Strategic Optimization of a Transportation Distribution Network

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Abstract
The transportation network of any large manufacturing or retail company is often initially designed and periodically refined to minimize some function of cost. In this paper, we develop an integer linear programming model for the distribution network of a large company from a strategic planning perspective. Specifically, we are interested in future network expansion and contraction opportunities and how these phenomena affect network utilization, the company's operational approaches, and transportation asset utilization. The model formulation is presented, followed by some preliminary results.

Keywords
Optimization, Transportation, Distribution

1. Introduction
Network-structured problems arise directly or indirectly in many practical scenarios where physical flow transfers occur between a set of origin points (supply nodes) and a set of destination points (demand nodes) [1]. This physical flow can be associated with modal types used to transfer a combination of goods and/or people via links to and from origin and destination points. The decisions to be made in network optimization problems include the location of supply nodes, transportation routing, inventory allocation, scheduling of flow between supply and demand nodes, and the design of links within the network [1].

Numerous studies have been conducted and documented to determine such decision variables in a distribution network. Akyilmaz [2] presents a framework of algorithms that route Less-than Truck Load (LTL) shipments via intermediate terminals. Pirkul and Jayaraman [3] develop a mixed integer programming (MIP) model for the plant and warehouse location problem. The objective to the model is to minimize total transportation and distribution costs, including fixed plant opening and operating costs, as well as warehouse costs. Sharma and Saxena [4] present a special case of the transshipment problem wherein flow occurs strictly between supplier plants and intermediate facilities and between intermediate facilities and the end customer market.

These previous studies have presented formulations for the optimal assignment of entities in various types of distribution networks for some single time period of interest. In this paper, we expand on the formulations of previous studies to include multiple time periods and to incorporate the refining or updating component of transportation distribution network strategic planning. In the transportation industry, transportation networks are often refined to minimize costs. This refinement can either be an expansion or a contraction within the existing network. Often, the complex network of suppliers, intermediate consolidation centers (ICC), and distribution centers (DCs) that exists throughout the country creates the need for periodic review of both supplier to ICC and ICC to DC assignments. Another question of interest often pertains to the choice of transportation mode that should be used to transport goods between ICCs and DCs (i.e., air, highway, rail, etc.). When faced with growing demand, companies frequently wish to identify strategic locations for future ICCs. Alternately, companies may also need to determine which ICC(s) should be closed under low volume conditions.

In this paper, we model the distribution network of a large company that is composed of a number of complex product distribution "subnetworks" that are used to transport various types of goods. In terms of our overall research agenda, we seek to identify what type(s) of strategic network updates/refinements could be performed to further minimize distribution and operating costs. Some preliminary results are presented for a scaled-down version of true distribution network complexity to promote understanding of our model outputs. Section 2 describes the distribution network of interest, followed by our MIP model formulation in Section 3. After preliminary experimental results are discussed in Section 4, preliminary conclusions and directions for future research are presented in Section 5.

2. Distribution Network Operations

Consider a transportation distribution network consisting of a set of I suppliers, a set of J ICCs, and a set of K DCs (Figure 1). For each load being shipped from ICC $j \in J$ to DC $k \in K$, the company utilizes up to three types of transportation: rail, full truckload (TL), and intermodal (i.e., a combination of rail and TL). Each load shipped from supplier $i \in I$ to ICC $j \in J$ is always made via LTL transportation.

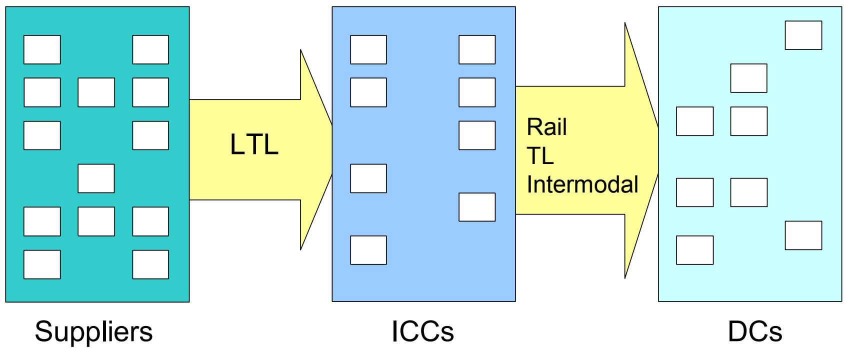

Figure 1. Transportation Options for Distribution Network Under Study

Distribution operations within the network are initiated when some quantity of demand, expressed in pounds, for a specific product type is generated by DC $k \in K$. Once appropriate suppliers are informed of the demand, they notify DC $k \in K$ of their ability to fulfill the demand for the requested product. Once supplier $i \in I$ is instructed to fulfill the DC's demand, supplier $i \in I$ ships the requested product to a nearby ICC via LTL transport. Demanded products get consolidated at each ICC $j \in J$ and then are shipped to the requesting DC by the minimum cost transportation option.

