The Cognitive Process of Decision Making

Yingxu Wang, University of Calgary, Canada
Guenther Ruhe, University of Calgary, Canada

ABSTRACT

Decision making is one of the basic cognitive processes of human behaviors by which a preferred option or a course of actions is chosen from among a set of alternatives based on certain criteria. Decision theories are widely applied in many disciplines encompassing cognitive informatics, computer science, management science, economics, sociology, psychology, political science, and statistics. A number of decision strategies have been proposed from different angles and application domains such as the maximum expected utility and Bayesian method. However, there is still a lack of a fundamental and mathematical decision model and a rigorous cognitive process for decision making. This article presents a fundamental cognitive decision-making process and its mathematical model, which is described as a sequence of Cartesian-product based selections. A rigorous description of the decision process in real-time process algebra (RTPA) is provided. Real-world decisions are perceived as a repetitive application of the fundamental cognitive process. The result shows that all categories of decision strategies fit in the formally described decision process. The cognitive process of decision making may be applied in a wide range of decision-based systems such as cognitive informatics, software agent systems, expert systems, and decision support systems.

Keywords: cognitive informatics; cognitive process; decision-making; expert systems; formal description; mathematical model; RTPA; software engineering

INTRODUCTION

Decision making is a process that chooses a preferred option or a course of actions from among a set of alternatives based on given criteria or strategies (Wang, Wang, Patel, & Patel, 2004; Wilson & Keil, 2001). Decision making is one of the 37 fundamental cognitive processes modeled in the layered reference model of the brain (LRMB) (Wang et al., 2004; Wang, 2007b). The study on decision making is interested in multiple disciplines such as cognitive informatics, cognitive science, computer science, psychology, management science, decision science, economics, sociology, political science, and statistics (Berger, 1990; Edwards & Fasolo, 2001; Hastie, 2001; Matlin, 1998; Payne & Wenger, 1998; Pinel, 1997; Wald, 1950; Wang et al., 2004; Wilson et al., 2001). Each of those disciplines has emphasized on a special aspect of decision making. It is recognized that there is a need to seek an axiomatic and rigorous model of the cognitive decision-
making process in the brain, which may be served as the foundation of various decision making theories.

Decision theories can be categorized into two paradigms: the descriptive and normative theories. The former is based on empirical observation and on experimental studies of choice behaviors; and the latter assumes a rational decision-maker who follows well-defined preferences that obey certain axioms of rational behaviors. Typical normative theories are the expected utility paradigm (Osborne & Rubinstein, 1994) and the Bayesian theory (Berger, 1990; Wald, 1950). Edwards developed a 19-step decision-making process (Edwards et al., 2001) by integrating Bayesian and multi-attribute utility theories. Zachary, Wherry, Glenn, and Hopson (1982) perceived that there are three constituents in decision making known as the decision situation, the decision maker, and the decision process. Although the cognitive capacities of decision makers may be greatly varying, the core cognitive processes of the human brain share similar and recursive characteristics and mechanisms (Wang, 2003a; Wang & Gafurov, 2003; Wang & Wang, 2004; Wang et al., 2004).

This article adopts the philosophy of the axiom of choice (Lipschutz, 1967). The three essences for decision making recognized in this article are the decision goals, a set of alternative choices, and a set of selection criteria or strategies. According to this theory, decision makers are the engine or executive of a decision making process. If the three essences of decision making are defined, a decision making process may be rigorously carried out by either a human decision maker or by an intelligent system. This is a cognitive foundation for implementing expert systems and decision supporting systems (Ruhe, 2003; Ruhe & An, 2004; Wang et al., 2004; Wang, 2007a).

In this article, the cognitive foundations of decision theories and their mathematical models are explored. A rigorous description of decisions and decision making is presented. The cognitive process of decision making is explained, which is formally described by using real-time process algebra (RTPA). The complexity of decision making in real-world problems such as software release planning is studied, and the need for powerful decision support systems are discussed.

