Mathematics education, democracy and development: Exploring connections

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Mathematics education and its links to democracy and development are explored in this article, with specific reference to the case of South Africa. This is done by engaging four key questions. Firstly, the question of whether mathematics education can be a preparation for democracy and include a concern for development, is discussed by drawing on conceptual tools of critical mathematics education and allied areas in a development context. Secondly, the question of how mathematics education is distributed in society and participates in shaping educational possibilities in addressing its development needs and goals is used to examine the issues emerging from mathematics performance in international studies and the national Grade 12 examination; the latter is explored specifically in respect of the South African mathematics curriculum reforms and teacher education challenges. Thirdly, the question of whether a mathematics classroom can be a space for democratic living and learning that equally recognises the importance of issues of development in contexts like South Africa, as a post-conflict society still healing from its apartheid wounds, continuing inequality and poverty, is explored through pedagogies of conflict, dialogue and forgiveness. Finally the question of whether democracy and development can have anything to do with mathematics content matters, is discussed by appropriating, as a metaphor, South Africa's Truth and Reconciliation Commission's framework of multiple 'truths', to seek links within and across the various forms and movements in mathematics and mathematics education that have emerged in the past few decades.

Introduction

There is no doubt that notions of democracy and development are highly contested in themselves and in education; hence, so too would be any exploration of their links to mathematics education. Whilst recent research, theory and practice, has emerged in the literature about connections between mathematics education and democracy and the related issues of equity and social justice, arguably much less has been written about mathematics education and development and related aspects such as poverty.

The four questions addressed in this article were inspired and have evolved from those first posed by Skovsmose (1994) with respect to general education and democracy. They were later reformulated to focus more directly on *mathematics* education (Vithal, 2003). Both have been framed within mathematics education from critical perspectives. This article brings in a further dimension, that of development, and attempts to connect the triad of mathematics education, democracy and development, through the four key questions, using the case of mathematics education in South Africa. This expansion enables a more explicit engagement with issues that are found across countries like South Africa, with developmental features such as high levels of poverty and inequality, when considering connections between mathematics education and democracy.

The article explores the following four questions:

1. Can mathematics education be a preparation for democracy that includes a concern for development?
2. How is mathematics education being distributed in society and thereby shaping educational possibilities?
3. Can mathematics education pedagogy be a space for democratic living and engaging development issues?
4. Can considerations about democracy and development in mathematics education have something to do with mathematics content matters?

This article begins, at most, a conversation about notions of development in deepening and broadening understandings of the many facets of mathematics education in a young democratic, post-conflict society. From the many gains and challenges emerging from post-apartheid South

Africa, a selection of issues are engaged in each of the questions being posed to reflect on the many aspects of mathematics education in a transforming society.

Mathematics education as a concern for democracy and development

The first question that is posed is: Can mathematics education, by itself and as part of general education, provide an introduction and preparation for democracy, and teach about democracy in ways that contribute to a society's development agenda? This question is discussed by drawing on the conceptual tools of critical mathematics education, in particular the formatting power of mathematics within a 'developmental state'.

Mathematics grows as it addresses questions and problems arising from within its own self-referential system(s), but increasingly it also advances as a discipline as it is applied to a diversity of problems in society, from everyday life to warfare and to poverty. D'Ambrosio (1994), points out the paradox in which mathematics is centrally implicated:

> In the last 100 years, we have seen enormous advances in our knowledge of nature and in the development of new technologies. … And yet this same century has shown us despicable human behaviour. Unprecedented means of mass destruction, of insecurity, new terrible diseases, unjustified famine, drug abuse, and moral decay are matched only by an irreversible destruction of the environment. Much of this paradox has to do with an absence of reflections and considerations of values in academics, particularly in the scientific disciplines, both in research and in education. Most of the means to achieve these wonders and also these horrors of science have to do with advances in mathematics. (p. 443)

Much of the concern in the developed contexts of the north is with how rapid advances in science and technology are fundamentally changing those societies and how these changes might pose a threat to democracy because of how they might limit the capacity of the electorate to participate meaningfully in understanding and influencing decisions that affect their lives. Another aspect of the concern is with the complexity of the science and technology and having to rely on experts (an 'expertocracy'), in particular the capacity of politicians and decision-makers to fully grasp the implications. In developing countries like South Africa, with 'emerging' or 'young' democracies, a much less literate electorate, lower levels of (science and technology) education amongst politicians and smaller pools of experts, this threat increases manyfold, especially with the global transfer and trade in science and technologies, which usually take place from developed to developing nations in view of their significant development challenges.

The question of how mathematics participates or is recruited in this is explained by Skovsmose (1994), through what he calls the 'formatting power of mathematics':

> [M]athematics produces new inventions in reality, not only in the sense that new insights may change interpretation, but also in the sense that mathematics colonises part of reality and reorders it. (p. 42)

The issue here is not that mathematics itself does anything, but rather it is about how mathematics is used by people, institutions or 'agencies' through all types of applications that come to produce and result in a formatting of society. Society today is increasingly mathematised. Keitel (1993; Keitel, Kotzmann & Skovsmose, 1993) has demonstrated the complexity of this through the relationship between social abstractions and thinking abstractions. As mathematics is applied to all sorts of processes, structures, problems and organisations, these in turn change and require further mathematisation. Whilst an increasing amount of implicit mathematics is found in all areas of life today, it requires a more sophisticated mathematically literate person to question the applications within a democracy. Paradoxically, at the same time less procedural mathematics is needed as technology takes over, for example the widespread availability of calculators.

This has serious implications for the mathematics education provided across the education system to strengthen democracy and fuel development. For countries like South Africa, if the notion of the formatting power of mathematics is accepted then it is imperative to educate those who will come to participate in that formatting power and address problems of development as well as those who will need to be able to react to it to ensure that fair, equitable and just solutions are found. It may be argued that a mathematics curriculum has an obligation to produce both 'insiders' and 'outsiders' to the formatting power of mathematics. The insiders are those who come to participate in the formatting (as high performers), be they 'constructors or producers' of mathematics or 'operators or users' of mathematics. The outsiders are those who must read and react to that formatting power as (mathematically literate or numerate) consumers of mathematics or as the marginalised of society (Skovsmose, 2003; Vithal, 2004a). The recently introduced new Mathematics and Mathematics Literacy curricula for Grades 10–12 in South Africa could be said to roughly match these two requirements (discussed later). What is evident in this line of argument is that access to and competence in mathematics serve very different purposes. Both the presence and absence of mathematics education has real consequences; it is neither neutral nor value free. Mathematics is used in a multitude of ways in society: to predict, control, interpret, describe and explain within a particular cultural, economic and sociopolitical context.

