Longitudinal gluons and Nambu-Goldstone bosons in a two-flavor color superconductor

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(July 3, 2003)

In a two-flavor color superconductor, the $SU(3)_c$ gauge symmetry is spontaneously broken by diquark condensation. The Nambu-Goldstone excitations of the diquark condensate mix with the gluons associated with the broken generators of the original gauge group. It is shown how one can decouple these modes with a particular choice of 't Hooft gauge. We then explicitly compute the spectral density for transverse and longitudinal gluons of adjoint color 8. The Nambu-Goldstone excitations give rise to a singularity in the real part of the longitudinal gluon self-energy. This leads to a vanishing gluon spectral density for energies and momenta located on the dispersion branch of the Nambu-Goldstone excitations.

I. INTRODUCTION

Cold, dense quark matter is a color superconductor [1]. For two massless quark flavors (say, up and down), Cooper pairs with total spin zero condense in the color-antitriplet, flavor-singlet channel. In this so-called two-flavor color superconductor, the $SU(3)_c$ gauge symmetry is spontaneously broken to $SU(2)_c$ [2]. If we choose to orient the (anti-) color charge of the Cooper pair along the (anti-) blue direction in color space, only red and green quarks form Cooper pairs, while blue quarks remain unpaired. Then, the three generators T_1, T_2, and T_3 of the original $SU(3)_c$ gauge group form the generators of the residual $SU(2)_c$ symmetry. The remaining five generators $T_4, \ldots, T_8$ are broken. (More precisely, the last broken generator is a combination of T_8 and the generator $\mathbf{1}$ of the global $U(1)$ symmetry of baryon number conservation, for details see Ref. [3] and below).

According to Goldstone's theorem, this pattern of symmetry breaking gives rise to five massless bosons, the so-called Nambu-Goldstone bosons, corresponding to the five broken generators of $SU(3)_c$. Physically, these massless bosons correspond to fluctuations of the order parameter, in our case the diquark condensate, in directions in color-flavor space where the effective potential is flat. For gauge theories (where the local gauge symmetry cannot truly be spontaneously broken), these bosons are "eaten" by the gauge bosons corresponding to the broken generators of the original gauge group, *i.e.*, in our case the gluons with adjoint colors $a = 4, \ldots, 8$. They give rise to a longitudinal degree of freedom for these gauge bosons. The appearance of a longitudinal degree of freedom is commonly a sign that the gauge boson becomes massive.

In a dense (or hot) medium, however, even *without* spontaneous breaking of the gauge symmetry the gauge bosons already have a longitudinal degree of freedom, the so-called *plasmon* mode [4]. Its appearance is related to the presence of gapless charged quasiparticles. Both transverse and longitudinal modes exhibit a mass gap, *i.e.*, the gluon energy $p_0 \to m_g > 0$ for momenta $p \to 0$. In quark matter with N_f massless quark flavors at zero temperature $T = 0$, the gluon mass parameter (squared) is [4]

$$m_g^2 = \frac{N_f}{6\,\pi^2}\, g^2\, \mu^2 \ , \tag{1}$$

where g is the QCD coupling constant and μ is the quark chemical potential.

It is *a priori* unclear how the Nambu-Goldstone bosons interact with these longitudinal gluon modes. In particular, it is of interest to know whether coupling terms between these modes exist and, if yes, whether these terms can be

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eliminated by a suitable choice of ('t Hooft) gauge. The aim of the present work is to address these questions. We shall show that the answer to both questions is "yes". We shall then demonstrate by focussing on the gluon of adjoint color 8, how the Nambu-Goldstone mode affects the spectral density of the longitudinal gluon.

Our work is partially based on and motivated by previous studies of gluons in a two-flavor color superconductor [5–7]. The gluon self-energy and the resulting spectral properties have been discussed in Ref. [7]. In that paper, however, the fluctuations of the diquark condensate have been neglected. Consequently, the longitudinal degrees of freedom of the gluons corresponding to the broken generators of $SU(3)_c$ have not been treated correctly. The gluon polarization tensor was no longer explicitly transverse (a transverse polarization tensor $\Pi^{\mu\nu}$ obeys $P_\mu \, \Pi^{\mu\nu} = \Pi^{\mu\nu} \, P_\nu = 0$), and it did not satisfy the Slavnov-Taylor identity. As a consequence, the plasmon mode exhibited a certain peculiar behavior in the low-momentum limit, which cannot be physical (cf. Fig. 5 (a) of Ref. [7]). It was already realized in Ref. [7] that the reason for this unphysical behavior is the fact that the mixing of the gluon with the excitations of the condensate was neglected. It was moreover suggested in Ref. [7] that proper inclusion of this mixing would amend the shortcomings of the previous analysis. The aim of the present work is to follow this suggestion and thus to correct the results of Ref. [7] with respect to the longitudinal gluon. Note that in Ref. [5] fluctuations of the color-superconducting condensate were taken into account in the calculation of the gluon polarization tensor. As a consequence, the latter is explicitly transverse. However, the analysis was done in the vacuum, at $\mu = 0$, not at (asymptotically) large chemical potential.

The outline of the present work is as follows. In Section II we derive the transverse and longitudinal gluon propagators including fluctuations of the diquark condensate. In Section III we use the resulting expressions to compute the spectral density for the gluon of adjoint color 8. Section IV concludes this work with a summary of our results.

Our units are $\hbar = c = k_B = 1$. The metric tensor is $g^{\mu\nu} = \mathrm{diag}\,(+,-,-,-)$. We denote 4-vectors in energy-momentum space by capital letters, $K^\mu = (k_0, \mathbf{k})$. Absolute magnitudes of 3-vectors are denoted as $k \equiv |\mathbf{k}|$, and the unit vector in the direction of $\mathbf{k}$ is $\hat{\mathbf{k}} \equiv \mathbf{k}/k$.

II. DERIVATION OF THE PROPAGATOR FOR TRANSVERSE AND LONGITUDINAL GLUONS

In this section, we derive the gluon propagator taking into account the fluctuations of the diquark condensate. A short version of this derivation can be found in Appendix C of Ref. [8] [see also the original Ref. [9]]. Nevertheless, for the sake of clarity and in order to make our presentation self-contained, we decide to present this once more in greater detail and in the notation of Ref. [7]. As this part is rather technical, the reader less interested in the details of the derivation should skip directly to our main result, Eqs. (56), (57), and (58).

We start with the grand partition function of QCD,

$$\mathcal{Z} = \int \mathcal{D}A \; e^{S_A} \; \mathcal{Z}_q[A] \; , \tag{2a}$$

where

$$\mathcal{Z}_q[A] = \int \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \; \exp \left[\int_x \bar{\psi} \left(i\gamma^\mu \partial_\mu + \mu\gamma_0 + g\gamma^\mu A^a_\mu T_a \right) \psi \right] \; . \tag{2b}$$

is the grand partition function for massless quarks in the presence of a gluon field A^μ_a. In Eq. (2), the space-time integration is defined as $\int_x \equiv \int_0^{1/T} d\tau \int_V d^3\mathbf{x}$, where V is the volume of the system, γ^μ are the Dirac matrices, and $T_a = \lambda_a/2$ are the generators of $SU(N_c)$. For QCD, $N_c = 3$, and λ_a are the Gell-Mann matrices. The quark fields ψ are $4N_cN_f$-component spinors, *i.e.*, they carry Dirac indices $\alpha = 1, \ldots, 4$, fundamental color indices $i = 1, \ldots, N_c$, and flavor indices $f = 1, \ldots, N_f$. The action for the gauge fields consists of three parts,

$$S_A = S_{F^2} + S_{\rm gf} + S_{\rm FPG} \; , \tag{3}$$

where

$$S_{F^2} = -\frac{1}{4} \int_x F^{\mu\nu}_a \, F^a_{\mu\nu} \tag{4}$$

is the gauge field part; here, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is the field strength tensor. The part corresponding to gauge fixing, $S_{\rm gf}$, and to Fadeev-Popov ghosts, $S_{\rm FPG}$, will be discussed later.