A MATHEMATICAL MODEL OF DECISIONS AND DECISION MAKING

Decision making is one of the fundamental cognitive processes of human beings (Wang et al., 2004; Wang, 2007a; Wang, 2007b) that is widely used in determining rational, heuristic, and intuitive selections in complex scientific, engineering, economical, and management situations, as well as in almost each procedure of daily life. Since decision making is a basic mental process, it occurs every few seconds in the thinking courses of human mind consciously or subconsciously.

This section explores the nature of selection, decision, and decision making, and their mathematical models. A rigorous description of decision making and its strategies is developed.

The Mathematical Model of Decision Making

The axiom of selection (or choice) (Lipschutz, 1967) states that there exists a selection function for any nonempty collection of nonempty disjoint sets of alternatives.

Definition 1. Let \(\{A_i | i \in I\}\) be a collection of disjoint sets, \(A_i \subseteq U\), and \(A_i \neq \emptyset\), a function

\[ c: \{A_i\} \rightarrow A_p,\ i \in I \]  

(1)

is a choice function if \(c(A_i) = a_i, a_i \in A_i\). Or an element \(a_i \in A_i\) may be chosen by \(c\), where \(A_i\) is called the set of alternatives, \(U\) the universal set, and \(I\) a set of natural numbers.

On the basis of the choice function and the axiom of selection, a decision can be rigorously defined as follows.
Definition 2. A decision, \( d \), is a selected alternative \( a \in \mathcal{A} \) from a nonempty set of alternatives \( \mathcal{A} \subseteq \mathcal{U} \), based on a given set of criteria \( \mathcal{C} \), i.e.:

\[
d = f (\mathcal{A}, \mathcal{C}) = f: \mathcal{A} \times \mathcal{C} \rightarrow \mathcal{A}, \quad \mathcal{A} \subseteq \mathcal{U}, \mathcal{A} \neq \emptyset
\] (2)

where \( \times \) represents a Cartesian product.

It is noteworthy that the criteria in \( \mathcal{C} \) can be a simple one or a complex one. The latter is the combination of a number of joint criteria depending on multiple factors.

Definition 3. Decision making is a process of decision selection from available alternatives against the chosen criteria for a given decision goal.

According to Definition 2, the number of possible decisions, \( n \), can be determined by the sizes of \( \mathcal{A} \) and \( \mathcal{C} \), for example:

\[
n = \# \mathcal{A} \cdot \# \mathcal{C}
\] (3)

where \( \# \) is the cardinal calculus on sets, and \( \mathcal{A} \cap \mathcal{C} = \emptyset \).

According to Eq.3, in case \( \# \mathcal{A} = 0 \) and/or \( \# \mathcal{C} = 0 \), no decision may be derived.

The previous definitions provide a generic and fundamental mathematical model of decision making, which reveal that the factors determining a decision are the alternatives \( \mathcal{A} \) and criteria \( \mathcal{C} \) for a given decision making goal. A unified theory on fundamental and cognitive decision making can be developed based on the axiomatic and recursive cognitive process elicited from the most fundamental decision-making categories as shown in Table 1.

Strategies and Criteria of Decision Making

According to Definition 2, the outcomes of a decision making process are determined by the decision-making strategies selected by decision makers when a set of alternative decisions has been identified. It is obvious that different decision making strategies require different decision selection criteria. There is a great variation of decision-making strategies developed in traditional decision and game theories, as well as cognitive science, system science, management science, and economics.

The taxonomy of strategies and corresponding criteria for decision making can be classified into four categories known as intuitive, empirical, heuristic, and rational as shown in Table 1. It is noteworthy in Table 1 that the existing decision theories provide a set of criteria \( \mathcal{C} \) for evaluating alternative choices for a given problem.

