INFLUENCE OF SECONDARY PATH ESTIMATION ERRORS ON THE PERFORMANCE OF ANC-MOTIVATED NOISE REDUCTION ALGORITHMS FOR HEARING AIDS

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ABSTRACT

Current noise reduction techniques for open-fitting hearing aids that only use the external microphones on the hearing aid typically disregard the occurrence of signal leakage through the open fitting, leading to a degraded noise reduction performance. Using an ear mould with an internal (so-called error) microphone provides information about the signal leakage and hence enables to improve the noise reduction performance. Recently, feedforward and combined feedforward-feedback active-noise-control-motivated (FF ANC and FF-FB ANC, respectively) algorithms for noise reduction have been presented for such open-fitting hearing aids.

The noise reduction filters of these ANC-motivated algorithms depend on an estimate of the so-called secondary path between the receiver and the error microphone. In this paper, we analyze the influence of secondary path estimation errors on the performance of the ANC-motivated algorithms. For the FF ANC algorithm it is possible to derive a closed-form expression of the filter as a function of the secondary path estimation error and to derive limit values for the allowable secondary path estimation errors. In addition, simulations show that even when estimation errors occur the FF-FB ANC algorithm still outperforms the FF ANC algorithm.

Index Terms— active noise control, secondary path estimation errors, open-fitting hearing aid, noise reduction, Multichannel Wiener Filter

1. INTRODUCTION

Over the past years the usage of open-fitting hearing aids has been steadily increasing, due to the fact that they largely alleviate occlusion-related problems. However, existing noise reduction (NR) algorithms such as the Multichannel Wiener Filter (MWF) [1] do not take into account the ambient noise leaking through the open fitting, therefore typically leading to a degraded noise reduction performance. To provide information about this leakage and hence improve the performance of NR algorithms, an internal (so-called error) microphone can be incorporated in the ear mould. In [2] a feedforward ANC-motivated (FF ANC) algorithm in open-fitting hearing aids has been introduced, which uses both external and internal microphones and hence takes into account the signal leakage for the filter optimization. In [3] a combined feedforward-feedback ANC-motivated (FF-FB ANC) algorithm for open-fitting hearing aids has been presented, which uses the signal leakage in the error microphone as an additional input signal together with the external microphone signals. It has been experimentally shown that the FF-FB ANC algorithm outperforms both the standard MWF and the FF ANC algorithms in terms of SNR improvement. In [4], a theoretical analysis has been performed for these algorithms, where it has been shown that if a perfect estimate of the so-called secondary path is available, the error microphone signal of the FF-FB ANC algorithm is independent of the signal leakage.

Since in practice the secondary path is typically unknown and can change over time, it needs to be estimated, e.g., using an adaptive filtering algorithm [5, 6]. In general, the performance of ANC-motivated algorithms is affected by errors in the secondary path estimation [7]. In this paper, we analyze the influence of secondary path estimation errors (amplitude and phase) on the performance of the FF ANC and FF-FB ANC algorithms. We will derive a closed-form expression for the filter of the FF ANC algorithm as a function of the secondary path estimation error. In addition, we will derive limit values for the allowable secondary path estimation errors. Simulations with a two-microphone behind-the-ear (BTE) hearing aid with a vent size of 2 mm show that even when estimation errors occur, the FF-FB ANC algorithm still outperforms the FF ANC algorithm in terms of SNR improvement and speech distortion.

2. CONFIGURATION AND BACKGROUND

2.1. Signal Model

Consider a hearing aid with \( M \) external microphones and an internal (error) microphone in the ear canal, as depicted in Figure 1. The \( m \)th microphone signal \( Y_m(k, n) \) in the short-time Fourier transform domain can be written as

\[
Y_m(k, n) = X_m(k, n) + V_m(k, n), \quad m = 1 \ldots M, \tag{1}
\]

with \( X_m(k, n) \) the speech component and \( V_m(k, n) \) the additive noise component, where \( k \) denotes the frequency index and \( n \) the block index. For conciseness the indices \( k \) and \( n \) will be omitted in the remainder of the paper, except where explicitly required. The \( M \)-dimensional stacked vector \( \mathbf{Y} \), consisting of all microphone signals, is defined as

\[
\mathbf{Y} = [Y_1 \; Y_2 \; \ldots \; Y_M]^T = \mathbf{X} + \mathbf{V}. \tag{2}
\]

Figure 1: Hearing aid configuration with external microphone signals \( \mathbf{Y} \), error microphone signal \( E \) and signal leakage \( L \).

