal{L}
\end{align}
\]
multicell downlink multiple-input single-output systems have been considered, which is known to be NP-hard even for the perfect CSI scenario. We proposed a solution method, based on semi-definite relaxation and branch and bound technique, which solves globally the noncovex robust WSRMax problem with an optimality certificate.

VI. CONCLUSION

The worst-case weighted sum-rate maximization (WSRMax) problem under imperfect channel state information in