As summarized in Table 1, the first two categories of decision-making, intuitive and empirical, are in line with human intuitive cognitive psychology and there is no specific rational model for explaining those decision criteria. The rational decision-making strategies can be described by two subcategories: the static and dynamic strategies and criteria. The heuristic decision-making strategies are frequently used by human beings as a decision maker. Details of the heuristic decision-making strategies may be referred to cognitive psychology and AI (Hastie, 2001; Matlin, 1998; Payne et al., 1998; Wang, 2007a).

It is interesting to observe that the most simple decision making theory can be classified into the intuitive category such as arbitrary and preference choices based on personal propensity, hobby, tendency, expectation, and/or common senses. That is, a naïve may still be able to make important and perhaps wise decisions every day, even every few seconds. Therefore, the elicitation of the most fundamental and core process of decision making shared in human cognitive processes is yet to be sought in the following sections. Recursive applications of the core process of decision making will be helpful to solve complicated decision problems in the real world.
Table 1. Taxonomy of strategies and criteria for decision-making

<table>
<thead>
<tr>
<th>No.</th>
<th>Category</th>
<th>Strategy</th>
<th>Criterion (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Intuitive</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td></td>
<td>Arbitrary</td>
<td>Based on the most easy or familiar choice</td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td>Preference</td>
<td>Based on propensity, hobby, tendency, expectation</td>
</tr>
<tr>
<td>1.3</td>
<td></td>
<td>Common senses</td>
<td>Based on axioms and judgment</td>
</tr>
<tr>
<td>2</td>
<td>Empirical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td></td>
<td>Trial and error</td>
<td>Based on exhaustive trial</td>
</tr>
<tr>
<td>2.2</td>
<td></td>
<td>Experiment</td>
<td>Based on experiment results</td>
</tr>
<tr>
<td>2.3</td>
<td></td>
<td>Experience</td>
<td>Based on existing knowledge</td>
</tr>
<tr>
<td>2.4</td>
<td></td>
<td>Consultant</td>
<td>Based on professional consultation</td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td>Estimation</td>
<td>Based on rough evaluation</td>
</tr>
<tr>
<td>3</td>
<td>Heuristic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td></td>
<td>Principles</td>
<td>Based on scientific theories</td>
</tr>
<tr>
<td>3.2</td>
<td></td>
<td>Ethics</td>
<td>Based on philosophical judgment and belief</td>
</tr>
<tr>
<td>3.3</td>
<td></td>
<td>Representative</td>
<td>Based on common rules of thumb</td>
</tr>
<tr>
<td>3.4</td>
<td></td>
<td>Availability</td>
<td>Based on limited information or local maximum</td>
</tr>
<tr>
<td>3.5</td>
<td></td>
<td>Anchoring</td>
<td>Based on presumption or bias and their justification</td>
</tr>
<tr>
<td>4</td>
<td>Rational</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Static</td>
<td>Minimum cost</td>
<td>Based on minimizing energy, time, money</td>
</tr>
<tr>
<td>4.2</td>
<td></td>
<td>Maximum benefit</td>
<td>Based on maximizing gain of usability, functionality, reliability, quality, dependability</td>
</tr>
<tr>
<td>4.3</td>
<td></td>
<td>Maximum utility</td>
<td>Based on cost-benefit ratio</td>
</tr>
<tr>
<td>4.3.1</td>
<td></td>
<td>- Certainty</td>
<td>Based on maximum probability, statistic data</td>
</tr>
<tr>
<td>4.3.2</td>
<td></td>
<td>- Risks</td>
<td>Based on minimum loss or regret</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Uncertainty</td>
<td></td>
</tr>
<tr>
<td>4.3.3</td>
<td></td>
<td>- Pessimist</td>
<td>Based on maximin</td>
</tr>
<tr>
<td>4.3.4</td>
<td></td>
<td>- Optimist</td>
<td>Based on maximax</td>
</tr>
<tr>
<td>4.3.5</td>
<td></td>
<td>- Regretist</td>
<td>Based on minimax of regrets</td>
</tr>
<tr>
<td>4.2</td>
<td>Dynamic</td>
<td>Interactive events</td>
<td>Based on automata</td>
</tr>
<tr>
<td>4.2</td>
<td></td>
<td>Games</td>
<td>Based on conflict</td>
</tr>
<tr>
<td>4.2.1</td>
<td></td>
<td>- Zero sum</td>
<td>Based on $\sum (\text{gain} + \text{loss}) = 0$</td>
</tr>
<tr>
<td>4.2.2</td>
<td></td>
<td>- Non zero sum</td>
<td>Based on $\sum (\text{gain} + \text{loss}) \neq 0$</td>
</tr>
<tr>
<td>4.2</td>
<td></td>
<td>Decision grids</td>
<td>Based on a series of choices in a decision grid</td>
</tr>
</tbody>
</table>
The Framework of Rational Decision Making
According to Table 1, rational and complex decision making strategies can be classified into the static and dynamic categories. Most existing decision-making strategies are static because the changes of environments of decision makers are independent of the decision makers’ activities. Also, different decision strategies may be selected in the same situation or environment based on the decision makers’ values and attitudes towards risk and their prediction on future outcomes. When the environment of a decision maker is interactive with his or her decisions or the environment changes according to the decision makers’ activities and the decision strategies and rules are predetermined, this category of decision making needs are classified into the category of dynamic decisions such as games and decision grids (Matlin, 1998; Payne et al., 1998; Pinel, 1997; Wang, 2005a,b).