For fermions at finite chemical potential it is advantageous to introduce the charge-conjugate degrees of freedom explicitly. This restores the symmetry of the theory under $\mu \to -\mu$. Therefore, in Ref. [6], a kind of replica method was applied, in which one first artificially increases the number of quark species, and then replaces half of these species of quark fields by charge-conjugate quark fields. More precisely, first replace the quark partition function $\mathcal{Z}_q[A]$ by $\mathcal{Z}_M[A] \equiv \{\mathcal{Z}_q[A]\}^M$, M being some large integer number. (Sending $M \to 1$ at the end of the calculation reproduces the original partition function.) Then, take M to be an even integer number, and replace the quark fields by charge-conjugate quark fields in $M/2$ of the factors $\mathcal{Z}_q[A]$ in $\mathcal{Z}_M[A]$. This results in

$$\mathcal{Z}_M[A] = \int \prod_{r=1}^{M/2} \mathcal{D}\bar{\Psi}_r \, \mathcal{D}\Psi_r \, \exp\left\{ \sum_{r=1}^{M/2} \left[\int_{x,y} \bar{\Psi}_r(x) \, \mathcal{G}_0^{-1}(x,y) \, \Psi_r(y) + \int_x g \, \bar{\Psi}_r(x) \, A_\mu^a(x) \, \hat{\Gamma}_a^\mu \, \Psi_r(x) \right] \right\} . \tag{5}$$

Here, r labels the quark species and Ψ_r, $\bar{\Psi}_r$ are $8N_cN_f$-component Nambu-Gor'kov spinors,

$$\Psi_r \equiv \left(\begin{array}{c} \psi_r \\ \psi_{Cr} \end{array} \right) \;\; , \;\; \bar{\Psi}_r \equiv (\bar{\psi}_r \, , \, \bar{\psi}_{Cr}) \; , \tag{6}$$

where $\psi_{Cr} \equiv C\bar{\psi}_r^T$ is the charge conjugate spinor and $C = i\gamma^2\gamma_0$ is the charge conjugation matrix. The inverse of the $8N_cN_f \times 8N_cN_f$-dimensional Nambu-Gor'kov propagator for non-interacting quarks is defined as

$$\mathcal{G}_0^{-1} \equiv \left(\begin{array}{cc} [G_0^+]^{-1} & 0 \\ 0 & [G_0^-]^{-1} \end{array} \right) \; , \tag{7}$$

where

$$[G_0^\pm]^{-1}(x,y) \equiv -i \, (i\gamma_\mu \partial_x^\mu \pm \mu\gamma_0) \, \delta^{(4)}(x-y) \tag{8}$$

is the inverse propagator for non-interacting quarks (upper sign) or charge conjugate quarks (lower sign), respectively. The Nambu-Gor'kov matrix vertex describing the interaction between quarks and gauge fields is defined as follows:

$$\hat{\Gamma}_a^\mu \equiv \left(\begin{array}{cc} \Gamma_a^\mu & 0 \\ 0 & \bar{\Gamma}_a^\mu \end{array} \right) \; , \tag{9}$$

where $\Gamma_a^\mu \equiv \gamma^\mu T_a$ and $\bar{\Gamma}_a^\mu \equiv C(\gamma^\mu)^T C^{-1} T_a^T \equiv -\gamma^\mu T_a^T$.

Following Ref. [1] we now add the term $\int_{x,y} \bar{\psi}_{Cr}(x) \, \Delta^+(x,y) \, \psi_r(y)$ and the corresponding charge-conjugate term $\int_{x,y} \bar{\psi}_r(x) \, \Delta^-(x,y) \, \psi_{Cr}(y)$, where $\Delta^- \equiv \gamma_0 \, (\Delta^+)^\dagger \, \gamma_0$, to the argument of the exponent in Eq. (5). This defines the quark (replica) partition function in the presence of the gluon field A_a^μ *and* the diquark source fields Δ^+, Δ^-:

$$\mathcal{Z}_M[A,\Delta^+,\Delta^-] \equiv \int \prod_{r=1}^{M/2} \mathcal{D}\bar{\Psi}_r \, \mathcal{D}\Psi_r \, \exp\left\{ \sum_{r=1}^{M/2} \left[\int_{x,y} \bar{\Psi}_r(x) \, \mathcal{G}^{-1}(x,y) \, \Psi_r(y) + \int_x g \, \bar{\Psi}_r(x) \, A_\mu^a(x) \, \hat{\Gamma}_a^\mu \, \Psi_r(x) \right] \right\} \; , \tag{10}$$

where

$$\mathcal{G}^{-1} \equiv \left(\begin{array}{cc} [G_0^+]^{-1} & \Delta^- \\ \Delta^+ & [G_0^-]^{-1} \end{array} \right) \tag{11}$$

is the inverse quasiparticle propagator.

Inserting the partition function (10) into Eq. (2a), the (replica) QCD partition function is then computed in the presence of the (external) diquark source terms $\Delta^\pm(x,y)$, $\mathcal{Z} \to \mathcal{Z}[\Delta^+,\Delta^-]$. In principle, this is not the physically relevant quantity, from which one derives thermodynamic properties of the color superconductor. The diquark condensate is not an external field, but assumes a nonzero value because of an intrinsic property of the system, namely the attractive gluon interaction in the color-antitriplet channel, which destabilizes the Fermi surface.

The proper functional from which one derives thermodynamic functions is obtained by a Legendre transformation of $\ln \mathcal{Z}[\Delta^+,\Delta^-]$, in which the functional dependence on the diquark source term is replaced by that on the corresponding canonically conjugate variable, the diquark condensate. The Legendre-transformed functional is the effective action for the diquark condensate. If the latter is *constant*, the effective action is, up to a factor of V/T, identical to the effective potential. The effective potential is simply a function of the diquark condensate. Its explicit form for large-density QCD was derived in Ref. [10]. The value of this function at its maximum determines the pressure. The maximum is determined by a Dyson-Schwinger equation for the diquark condensate, which is identical to the standard gap equation for the color-superconducting gap. It has been solved in the mean-field approximation in Refs. [11–13]. In the mean-field approximation [14],

$$\Delta^+(x,y) \sim \langle\, \psi_{Cr}(x)\, \bar{\psi}_r(y)\,\rangle \quad , \quad \Delta^-(x,y) \sim \langle\, \psi_r(x)\, \bar{\psi}_{Cr}(y)\,\rangle \; . \tag{12}$$

In this work, we are interested in the gluon propagator, and the derivation of the pressure via a Legendre transformation of $\ln \mathcal{Z}[\Delta^+,\Delta^-]$ is of no concern to us. In the following, we shall therefore continue to consider the partition function in the presence of (external) diquark source terms $\Delta^\pm$.

The diquark source terms in the quark (replica) partition function (10) could in principle be chosen differently for each quark species. This could be made explicit by giving $\Delta^\pm$ a subscript r, $\Delta^\pm \to \Delta^\pm_r$. However, as we take the limit $M \to 1$ at the end, it is not necessary to do so, as only $\Delta^\pm_1 \equiv \Delta^\pm$ will survive anyway. In other words, we use the *same* diquark sources for *all* quark species.

The next step is to explicitly investigate the fluctuations of the diquark condensate around its expectation value. These fluctuations correspond physically to the Nambu-Goldstone excitations (loosely termed "mesons" in the following) in a color superconductor. As mentioned in the introduction, there are five such mesons in a two-flavor color superconductor, corresponding to the generators of $SU(3)_c$ which are broken in the color-superconducting phase. If the condensate is chosen to point in the (anti-) blue direction in fundamental color space, the broken generators are $T_4,\ldots,T_7$ of the original $SU(3)_c$ group and the particular combination $B \equiv (\mathbf{1} + \sqrt{3}T_8)/3$ of generators of the global $U(1)_B$ and local $SU(3)_c$ symmetry [3].

The effective action for the diquark condensate and, consequently, for the meson fields as fluctuations of the diquark condensate, is derived via a Legendre transformation of $\ln \mathcal{Z}[\Delta^+,\Delta^-]$. In this work, we are concerned with the properties of the gluons and thus refrain from computing this effective action explicitly. Consequently, instead of considering the physical meson fields, we consider the variables in $\mathcal{Z}[\Delta^+,\Delta^-]$, which correspond to these fields. These are the fluctuations of the diquark source terms $\Delta^\pm$. We choose these fluctuations to be complex phase factors multiplying the magnitude of the source terms,

$$\Delta^+(x,y) = \mathcal{V}^*(x)\, \Phi^+(x,y)\, \mathcal{V}^\dagger(y) \; , \tag{13a}$$

$$\Delta^-(x,y) = \mathcal{V}(x)\, \Phi^-(x,y)\, \mathcal{V}^T(y) \; , \tag{13b}$$

where

$$\mathcal{V}(x) \equiv \exp\left[i \left(\sum_{a=4}^{7} \varphi_a(x) T_a + \frac{1}{\sqrt{3}}\, \varphi_8(x) B \right) \right] \; . \tag{14}$$

The extra factor $1/\sqrt{3}$ in front of φ_8 as compared to the treatment in Ref. [8] is chosen to simplify the notation in the following.

Although the fields φ_a are not the meson fields themselves, but external fields which, after a Legendre transformation of $\ln \mathcal{Z}[\Delta^+,\Delta^-]$, are replaced by the meson fields, we nevertheless (and somewhat imprecisely) refer to them as meson fields in the following. After having explicitly introduced the fluctuations of the diquark source terms in terms of phase factors, the functions $\Phi^\pm$ are only allowed to fluctuate in magnitude. For the sake of completeness, let us mention that one could again have introduced different fields φ_{ar} for each replica r, but this is not really necessary, as we shall take the limit $M \to 1$ at the end of the calculation anyway.