Mathematics for those who will come to participate as the 'formatters' or 'high performers' in positions of power (as government or experts), especially in developing countries with scarce high-level human resources, needs to include critical engagement about development and poverty and integrate an ethics and social responsibility as part of the multiple goals of mathematics education. This is important since a democratic competence cannot be assumed and nor does 'high-level' or abstract mathematics necessarily produce an integrated critical mathematical literacy competence. This is because the thinking tools and language of mathematics do not by themselves provide the full means for criticising its applications in society. Similarly, a mathematical literacy for the majority has to be more than a functional or practical literacy. It is expected to integrate a critical and democratic competence with a mathematical competence if a citizenry is to participate meaningfully in a young democracy and growing economy and be able to grasp the mathematical basis implicit in the decisions taken for or against them. Mathematics needs to be responsive to a diversity of contexts, avoiding a ghettoisation of the mathematics curriculum and yet providing a mathematics that is inclusive of the consumers of mathematics and those who do not enter further and higher education or the world of work. It is not only necessary to know when a problem is dealt with using mathematics, whether the correct mathematics has been chosen, whether the mathematics has been correctly executed and whether the result can be relied on; it is also important to include reflections about how the use of mathematics to solve the problem relates to the broader (social, political, economic) context and its possible consequences, whether the problem could have been solved without mathematics and whether the evaluation or reflections could have been done differently (Keitel et al., 1993; Skovsmose, 1994).

The developmental challenge for mathematics education is not confined to particular parts of the world. It is a global challenge. At the Centennial Symposium of the International Commission of Mathematics Instruction (ICMI), at which mathematics educators reflected on mathematics education of the past hundred years and attempted to map out directions for the future, Setati (2008) proposed a significant new role in contributing to the acceleration and attainment of the United Nations Millennium Development Goals of eradicating poverty, promoting gender equality and universal primary education. She argued that mathematicians and mathematics educators need to work together – from different levels of the education system, in different aspects of research and practice, from different perspectives, and from different parts of the world – to address poverty, injustices, inequity, illiteracy and access to education. In proposing a shift to the developing world, she recommended ICMI studies of mathematics education within contexts of poverty, multilingualism and multiculturalism and a focus on mathematical literacy. Supporting development goals through general education in order to deepen and strengthen democracy is well understood, well known and accepted. However the key role *mathematics education* has in this, and how it can provide an introduction to democratic life and values and thereby to a better life for all, is arguably less so.

Within developing and poorer countries, mathematics education has an explicit and critical role. Lubisi (2008) spells out the direct connection between mathematics education and notions of the 'developmental state' as it is being deliberated in South Africa. For him, mathematics education is the key to empowering people with knowledge and skills that are necessary to reach the targeted economic growth rates, to create employment and to fight poverty. He argues that analyses of most of the skills areas of the economic sectors that are being targeted to ensure that this growth is achieved require mathematics. In particular, the shortage of artisans and technicians in South Africa, that is, those who are involved in various kinds of applications of mathematics as the 'users or operators', has become evident and needs to be taken seriously. Therefore, strengthening mathematics (and science) teaching in schools is important in order to reach development goals and the needs of economic growth. For developing countries, improving the basic conditions of peoples' lives, including schooling and the quality of all aspects of mathematics education is crucial to sustaining democracy.

Mathematics education provides not only access to mathematical knowledge and skills, which is important for living in the 21st century, but in many countries, performance in mathematics determines access to jobs and further or higher education studies in a range of areas, from the natural and physical sciences to economics and technology. It is for this reason that mathematics is on the one hand regarded as a gateway subject to a large number of these high-status, high-paying professions, but on the other hand also functions as a gatekeeper for the many who fail to learn and perform at the requisite levels or are failed by the education system. In this respect, mathematics education functions implicitly to stratify society. How it does this is important to analyse if it is to be addressed so as to open more and better life opportunities for all students, whatever role they come to fill as producers, users or consumers of mathematics. As a high-stakes subject, it is not surprising that that there is much concern about who gets access to and performs well in mathematics and who gets excluded.

Whilst mathematics educators have arguably not focused on developmental challenges, research and literature on the nature and extent of such challenges is emerging from outside mathematics education. The World Bank's *World Development Report* on equity and development (World Bank, 2006) offered the following South African narrative, which illustrates the complexity of engaging development contexts:

> Consider two South African children born on the same day in 2000. Nthabiseng is black, born to a poor family in a rural area in the Eastern Cape province, about 700 kilometers from Cape Town. Her mother had no formal schooling. Pieter is white, born to a wealthy family in Cape Town. His mother completed a college education at the nearby prestigious Stellenbosch University.
>
> On the day of their birth, Nthabiseng and Pieter could hardly be held responsible for their family circumstances: their race, their parents' income and education, their urban or rural location, or indeed their sex. Yet statistics suggest that those predetermined background variables will make a major difference for the lives they lead. Nthabiseng has a 7.2 percent chance of dying in the first year of her life, more than twice Pieter's 3 percent. Pieter can look forward to 68 years of life, Nthabiseng to 50. Pieter can expect to complete 12 years of formal schooling, Nthabiseng less than 1 year. … Nthabiseng is likely to be considerably poorer than Pieter throughout her life. … Growing up, she is less likely to have access to clean water and sanitation, or to good schools.

> So the opportunities these two children face to reach their full human potential are vastly different from the outset, through no fault of their own.
>
> Such disparities in opportunity translate into different abilities to contribute to South Africa's development. …
>
> As striking as the differences in life chances are between Pieter and Nthabiseng in South Africa, they are dwarfed by the disparities between average South Africans and citizens of more developed countries. Consider the cards dealt to Sven – born on that same day to an average Swedish household. His chances of dying in the first year of life are very small (0.3 percent) and he can expect to live to the age of 80, 12 years longer than Pieter, and 30 years more than Nthabiseng. He is likely to complete 11.4 years of schooling – 5 years more than the average South African. These differences in the quantity of schooling are compounded by differences in quality: in the eighth grade, Sven can expect to obtain a score of 500 on an internationally comparable math test, while the average South African student will get a score of only 264 – more than two standard deviations below the Organisation for Economic Cooperation and Development (OECD) median. Nthabiseng most likely will never reach that grade and so will not take the test. (pp. 1–2)

The above narrative is reproduced in detail to illustrate how mathematics education operates as a system to open or limit educational possibilities for students. It also points to the necessity for locating mathematics education research, practice, policy and theory within a broader landscape in relation to other aspects that impact and shape (mathematics) educational opportunities. It shows the need to link mathematics education to multidisciplinary development studies, addressing issues of concern to developing countries, and especially to social and economic development.

South Africa is a useful case as a young democracy of almost two decades within which four waves of curriculum reforms have occurred, and in which mathematics educators, mathematicians, and a range of other 'stakeholders' have had the possibility to participate in shaping the mathematics curriculum. It has been observed that there are two main imperatives driving and shaping curriculum debates in South Africa: one is the post-apartheid challenge for greater equity and social justice, to redress decades of deliberate inequalities and to entrench and deepen democratic life; and the other is the global competitive and development challenge to provide opportunities to learn and access knowledge and skills to participate effectively in the internationalised and globalised economy of the 21st century (Vithal & Volmink, 2005).