**Definition 4.** The *dynamic strategies and criteria* of decision-making are those that all alternatives and criteria are dependent on both the environment and the effect of the historical decisions made by the decision maker.

Classic dynamic decision making methods are decision trees (Edwards et al., 2001). A new theory of decision grid is developed in Wang (2005a,b) for serial decision making. Decision making under interactive events and competition is modeled by games (Matlin, 1998; Payne et al., 1998; von Neumann & Morgenstern, 1980; Wang, 2005a). Wang (2005a) presents a formal model of games, which rigorously describes the architecture or layout of games and their dynamic behaviors.

**Figure 1. A framework of decisions and strategies**
An overview of the classification of decisions and related rational strategies is provided in Figure 1. It can be seen that games are used to deal with the most complicated decision problems, which are dynamic, interactive, and under uncontrollable competitions. Further discussion on game theories and its formal models may be referred to von Neumann et al. (1980), Berger (1990), and Wang (2005a,b). Decision models may also be classified among others point of views such as structures, constraints, degrees of uncertainty, clearness and scopes of objectives, difficulties of information processing, degrees of complexity, utilities and beliefs, ease of formalization, time constraints, and uniqueness or novelty.

Typical Theories of Decision Making

Decision making is the process of constructing the choice criteria (or functions) and strategies and use them to select a decision from a set of possible alternatives. In this view, existing decision theories are about how a choice function may be created for finding a good decision. Different decision theories provide different choice functions. The following are examples from some of the typical decision paradigms as shown in Table 1.

(a) The Game Theory

In game theory (Osborne et al., 1994), a decision problem can be modeled as a triple, for example:

\[ d = (\Omega, C, A) \]  \hspace{1cm} (4)

where \( \Omega \) is a set of possible states of the nature, \( C \) is a set of consequences, and \( A \) is a set of actions, \( A \subseteq C^\Omega \).

If an action \( a \in A \) is chosen, and the prevailing state is \( \omega \in \Omega \), then a certain consequence \( \alpha(\omega) \in C \) can be obtained. Assuming a probability estimation and a utility function be defined for a given action \( a \) as \( p(a) : A \rightarrow \Re \) and \( u : C \rightarrow \Re \), respectively, a choice function based on the utility theory can be expressed as follows:

\[ d = \{ a | \sum_{\omega} u[\alpha(\omega)] p(a) = \max \{ \sum_{x} u[x(\omega)] p(x) \} \land x \in A \} \]  \hspace{1cm} (5)

(b) The Bayesian Theory

In Bayesian theory (Wald, 1950; Berger, 1990) the choice function is called a decision rule. A loss function, \( L \), is adopted to evaluate the consequence of an action as follows:

\[ L: \Omega \times A \rightarrow \Re \]  \hspace{1cm} (6)

where \( \Omega \) is a set of all possible states of nature, \( A \) is a set of actions, and \( \Omega \times A \) denotes a Cartesian product of choice.