It is advantageous to also subject the quark fields ψ_r to a nonlinear transformation, introducing new fields χ_r via

$$\psi_r = \mathcal{V}\, \chi_r \quad , \quad \bar{\psi}_r = \bar{\chi}_r\, \mathcal{V}^\dagger \; . \tag{15}$$

Since the meson fields are real-valued and the generators $T_4,\ldots,T_7$ and B are hermitian, the (matrix-valued) operator $\mathcal{V}$ is unitary, $\mathcal{V}^{-1} = \mathcal{V}^\dagger$. Therefore, the measure of the Grassmann integration over quark fields in Eq. (10) remains unchanged. From Eq. (15), the charge-conjugate fields transform as

$$\psi_{Cr} = \mathcal{V}^*\, \chi_{Cr} \quad , \quad \bar{\psi}_{Cr} = \bar{\chi}_{Cr}\, \mathcal{V}^T \; , \tag{16}$$

The advantage of transforming the quark fields is that this preserves the simple structure of the terms coupling the quark fields to the diquark sources,

$$\bar{\psi}_{Cr}(x)\, \Delta^+(x,y)\, \psi_r(y) \equiv \bar{\chi}_{Cr}(x)\, \Phi^+(x,y)\, \chi_r(y) \quad , \quad \bar{\psi}_r(x)\, \Delta^-(x,y)\, \psi_{Cr}(y) \equiv \bar{\chi}_r(x)\, \Phi^-(x,y)\, \chi_{Cr}(y) \; . \tag{17}$$

In mean-field approximation, the diquark source terms are proportional to

$$\Phi^+(x,y) \sim \langle\, \chi_{Cr}(x)\,\bar{\chi}_r(y)\,\rangle \quad , \quad \Phi^-(x,y) \sim \langle\, \chi_r(x)\,\bar{\chi}_{Cr}(y)\,\rangle\ . \tag{18}$$

The transformation (15) has the following effect on the kinetic terms of the quarks and the term coupling quarks to gluons:

$$\bar{\psi}_r\ (i\,\gamma^\mu\partial_\mu + \mu\,\gamma_0 + g\,\gamma_\mu\,A_a^\mu T_a)\ \psi_r = \bar{\chi}_r\ (i\,\gamma^\mu\partial_\mu + \mu\,\gamma_0 + \gamma_\mu\,\omega^\mu)\ \chi_r\ , \tag{19a}$$

$$\bar{\psi}_{Cr}\ \left(i\,\gamma^\mu\partial_\mu - \mu\,\gamma_0 - g\,\gamma_\mu\,A_a^\mu T_a^T\right)\ \psi_{Cr} = \bar{\chi}_{Cr}\ (i\,\gamma^\mu\partial_\mu - \mu\,\gamma_0 + \gamma_\mu\,\omega_C^\mu)\ \chi_{Cr}\ , \tag{19b}$$

where

$$\omega^\mu \equiv \mathcal{V}^\dagger\ (i\,\partial^\mu + g\,A_a^\mu T_a)\ \mathcal{V} \tag{20a}$$

is the $N_cN_f \times N_cN_f$-dimensional Maurer-Cartan one-form introduced in Ref. [15] and

$$\omega_C^\mu \equiv \mathcal{V}^T\ \left(i\,\partial^\mu - g\,A_a^\mu T_a^T\right)\ \mathcal{V}^* \tag{20b}$$

is its charge-conjugate version. Note that the partial derivative acts only on the phase factors $\mathcal{V}$ and $\mathcal{V}^*$ on the right.

Introducing the Nambu-Gor'kov spinors

$$X_r \equiv \begin{pmatrix} \chi_r \\ \chi_{Cr} \end{pmatrix} \quad , \quad \bar{X}_r \equiv (\bar{\chi}_r\,,\,\bar{\chi}_{Cr}) \tag{21}$$

and the $2N_cN_f \times 2N_cN_f$-dimensional Maurer-Cartan one-form

$$\Omega^\mu(x,y) \equiv -i\begin{pmatrix} \omega^\mu(x) & 0 \\ 0 & \omega_C^\mu(x) \end{pmatrix}\ \delta^{(4)}(x-y)\ , \tag{22}$$

the quark (replica) partition function becomes

$$\mathcal{Z}_M[\Omega,\Phi^+,\Phi^-] \equiv \int \prod_{r=1}^{M/2} \mathcal{D}\bar{X}_r\,\mathcal{D}X_r\ \exp\left\{\sum_{r=1}^{M/2}\int_{x,y} \bar{X}_r(x)\ \left[\,\mathcal{S}^{-1}(x,y) + \gamma_\mu\Omega^\mu(x,y)\,\right]\ X_r(y)\right\}\ , \tag{23}$$

where

$$\mathcal{S}^{-1} \equiv \begin{pmatrix} [G_0^+]^{-1} & \Phi^- \\ \Phi^+ & [G_0^-]^{-1} \end{pmatrix}\ . \tag{24}$$

We are interested in the properties of the gluons, and thus may integrate out the fermion fields. This integration can be performed analytically, with the result

$$\mathcal{Z}_M[\Omega,\Phi^+,\Phi^-] \equiv \left[\,\det\left(\mathcal{S}^{-1} + \gamma_\mu\Omega^\mu\right)\,\right]^{M/2}\ . \tag{25}$$

The determinant is to be taken over Nambu-Gor'kov, color, flavor, spin, and space-time indices. Finally, letting $M \to 1$, we obtain the QCD partition function (in the presence of meson, φ_a, and diquark, $\Phi^\pm$, source fields)

$$\mathcal{Z}[\varphi,\Phi^+,\Phi^-] = \int \mathcal{D}A\ \exp\left[S_A + \frac{1}{2}\,\mathrm{Tr}\ln\left(\mathcal{S}^{-1} + \gamma_\mu\Omega^\mu\right)\right]\ . \tag{26}$$

Remembering that Ω^μ is linear in A_a^μ, cf. Eq. (22) with (20), in order to derive the gluon propagator it is sufficient to expand the logarithm to second order in Ω^μ,

$$\begin{aligned}\frac{1}{2}\,\mathrm{Tr}\ln\left(\mathcal{S}^{-1} + \gamma_\mu\Omega^\mu\right) &\simeq \frac{1}{2}\,\mathrm{Tr}\ln\mathcal{S}^{-1} + \frac{1}{2}\,\mathrm{Tr}\left(\mathcal{S}\,\gamma_\mu\Omega^\mu\right) - \frac{1}{4}\mathrm{Tr}\left(\mathcal{S}\,\gamma_\mu\Omega^\mu\,\mathcal{S}\,\gamma_\nu\Omega^\nu\right) \\ &\equiv S_0[\Phi^+,\Phi^-] + S_1[\Omega,\Phi^+,\Phi^-] + S_2[\Omega,\Phi^+,\Phi^-]\ ,\end{aligned} \tag{27}$$

with obvious definitions for the S_i. The quasiparticle propagator is

$$\mathcal{S} \equiv \begin{pmatrix} G^+ & \Xi^- \\ \Xi^+ & G^- \end{pmatrix}\ , \tag{28}$$

with

$$G^{\pm} = \left\{ [G_0^{\pm}]^{-1} - \Sigma^{\pm} \right\}^{-1} \quad , \quad \Sigma^{\pm} = \Phi^{\mp} \, G_0^{\mp} \, \Phi^{\pm} \quad , \quad \Xi^{\pm} = -G_0^{\mp} \, \Phi^{\pm} \, G^{\pm} \; . \tag{29}$$

To make further progress, we now expand ω^μ and ω_C^μ to linear order in the meson fields,

$$\omega^\mu \simeq g \, A_a^\mu \, T_a - \sum_{a=4}^{7} (\partial^\mu \varphi_a) \; T_a - \frac{1}{\sqrt{3}} \, (\partial^\mu \varphi_8) \; B \; , \tag{30a}$$

$$\omega_C^\mu \simeq -g \, A_a^\mu \, T_a^T + \sum_{a=4}^{7} (\partial^\mu \varphi_a) \; T_a^T + \frac{1}{\sqrt{3}} \, (\partial^\mu \varphi_8) \; B^T \; . \tag{30b}$$

The term S_1 in Eq. (27) is simply a tadpole source term for the gluon fields. This term does not affect the gluon propagator, and thus can be ignored in the following.