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The correlation matrices of the signal components are defined as $R_y = E\{Y Y^H\}$, $R_e = E\{X X^H\}$ and $R_v = E\{V V^H\}$. The error microphone signal $E$ is equal to

$$E = C Z + L,$$

with $C$ the secondary path and $L$ the signal leakage through the open fitting. The receiver signal $Z$ is given by

$$Z = G W^H Y,$$

with $G$ the (broadband) gain of the hearing aid and $W$ the $M$-dimensional filter on the microphone signals, i.e.,

$$W = [W_1, W_2, ..., W_M]^T.$$

### 2.2. Multichannel Wiener Filter (MWF)

The multichannel Wiener filter [1] produces a minimum-mean-square-error (MMSE) estimate of the (unknown) speech component in a reference microphone (e.g., the first microphone). The MSE cost function is given by

$$J_{\text{MSE}}(W) = E\{|C Z - D|^2\} = E\{|GCW^H Y - D|^2\},$$

where the desired signal $D$ is chosen to be equal to the speech component in the reference microphone, multiplied with the hearing aid gain and filtered with the secondary path, i.e.,

$$D = GCX_1.$$

The filter minimizing the cost function in (6) is equal to

$$W_{MWF} = R_y^{-1} R_e e_1,$$

with $e_i$, a vector whose $i$th element is equal to 1 and all other elements are equal to 0. Note that this filter does not take into account the signal leakage $L$ through the open fitting, such that the performance of the MWF will typically be degraded by this signal leakage [3].

### 3. EFFECT OF SECONDARY PATH ESTIMATION ERRORS

In order to take into account the signal leakage, the feedforward and the combined feedforward-feedback ANC-motivated algorithms proposed in [2] and [3] use both the external microphone signals and the error microphone signal $E$, providing information about the signal leakage $L$. The aim is then to minimize the MSE between the error microphone signal $E$ (including leakage) and the desired signal $D$ defined in (7).

If a perfect estimate of the secondary path is available (e.g., using a calibration measurement procedure), then a closed-form expression for the FF ANC and the FF-FB ANC filters can be derived [3]. In practice the secondary path is typically estimated, e.g., using an adaptive filtering algorithm [5, 6], such that estimation errors may occur. The estimated secondary path $\hat{C}$ can be written as

$$\hat{C} = CAe^{-j\varphi}$$

where $A$ and $\varphi$ represent the amplitude and the phase error in the secondary path estimate, respectively. In this paper, we will analyze the effect of secondary path estimation errors on the performance of the ANC-motivated algorithms. For the FF ANC algorithm it is possible to derive a closed-form expression of the filter as a function of the secondary path estimation error and to derive limit values for the allowable secondary path estimation errors ($A > 0.5$, $|\varphi| < 60^\circ$). On the other hand, for the FF-FB ANC it is unfortunately not possible to derive such a closed-form expression for the filter.

#### 3.1. Feedforward ANC-Motivated (FF ANC) Algorithm

In contrast to the MSE cost function in (6), the FF ANC algorithm proposed in [2] minimizes the cost function

$$J_{\text{FF ANC}}(W) = E\{|E - D|^2\} = E\{|C Z + L - D|^2\},$$

taking into account the signal leakage $L$. The filter minimizing the cost function in (10) is given by

$$W_{\text{FF opt}} = (GC^* R_y)^{-1} (GC^* R_y e_1 - r_{gd}),$$

with $r_{gd} = E\{Y L^*\}$. The filter in (11) can be related to the MWF in (8) as

$$W_{\text{FF opt}} = W_{MWF} - (GC^*)^{-1} R_y^{-1} r_{gd}. $$

In practice we typically do not have a perfect estimate of the secondary path and the signal leakage. The filter of the FF ANC algorithm with the estimated secondary path and the estimated signal leakage $\hat{L}_{\text{FF}}$ can be expressed as