Using the loss function for determining possible risks, a choice function for decision making can be derived as follows:

\[ d = \{ a | p[L(\omega, \alpha)] = \min_{x \in A} (p[L(\omega, x)]) \} \]  \hspace{1cm} (7)

where \( p[L(\omega, \alpha)] \) is the expected probability of loss for action \( x \) on \( \omega \in \Omega \).

Despite different representations in the utility theory and Bayesian theory, both of them provide alternative decision making criteria from different angles where loss in the latter is equivalent to the negative utility in the former. Therefore, it may be perceived that a decision maker who uses the utility theory is seeking optimistic decisions; and a decision maker who uses the loss or risk-based theory is seeking pessimistic or conservative decisions.

THE COGNITIVE PROCESS OF DECISION MAKING

The LRMB model has revealed that there are 37 interacting cognitive processes in the brain (Wang et al., 2004). Relationships between the decision-making process and other major ones in LRMB are shown in Figure 2. Figure 2 indicates that, according to UML semantics, the decision-making process inherits the problem-solving process. In other end, it functions by aggregations of or supported by
the layer 6 processes comprehension, qualification, and quantification, as well as the layer 5 processes of search, representation, and memorization. Formal descriptions of these related cognitive processes in LRMB may be referred to in Wang (2003b), Wang et al. (2003), and Wang et al. (2003, 2004).

In contrary to the traditional container metaphor, the human memory mechanism can be described by a relational metaphor, which perceives that memory and knowledge are represented by the connections between neurons in the brain, rather than the neurons themselves as information containers. Therefore, the cognitive model of human memory, particularly the long-term memory (LTM) can be described by two fundamental artifacts (Wang et al., 2003): (a) Objects: The abstraction of external entities and internal concepts. There are also sub-objects known as attributes, which are used to denote detailed properties and characteristics of an object. (b) Relations: Connections and relationships between object-object, object-attributes, and attribute-attribute.

Based on the previous discussion, an object-attribute-relation (OAR) model of memory can be described as a triple (Wang & Wang, 2004; Wang et al., 2003), for example:

\[ OAR = (O, A, R) \]  

(8)

where O is a given object identified by an abstract name, A is a set of attributes for characterizing the object, and R is a set of relations between the object and other objects or attributes of them.

On the basis of the LRMB and OAR models developed in cognitive informatics (Wang, 2003a, 2007b), the cognitive process of decision making may be informally described by the following courses:

1. To comprehend the decision making problem and to identify the decision goal in terms of Object (O) and its attributes (A).
2. To search in the abstract layer of LTM (Squire, Knowlton, & Musen et al. 1993; Wang & Wang, 2004) for alternative solutions (A) and criteria for useful decision strategies (C).
3. To quantify A and C and determine if the search should be go on.
4. To build a set of decisions by using A and C as obtained in previous searches.
5. To select the preferred decision(s) on the basis of satisfaction of decision makers.
6. To represent the decision(s) in a new sub-OAR model.
7. To memorize the sub-OAR model in LTM.

Figure 2. Relationships between decision-making process and other processes in LRMB

![Diagram of relationships between decision-making process and other processes in LRMB](image-url)
A detailed cognitive process model of decision making is shown in Figure 3 where a double-ended rectangle block represents a function call that involves a predefined process as provided in the LRMB model.