The development and democracy challenges for mathematics education are captured in the lives of Nthabiseng and Pieter. Despite each successive Minister of Education effecting some or other official curriculum change since the advent of democracy, with incremental improvements in school infrastructure and resourcing, Nthabiseng and Pieter continue to experience very different and substantially unequal implemented and attained mathematics curricula. However, in recognising the formatting power of mathematics, this discussion points to the potential of mathematics education to transform both Nthabiseng's and Pieter's lives in contributing to strengthening both democracy and development imperatives of South Africa.

Mathematics education, its distribution and educational possibilities

The second question is: How is mathematics education, in terms of mathematical knowledge, skills, values and attitudes, distributed in society and thereby shaping educational possibilities? This question is engaged through South African learners' mathematics performance in the much-publicised international studies and the national Grade 12 examination results. Possible explanations for these are then explored in terms of the mathematics curriculum, its recent reforms and related teacher education challenges.

One of the ways in which the distribution of mathematics education is made visible and public is through international studies of student mathematics performance and national tests and assessments. In the public imagination, shaped by the media, mathematics education is reduced to league tables of student mathematics performance scores. South Africa's repeated ranking at the very bottom of international studies of student mathematics performance and equally poor outcomes in the annual high-stakes national Grade 12 matric examination results, when each are released, follow with endless speculation about the reasons and causes of South Africa's continued poor mathematics performance.

The mathematics performance of Nthabiseng and Pieter and their consequent educational possibilities and life journeys allude to the deeply unequal conditions of schooling and opportunity to learn which have endured almost two decades since the advent of democracy. Inequities in the quality of South African schooling and living conditions are reflected in the test and assessment outcomes. However, an aspect that has not received much public attention is whether these studies, tests and assessments do indeed offer an accurate account of the mathematics knowledge and skills of learners. If all learners are deemed to have some mathematical knowledge by virtue of having lived in particular communities or cultures, as ethnomathematics for example argues, what do the tests in fact reveal, if anything, about what mathematics learners know and understand?

These large-scale assessments, which are costly to mount, are often driven as much by political imperatives as they are by educational ones and conducted within funding and other constraints. Methodological issues about the language in which the tests are conducted, familiarity with the format of the test items and the reliance on only paper and pencil assessments are seldom discussed publically to qualify the outcomes and findings. Even though these international studies have long come under criticism by mathematics educators and the use and misuse of their results cautioned against (e.g. Kaiser, Luna & Huntley, 1999), they have been latched onto by governments, including South Africa's, and used to introduce wide-scale national testing regimes. In the UK, which has a history of national testing, studies based on these national tests to explain leaner performance demonstrate the caution with which these results need to be interpreted. Cooper and Dunne (2000) showed, by comparing test and interview data, that many children fail to demonstrate in tests mathematical knowledge and understanding that they actually possess. They showed learners' confusion over the requirements of 'realistic' test items as compared to 'esoteric' items, and how this was particularly the case for children from working-class backgrounds. This is an important finding, relevant to the new South African school curriculum, which foregrounds applications and context in mathematics, as the newly introduced annual national assessments are being implemented at all levels of the education system. It raises serious questions about what the assessments really reveal and about whom or what.

The point here is not to suggest that testing should not take place, but it is necessary to understand how such test outcomes have become a public window to the mathematics classroom and have come to generate a particular discourse about the distribution of mathematical knowledge to which politicians and policymakers are particularly responsive. The outcomes of such studies need to be qualified with reference to issues of methodology and contextualised historically, taking into account the sociopolitical, economic, urban–rural and cultural dynamics. In the Trends in International Mathematics and Science Study, which yet again confirmed South Africa's poor performance, Reddy (2006) categorised learner performance scores according to the previously racially segregated schools and showed, not surprisingly, that despite their levels of resourcing, the former White schools (Model C) performed only at the international average, whilst former African schools performed at half the average of the White schools. African schooling has come under intense scrutiny, has had a myriad of interventions and is being researched for its lack of effectiveness. White schooling, however, has remained outside the research gaze and has not been interrogated, for example, for its failure to exceed international averages given their disproportionate share of considerable resources and other advantages. Much less is known about why Pieter is not performing as well as Sven, given comparable educational contexts and advantages.

Another very public lens on the distribution of mathematical knowledge and skills is the high-stakes annual Grade 12 mathematics matric examination, written by some half a million students each year, which plays one of the most important direct roles in apportioning further educational opportunities. These results are released at the end of each year amidst much fanfare and commentary.

We could well ask: What are the chances Nthabiseng and Peter would be amongst those to have studied and passed mathematics? Issues of race and gender have been foregrounded in post-apartheid South Africa in considerations of who gets access to higher education. Analyses by Kahn (2005) demonstrate a steady increase in the numbers of African students studying and passing mathematics at the Higher Grade (HG), which provides eligibility to access university. The 1991 figures have increased tenfold by 2005, with just under 10 000 African students successfully passing HG matric mathematics. This is, however, a miniscule proportion relative to both the numbers of African learners that enter schooling in Grade 1 and those who make it to Grade 12 each year. According to Kahn (2005) 'the white community generates science based skills at something close to saturation level' (p. 142). Pieter is assured of a pass in mathematics to enter university but what of Nthabiseng?

Although overall gender differences in participation and performance in matric mathematics are not significant, when race is intersected with gender, major differences are found between African females and other female candidates. Restricting analyses of performance to only gender has been found to mask large disparities in matric HG mathematics pass rates for African females, which in 2002 were found to be only a quarter of the pass rates of White females (Centre for Development and Enterprise, 2004). It is possible to speculate that even amongst her White, Indian and Coloured sisters, Nthabiseng has had the system odds stacked against her passing mathematics to secure entry into higher education. The issue of race, however, is a vexed one and the Department of Basic Education, in releasing matric performance statistics, has in recent years reported on gender but not race data. Notwithstanding the dangers and arguments against entrenching apartheid constructed racial categories, this analysis demonstrates that such information is crucial to developing appropriate and targeted interventions that can impact the most marginalised in society. Being able to identify the Nthabisengs of South Africa for redress to ensure she does not continue to carry a disproportionate burden of apartheid's damage.

The South African national mathematics curriculum underwent a major reform for the Grade 10–12 band in which all Grade 12 students from 2008 wrote either a Mathematics or a Mathematical Literacy examination. These changes were made to address two main problems that directly relate to the mathematical knowledge distribution and education opportunity challenges. The first problem was that although the numbers taking mathematics (whether HG or SG [Standard Grade]) in the period from 1995 to 2007 had risen to above 60%, much of this increase was due to an increase in SG mathematics enrolments but it was a pass in HG mathematics that typically gave entry to higher education opportunities. The numbers studying and passing the sought-after and much-needed HG remained low and appeared to have reached a ceiling that remained below 30 000. The mathematics taught at SG, which was largely procedural and excluded key topics that were found in the HG curriculum, did not provide the kinds of mathematical competencies needed for further higher education studies to fuel the high-level science, economic and technology needs of the country and sustain the supply of constructors or producers of mathematics or indeed even the users or operators of mathematics (Skovsmose, 2003; Vithal, 2004a). A second problem that contributed directly to inequities in the distribution of mathematical knowledge and skills was that at least 40% of matriculants each year did not take any mathematics at all and hence were not taught any mathematics in schools between Grades 10 and 12. This meant they were not provided with even the competence to be consumers of mathematics that a citizenry in a democratic South Africa of the 20th century should at least have acquired through schooling. Mathematical Literacy was intended for this group.