3. Optimization Model Formulation

Demand at each DC can only be fulfilled by those suppliers that stock the requested product type. The objective of the proposed optimization model is to determine, for a given demand profile (by DC in pounds), which ICC should service which DC, and by which mode of transportation this service should occur, so that distribution costs are minimized. This section details the optimization model formulation by first describing model sets and parameters, then decision variables, followed by the objective function and corresponding problem constraints.

Sets and Indices

I	suppliers ($i = 1 \ldots \|I\|$)
J	ICCs ($j = 1 \ldots \|J\|$)
K	DCs ($k = 1 \ldots \|K\|$)
M	modes of transportation ($m = 1 \ldots \|M\|$)
T	time periods ($t = 1 \ldots \|T\|$)

Parameters

S_{ij}	Distance from supplier i to ICC j in miles
T_{jk}	Distance from ICC j to DC k in miles
U_{ij}	Supplier i to ICC j transportation cost in \$/pound/mile
V_{jkm}	ICC j to DC k transportation cost by transportation mode m in \$/pound/mile

N_{jt}	Number of dock doors at ICC j in time period t
n	Number of ICCs (i.e., $n = \|J\|$)
C_{jt}	Capacity per dock door in ICC j in time period t in pounds/door
D_{ikt}	Demand requested from supplier i by DC k in period t in pounds
A_{it}	Supply that is available from supplier i in period t in pounds
Mo_{jt}	Cost of opening ICC j in time period t
Mc	Cost of closing an ICC
B_j	Initial status of ICC j; $B_j = 1$ if ICC j is initially open; $B_j = 0$ otherwise
Tr_{lbs}	Truck capacity in pounds
Ra_{lbs}	Rail capacity in pounds
M_{demand}	Minimum required demand required to keep an ICC open

Variables

Vo_{jt}	$Vo_{jt} = 1$ when ICC j opens in period t; $Vo_{jt} = 0$ otherwise
Vc_{jt}	$Vc_{jt} = 1$ when ICC j closes in period t; $Vc_{jt} = 0$ otherwise
Oc_t	Total opening and closing cost of ICCs in period t
Tc_t	Total transportation cost of ICCs in period t
X_{ijkmt}	Pounds of demand shipped from supplier i through ICC j to DC k by transportation mode m in time period t
Y_{jkt}	$Y_{jkt} = 1$ if ICC j services DC k in time period t; $Y_{jkt} = 0$ otherwise
Z_{jt}	$Z_{jt} = 1$ if ICC j is open in time period t; $Z_{jt} = 0$ otherwise

The model's objective function attempts to minimize the total transportation cost and the total cost of opening and closing ICCs across all time periods:

$$\sum_t \left(Tc_t + Oc_t\right) \tag{1}$$

In (1),

$$Tc_t = \sum_i \sum_j \sum_k \sum_m X_{ijkmt}(U_{ij} * S_{ij} + V_{jkm} * T_{jk}) \ \ \forall t \in T \tag{2}$$

$$Oc_t = \sum_j (Vo_{jt} * Mo_{jt} + Vc_{jt} * Mc\ ) \ \ \forall t \in T \tag{3}$$

Constraints (2) and (3) calculate the total transportation cost and the total cost of opening and closing ICCs in period t, respectively. Care must be taken to ensure that ICCs are not overloaded:

$$\sum_i \sum_k \sum_m X_{ijkmt} \le N_{jt} * C_{jt} * Z_{jt} \ \ \forall j \in J, t \in T \tag{4}$$

However, demand at each DC must be met in every time period:

$$\sum_j \sum_m X_{ijkmt} \ge D_{ikt} \ \ \forall i \in I, k \in K, t \in T \tag{5}$$

Truck (Constraints 6) and rail (Constraints 7) shipments must not exceed the corresponding vehicles payload capacity in any time period:

$$\sum_i \sum_k \sum_m X_{ijkmt} \le Tr_{lbs} \ \ \forall j \in J, t \in T \tag{6}$$

$$\sum_i \sum_k \sum_m X_{ijkmt} \le Ra_{lbs} \ \ \forall j \in J, t \in T \tag{7}$$

As the company does not wish to split a given DC's demand across multiple suppliers (Constraints 8), we must make sure that the selected supplier has at least the total pounds demanded from the DC available in the corresponding time period (Constraints 9):

$$\sum_{j} Y_{jkt} = 1 \quad \forall k \in K, t \in T \tag{8}$$

$$\sum_{j}\sum_{k}\sum_{m} X_{ijkmt} \leq A_{it} \quad \forall i \in I, t \in T \tag{9}$$