The quadratic term S_2 represents the contribution of a fermion loop to the gluon self-energy. Its computation proceeds by first taking the trace over Nambu-Gor'kov space,

$$S_2 = -\frac{1}{4} \int_{x,y} \mathrm{Tr}_{c,f,s} \left[G^+(x,y) \, \gamma_\mu \omega^\mu(y) \, G^+(y,x) \, \gamma_\nu \omega^\nu(x) + G^-(x,y) \, \gamma_\mu \omega_C^\mu(y) \, G^-(y,x) \, \gamma_\nu \omega_C^\nu(x) \right.$$
$$\left. + \, \Xi^+(x,y) \, \gamma_\mu \omega^\mu(y) \, \Xi^-(y,x) \, \gamma_\nu \omega_C^\nu(x) + \Xi^-(x,y) \, \gamma_\mu \omega_C^\mu(y) \, \Xi^+(y,x) \, \gamma_\nu \omega^\nu(x) \right] \; . \tag{31}$$

The remaining trace runs only over color, flavor, and spin indices. Using translational invariance, the propagators and fields are now Fourier-transformed as

$$G^{\pm}(x,y) = \frac{T}{V} \sum_K e^{-iK\cdot(x-y)} \, G^{\pm}(K) \; , \tag{32a}$$

$$\Xi^{\pm}(x,y) = \frac{T}{V} \sum_K e^{-iK\cdot(x-y)} \, \Xi^{\pm}(K) \; , \tag{32b}$$

$$\omega^\mu(x) = \sum_P e^{-iP\cdot x} \, \omega^\mu(P) \; , \tag{32c}$$

$$\omega_C^\mu(x) = \sum_P e^{-iP\cdot x} \, \omega_C^\mu(P) \; . \tag{32d}$$

Inserting this into Eq. (31), we arrive at Eq. (C16) of Ref. [8], which in our notation reads

$$S_2 = -\frac{1}{4} \sum_{K,P} \mathrm{Tr}_{c,f,s} \left[G^+(K) \, \gamma_\mu \omega^\mu(P) \, G^+(K-P) \, \gamma_\nu \omega^\nu(-P) + G^-(K) \, \gamma_\mu \omega_C^\mu(P) \, G^-(K-P) \, \gamma_\nu \omega_C^\nu(-P) \right.$$
$$\left. + \, \Xi^+(K) \, \gamma_\mu \omega^\mu(P) \, \Xi^-(K-P) \, \gamma_\nu \omega_C^\nu(-P) + \Xi^-(K) \, \gamma_\mu \omega_C^\mu(P) \, \Xi^+(K-P) \, \gamma_\nu \omega^\nu(-P) \right] \; . \tag{33}$$

The remainder of the calculation is straightforward, but somewhat tedious. First, insert the (Fourier-transform of the) linearized version (30) for the fields ω^μ and ω_C^μ. This produces a plethora of terms which are second order in the gluon and meson fields, with coefficients that are traces over color, flavor, and spin. Next, perform the color and flavor traces in these coefficients. It turns out that some of them are identically zero, preventing the occurrence of terms which mix gluons of adjoint colors 1, 2, and 3 (the unbroken $SU(2)_c$ subgroup) among themselves and with the other gluon and meson fields. Furthermore, there are no terms mixing the meson fields φ_a, $a = 4, \ldots 7$, with φ_8. There are mixed terms between gluons and mesons with adjoint color indices $4, \ldots, 7$, and between the gluon field A_8^μ and the meson field φ_8.

Some of the mixed terms (those which mix gluons and mesons of adjoint colors 4 and 5, as well as 6 and 7) can be eliminated via a unitary transformation analogous to the one employed in Ref. [6], Eq. (80). Introducing the tensors

$$\Pi_{11}^{\mu\nu}(P) \equiv \Pi_{22}^{\mu\nu}(P) \equiv \Pi_{33}^{\mu\nu}(P) = \frac{g^2}{2} \, \frac{T}{V} \sum_K \mathrm{Tr}_s \left[\gamma^\mu \, G^+(K) \, \gamma^\nu \, G^+(K-P) + \gamma^\mu \, G^-(K) \, \gamma^\nu \, G^-(K-P) \right.$$
$$\left. + \, \gamma^\mu \, \Xi^-(K) \, \gamma^\nu \, \Xi^+(K-P) + \gamma^\mu \, \Xi^+(K) \, \gamma^\nu \, \Xi^-(K-P) \right] \; , \tag{34a}$$

cf. Eq. (78a) of Ref. [6],

$$\Pi_{44}^{\mu\nu}(P) \equiv \Pi_{66}^{\mu\nu}(P) = \frac{g^2}{2}\frac{T}{V}\sum_K \mathrm{Tr}_s\left[\gamma^\mu\, G_0^+(K)\,\gamma^\nu\, G^+(K-P) + \gamma^\mu\, G^-(K)\,\gamma^\nu\, G_0^-(K-P)\right] \,, \tag{34b}$$

cf. Eq. (83a) of Ref. [6],

$$\Pi_{55}^{\mu\nu}(P) \equiv \Pi_{77}^{\mu\nu}(P) = \frac{g^2}{2}\frac{T}{V}\sum_K \mathrm{Tr}_s\left[\gamma^\mu\, G^+(K)\,\gamma^\nu\, G_0^+(K-P) + \gamma^\mu\, G_0^-(K)\,\gamma^\nu\, G^-(K-P)\right] \,. \tag{34c}$$

cf. Eq. (83b) of Ref. [6], as well as

$$\Pi_{88}^{\mu\nu}(P) = \frac{2}{3}\,\Pi_0^{\mu\nu}(P) + \frac{1}{3}\,\tilde{\Pi}^{\mu\nu}(P) \,, \tag{34d}$$

$$\begin{aligned}\tilde{\Pi}^{\mu\nu}(P) = \frac{g^2}{2}\frac{T}{V}\sum_K \mathrm{Tr}_s\big[&\gamma^\mu\, G^+(K)\,\gamma^\nu\, G^+(K-P) + \gamma^\mu\, G^-(K)\,\gamma^\nu\, G^-(K-P) \\ &- \gamma^\mu\, \Xi^-(K)\,\gamma^\nu\, \Xi^+(K-P) - \gamma^\mu\, \Xi^+(K)\,\gamma^\nu\, \Xi^-(K-P)\big] \,,\end{aligned} \tag{34e}$$

cf. Eq. (78c) of Ref. [6], where $\Pi_0^{\mu\nu}$ is the gluon self-energy in a dense, but normal-conducting system,

$$\Pi_0^{\mu\nu}(P) = \frac{g^2}{2}\frac{T}{V}\sum_K \mathrm{Tr}_s\left[\gamma^\mu\, G_0^+(K)\,\gamma^\nu\, G_0^+(K-P) + \gamma^\mu\, G_0^-(K)\,\gamma^\nu\, G_0^-(K-P)\right] \,, \tag{34f}$$

cf. Eq. (27b) of Ref. [6], the final result can be written in the compact form (cf. Eq. (C19) of Ref. [8])

$$S_2 = -\frac{1}{2}\frac{V}{T}\sum_P\sum_{a=1}^{8}\left[A_\mu^a(-P) - \frac{i}{g}\,P_\mu\,\varphi^a(-P)\right]\Pi_{aa}^{\mu\nu}(P)\left[A_\nu^a(P) + \frac{i}{g}\,P_\nu\,\varphi^a(P)\right] \,. \tag{35}$$

In deriving Eq. (35), we have made use of the transversality of the polarization tensor in the normal-conducting phase, $\Pi_0^{\mu\nu}(P)\,P_\nu = P_\mu\,\Pi_0^{\mu\nu}(P) = 0$. Note that the tensors $\Pi_{aa}^{\mu\nu}$ for $a = 1$, 2, and 3 are also transverse, but those for $a = 4, \ldots, 8$ are not. This can be seen explicitly from the expressions given in Ref. [7]. The compact notation of Eq. (35) is made possible by the fact that $\varphi^a \equiv 0$ for $a = 1, 2, 3$, and because we introduced the extra factor $1/\sqrt{3}$ in Eq. (14) as compared to Ref. [8].

To make further progress, it is advantageous to tensor-decompose $\Pi_{aa}^{\mu\nu}$. Various ways to do this are possible [8]; here we follow the notation of Ref. [4]. First, define a projector onto the subspace parallel to P^μ,

$$\mathrm{E}^{\mu\nu} = \frac{P^\mu\, P^\nu}{P^2} \,. \tag{36}$$

Then choose a vector orthogonal to P^μ, for instance

$$N^\mu \equiv \left(\frac{p_0\, p^2}{P^2}, \frac{p_0^2\,\mathbf{p}}{P^2}\right) \equiv \left(g^{\mu\nu} - \mathrm{E}^{\mu\nu}\right)\, f_\nu \,, \tag{37}$$

with $f^\mu = (0, \mathbf{p})$. Note that $N^2 = -p_0^2\, p^2/P^2$. Now define the projectors

$$\mathrm{B}^{\mu\nu} = \frac{N^\mu\, N^\nu}{N^2} \quad , \quad \mathrm{C}^{\mu\nu} = N^\mu\, P^\nu + P^\mu\, N^\nu \quad , \quad \mathrm{A}^{\mu\nu} = g^{\mu\nu} - \mathrm{B}^{\mu\nu} - \mathrm{E}^{\mu\nu} \,. \tag{38}$$

Using the explicit form of N^μ, one convinces oneself that the tensor $\mathrm{A}^{\mu\nu}$ projects onto the spatially transverse subspace orthogonal to P^μ,

$$\mathrm{A}^{00} = \mathrm{A}^{0i} = 0 \quad , \quad \mathrm{A}^{ij} = -\left(\delta^{ij} - \hat{p}^i\,\hat{p}^j\right) \,. \tag{39}$$

(Reference [4] also uses the notation $P_T^{\mu\nu}$ for $\mathrm{A}^{\mu\nu}$.) Consequently, the tensor $\mathrm{B}^{\mu\nu}$ projects onto the spatially longitudinal subspace orthogonal to P^μ,

$$\mathrm{B}^{00} = -\frac{p^2}{P^2} \quad , \quad \mathrm{B}^{0i} = -\frac{p_0\, p^i}{P^2} \quad , \quad \mathrm{B}^{ij} = -\frac{p_0^2}{P^2}\,\hat{p}^i\,\hat{p}^j \,. \tag{40}$$