One of the difficulties that the new Mathematics and Mathematical Literacy curricula have faced is a lack of consensus and clarity about what each of these are, their relation to the previous HG and SG mathematics curricula and their relation to each other. As a new subject, Mathematical Literacy faces a particular difficulty in escaping the image of a practical or functional lower order mathematics (for those who were deemed incapable of or uninterested in doing mathematics) rather than being conceptualised as a different integrated contextualised competence. Deriving from a broader 'mathematics for all' movement, it has been variously labelled within policy, theory, research and practice as numeracy, quantitative literacy (Steen, 2001) and mathemacy (Skovsmose, 1994), amongst others. This points to different ideological orientations, intentions and goals of such a mathematics, which extends from a concern with acquiring basic mathematics to a sophisticated critical integrated competence (Vithal, 2004a). The most recent South African Mathematical Literacy curriculum policy describes some of the key elements as involving 'the use of *elementary* mathematics', '*authentic* real-life contexts', decision-making and communication and 'the use of integrated content and/or skills in solving problems' so that they become 'participating citizens in a developing democracy' and 'astute consumers of mathematics in the media' (Department of Basic Education, 2011b, pp. 8–10, [*emphasis in original*]).

On the introduction of the new curricula, fears that large numbers of students would choose Mathematical Literacy rather than Mathematics were not fully founded. The numbers who followed the new Mathematics curriculum in 2008 was just over half of all matriculants. However, the more troubling observation since then is that the number choosing Mathematics has decreased year on year. From a high of 300 000 taking Mathematics in 2008 the number had decreased by 25% in 2011(Department of Basic Education, 2011a). Although many more students are deemed to be succeeding in Mathematics, the number passing at 40% relative to those studying Mathematics have stabilised at a low of about 30% whilst the number passing at 30% has hovered between 45% to 47% (Department of Basic Education, 2011a). The new Mathematics curriculum (Department of Basic Education, 2011c), it would appear, is being differentiated by assessment rather than by content and levels of difficulty. These changes have, however, increased the numbers of students eligible for entry into university and opened another debate about their readiness to pursue and succeed in higher education programmes.

Which learners are allowed to do mathematics and the quality of mathematics education learners receive in school are shaped by many factors. Although all secondary schools must now offer Mathematical Literacy and system-wide interventions have taken place for its implementation, the same has not been obtained for the delivery of the new Mathematics curriculum, which is much more demanding than the previous SG curriculum and is much more application oriented, with several new areas and topics, compared to the former HG Mathematics curriculum. The new Mathematics curriculum has been implemented in a context in which only half of all secondary schools who previously offered mathematics offered it at the HG (Centre for Development and Enterprise, 2004). This means that the opportunity to learn mathematics is limited in real terms for those learners who find themselves in schools that do not offer the new curriculum or, where it is offered, do not have appropriately educated and trained teachers to deliver it. We could ask: In which schools are Pieter and Nthabiseng likely to find themselves and what teachers are they likely to encounter?

By all accounts, teachers are critical to the delivery of any curriculum. From this perspective the success or failure of the new Mathematics curriculum hinges on the question of what further education and training provisions are being made at a system level for the substantial cohort of teachers that were teaching the 300 000 SG Mathematics students in 2007 and were then required from 2008 to deliver a new, different and more demanding Mathematics curriculum. The new official intended Mathematics curriculum has been found to compare well internationally, regarded to be as difficult as or more so than the previous HG Mathematics and as embodying best practices and knowledge about pedagogy and content that should go some way toward preparing students for the scientifically, technologically and mathematically advancing society of the 21st century. It is, however, in the implemented curriculum, at the school and at the classroom level, that the challenges are to be found. No doubt many different kinds and levels of resources and infrastructure are needed for the successful implementation of the new curricula; however, the main lever is the quantity and quality of competent and confident teachers who can deliver the new Mathematics curriculum and thereby shape South Africa's democratic ideals and contribute to its development goals.

To understand the extent of this challenge, of mathematics knowledge and skills of teachers, it is necessary to appreciate the historical legacy of mathematics teacher education from a system point of view. In South Africa, through the deliberate underdevelopment of apartheid, the education system has inherited a substantial core of teachers with diplomas as opposed to degrees, and an uneven preparation in the core content knowledge of mathematics. This legacy remains intact and must be addressed for any radical break with the past and for substantial improvements in providing learners with adequate and appropriately qualified mathematics teachers to acquire the kinds the mathematical knowledge and skills the official curriculum promises.

The magnitude of the task relates to both the issue of supply of new teachers and the continuous education of existing teachers. In the past decade, the number of students seeking to become senior secondary teachers of mathematics has not kept pace with demand as teaching is unable to compete with the status, remuneration and prestige of other expanding career options in science and technology, given the small pool of successful candidates in matric mathematics. This problem may acerbated the policies introduced in the late nineties to redistribute teachers, which resulted in a number of qualified mathematics teachers exiting the system. Furthermore, there has been no systemic state intervention for upgrading mathematical content knowledge at these higher levels, for example, systematically targeting *all* former mathematics teachers who were only able to teach SG mathematics. More than a decade since the first mathematics teacher audit (Arnott, Kubeka, Rice & Hall, 1997), approximately 20% of Grade 10–12 mathematics teachers are professionally unqualified and of those that are qualified, still only 21% have some university level courses (Parker, 2010). There is also evidence to suggest that qualified mathematics teachers in the system are either not teaching mathematics or not teaching it at the level at which they are qualified (Parker, 2010; Peltzer et al., 2005), for a number of different reasons, showing that a limited and scarce resource is being poorly utilised.

In South Africa, every new minister of education since 1994 has introduced curriculum reforms, resulting in several waves of curriculum changes. In this context, the fragile and weaker parts of the system are more likely to become dysfunctional. Teachers require time to be inducted into new or changing content and pedagogy. For teachers who may be struggling with mathematical content knowledge, forms of assessments and their associated pedagogical reforms, this may acerbate the problem especially in poorer and under-resourced schools and classrooms, the kind of classroom in which Nthabiseng is likely to find herself. A vicious cycle persists if a curriculum reform is evaluated too soon, when it is more likely to show a dip in performance areas as the new curriculum is still bedding down; and if further changes are introduced before they are thoroughly understood and institutionalised. It is in this respect that stability in the official curriculum is crucial so that teachers are given a chance to interpret and give effect to the curriculum. In this context, it is assumed that other foundational infrastructure is in place, such as adequate and timely provision of core mathematical teaching and learning resources, for example, appropriate quality textbooks relevant to and necessary for each curriculum reform.