The first step in the cognitive process of decision making is to understand the given decision-making problem. According to the cognitive process of comprehension (Wang et al., 2003), the object (goal) of decision will be identified and an initial OAR model will be created. The object, its attributes, and known relations are retrieved and represented in the OAR model. Then, alternatives and strategies are searched, which result in two sets of A and C, respectively. The results of search will be quantified in order to form a decision as given in Eq. 2, for example: \( d = f: A \times C \rightarrow A \), where \( A \subseteq U \) and \( A \neq \emptyset \).

When the decision \( d \) is derived, the previous OAR model will be updated with \( d \) and related information. Then, the decision maker may consider whether the decision is satisfied according to the current states of nature and personal judgment. If yes, the OAR model for the decision is memorized in the LTM. Otherwise, the decision-making process has to be repeated until a satisfied decision is found.

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**Figure 3. The cognitive process of decision making**

```
Begin

Identify
(Object - O)

Identify
(Attributes - A)

Search (Alternatives of choices - A)

Quantify (A)

No

Evaluate (Adequacy of A)

Yes

Select (Decision - d)
\( d = f(A, C) \)

No

Evaluate (Satisfaction of d)

Yes

Represent (OAR)

Memorize (OAR)

End

Search (Criteria of choices - C)

Quantify (C)

No

Evaluate (Adequacy of C)

Yes
```

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or the decision maker chooses to quit without a final decision. During the decision making process, both the mind state of the decision maker and the global OAR model in the brain change from time to time. Although the state of nature will not be changed in a short period during decision making, the perception towards it may be changed with the effect of the updated OAR model.

As described in the LRMB model (Wang et al., 2004), the process of decision making is a higher-layer cognitive process defined at Layer 6. The decision making process interacts with other processes underneath this layer such as search, representation, and memorization, as well as the processes at the same layer such as comprehension, qualification, quantification, and problem solving. Relationships between the decision-making process and other related processes have been described in Figure 1 and in Wang and Wang (2004) and Wang et al. (2004).

FORMAL DESCRIPTION OF THE COGNITIVE DECISION MAKING PROCESS

On the basis of the cognitive model of decision making as described in Figure 3, a rigorous cognitive process can be specified by using RTPA (Wang, 2002; Wang, 2003b). RTPA is designed for describing the architectures, static and dynamic behaviors of software systems (Wang, 2002), as well as human cognitive behaviors and sequences of actions (Wang, 2003b; Wang et al., 2003).

The formal model of the cognitive process of decision making in RTPA is presented in Figure 4. According to LRMB and the OAR model of internal knowledge representation in the brain, the result of a decision in the mind of the decision maker is a new sub-OAR model, or an updated version of the global OAR model of knowledge in the human brain.

As shown in Figure 4, a decision-making process (DMP) is started by defining the goal of decision in terms of the object attributes. Then, an exhaustive search of the alternative decisions ($A$) and useful criteria ($C$) are carried out in parallel. The searches are conducted in both the brain of a decision maker internally, and through external resources based on the knowledge, experiences, and goal expectation. The results of searches are quantitatively evaluated until the searching for both $A$ and $C$ are satisfied. If nonempty sets are obtained for both $A$ and $C$, the $n$ decisions in $d$ have already existed as determinable by Eqs. 2 and 3.

It is noteworthy that learning results, experiences, and skills of the decision maker may dramatically reduce the exhaustive search process in DMP based on known heuristic strategies.

When one or more suitable decisions are selected from the set of $d$ by decision makers via evaluating the satisfaction levels, satisfied decisions will be represented in a sub-OAR model, which will be added to the entire knowledge of the decision maker in LTM.

SOLVING COMPLEX PLANNING PROBLEMS BY DECISION SUPPORT SYSTEMS

The decision-making models and the formal description of the cognitive decision-making process as presented in the second through fourth sections, can be used to address the solution of wicked planning problems in software engineering. Wicked planning problems are not only difficult to solve but also difficult to be explicitly formulated. The notion of a wicked planning problem was introduced by Rittel and Webber (1984), where several characteristics were given to classify a problem as wicked. One of them states that there is no definite formulation of the problem. Another one states that wicked problems have no stopping rule. So, in these cases, does it make sense to look into a more systematic approach at all or shouldn’t we just rely on human intuition and personnel experience to figure out a decision?