(Reference [4] also employs the notation $P_L^{\mu\nu}$ for $\mathrm{B}^{\mu\nu}$.) With these tensors, the gluon self-energy can be written in the form

$$\Pi_{aa}^{\mu\nu}(P) = \Pi_{aa}^{\mathrm{a}}(P)\,\mathrm{A}^{\mu\nu} + \Pi_{aa}^{\mathrm{b}}(P)\,\mathrm{B}^{\mu\nu} + \Pi_{aa}^{\mathrm{c}}(P)\,\mathrm{C}^{\mu\nu} + \Pi_{aa}^{\mathrm{e}}(P)\,\mathrm{E}^{\mu\nu} \; . \tag{41}$$

The polarization functions Π_{aa}^{a}, Π_{aa}^{b}, Π_{aa}^{c}, and Π_{aa}^{e} can be computed by projecting the tensor $\Pi_{aa}^{\mu\nu}$ onto the respective subspaces of the projectors (36) and (38). Introducing the abbreviations

$$\Pi_{aa}^{t}(P) \equiv \frac{1}{2}\left(\delta^{ij} - \hat{p}^i\,\hat{p}^j\right)\,\Pi_{aa}^{ij}(P) \quad , \quad \Pi_{aa}^{\ell}(P) \equiv \hat{p}_i\,\Pi_{aa}^{ij}(P)\,\hat{p}_j \; . \tag{42}$$

these functions read

$$\Pi_{aa}^{\mathrm{a}}(P) = \frac{1}{2}\,\Pi_{aa}^{\mu\nu}(P)\,\mathrm{A}_{\mu\nu} = -\Pi_{aa}^{t}(P) \; , \tag{43a}$$

$$\Pi_{aa}^{\mathrm{b}}(P) = \Pi_{aa}^{\mu\nu}(P)\,\mathrm{B}_{\mu\nu} = -\frac{p^2}{P^2}\left[\Pi_{aa}^{00}(P) + 2\,\frac{p_0}{p}\,\Pi_{aa}^{0i}(P)\,\hat{p}_i + \frac{p_0^2}{p^2}\,\Pi_{aa}^{\ell}(P)\right] \; , \tag{43b}$$

$$\Pi_{aa}^{\mathrm{c}}(P) = \frac{1}{2\,N^2\,P^2}\,\Pi_{aa}^{\mu\nu}(P)\,\mathrm{C}_{\mu\nu} = -\frac{1}{P^2}\left[\Pi_{aa}^{00}(P) + \frac{p_0^2+p^2}{p_0\,p}\,\Pi_{aa}^{0i}(P)\,\hat{p}_i + \Pi_{aa}^{\ell}(P)\right] \; , \tag{43c}$$

$$\Pi_{aa}^{\mathrm{e}}(P) = \Pi_{aa}^{\mu\nu}(P)\,\mathrm{E}_{\mu\nu} = \frac{1}{P^2}\left[p_0^2\,\Pi_{aa}^{00}(P) + 2\,p_0\,p\,\Pi_{aa}^{0i}(P)\,\hat{p}_i + p^2\,\Pi_{aa}^{\ell}(P)\right] \; . \tag{43d}$$

For the explicitly transverse tensor $\Pi_{11}^{\mu\nu}$, the functions $\Pi_{11}^{\mathrm{c}} = \Pi_{11}^{\mathrm{e}} \equiv 0$. The same holds for the HDL polarization tensor $\Pi_0^{\mu\nu}$. For the other gluon colors $a = 4, \ldots, 8$, the functions Π_{aa}^{c} and Π_{aa}^{e} do not vanish. Note that the dimensions of Π_{aa}^{a}, Π_{aa}^{b}, and Π_{aa}^{e} are $[\mathrm{MeV}^2]$, while Π_{aa}^{c} is dimensionless.

Now let us define the functions

$$A_{\perp\,\mu}^{a}(P) = \mathrm{A}_{\mu}{}^{\nu}\,A_\nu^a(P) \quad , \quad A_{\|}^{a}(P) = \frac{P^\mu\,A_\mu^a(P)}{P^2} \quad , \quad A_N^a(P) = \frac{N^\mu\,A_\mu^a(P)}{N^2} \; . \tag{44}$$

Note that $A_{\|}^{a}(-P) = -P^\mu\,A_\mu^a(-P)/P^2$, and $A_N^a(-P) = -N^\mu\,A_\mu^a(-P)/N^2$, since N^μ is odd under $P \rightarrow -P$. The fields $A_{\|}^a(P)$ and $A_N^a(P)$ are dimensionless. With the tensor decomposition (41) and the functions (44), Eq. (35) becomes

$$\begin{aligned} S_2 = -\frac{1}{2}\,\frac{V}{T}\sum_P\sum_{a=1}^{8}\Bigg\{ & A_{\perp\,\mu}^{a}(-P)\,\Pi_{aa}^{\mathrm{a}}(P)\,\mathrm{A}^{\mu\nu}\,A_{\perp\,\nu}^{a}(P) - A_N^a(-P)\,\Pi_{aa}^{\mathrm{b}}(P)\,N^2\,A_N^a(P) \\ & - \left[A_{\|}^a(-P) + \frac{i}{g}\,\varphi^a(-P)\right]\Pi_{aa}^{\mathrm{c}}(P)\,N^2P^2\,A_N^a(P) - A_N^a(-P)\,\Pi_{aa}^{\mathrm{c}}(P)\,N^2P^2\left[A_{\|}^a(P) + \frac{i}{g}\,\varphi^a(P)\right] \\ & - \left[A_{\|}^a(-P) + \frac{i}{g}\,\varphi^a(-P)\right]\Pi_{aa}^{\mathrm{e}}(P)\,P^2\left[A_{\|}^a(P) + \frac{i}{g}\,\varphi^a(P)\right]\Bigg\} \; . \end{aligned} \tag{45}$$

In any spontaneously broken gauge theory, the excitations of the condensate mix with the gauge fields corresponding to the broken generators of the underlying gauge group. The mixing occurs in the components orthogonal to the spatially transverse degrees of freedom, *i.e.*, for the spatially longitudinal fields, A_N^a, and the fields parallel to P^μ, $A_{\|}^a$. For the two-flavor color superconductor, these components mix with the meson fields for gluon colors $4, \ldots, 8$. The mixing is particularly evident in Eq. (45).

The terms mixing mesons and gauge fields can be eliminated by a suitable choice of gauge. The gauge to accomplish this goal is the 't Hooft gauge. The "unmixing" procedure of mesons and gauge fields consists of two steps. First, we eliminate the terms in Eq. (45) which mix A_N^a and $A_{\|}^a$. This is achieved by substituting

$$\hat{A}_{\|}^a(P) = A_{\|}^a(P) + \frac{\Pi_{aa}^{\mathrm{c}}(P)\,N^2}{\Pi_{aa}^{\mathrm{e}}(P)}\,A_N^a(P) \; . \tag{46}$$

(We do not perform this substitution for $a = 1, 2, 3$; for these gluon colors $\Pi_{aa}^{\mathrm{c}} \equiv 0$, such that there are no terms in Eq. (45) which mix $A_{\|}^a$ and A_N^a). This shift of the gauge field component $A_{\|}^a$ is completely innocuous for the following reasons. First, the Jacobian $\partial(\hat{A}_{\|}, A_N)/\partial(A_{\|}, A_N)$ is unity, so the measure of the functional integral over gauge fields is not affected. Second, the only other term in the gauge field action, which is quadratic in the gauge fields and thus relevant for the derivation of the gluon propagator, is the free field action

$$S^{(0)}_{F^2} \equiv -\frac{1}{2}\,\frac{V}{T}\sum_P\sum_{a=1}^{8} A^a_\mu(-P)\,\left(P^2\,g^{\mu\nu} - P^\mu\,P^\nu\right)\,A^a_\nu(P) \equiv -\frac{1}{2}\,\frac{V}{T}\sum_P\sum_{a=1}^{8} A^a_\mu(-P)\,P^2\,\left(\mathrm{A}^{\mu\nu} + \mathrm{B}^{\mu\nu}\right)\,A^a_\nu(P)\;, \tag{47}$$

and it does not contain the parallel components $A^a_\|(P)$. It is therefore also not affected by the shift of variables (46).