Attempting to increase and better distribute educational opportunities for many more learners to effectively break the glass ceiling of mathematics performance, particularly for those at the margins, in the most impoverished parts of the schooling system, requires a targeted, systemic and systematic long-term mathematics teacher development intervention, a stable curriculum policy environment, and, at the very least, a critical level of resourcing and schooling infrastructure for the mathematics education system to function.

Mathematics education pedagogy as democratic living and engagement with development

The third question is: Can democracy in mathematics education refer to the very life of a mathematics classroom, learning democratic values, democratic attitude and democratic competence in a context that recognises and seeks to address issues of development? This question is explored through a discussion on pedagogies of conflict and dialogue, and of forgiveness.

The new mathematics national curriculum policy reforms in all their different waves in post-apartheid South Africa, take as their point of departure quite explicitly the new constitution and provide the imperative for teachers to explicitly make connections between mathematics and the real world. The question of whose world gets selected, by whom and for what purpose in a mathematics classroom then becomes important. Teachers make selections of content, context and pedagogy and realise different kinds of actual and hidden curricula, for instance, in choosing to teach about mathematical modelling through the context of HIV and AIDS or some other development challenge or inequity in society.

In his study on teaching and learning mathematics for social justice in an urban Latino school, Gutstein (2003) showed how mathematics can be taught in a way that develops learners' sociopolitical consciousness and sense of agency, develops positive social and cultural identities through a classroom pedagogy that assists them to 'read the world (understand complex issues involving justice and equity) using mathematics' (p. 37), develops mathematical power in the ways in which they do and think mathematically, and thereby changes their disposition and orientation toward mathematics. Much of the foundation for this kind of pedagogy was laid in the eighties and early nineties and has spawned a diverse literature in mathematics education describing and analysing activities and theoretical ideas that explored a political mathematics education (e.g. Mellin-Olsen, 1987) or critical mathematical literacy (e.g. Frankenstein, 1987). It has continued in different forms and endured to the present in debates particularly about equity, quality and social justice in mathematics education (e.g. Atweh, Graven, Secada & Valero, 2011)

An overtly political approach to mathematics education also has early roots in South Africa. 'People's mathematics for people's power' was a part of the broader phenomenon of the People's Education movement that arose during the apartheid era, which viewed schools and classrooms, including the mathematics classroom, as important sites for the struggle against apartheid. A number of mathematics educators engaged these early ideas in their teacher education programmes at the time (Adler, 1991; Breen, 1986; Julie, 1991). Suffice it to say here, not surprisingly there was deep contestation and resistance as any historical account of the people's mathematics movement demonstrates (Vithal, 2003). However, as a society still struggling with deep inequalities and continuing injustices, the question of whether mathematics education can participate in moving us toward more humanitarian goals – democracy, equity, social justice, non-racism, non-sexism – is as relevant today as, or perhaps more so than, it was all those years ago. The new South African mathematics curriculum provides a policy space for such engagement but the question of teachers' implementation of such a pedagogy remains open.

It was with this explicit concern and ideological orientation that I undertook my doctoral study in the mid-nineties (Vithal, 2003) and from which this section of the article draws in pointing to a pedagogy that emerged from empirical work, and which I reflect on and extend. I explored the question of what happens in mathematics classrooms when student teachers attempt to realise what could be called a social, cultural, political approach to the mathematics curriculum that integrates a critical perspective in practice. Although the student teachers were introduced to diverse practices related to this broad approach, the dominant curriculum practice engaged by them was that of project work (Vithal, 2004b, 2006). The particular conception of project work employed was one that is well developed within the Scandinavian context (Olesen & Jensen, 1999) and widely implemented and researched in mathematics teaching and learning from primary to university education (Christiansen, 1996; Nielsen, Patronis, & Skovsmose, 1999; Niss, 2001; Vithal, Christiansen & Skovsmose, 1995). It specifically seeks to develop a critical perspective through an approach that is problem orientated, interdisciplinary and participant directed. By choosing exemplary problems of societal relevance to investigate, learners develop both knowledge and skills and the means for critiquing that very knowledge and skills.

An in-depth study and detailed description of how one student teacher, Sumaiya, taught mathematics in a Grade 6 mathematics classroom of African and Indian learners, in a school located in a predominantly Indian suburb, enabled an interrogation of the theory and practices advocated in the literature that seek to construe mathematics classrooms and schools as spaces for enacting democratic life. The student teacher, class teacher and learners engaged in groups a range of projects that required enacting democratic life as they dealt with issues of development – from creating a mathematics newsletter and questioning the school's use of their school funds and provision of facilities to the inherent inequalities and gendering of time for mathematics homework. Sumaiya brought a deeply reflective perspective as she grappled with introducing what were considered radical ideas about teaching mathematics in which democratic approaches were engaged in dealing with real micro local problems of development selected by learners.

The study generated a thesis of a pedagogy of conflict and dialogue embedding five dual-concept themes: freedom and structure, democracy and authority, mathematics and context, equity and differentiation, potentiality and actuality. These concept pairs capture the multifaceted and multidimensional nature of mathematics classrooms that choose to engage matters of democracy and development in this direct way. Conflict and dialogue and the dual-concepts are themselves explained as being in a relationship of complementarity, a complex relationship of cooperation and opposition. Drawing specifically on the interpretation offered by Otte (1990), complementarity allows one to see concepts (such as object/content and tool) firstly as woven together, each presupposing the other, where the one cannot be defined or described without the other, and secondly as contradictory to each other, opposing each other, where the one does not directly show itself in the other. The mathematics classroom in this framing is seen as a functional whole, not only in the school but also linked to the broader societal setting in which it is located. It is a space fraught with conflict and contradictions, but also containing all the possibilities and hope for their engagement and potential for resolution.

In a pedagogy of conflict and dialogue, in particular, not only are conflict as content and dialogue as tool in a relation of complementarity, but there are also complementarities within each. If mathematics classrooms are to be spaces not only for learning about democracy but also for enacting democratic life, then conflicts and crises of society will become part of classroom life and dialogue is needed, even as it is resisted. Dialogue as an epistemic or didactic tool (Mellin-Olsen, 1993) to deal with different kinds of (knowing/knowledge) conflicts functions to provide a better understanding not only *of* each other but also *with* each other. For Mellin-Olsen, dialogue is not a search for consensus or compromises as much as it is a search for deeper insight with the partners of the dialogue. The disagreement or conflict has to be engaged in a way that does not destroy the dialogue, which creates a paradox: 'confrontation and disagreement … have to be developed in a context of agreement and co-operation' (Mellin-Olsen, 1993, p. 256). Hence, such a mathematics education legitimates not only engaging with different kinds of knowledge conflicts but also learning to dialogue and about dialogue as a method of confrontation and cooperation.