$$W_{\text{FF}} = W_{\text{MWF}} - (GC)^* R_y^{-1} r_{gd},$$

with $r_{gd} = E\{Y \hat{L}_{\text{FF}}^*\}$. In order to estimate the signal leakage in the error microphone, the receiver signal is filtered with the estimated secondary path and subtracted from the error signal, i.e.,

$$\hat{L}_{\text{FF}} = E - \hat{C} Z = G (C - \hat{C}) W_{\text{MWF}} Y + L.$$

Using (14), the cross correlation vector $r_{gd}$ can then be written as

$$\hat{r}_{gd} = E\{Y \hat{L}_{\text{FF}}^*\} = G(C - \hat{C}) R_y W_{\text{MWF}} + r_{gd}. $$

Inserting (15) in (13) yields a closed-form expression for the filter:

$$W_{\text{FF}} = W_{\text{MWF}} \left( C \right)^* y - (GC)^* r_{gd}. $$

Note that only the first part of the filter in (16) depends on the secondary path estimation error.

Alternatively, the closed-form expression for the filter $W_{\text{FF}}$ can be derived iteratively, allowing to compute limit values for the maximum allowable secondary path estimation error.

I. Starting from an initial value for the estimated signal leakage $\hat{L}_{\text{FF}}^{(0)}$, $W_{\text{FF}}^{(i)}$ can be computed using (13) as

$$W_{\text{FF}}^{(i)} = W_{\text{MWF}} - (GC^*)^{-1} R_y^{-1} r_{gd}^{(i)}$$

with $r_{gd}^{(i)} = E\{Y \hat{L}_{\text{FF}}^{(i)}^*\}$.

II. By plugging $W_{\text{FF}}^{(i)}$ into (14) a new estimated signal leakage $\hat{L}_{\text{FF}}^{(i+1)}$ can be found.

III. By repeating this procedure, the $(i+1)$th recursion for the filter can be written as

$$W_{\text{FF}}^{(i+1)} = W_{\text{FF}}^{(i)} \sum_{r=0}^{i} (1 - \frac{C^* C}{C^* C})^r$$

Only when $|1 - \frac{C^* C}{C^* C}| < 1$, the geometric sequence in (18) converges to

$$W_{\text{FF}} = \left( \frac{C^* C}{C^* C} \right)^{i} W_{\text{FF}}^{(i)}$$

Note that the filter in (19) is independent of the initial value for the estimated signal leakage $\hat{L}_{\text{FF}}^{(0)}$. By plugging (17) into (19) the same closed-form expression as in (16) can be found.

From the necessary condition $|1 - \frac{C^* C}{C^* C}| = |1 - \frac{1}{A^2} e^{-j\varphi}| < 1$ it follows that if there is no phase error ($\varphi = 0^\circ$) the filter of the FF ANC algorithm converges for amplitude errors $A > 0.5$. On the other hand, if there is no amplitude error ($A = 1$) the filter converges for phase errors $|\varphi| < 60^\circ$. 

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3.2. Combined Feedforward-Feedback ANC-Motivated (FF-FB ANC) Algorithm

In the FF-FB ANC algorithm proposed in [3], the signal leakage in the error microphone – unlike in the FF ANC algorithm – is used as an additional input signal together with the external microphones (cf. Figure 2), i.e.,

$$Z_{\text{FF-FB}} = G W_{\text{FF-FB}}^H Y_{\text{FF-FB}}$$

and

$$Y_{\text{FF-FB}} = Y_L,$$

with $W_{\text{FF-FB}}$ an $(M + 1)$-dimensional filter and

$$E_{\text{FF-FB}} = C Z_{\text{FF-FB}} + L.$$  

(20)

The cost function for the FF-FB ANC algorithm is given by

$$J_{\text{FF-FB opt}}(W) = \mathcal{E}\{|E_{\text{FF-FB}} - D|^2\} = \mathcal{E}\{|CZ_{\text{FF-FB}} + L - D|^2\}.  \quad (21)$$

The filter minimizing the cost function in (21) is equal to

$$W_{\text{FF-FB}} = (G^* R_{y^*}^{-1} G C^*)^{-1}(G^* R_{y^*} e_1 - \hat{r}_{y^G}^L),$$

(22)

with $\hat{r}_{y^G}^L = \mathcal{E}\{Y_{\text{FF-FB}} L^*\}$ and $R_{y^*}$, $R_{y^G}$ the correlation matrices of the microphone signals $Y_{\text{FF-FB}}$ and their speech components, respectively.