A systematic approach for solving the wicked planning problem of software release planning was given in Ngo-The and Ruhe (2006). Release planning is known to be cog-
Figure 4. Formal description of the cognitive process of decision-making in RTPA

\[
\text{The Decision Making Process (DMP)}
\]

\[
\text{DMP\_Process(I:: OS, O:: OAR(d\text{ST}) \triangleq}
\]

\[
\begin{align*}
&\{ \\
&\quad \text{// I. Form decision goal(s)} \\
&\quad \rightarrow \text{Identify (O)} \quad // \text{The decision making goal} \\
&\quad \rightarrow \text{Identify (A)} \quad // \text{Sub decision making goals} \\
&\quad \rightarrow \{ \\
&\quad \quad \text{Satisfaction of } A = f \\
&\quad \quad ( \rightarrow \text{Search (A)} \\
&\quad \quad \quad \rightarrow \text{Quantify (A)} \\
&\quad \quad \quad \rightarrow \text{Evaluate (A)} \\
&\quad \quad ) \\
&\quad \rightarrow \{ \\
&\quad \quad \text{Satisfaction of } C = f \\
&\quad \quad ( \rightarrow \text{Search (C)} \\
&\quad \quad \quad \rightarrow \text{Quantify (C)} \\
&\quad \quad \quad \rightarrow \text{Evaluate (C)} \\
&\quad \quad ) \\
&\quad \}
\end{align*}
\]

\[
\begin{align*}
&\{ \\
&\quad \text{// II. Select decisions} \\
&\quad \rightarrow d = f: \mathcal{A} \times \mathcal{C} \rightarrow \mathcal{A} \quad // \text{Refers to Eq. 2} \\
&\quad \rightarrow \text{Evaluate (d)} \\
&\quad \rightarrow (\bullet s(d) \geq k) \\
&\quad \quad \rightarrow \text{Memorize (OAR\text{ST})} \\
&\quad \quad \rightarrow \emptyset \quad // \text{Otherwise} \\
&\quad \quad \rightarrow (\bullet \text{GiveUpBL} = f) \\
&\quad \quad \rightarrow \text{DMP\_Process(I:: OS, O:: OAR(d\text{ST}))} \\
&\quad \quad (\bullet \rightarrow) \\
\end{align*}
\]

\[
\begin{align*}
&\{ \\
&\quad \text{// III. Represent decisions} \\
&\quad \rightarrow R = <d, \mathcal{A}, C> \quad // \text{Form new relation on } d \\
&\quad \rightarrow \text{OAR\text{ST} = }<\emptyset, \mathcal{A}, R> \quad // \text{Form new OAR model for } d
\end{align*}
\]

...it provides a maximum utility value from offering a best possible blend of features in the right sequence of releases.

- It is feasible to the existing hard constrains that have to be fulfilled.
- It satisfies some additional soft constraints sufficiently well. These soft constraints, for example, can be related to stakeholder satisfaction, consideration of the risk of implementing the suggested releases, balancing of resources or other aspects which are either hard to formalize or not known in advance.

It seems that uncertain software engineering decision problems are difficult to be...
explicitly modeled and completely formalized, since the constraints of organizations, people, technology, functionality, time, budget, and resources. Therefore, all spectrum of decision strategies as identified in Table 1 and Figure 1 need to be examined. This is a typical case where the idea of decision support arises when human decisions have to be made in complex, uncertain, and/or dynamic environments. Carlsson and Turban (2002) point out that the acceptance of these systems is primarily limited by human related factors: (1) cognitive constraints, (2) understanding the support of such a model, (3) difficulty in handling large amounts of information and knowledge, and (4) frustration caused by complicated theories.