Both conflict and dialogue are needed in a pedagogy that attends to issues of democracy and development from a critical perspective. In South Africa, Nthabiseng and Pieter may well find themselves in the same mathematics classroom. However, they are arguably more likely to find themselves in a school where one or the other dominates:

> We could imagine a pedagogy of conflict without dialogue degenerating into anarchy and chaos or dictatorship. In

> current actual situations in South African schooling [*more likely Nthabiseng's school*] we have seen pupils' expression of dissatisfaction with the school, its authority, structures and dimensions of differentiation, expressed through violent means and then curbed through enacting stronger forms of autocracy rather than democracy. … Here, we can see how a pedagogy of dialogue is essential to a pedagogy of conflict especially if democracy, freedom, context and equity are to be valued in schools.
>
> We could also imagine a pedagogy of dialogue without a pedagogy of conflict reduced to benign endless rounds of entertaining, interesting safe talk and action. Teachers in more advantaged schools [*more likely Pieter's school*] could talk about the inequalities and injustices brought about through apartheid in a pedagogy of dialogue without conflict. The pupils in this pedagogy could never really come to make connections between the apartheid past and the present, or question or act on the conflicts immediately around them. …
>
> A pedagogy of conflict and dialogue means therefore that each, conflict and dialogue, presuppose the other in a mathematics curriculum approach that seeks to focus on social, cultural, economic and political aspects of society. They are separate and each must be developed independently, conflict as content and dialogue as tool. But they are also connected, and therefore must be realised in relation to each other in a classroom. (Vithal, 2003, p. 356)

In a mathematics education that embeds a critical perspective, there is no doubt a level of risk and uncertainty that attempting a pedagogy of conflict and dialogue invokes, especially when the inequalities and injustices do not reside in some distant place or time but are embodied in the very students and teachers in the classroom and each is somehow seen as implicated as 'victim' or 'perpetrator'. Even though not all mathematics can be taught with reference to context, given its abstract nature, creating some spaces in the curriculum for critical societal issues of development, diversity, equity and social justice must be argued for. Yet it is likely to be resisted in much the same way as People's Mathematics was, though for different reasons, by different parts of the schooling system.

Schools in general and mathematics classrooms in particular need to become spaces for learning about and through democracy or we risk repeating past failures. That so many members of the White community in South Africa continue to claim today not to have known about the huge suffering of Black people perpetrated in their interest can be analysed as a most serious failure of White schooling and mathematics education. That young White learners who come through White schooling today fail to appreciate their positionality and privileged inheritance from the injustices of the past continues to be a failure of White schooling. The 'whiteness' of White schooling is to be understood in terms of not only its demographics, but also its rituals, rules and traditions that enculturate its members into a culture of 'whiteness' and has remained largely outside the mathematics educational research gaze. Much of the attention has been focused on the deficits of Black education and much less on the pathologies of 'whiteness' in how it is being reproduced or transformed in post-apartheid South Africa. Not surprisingly a pedagogy of conflict and dialogue is less likely to be enacted in diverse settings, where it can be a dangerous if not painful path to tread, where White teachers and learners cannot escape being seen as 'perpetrators' and Black teachers and learners cannot escape the feelings of 'victimhood' and suffering.

It is in this respect that a pedagogy of conflict and dialogue must also integrate a pedagogy of forgiveness (Waghid, 2005). A mathematics that reveals inequities and injustices of the past or present is likely to produce feelings of resentment and hate. In such contexts, as Waghid (2005, p. 226) notes, 'learning about forgiveness can become useful in enhancing pedagogical relation' and when teachers and learners 'cultivate forgiveness', it becomes a way to 'engender possibilities whereby people are attentive to one another' and can engage 'imaginative action' to move forward. A pedagogy of conflict and dialogue for a mathematics education for equity and social justice invariably opens wounds so that the 'truth' can be known, even relived, and understood. Each learns by being in the place and experience of the 'Other'. But if such a pedagogy is not to run the risk of deepening divides and difference then it must provide a means to heal. A pedagogy of forgiveness integrates into conflict and dialogue, a point of hope and creative action. The principle of hope, Skovsmose (1994) argues, needs to be preserved in a critical mathematics education. It is not surprising that 'forgiveness pedagogies' have emerged and are being engaged and studied within educational settings in societies that have had histories of political conflict and trauma (Zembylas & Michaelidou, 2011) as part of processes of reconciliation. Such pedagogies are needed in any mathematics education enacted with a consciousness for issues of development and deepening democracy.

Both a pedagogy of conflict and dialogue and a pedagogy of forgiveness take their bearing from South Africa's own post-apartheid processes of the Truth and Reconciliation Commission (TRC) (1998). In doing so, these pedagogies require the creation of spaces for 'truth' to be told so that reconciliation can occur, and mathematics by its power and status in society opens a unique and special way for such truths to be told. Only then can dignity be reclaimed, compassion shown and respect and friendship built. Critical, feminist and social justice mathematics pedagogies have sought to mobilise the power of mathematics knowledge and skills to overt political and social agendas. But in order for restoration, healing and peace to emerge, such pedagogies will have to attend not only to mathematics education pedagogies but also to mathematical knowledge itself.

Democracy, development and mathematics content issues

The fourth question is: Can mathematics education, democracy and development have something to do with mathematics content matters? This question is discussed by appropriating, as a metaphor, South Africa's TRC's multiple notions of truth in seeking a similar framing for linking the various forms and movements in mathematics and mathematics education that have emerged in the last few decades.

The myth that mathematics and mathematics education are neutral and value free has long been exploded. D'Ambrosio (1994) implicates mathematics in both the beauty and the devastation brought by advances in science and technology and raises serious questions for mathematics education for the 21st century. Ethnomathematics, critical mathematics education, mathematics for equity and social justice are areas of study and practice that have grown rapidly in the last few decades, forcing a re-examination of what constitutes and counts as mathematical knowledge, questioning how it has been and continues to be produced and legitimated, raising issues about who is recognised for its production and problematising mathematics curricula for their purpose, relevance and appropriateness for different groups in society they are intended to serve.

These different orientations to mathematical content, which appear discrete and disconnected, may be brought into relation with each other by appropriating, as a metaphor, a framework from South Africa's TRC (1998). The TRC enabled South Africans in post-apartheid society to confront the truth about apartheid by acknowledging, legitimating and validating multiple forms of 'truth'. It may be similarly proposed that there are different forms of 'truth' constituting mathematics and that engaging more than one 'mathematical truth' in a mathematics classroom, especially in a pedagogy of conflict and dialogue and of forgiveness, is necessary for both learning and reconciliation to occur.

The TRC dealt with the complexity about what constituted truth and whose truth by developing a conceptual framework comprising four notions of truth: factual or forensic truth, dialogue or social truth, personal or narrative truth, and healing or restorative truth. Each of these truths in turn provides a means for presenting recent challenges to mathematical content questions as multiple forms of 'mathematical truths'. This framework is useful to engage questions of what is taken to mean and count as 'mathematical truth' and whose 'mathematical truth' is privileged in mathematics education. It allows for a way of bringing divergent notions of mathematics content that have emerged into a single framing that enables these to coexist.

The first kind of truth recognised in the TRC (1998, p. 111) was *forensic or factual truth based on 'objective information and evidence'* that could be corroborated. This was truths that could be validated through impartial objective procedures. They were considered 'scientific truths' which utilised 'empirical processes' and were also regarded as 'legal truths'.