The filter of the FF-FB ANC algorithm with the estimated secondary path and the estimated signal leakage $L_{\text{FF-FB}}$ hence can be expressed as

$$W_{\text{FF-FB}} = (G^* R_{y^*}^{-1} G C^*)^{-1}(G^* R_{y^*} e_1 - \hat{r}_{y^G}^L),$$

(23)

with $\hat{r}_{y^G}^L = \mathcal{E}\{Y_{\text{FF-FB}} L^*\}$ and $R_{y^*}$, $R_{y^G}$ the correlation matrices of the input signals

$$\hat{Y}_{\text{FF-FB}} = \begin{bmatrix} Y \\ L_{\text{FF-FB}} \end{bmatrix},$$

(24)

and their speech components, respectively. Using the fact that $\hat{L}_{\text{FF-FB}} = e_{\text{MWF}}^T \hat{Y}_{\text{FF-FB}}$, it follows that

$$\hat{r}_{y^G}^L = \mathcal{E}\{\hat{Y}_{\text{FF-FB}} L_{\text{FF-FB}}^*\} = \hat{r}_{y^G} - \hat{r}_{y^G}^L e_{\text{MWF}},$$

(25)

Using (25) the filter $W_{\text{FF-FB}}$ in (23) can be written as

$$W_{\text{FF-FB}} = W_{\text{MWF}} - (G^* C^*)^{-1} e_{\text{MWF}},$$

(26)

with $W_{\text{FF-FB}} = (G^* R_{y^*}^{-1} G C^*)^{-1}(G^* R_{y^*} e_1$. Note that the first part $W_{\text{MWF}}$ depends on the estimated signal leakage $L_{\text{FF-FB}}$ and can be interpreted as the MWF with $\hat{L}_{\text{FF-FB}}$ as an additional input signal. The second part is a vector where only the last element is not equal to zero and only depends on the gain and the estimated secondary path.

For estimating the signal leakage in the error microphone, the receiver signal is filtered with the estimated secondary path and subtracted from the error signal (cf. (14)), i.e.

$$\hat{L}_{\text{FF-FB}} = E_{\text{FF-FB}} - C Z_{\text{FF-FB}} = G(C - \bar{C}) W_{\text{FF-FB}}^H Y_{\text{FF-FB}} + L.$$  

(27)

However, by plugging (27) into (26), it is unfortunately not possible to derive a closed-form expression for the filter $W_{\text{FF-FB}}$ as a function of the secondary path estimation error.

Nevertheless the same iterative procedure used for computing the limit values for the filter of the FF ANC algorithm can be used here to compute the filter $W_{\text{FF-FB}}$. Starting from an initial value for the estimated signal leakage $L_{\text{FF-FB}}$, $W_{\text{FF-FB}}$ can be computed using (26). By plugging the computed $W_{\text{FF-FB}}$ into (27) a new estimated signal leakage can be found. These iterations are then performed until the filter converges.

4. EXPERIMENTAL RESULTS

4.1. Setup and performance measures

Simulations were performed using anechoic room recordings obtained with a KEMAR head and torso, a two-microphone behind-the-ear (BTE) hearing aid, an external receiver (Knowles, TWFK-30017-000) and an active ear mould with an internal microphone (Knowles, FG-23329-P07) and a vent size of 2 mm.

The sound sources were positioned at a distance of 3 m from the center of the head. The BTE was worn on the right ear. The speech source was located at $0^\circ$ and multiple noise sources at $90^\circ$, $180^\circ$ and $270^\circ$ were considered. The noise signal was multilalker babble noise and the speech signal was taken from the HINT database [8].