After renaming $\hat{A}^a_\| \to A^a_\|$, the final result for S_2 reads:

$$S_2 = -\frac{1}{2}\,\frac{V}{T}\sum_P\sum_{a=1}^{8}\left\{ A^a_{\perp\,\mu}(-P)\,\Pi^{\mathrm{a}}_{aa}(P)\,A^{\mu\nu}\,A^a_{\perp\,\nu}(P) - A^a_N(-P)\,\hat{\Pi}^{\mathrm{b}}_{aa}(P)\,N^2\,A^a_N(P) \right.$$
$$\left. - \left[A^a_\|(-P) + \frac{i}{g}\,\varphi^a(-P)\right]\,\Pi^{\mathrm{e}}_{aa}(P)\,P^2\,\left[A^a_\|(P) + \frac{i}{g}\,\varphi^a(P)\right]\right\}\;, \tag{48}$$

where we introduced

$$\hat{\Pi}^{\mathrm{b}}_{aa}(P) \equiv \Pi^{\mathrm{b}}_{aa}(P) - \frac{\left[\Pi^{\mathrm{c}}_{aa}(P)\right]^2 N^2 P^2}{\Pi^{\mathrm{e}}_{aa}(P)}\;. \tag{49}$$

The 't Hooft gauge fixing term is now chosen to eliminate the mixing between $A^a_\|$ and φ^a:

$$S_{\mathrm{gf}} = \frac{1}{2\,\lambda}\,\frac{V}{T}\sum_P\sum_{a=1}^{8}\left[P^2\,A^a_\|(-P) - \lambda\,\frac{i}{g}\,\Pi^{\mathrm{e}}_{aa}(P)\,\varphi^a(-P)\right]\,\left[P^2\,A^a_\|(P) - \lambda\,\frac{i}{g}\,\Pi^{\mathrm{e}}_{aa}(P)\,\varphi^a(P)\right]\;. \tag{50}$$

This gauge condition is non-local in coordinate space, which seems peculiar, but poses no problem in momentum space. Note that $P^2\,A^a_\|(P) \equiv P^\mu\,A^a_\mu(P)$. Therefore, in various limits the choice of gauge (50) corresponds to covariant gauge,

$$S_{\mathrm{cg}} = \frac{1}{2\,\lambda}\,\frac{V}{T}\sum_P\sum_{a=1}^{8} A^a_\mu(-P)\,P^\mu\,P^\nu\,A^a_\nu(P)\;. \tag{51}$$

The first limit we consider is $T,\mu \to 0$, *i.e.* the vacuum. Then, $\Pi^{\mathrm{e}}_{aa} \equiv 0$, and Eq. (50) becomes (51). The second case is the limit of large 4-momenta, $P \to \infty$. As shown in Ref. [7], in this region of phase space the effects from a color-superconducting condensate on the gluon polarization tensor are negligible. In other words, the gluon polarization tensor approaches the HDL limit. The physical reason is that gluons with large momenta do not see quark Cooper pairs as composite objects, but resolve the individual color charges inside the pair. Consequently, $\Pi^{\mathrm{e}}_{aa}(P)\,P^2 \to P_\mu\,\Pi^{\mu\nu}_0(P)\,P_\nu \equiv 0$ for $P \to \infty$ and, for large P, the individual terms in the sum over P in Eqs. (50) and (51) agree. Finally, for gluon colors $a = 1, 2, 3$, $\Pi^{\mathrm{e}}_{aa} \equiv 0$, since the self-energy $\Pi^{\mu\nu}_{11}$ is transverse. Thus, for $a = 1, 2, 3$ the terms in Eqs. (50) and (51) are identical.

The decoupling of mesons and gluon degrees of freedom becomes obvious once we add (50) to (48) and (47),

$$S^{(0)}_{F^2} + S_2 + S_{\mathrm{gf}} = -\frac{1}{2}\,\frac{V}{T}\sum_P\sum_{a=1}^{8}\left\{ A^a_{\perp\,\mu}(-P)\,\left[P^2 + \Pi^{\mathrm{a}}_{aa}(P)\right]\,\mathrm{A}^{\mu\nu}\,A^a_{\perp\,\nu}(P)\right.$$
$$- A^a_N(-P)\,\left[P^2 + \hat{\Pi}^{\mathrm{b}}_{aa}(P)\right]\,N^2\,A^a_N(P)$$
$$- A^a_\|(-P)\,\left[\frac{1}{\lambda}\,P^2 + \Pi^{\mathrm{e}}_{aa}(P)\right]\,P^2\,A^a_\|(P)$$
$$\left. + \frac{\lambda}{g^2}\,\varphi^a(-P)\,\left[\frac{1}{\lambda}\,P^2 + \Pi^{\mathrm{e}}_{aa}(P)\right]\,\Pi^{\mathrm{e}}_{aa}(P)\,\varphi^a(P)\right\}\;. \tag{52}$$

Consequently, the inverse gluon propagator is

$$\Delta^{-1\,\mu\nu}_{\;\;aa}(P) = \left[P^2 + \Pi^{\mathrm{a}}_{aa}(P)\right]\,\mathrm{A}^{\mu\nu} + \left[P^2 + \hat{\Pi}^{\mathrm{b}}_{aa}(P)\right]\,\mathrm{B}^{\mu\nu} + \left[\frac{1}{\lambda}\,P^2 + \Pi^{\mathrm{e}}_{aa}(P)\right]\,\mathrm{E}^{\mu\nu}\;. \tag{53}$$

Inverting this as discussed in Ref. [4], one obtains the gluon propagator for gluons of color a,

$$\Delta^{\mu\nu}_{aa}(P) = \frac{1}{P^2 + \Pi^{\mathrm{a}}_{aa}(P)}\,\mathrm{A}^{\mu\nu} + \frac{1}{P^2 + \hat{\Pi}^{\mathrm{b}}_{aa}(P)}\,\mathrm{B}^{\mu\nu} + \frac{\lambda}{P^2 + \lambda\,\Pi^{\mathrm{e}}_{aa}(P)}\,\mathrm{E}^{\mu\nu}\;. \tag{54}$$

For any $\lambda \neq 0$, the gluon propagator contains unphysical contributions parallel to P^μ, which have to be cancelled by the corresponding Faddeev-Popov ghosts when computing physical observables. Only for $\lambda = 0$ these contributions vanish and the gluon propagator is explicitly transverse, *i.e.*, $P_\mu \, \Delta^{\mu\nu}_{aa}(P) = \Delta^{\mu\nu}_{aa}(P)\, P_\nu = 0$. Also, in this case the ghost propagator is independent of the chemical potential μ. The contribution of Fadeev-Popov ghosts to the gluon polarization tensor is then $\sim g^2 T^2$ and thus negligible at $T = 0$. We shall therefore focus on this particular choice for the gauge parameter in the following. Note that for $\lambda = 0$, the inverse meson field propagator is

$$D^{-1}_{aa}(P) \equiv \Pi^{\rm e}_{aa}(P)\, P^2 = P_\mu \, \Pi^{\mu\nu}_{aa}(P)\, P_\nu \;, \tag{55}$$

and the dispersion relation for the mesons follows from the condition $D^{-1}_{aa}(P) = 0$, as demonstrated in Ref. [16] for a three-flavor color superconductor in the color-flavor-locked phase.

The gluon propagator for transverse and longitudinal modes can now be read off Eq. (54) as coefficients of the corresponding tensors $\mathrm{A}^{\mu\nu}$ (the projector onto the spatially transverse subspace orthogonal to P^μ) and $\mathrm{B}^{\mu\nu}$ (the projector onto the spatially longitudinal subspace orthogonal to P^μ). For the transverse modes one has [4]

$$\Delta^t_{aa}(P) \equiv \frac{1}{P^2 + \Pi^{\rm a}_{aa}(P)} = \frac{1}{P^2 - \Pi^t_{aa}(P)} \;, \tag{56}$$

where we used Eq. (43a). Multiplying the coefficient of $\mathrm{B}^{\mu\nu}$ in Eq. (54) with the standard factor $-P^2/p^2$ [4], one obtains for the longitudinal modes

$$\hat{\Delta}^{00}_{aa}(P) \equiv -\frac{P^2}{p^2}\, \frac{1}{P^2 + \hat{\Pi}^{\rm b}_{aa}(P)} = -\frac{1}{p^2 - \hat{\Pi}^{00}_{aa}(P)} \;, \tag{57}$$

where the longitudinal gluon self-energy

$$\hat{\Pi}^{00}_{aa}(P) \equiv p^2 \, \frac{\Pi^{00}_{aa}(P)\, \Pi^\ell_{aa}(P) - \left[\Pi^{0i}_{aa}(P)\, \hat{p}_i\right]^2}{p_0^2\, \Pi^{00}_{aa}(P) + 2\, p_0\, p\, \Pi^{0i}_{aa}(P)\, \hat{p}_i + p^2\, \Pi^\ell_{aa}(P)} \tag{58}$$

follows from the definition of $\hat{\Pi}^{\rm b}_{aa}$, Eq. (49), and the relations (43). The longitudinal gluon propagator $\hat{\Delta}^{00}_{aa}$ *must not be confused* with the the 00-component of $\Delta^{\mu\nu}_{aa}$. We deliberately use this (slightly ambiguous) notation to facilitate the comparison of our new and correct results with those of Ref. [7], which were partially incorrect. The results of that paper were derived in Coulomb gauge, where the 00-component of the propagator is indeed *identical* to the longitudinal propagator (57). We were not able to find a 't Hooft gauge that converged to the Coulomb gauge in the various limits discussed above, and consequently had to base our discussion on the covariant gauge (51) as limiting case of Eq. (50).