In mathematics education, for many there is only one objective mathematics variously described as academic mathematics or school mathematics, a canonical mathematics knowledge, free of context or social, political or cultural bias and unambiguously identifiable and articulated in the official curriculum at all levels of formal mathematics education. 'Mathematical truths' are in the main those truths that can be proved and there are universally agreed ways for establishing these truths. It is a powerful mathematics underpinning and manifest in much of the material, technological, scientific and social world today. This conventional mathematics is discernible through its own signs and symbols and its own discourse even when written in different natural languages.

However, this mathematics has increasingly been challenged, paradoxically referred to as 'Western' mathematics and seen as a product largely of Western culture. It is regarded as a paradox since many nationalities and cultures have and continue to contribute to its development. For Bishop (1990, p. 51), mathematics, 'as one of the most powerful weapons in the imposition of Western culture', has participated in 'the process of cultural invasion in colonized countries' through at least three agents: trade and commerce, for example, units, numbers, currency; mechanisms of administration and government, for example, computation systems; and imported systems of education, for example, mathematical curricula for the elite few. Despite being seen as most outside the influence and realm of the social or cultural, this mathematics is deeply implicated in the distribution and enactment of political power. This is also one of the major criticisms launched by ethnomathematics. Powell and Frankenstein (1997) outline the main goal of ethnomathematics as challenging the particular ways in which Eurocentrism permeates mathematics education

> [*in*] that the academic mathematics taught in schools world-wide was created solely by European males and diffused to the Periphery; that mathematics knowledge exists outside of and unaffected by culture; and that only a narrow part of human activity is mathematical. (p. 2)

Bishop (1988) concludes that

> mathematics must now be understood as a kind of cultural knowledge. … Just as all human cultures generate language, religious beliefs, rituals, food producing techniques, etc., so it seems do all human cultures generate mathematics. (p. 180)

Although mathematics can be thought of as a cultural product generated by different cultures in different social, political, economic environments, this does not mean that the forms such mathematical knowledge take are completely indistinguishable from each other. Bishop identified five fundamental activities that he argues are universal across all cultures that have been studied: counting, locating, measuring, designing, playing and explaining. All cultures have, for example, developed systems for counting, but how these are organised, the number words and symbols used differ and are tied to the contextual needs and conditions of different peoples. Numeration systems in Africa range from a few number words of some San people who live in desert areas to complex systems developed by those who have a long history of commerce, such as the Yoruba of Nigeria who have been urbanised farmers and traders for many centuries before colonialism and use a vegesimal system that requires both addition and subtraction to express a number, for example, $525 = (200 \times 3) - (20 \times 40) + 5$. The new South African mathematics curricula explicitly recognise mathematics as a cultural product and this has relevance for how access is provided to academic mathematics and also for valuing the different mathematics that learners bring into the classroom by virtue of the knowledge and skills they acquire from their community and life experiences.

D'Ambrosio (1985), as a founder of ethnomathematics, has argued that there are many mathematics, of which academics mathematics is but one, and these mathematics are developed by different sociocultural groups – from engineering mathematics to the mathematics of basket weaving. For him, 'Mathematics … are epistemological systems in their socio-cultural and historical perspectives' (D'Ambrosio, 1991, p. 374):

> This is a very broad range of human activities which, throughout history, have been appropriated by the scholarly establishment, formalised and codified and incorporated into what we call academic mathematics. But which remain alive in culturally identifiable groups and constitute routines in their practices. (D'Ambrosio, 1985, p. 45)

A mathematics education TRC, if it were to be held, would from the evidence led by ethnomathematicians point to a second kind of truth conceptualised by the TRC, that is, *social or dialogue truth*. Whilst 'the first (truth) is factual, verifiable and can be documented and proved', social or dialogue truth according to the TRC, is 'the truth of experience established through interaction, discussion and debate' (p. 113). This kind of truth acknowledges the importance of participation, of listening carefully, and in which 'all possible views could be considered and weighed one against the other' (p. 113). The TRC argues that social truths established through dialogue promote transparency and democracy as a basis for affirming human dignity and integrity. The process of establishing the truth is as important as the truth itself.

If mathematics is understood as a cultural activity and product then it follows that different groups in society come to develop different kinds of mathematics to deal with problems and needs they face, whether or not they refer to this as conventional mathematical knowledge. Moreover, the process by which a 'mathematical truth' comes to be established as a truth is as important as the truth itself, if we follow this notion of the TRC truth. In this regard, it is possible to refer to two broad areas: the mathematics of traditional societies, of indigenous peoples in both the developed and developing worlds, and the mathematics of different social and cultural groups in societies of today, of adults and children. These could be deemed 'dialogue or social truths' constituting a form of 'mathematical truths' that have come to be established over time by particular peoples or communities.

Although mathematics as a category is often not found in traditional or indigenous cultures, those who study mathematics in such contexts draw on a range of methodologies and disciplines, such as anthropology, archaeology, history, linguistics, economics, art, literature and oral traditions. Ongoing ethnomathematics research in particular has demonstrated that a wide variety of mathematical ideas are found in traditional cultures. These ideas have been elaborated through games, patterns, art, architecture, systems of time and money, logic, kinship relations, and practices and artefacts used in everyday and traditional life. In each culture or community, certain groups or individuals share a mathematical disposition and are in a sense custodians of mathematical ideas that evolve over time (Ascher, 1991), making comparisons across cultures difficult. It is however by linking mathematical developments to broader social, cultural, historical and political changes that descriptions of the mathematics of different groups (Joseph, 1991) and the 'mathematical truths' of different peoples may be valued and can be understood in an authentic and unprejudiced way.

A large and growing body of research has also shown that mathematical knowledge is generated by different groups of adults and children in a wide variety of contexts outside formal schooling. Studies involving dairy workers, carpenters, bookies, builders, fisherman, farmers, street vendors, shoppers, market sellers, dressmakers and many others have all been shown to develop efficient strategies for solving mathematical problems in their everyday life and work situations. Informal mathematical concepts and skills have been observed in children across nationalities, social classes and cultures. However, the mathematical understanding that children acquire has been explained to be rooted in their social and cultural experiences and may not resemble those expected or required in mathematics classrooms. This 'distance' between school culture and different groups in society has been analysed not only with respect to learners from traditional or indigenous cultures, but also with respect to other marginalised groups within Western society, such as women and the working class. Differences and similarities between school mathematics and out-of-school mathematics have been documented. For example, school mathematics is predominantly written, whilst oral forms have been found to characterise out-of-school mathematics (Nunes, Schliemann & Carraher, 1993).

The third kind of truth set out in the TRC framework, *personal or narrative truth*, is one in which each person is 'given a chance to say his or her truth as he or she sees it' (p. 112). It is a truth based on the lived experiences of the individual who is reporting, a form of truth that reflects the 'constructed nature of meaning-making' (Dhunpath & Samuel, 2009, p. x). This form of truth was recognised as recovering national memory that had been officially ignored, a 'validation of the individual subjective experience of people who had previously been silenced or voiceless' (TRC, 2008, p. 112).