The signals were processed at $f_s = 16$ kHz using a weighted overlap-add (WOLA) framework with a block size of 256 samples, an overlap of 75% between successive blocks and a Hann window. The correlation matrices $R_y(k)$. $R_x(k)$ and $R_z(k)$ in frequency-bin $k$ are estimated as

$$R_y(k) = \frac{1}{N_d} \sum_{i=1}^{N_d} Y(k, i) Y^H(k, i) \quad \text{speech present} \quad (28)$$

$$R_z(k) = \frac{1}{N_v} \sum_{i=1}^{N_v} V(k, i) V^H(k, i) \quad \text{speech absent} \quad (29)$$

with $N_d$ the number of blocks when speech is present and $N_v$ the number of blocks when speech is absent, determined by a perfect voice activity detector (VAD). Assuming speech and noise are uncorrelated, the speech correlation matrix is estimated as

$$R_x(k) = R_y(k) - R_z(k).$$

(30)

Using a calibration measurement procedure, the (perfect) secondary path $C$ has been identified as an FIR filter with $L_c = 128$ taps. The broadband gain is set to $G = 0$ dB. In the simulations in this paper, the amplitude and phase errors in the secondary path estimation are assumed to be frequency-independent.

To quantify the performance of the considered noise reduction algorithms, the (broadband) speech intelligibility-weighted SNR improvement [9] is computed as

$$\Delta \text{SNR}_{\text{int}} = \sum_{k=1}^{K} I(k) \Delta \text{SNR}(k),$$

(31)

where the weighting function $I(k)$ takes the importance of the $k$-th frequency band for speech intelligibility into account and

$$\Delta \text{SNR}(k) = 10 \log_{10} \frac{P_{se}(k)}{P_{se}(k)} \cdot 10 \log_{10} \frac{P_{se}(k)}{P_{se}(k)},$$

(32)
where \( P_{se}(k) \) and \( P_{sn}(k) \) denote the power spectral density (PSD) of the speech and noise components of the output signal and \( P_{s}(k) \) and \( P_{n}(k) \) are similarly defined for the reference microphone signal. In order to evaluate the amount of speech distortion, we use the speech intelligibility-weighted speech distortion measure \( SD_{int} \), i.e.

\[
SD_{int} = \sum_{k=1}^{K} I(k)SD(k), \tag{33}
\]

with

\[
SD(k) = |10 \log_{10} \frac{P_{se}(k)}{P_{s}(k)}|, \tag{34}
\]

where \( P_{d}(k) \) denote the PSD of the desired signal \( D \) in (7).

### 4.2. Results

Figure 3 depicts \( \Delta SNR_{int} \) and \( SD_{int} \) for the FF ANC and FF-FB ANC algorithms, as a function of frequency-independent amplitude errors in the secondary path estimate. In [4] it had been already shown that for a perfect estimate of the secondary path (i.e., \( A = 0 \text{ dB}, \varphi = 0 \) ) the FF-FB ANC algorithm yields a better SNR improvement than the FF ANC algorithm. Figure 3 shows that for secondary path amplitude estimation errors the SNR improvement of the FF-FB ANC algorithm is hardly degraded. Moreover, even when amplitude estimation errors occur the FF-FB ANC algorithm still outperforms the FF ANC algorithm. From this figure it can also be observed that the FF-FB ANC algorithm causes less speech distortion than the FF ANC algorithm for nearly all considered amplitude errors.

Figure 4 shows \( \Delta SNR_{int} \) and \( SD_{int} \) for the FF-FB ANC and FF ANC algorithms, as a function of frequency-independent phase estimation errors. As can be seen from this figure, even when phase errors occur the FF-FB ANC algorithm still yields a better SNR improvement than the FF ANC algorithm. Figure 4 illustrates that for phase errors the FF-FB ANC algorithm causes less speech distortion than the FF ANC algorithm. Comparing Figures 3 and 4, the performance of the FF-FB ANC algorithm appears to be more affected by phase errors than by amplitude errors.

### 5. CONCLUSION

In this paper, we have derived a closed-form expression for the filter of the FF ANC algorithm as a function of the secondary path estimation error and we have derived limit values for the allowable secondary path estimation errors. Moreover, we have shown that even when secondary path (amplitude and phase) estimation errors occur the FF-FB ANC algorithm still outperforms the FF ANC algorithm.

### 6. REFERENCES