To summarize this section, we have computed the gluon propagator for gluons in a two-flavor color superconductor. Due to condensation of quark Cooper pairs, the $SU(3)_c$ gauge symmetry is spontaneously broken to $SU(2)_c$, leading to the appearance of five Nambu-Goldstone bosons. In general, these bosons mix with some components of the gauge fields corresponding to the broken generators. To "unmix" them we have used a form of 't Hooft gauge which smoothly converges to covariant gauge in the vacuum, as well as for large gluon momenta, and when the gluon polarization tensor is explicitly transverse. Finally, choosing the gauge fixing parameter $\lambda = 0$ we derived the gluon propagator for transverse, Eq. (56), and longitudinal modes, Eq. (57) with (58).

III. SPECTRAL PROPERTIES OF THE EIGHTH GLUON

In this section, we explicitly compute the spectral properties of the eighth gluon. We shall confirm the results of Ref. [7] for the transverse mode and amend those for the longitudinal mode, which have not been correctly computed in Ref. [7]. In particular, we shall show that the plasmon dispersion relation now has the correct behavior $p_0 \to m_g$ as $p \to 0$. Furthermore, the longitudinal spectral density vanishes for gluon energies and momenta located on the dispersion branch of the Nambu-Goldstone bosons, *i.e.*, for energies and momenta given by the roots of Eq. (55). For the eighth gluon, this condition can be written in the form $P_\mu \, \tilde{\Pi}^{\mu\nu}(P)\, P_\nu = 0$ [9,16], since the HDL self-energy is transverse, $P_\mu \, \Pi^{\mu\nu}_0(P)\, P_\nu \equiv 0$.

A. Polarization tensor

We first compute the polarization tensor for the transverse and longitudinal components of the eighth gluon. To this end, it is convenient to rewrite the longitudinal gluon self-energy (58) in the form

$$\hat{\Pi}^{00}_{88}(P) \equiv \frac{2}{3}\,\Pi^{00}_0(P) + \frac{1}{3}\,\hat{\Pi}^{00}(P)\;, \tag{59}$$

$$\hat{\Pi}^{00}(P) \equiv p^2\,\frac{\tilde{\Pi}^{00}(P)\,\tilde{\Pi}^{\ell}(P) - \left[\tilde{\Pi}^{0i}(P)\,\hat{p}_i\right]^2}{p_0^2\,\tilde{\Pi}^{00}(P) + 2\,p_0\,p\,\tilde{\Pi}^{0i}(P)\,\hat{p}_i + p^2\,\tilde{\Pi}^{\ell}(P)}\;, \tag{60}$$

with $\tilde{\Pi}^{\ell}(P) \equiv \hat{p}_i\,\tilde{\Pi}^{ij}(P)\,\hat{p}_j$.

Let us now explicitly compute the polarization functions. As in Ref. [7] we take $T = 0$, and we shall use the identity

$$\frac{1}{x + i\eta} \equiv \mathcal{P}\,\frac{1}{x} - i\pi\,\delta(x)\;, \tag{61}$$

where $\mathcal{P}$ stands for the principal value description, in order to decompose the polarization tensor into real and imaginary parts. The imaginary parts can then be straightforwardly computed, while the real parts are computed from the dispersion integral

$$\mathrm{Re}\,\Pi(p_0, \mathbf{p}) \equiv \frac{1}{\pi}\,\mathcal{P}\int_{-\infty}^{\infty} d\omega\,\frac{\mathrm{Im}\,\Pi(\omega, \mathbf{p})}{\omega - p_0} + C\;, \tag{62a}$$

where C is a (subtraction) constant. If $\mathrm{Im}\,\Pi(\omega, \mathbf{p})$ is an odd function of ω, $\mathrm{Im}\,\Pi(-\omega, \mathbf{p}) = -\mathrm{Im}\,\Pi(\omega, \mathbf{p})$, Eq. (62a) becomes Eq. (39) of Ref. [7],

$$\mathrm{Re}\,\Pi(p_0, \mathbf{p}) \equiv \frac{1}{\pi}\,\mathcal{P}\int_0^{\infty} d\omega\,\mathrm{Im}\,\Pi_{\mathrm{odd}}(\omega, \mathbf{p})\left(\frac{1}{\omega + p_0} + \frac{1}{\omega - p_0}\right) + C\;, \tag{62b}$$

and if it is an even function of ω, $\mathrm{Im}\,\Pi(-\omega, \mathbf{p}) = \mathrm{Im}\,\Pi(\omega, \mathbf{p})$, we have instead

$$\mathrm{Re}\,\Pi(p_0, \mathbf{p}) \equiv \frac{1}{\pi}\,\mathcal{P}\int_0^{\infty} d\omega\,\mathrm{Im}\,\Pi_{\mathrm{even}}(\omega, \mathbf{p})\left(\frac{1}{\omega - p_0} - \frac{1}{\omega + p_0}\right) + C\;, \tag{62c}$$

Since the polarization tensor for the transverse gluon modes, $\Pi^t_{88} \equiv \frac{2}{3}\,\Pi^t_0 + \frac{1}{3}\,\tilde{\Pi}^t$, has already been computed in Ref. [7], we just cite the results. The imaginary part of the transverse HDL polarization function reads (cf. Eq. (22b) of Ref. [7])

$$\mathrm{Im}\,\Pi^t_0(P) = -\pi\,\frac{3}{4}\,m_g^2\,\frac{p_0}{p}\left(1 - \frac{p_0^2}{p^2}\right)\theta(p - p_0)\;. \tag{63a}$$

The corresponding real part is computed from Eq. (62b), with the result (cf. Eqs. (40b) and (41) of Ref. [7])

$$\mathrm{Re}\,\Pi^t_0(P) = \frac{3}{2}\,m_g^2\left[\frac{p_0^2}{p^2} + \left(1 - \frac{p_0^2}{p^2}\right)\frac{p_0}{2\,p}\,\ln\left|\frac{p_0 + p}{p_0 - p}\right|\right]\;. \tag{63b}$$

We have used the fact that the value of the subtraction constant is $C^t_0 = m_g^2$, which can be derived from comparing a direct calculation of $\mathrm{Re}\,\Pi^t_0$ using Eq. (19b) of Ref. [7] with the above computation via the dispersion formula (62b).

The imaginary part of the tensor $\tilde{\Pi}^t$ is given by (cf. Eq. (36) of Ref. [7])

$$\begin{aligned}\mathrm{Im}\,\tilde{\Pi}^t(P) = -\pi\,\frac{3}{4}\,m_g^2\,\theta(p_0 - 2\,\phi)\,\frac{p_0}{p}\Bigg\{ &\theta(E_p - p_0)\left[\left(1 - \frac{p_0^2}{p^2}\,(1 + s^2)\right)\mathbf{E}(t) - s^2\left(1 - 2\,\frac{p_0^2}{p^2}\right)\mathbf{K}(t)\right] \\ + \theta(p_0 - E_p)&\left[\left(1 - \frac{p_0^2}{p^2}\,(1 + s^2)\right)E(\alpha, t) - \left(1 - \frac{p_0^2}{p^2}\right)\frac{p}{p_0}\sqrt{1 - \frac{4\,\phi^2}{p_0^2 - p^2}} - s^2\left(1 - 2\,\frac{p_0^2}{p^2}\right)F(\alpha, t)\right]\Bigg\}\;,\end{aligned} \tag{64}$$

where ϕ is the value of the color-superconducting gap, $E_p = \sqrt{p^2 + 4\phi^2}$, $t = \sqrt{1 - 4\phi^2/p_0^2}$, $s^2 = 1 - t^2$, $\alpha = \arcsin[p/(t p_0)]$, and $F(\alpha, t)$, $E(\alpha, t)$ are elliptic integrals of the first and second kind, while $\mathbf{K}(t) \equiv F(\pi/2, t)$ and

$\mathbf{E}(t) \equiv E(\pi/2,t)$ are the corresponding complete elliptic integrals. The real part is again computed from Eq. (62b). The integral has to be done numerically, see Appendix A of Ref. [7] for details. The subtraction constant is, for reasons discussed at length in Ref. [7], identical to the one in the HDL limit, $C^t \equiv C_0^t = m_g^2$. Finally, taking the linear combination $\Pi^t_{88} \equiv \frac{2}{3}\,\Pi^t_0 + \frac{1}{3}\,\tilde{\Pi}^t$ completes the calculation of the transverse polarization function Π^t_{88}.