In mathematics education, it has long been recognised that each person develops mathematics ideas, knowledge and skills by virtue of their individual thinking processes and schemas. More recently, it has been recognised that this knowledge is also acquired by an individual by virtue of the community in which that person lives, works and functions. Whilst the former has derived largely from psychological perspectives, the latter has arisen from more sociological ones.

A substantial and well-established body of research in the area of (socio) constructivism demonstrates how individuals make sense and meaning of new mathematical ideas in terms of the frameworks each person has and how each develops strategies for dealing with mathematics that they are confronted by. Many of the studies of groups identified above have come out of in-depth research into how individual learners (adults and children), in varying contexts, reason and think mathematically and do mathematics. A key challenge for teachers of mathematics is to be able to discern the kinds of mathematical ideas, knowledge, skills and even attitudes each of their learners brings to school to provide access into school mathematics. Each learner develops their own 'mathematical truths', by virtue of their personal life trajectory, conditions and opportunities for learning, which have to cope with differences and conflicts between mathematical practices in school and in out-of-school contexts, for example, when they migrate from rural to urban areas.

The official South African mathematics curricula give due recognition to the unique experiences and knowledge of the individual but the question that remains is accessing the 'mathematical truths' of learners. The challenge is to avoid stereotyping students, for instance choosing a mathematics problem involving traditional Zulu home building if there are African learners in a diverse classroom, even though none of the learners may have lived in or experienced a rural context. These personal 'mathematical truths' carried by each learner are also rendered invisible and not accessed or accommodated in national and large-scale mathematical assessments.

A fourth kind of truth in the TRC is *healing and restorative truth*. It is 'the kind of truth that places facts and what they mean in the context of human relationships – both amongst citizens and between the state and its citizens' (p. 114). In the TRC it was not enough to establish what was the truth, as objective and factual, but it was equally important to see it as connected to how it was acquired and the purpose it was to serve. The role of 'acknowledgement' was highlighted as a form of affirmation of dignity by placing information on record and publically recognising it.

If what counts as mathematical knowledge and truth in mathematics is broadened then it should be possible to admit and accept that there are different mathematics within and across societies. However if the power of mathematics as an abstract knowledge is to be maintained and owned by all, then the relations between academic, Western or conventional mathematics and the different mathematical knowledges and practices of different groups and individuals have to be brought into dialogue with each other, to be connected and contextualised. By valuing different kinds of mathematics and ways of knowing (and doing) mathematics, different peoples are valued and respected. Notwithstanding that the playing field of the different mathematics are not level, for mathematics to have a restorative power in situations of conflict, there has to be at the very least, recognition that there are different ways of knowing the world mathematically, which may be relevant, useful and appropriate in different contexts. The enormous power of academic mathematics to cast its gaze on almost any human activity today and re-present or appropriate it through its discourse gives healing and restorative 'mathematical truths' a particularly important place in mathematics classrooms. The legacy of colonialism and apartheid, which damaged the growth of indigenous knowledge systems, must be addressed both for its own sake to reclaim lost and hidden 'mathematical truths' and also because it provides possibilities for new knowledge, even if defined in terms of academic or Western knowledge systems.

The role of 'acknowledgement' in restoring dignity lies in the recognition that different cultures on every continent, in different periods of its history, have contributed mathematical knowledge. Acknowledging multiple histories is part of healing. The hegemony of Western or academic mathematics has been challenged for the ways in which conventional histories of mathematics have ignored, marginalised, devalued or distorted the contributions of peoples and cultures outside Europe – of China, India, North Africa and the Arab world – to that mathematics that is referred to as academic or Western mathematics. Joseph (1991) points out that

> [s]cientific knowledge which originated in India, China and the Hellenic world was sought out by Arab scholars and then translated, refined synthesised and augmented at different centres of learning… from where this knowledge spread to Western Europe. (p. 10)

However, Eurocentric historiographies of mathematics have also been criticised from another perspective: for failing to acknowledge the independent histories of mathematics of peoples who have developed their own mathematics, particularly the indigenous peoples of different regions of Africa, America and Australia (Ascher, 1991). A healing and restorative mathematics would therefore be one that recognises the rich mathematical histories of peoples not only in terms of conventional mathematics but on its own terms and its own forms, which may or may not be easily distinguishable as mathematics, and would be dignified by being given a proper space and engagement in mathematical curricula.

Recognising multiple 'mathematical truths', as well as the processes by which these truths come to be constructed, allows for improved possibilities for the critique of truths in mathematics to be found within mathematics. In particular, these varied forms of 'mathematical truths' have the potential to make visible and more explicit the formatting power of mathematics, which acknowledges each 'formatter', from the constructors or producers of mathematics to the consumers and those marginalised, because each kind of personal, social or academic 'mathematical truth' is part of a network of truths in mathematics and each is seen to have value.

Within this framework of 'truths', 'mathematical truth' as factual, objective, invariant and decontexualised may be deemed but one kind of truth within a framework of 'truths' that need to get expression in a mathematics classroom. It alludes to how conflicts and dialogues that take place in such classrooms need to be handled if mathematics education is not only about increasing knowledge and awareness of inequities and injustices, but also a means for forgiveness and healing. Often mathematics is presented as a one and only truth, the most objective or neutral and this one truth is to be most valued whilst social or personal mathematical knowledge, skills and practices are subordinated or silenced. It is a healing and restorative 'mathematic truth' that gives meaning to a pedagogy of forgiveness. If these 'mathematical truths' are seen to be in a relation of complementarity with each other, then it is possible to acknowledge: firstly, that all kinds of truth in mathematics coexist, even though not all forms of mathematical truths find expression at any one moment in a classroom; secondly, that they need not be in harmony with each other because they exist in relations of cooperation and opposition; and thirdly, that this may be necessary for the growth and development of each.

Conclusion

The triad of mathematics education, democracy and development was explored with reference to the mathematisation of society through the first question. This societal focus drew attention to the formatting power of mathematics and the developmental challenges faced in a country like South Africa, pointing to the potential powerful role of mathematics education in addressing these for both Nthabiseng and Pieter. The second question threw a spotlight on the mathematics education system. The distribution of mathematics education and its associated educational possibilities was brought into sharp relief through a discussion on international studies and the national Grade 12 mathematics assessments and performance, and demonstrated how mathematics education becomes complicit in the inequities that are reproduced in society through mathematics curricula reforms and teacher education provisions. The third question moved the discussion into the school and classroom. It exemplified how mathematics classrooms can be places where democracy is learnt and development issues are engaged through a mathematics education pedagogy of conflict and dialogue that embodies forgiveness. The TRC truth framework metaphor drawn on in the fourth and final question to elaborate mathematical content matter, shows how the very mathematisation of society can recognise different forms of 'mathematical truths' that can coexist and come to constitute a mathematical knowledge and a mathematics education that can be healing and restorative of the dignity of people. Just as human beings are connected in complex relations of cooperation and contradiction, so too are our knowledge forms, including mathematics.

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