Obviously, any DC's demand should only be assigned to an open ICC:

$$\sum_{j} Y_{jkt} \leq n * Z_{jt} \quad \forall j \in J, t \in T \tag{10}$$

Constraints (11) through (14) are intermediate constraints used to ensure that the cost of opening and closing ICCs is calculated appropriately:

$$Vo_{jt} \geq Z_{jt} - B_j \quad \forall j \in J, t = 1 \tag{11}$$

$$Vc_{jt} \geq Z_{jt} - B_j \quad \forall j \in J, t = 1 \tag{12}$$

$$Vo_{jt} \geq Z_{jt} - Z_{jt-1} \quad \forall j \in J, t > 1 \tag{13}$$

$$Vc_{jt} \geq Z_{jt-1} - Z_{jt} \quad \forall j \in J, t > 1 \tag{14}$$

Finally, an ICC is only allowed to remain open if a minimum amount of demand flows through it:

$$\sum_{i}\sum_{k}\sum_{m} X_{ijkmt} \geq M_{demand} * Z_{jt} \quad \forall i \in I, t \in T \tag{15}$$

4. Verifying the Model: An Example Instance

For the sake of clarity and illustration, let $\|I\| = 10$, $\|J\| = 3$, and $\|K\| = 6$. The number of dock doors per ICC and the capacity associated with each dock door at each ICC $j = 1...3$ is given for this example distribution network instance in Table 1. Initially, we consider $\|T\| = 3$ time periods, with 60 unique DC demands in total (i.e., 10 demands per DC) being generated from a discrete uniform distribution over the interval [0, 100] units for time period $t = 1$. In subsequent periods, this demand is increased by 10% in period 2, followed by a 15% increase over period 1 for the period 3 demand.

Table 1. ICC Information in Example Problem Instance

ICC	Dock Doors	Capacity/Dock Door
1	3	1000
2	2	2500
3	1	3000

The optimization model presented in Section 3 above was coded in AMPL, then analyzed using the MIP solver in CPLEX v8.1. After running the example problem instance in CPLEX, we observe the ICC assignments shown in Table 2. For $t = 1$, the model recommends that ICC #1 be closed, but the other two ICCs (#2 and #3) be open. However, as demand increases in the next two time periods under study, model results do vary by time period. The model recommends that ICCs #1 and #2 be open for $t = 2$, while ICC #3 should be closed during the same time period. Finally, when $t = 3$, the model suggests that all demand should flow through ICC #1. Observing these ICC opening and closing schedules results in the optimal, minimum cost solution for the example problem instance under study.

Table 2. Assignment of ICCs as Recommended by Optimization Model Output

Period	ICC	Z*
1	1	0
1	2	1
1	3	1
2	1	1
2	2	1
2	3	0
3	1	1
3	2	0
3	3	0

Table 3 displays representative model output with respect to transportation model decisions when $t = 2$. Table 3 shows total pounds shipped from which supplier through which ICC to which DC by what mode of transportation for each shipment. The decision variables for the other two time periods show comparable results/trends.

Table 3. Transportation Decisions Summary for Time Period 2

Pounds	Supplier	ICC	DC	Mode
68	1	1	1	LTL
104	1	1	3	LTL
81	1	1	5	LTL
81	9	1	1	LTL
49	9	1	3	LTL
88	9	1	5	LTL
19	3	1	1	LTL
67	3	1	2	LTL
106	3	1	5	LTL
41	7	1	1	LTL
84	7	1	3	LTL
1370	7	1	5	LTL
96	6	1	1	LTL
67	6	1	4	Rail
104	6	1	4	Rail
40	1	1	4	Rail
44	9	1	4	Rail
404	7	1	4	Rail
27	6	1	6	Rail
104	1	1	6	Truck
38	9	1	6	Truck
92	3	1	6	Truck
107	7	1	6	Truck
19	6	2	4	Truck
35	3	2	2	Truck
30	1	2	2	Truck
2706	9	2	2	Truck
85	3	2	2	Truck
83	7	2	2	Truck
61	6	2	2	Truck

Taking into consideration the high variability of demand, we examine the model's sensitivity to demand variation over time. As mentioned in the model formulation above, costs are incurred each time an ICC is opened and/or closed. Further, an ICC can remain open only if some minimum amount of demand (e.g., 3,000 pounds) flows through it for a single time period. Preliminary experiments validated the intuition that ICCs frequently oscillate between open and closed states under variable demand. An important part of validating the ICC opening and closing components of the model is to meet with company personnel to determine if they wish to restrict to some acceptable level the number of ICC openings and/or closings per time period.

5. Conclusions and Future Research

In this paper, we present a mixed-integer programming model for analyzing the distribution network of a large company from a strategic planning perspective. Using the proposed model, optimal transportation route assignments can be made with minimum total cost. In addition, the status of each ICC in the network can be determined in each time period in terms of whether or not it is/should be open for business.

The example problem instance presented in this paper was selected only for illustrative purposes. An extension of the proposed model would be to include real data from the company, and then analyze the entire network distribution problem for the company. In the future, we will also continue to embellish and validate the optimization model presented in this paper. In parallel, the research team will analyze the dynamic ramifications of the optimization model's solution in terms of inventory build-up and required order cycle time via Monte Carlo simulation and/or discrete event simulation.

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