The solution approach presented in Ngo-The et al. (2006) address the inherent cognitive and computational complexity by (1) an evolutionary problem solving method combining rigorous solution methods to solve the actual formalization of the problem combined with the interactive involvement of the human experts in this process; (2) offering a portfolio of diversified and qualified solution at all iterations of the solution process; and (3) using the multi-criteria decision aid method ELECTRE (Roy, 1991) to assist the project manager in the selection of the final solution from the set of qualified solutions. Further research is ongoing to integrate these results with the framework of the decision-making models and the improved understanding of the cognitive process of decision-making as developed in this article.

**CONCLUSION**

Decision-making is one of the basic cognitive processes of human behaviors by which a preferred option or a course of actions is chosen from among a set of alternatives based on certain criteria. The interest in the study of decision-making has been widely shared in various disciplines because it is a fundamental process of the brain.

This article has developed an axiomatic and rigorous model for the cognitive decision-making process, which explains the nature and course in human and machine-based decision-making on the basis of recent research results in cognitive informatics. A rigorous description of the decision process in real-time process algebra (RTPA) has been presented. Various decision-making theories have been comparatively analyzed and a unified decision-making model has been obtained, which shows that existing theories and techniques on decision-making are well fit in the formally described decision process.

One of the interesting findings of this work is that the most fundamental decision that is recurrently used in any complex decision system and everyday life is a Cartesian product of a set of alternatives and a set of selection criteria. The larger both the sets, the more ideal the decisions generated. Another interesting finding of this work is that, although the cognitive complexities of new decision problems are always extremely high, they become dramatically simpler when a rational or formal solution is figured out. Therefore, the reducing of cognitive complexities of decision problems by heuristic feedbacks of known solutions in each of the categories of decision strategies will be further studied in intelligent decision support systems. According to case studies related to this work, the models and cognitive processes of decision-making provide in this article can be applied in a wide range of decision-support and expert systems.

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Yingxu Wang is professor of cognitive informatics and software engineering, director of International Center for Cognitive Informatics (ICfCI), and director of Theoretical and Empirical Software Engineering Research Center (TESERC) at the University of Calgary. He received a PhD in software engineering from The Nottingham Trent University, UK (1997), and a BSc in electrical engineering from Shanghai Tiedao University (1983). He was a visiting professor in the Computing Laboratory at Oxford University (1995), and has been a full professor since 1994. He is editor-in-chief of *International Journal of Cognitive Informatics and Natural Intelligence (IJCIINI),* editor-in-chief of World Scientific Book Series on Cognitive Informatics, and editor of CRC Book Series in Software Engineering. He has published more than 290 papers and 10 books in software engineering and cognitive informatics, and won dozens of research achievement, best paper, and teaching awards in the last 28 years, particularly the IBC 21st Century Award for Achievement “in recognition of outstanding contribution in the field of cognitive informatics and software science.”

Guenther Ruhe (http://sern.cpsc.ucalgary.ca/~ruhe/) received a doctorate in mathematics with emphasis on operations research from Freiberg University, Germany and a doctorate degree from both the Technical University of Leipzig and University of Kaiserslautern, Germany. From 1996 until 2001, he was deputy director of the Fraunhofer Institute for Experimental Software Engineering Fh IESE. Ruhe holds an Industrial Research Chair in Software Engineering at University of Calgary. This is a joint position between department of Computer Science and department of Electrical and Computer Engineering. His laboratory for Software Engineering Decision Support (see www.seng-decisionsupport.ucalgary.ca) is focusing on research in the area of intelligent support for the early phases of software system development, analysis of software requirements, empirical evaluation of software technologies, and selection of components-of-the-shelf (COTS) software products. He is the main inventor of a new generation of intelligent decision support tool for software release planning and prioritization. ReleasePlanner® (www.releaseplanner.com). Ruhe has published more than 155 reviewed research papers at journals, workshops, and conferences. Ruhe is a member of the ACM, the IEEE Computer Society, and the German Computer Society GI.