In order to compute the polarization function for the longitudinal gluon, $\hat{\Pi}^{00}_{88}$, we have to know the functions $\Pi^{00}_0(P)$, $\tilde{\Pi}^{00}(P)$, $\tilde{\Pi}^{0i}(P)\,\hat{p}_i$, and $\tilde{\Pi}^{\ell}(P)$. The first two functions, $\Pi^{00}_0(P)$ and $\tilde{\Pi}^{00}(P)$ have also been computed in Ref. [7]. The imaginary part of the longitudinal HDL polarization function is (cf. Eq. (22a) of Ref. [7])

$$\mathrm{Im}\,\Pi^{00}_0(P) = -\pi\,\frac{3}{2}\,m_g^2\,\frac{p_0}{p}\,\theta(p-p_0)\;. \tag{65a}$$

The real part is computed from Eq. (62b), with the result (cf. Eqs. (40a) and (41) of Ref. [7])

$$\mathrm{Re}\,\Pi^{00}_0(P) = -3\,m_g^2\,\left(1-\frac{p_0}{2\,p}\,\ln\left|\frac{p_0+p}{p_0-p}\right|\right)\;. \tag{65b}$$

Here, the subtraction constant is $C^{00}_0 = 0$.

The imaginary part of the function $\tilde{\Pi}^{00}$ is (cf. Eq. (35) of Ref. [7])

$$\mathrm{Im}\,\tilde{\Pi}^{00}(P) = -\pi\,\frac{3}{2}\,m_g^2\,\theta(p_0-2\,\phi)\,\frac{p_0}{p}\left\{\theta(E_p-p_0)\,\mathbf{E}(t)+\theta(p_0-E_p)\left[E(\alpha,t)-\frac{p}{p_0}\sqrt{1-\frac{4\,\phi^2}{p_0^2-p^2}}\right]\right\}\;. \tag{66}$$

The real part is computed from Eq. (62b), with the subtraction constant $C^{00} \equiv C^{00}_0 = 0$. Again, the integral has to be done numerically.

It remains to compute the functions $\tilde{\Pi}^{0i}(P)\,\hat{p}_i$ and $\tilde{\Pi}^{\ell}(P)$. First, one performs the spin traces in Eq. (34e) to obtain Eqs. (102b) and (102c) of Ref. [6]. Then, taking $T=0$,

$$\begin{aligned}\tilde{\Pi}^{0i}(P)\,\hat{p}_i &= \frac{g^2}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\sum_{e_1,e_2=\pm}\left(e_1\,\hat{\mathbf{k}}_1\cdot\mathbf{p}+e_2\,\hat{\mathbf{k}}_2\cdot\mathbf{p}\right)\left(\frac{\xi_2}{2\epsilon_2}-\frac{\xi_1}{2\epsilon_1}\right)\\ &\quad\times\left(\frac{1}{p_0+\epsilon_1+\epsilon_2+i\eta}+\frac{1}{p_0-\epsilon_1-\epsilon_2+i\eta}\right)\;,\end{aligned} \tag{67a}$$

$$\begin{aligned}\tilde{\Pi}^{\ell}(P) &= -\frac{g^2}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\sum_{e_1,e_2=\pm}\left[\left(1-e_1e_2\,\hat{\mathbf{k}}_1\cdot\mathbf{k}_2\right)+2\,e_1e_2\,\hat{\mathbf{k}}_1\cdot\mathbf{p}\,\hat{\mathbf{k}}_2\cdot\mathbf{p}\right]\,\frac{\epsilon_1\epsilon_2-\xi_1\xi_2-\phi_1\phi_2}{2\,\epsilon_1\epsilon_2}\\ &\quad\times\left(\frac{1}{p_0+\epsilon_1+\epsilon_2+i\eta}-\frac{1}{p_0-\epsilon_1-\epsilon_2+i\eta}\right)\;,\end{aligned} \tag{67b}$$

where $\mathbf{k}_{1,2} = \mathbf{k}\pm\mathbf{p}/2$, $\phi_i \equiv \phi^{e_i}_{\mathbf{k}_i}$ is the gap function for quasiparticles ($e_i=+1$) or quasi-antiparticles ($e_i=-1$) with momentum $\mathbf{k}_i$, $\xi_i \equiv e_ik_i-\mu$, and $\epsilon_i \equiv \sqrt{\xi_i^2+\phi_i^2}$.

One now repeats the steps discussed in detail in Section II.A of Ref. [7] to obtain (for $p_0 \geq 0$)

$$\begin{aligned}\mathrm{Im}\,\tilde{\Pi}^{0i}(P)\,\hat{p}_i &= \pi\,\frac{3}{2}\,m_g^2\,\theta(p_0-2\,\phi)\,\frac{p_0^2}{p^2}\,\Big\{\,\theta(E_p-p_0)\,\left[\mathbf{E}(t)-s^2\,\mathbf{K}(t)\right]\\ &\quad+\theta(p_0-E_p)\left[E(\alpha,t)-\frac{p}{p_0}\sqrt{1-\frac{4\,\phi^2}{p_0^2-p^2}}-s^2\,F(\alpha,t)\right]\Big\}\;,\end{aligned} \tag{68a}$$

$$\begin{aligned}\mathrm{Im}\,\tilde{\Pi}^{\ell}(P) &= -\pi\,\frac{3}{2}\,m_g^2\,\theta(p_0-2\,\phi)\,\frac{p_0^3}{p^3}\,\Big\{\,\theta(E_p-p_0)\,\left[(1+s^2)\,\mathbf{E}(t)-2\,s^2\,\mathbf{K}(t)\right]\\ &\quad+\theta(p_0-E_p)\left[(1+s^2)\,E(\alpha,t)-\frac{p}{p_0}\sqrt{1-\frac{4\,\phi^2}{p_0^2-p^2}}-2\,s^2\,F(\alpha,t)\right]\Big\}\;.\end{aligned} \tag{68b}$$

One observes that in the limit $\phi\to 0$, the functions (68) approach the HDL result

$$\mathrm{Im}\,\Pi^{0i}_0(P)\,\hat{p}_i = \pi\,\frac{3}{2}\,m_g^2\,\frac{p_0^2}{p^2}\,\theta(p-p_0)\;, \tag{69a}$$

$$\mathrm{Im}\,\Pi^{\ell}_0(P) = -\pi\,\frac{3}{2}\,m_g^2\,\frac{p_0^3}{p^3}\,\theta(p-p_0)\;. \tag{69b}$$

Applying Eq. (61) to Eqs. (67) we immediately see that the imaginary part of $\tilde{\Pi}^{0i}(P)\,\hat{p}_i$ is *even*, while that of $\tilde{\Pi}^{\ell}(P)$ is *odd*. Thus, in order to compute the real part of $\tilde{\Pi}^{0i}(P)\,\hat{p}_i$, we have to use Eq. (62c), while the real part of $\tilde{\Pi}^{\ell}(P)$ has to be computed from Eq. (62b). When implementing the numerical procedure discussed in Appendix A of Ref. [7] for the integral in Eq. (62c), one has to modify Eq. (A1) of Ref. [7] appropriately.

Finally, one has to determine the values of the subtraction constants C^{0i} and C^{ℓ}. We again use the fact that $C^{0i} \equiv C_0^{0i}$ and $C^{\ell} \equiv C_0^{\ell}$, where the index "0" refers to the HDL limit. The corresponding constants are determined by first computing $\mathrm{Re}\,\Pi_0^{0i}(P)\,\hat{p}_i$ and $\mathrm{Re}\,\Pi_0^{\ell}(P)$ from the dispersion formulas (62b) and (62c). The result of this calculation is then compared to that of a direct computation using, for instance, the result (65b) for $\mathrm{Re}\,\Pi_0^{00}(P)$ and then inferring $\mathrm{Re}\,\Pi_0^{0i}(P)\,\hat{p}_i$ and $\mathrm{Re}\,\Pi_0^{\ell}(P)$ from the transversality of $\Pi_0^{\mu\nu}$. The result is $C^{0i} \equiv C_0^{0i} = 0$ and $C^{\ell} \equiv C_0^{\ell} = m_g^2$.

At this point, we have determined all functions entering the transverse and longitudinal polarization functions for the eighth gluon. In Fig. 1 we show the imaginary parts and in Fig. 2 the real parts, for a fixed gluon momentum $p = 4\,\phi$, as a function of gluon energy p_0 (in units of $2\,\phi$). The units for the imaginary parts are $-3\,m_g^2/2$, and for the real parts $+3\,m_g^2/2$. For comparison, in parts (a) and (g) of these figures, we show the results from Ref. [7] for the longitudinal and transverse polarization function of the gluon with adjoint color 1. In parts (d), (e), and (f) the functions $\tilde{\Pi}^{00}$, $-\tilde{\Pi}^{0i}\,\hat{p}_i$, and $\tilde{\Pi}^{\ell}$ are shown. According to Eq. (60) these are required to determine $\hat{\Pi}^{00}$, shown in part (b). Using Eq. (59), this result is then combined with the HDL polarization function Π_0^{00} to compute $\hat{\Pi}_{88}^{00}$, shown in part (c). Finally, the transverse polarization function for gluons of color 8 is shown in part (i). This function is given by the linear combination $\Pi_{88}^t = \frac{2}{3}\,\Pi_0^t + \frac{1}{3}\,\tilde{\Pi}^t$ of the transverse HDL polarization function Π_0^t and the function $\tilde{\Pi}^t$, both of which are shown in part (h). In all figures, the results for the two-flavor color superconductor are drawn as solid lines, while the dotted lines correspond to those in a normal conductor, $\phi \to 0$ (the HDL limit).

Note that parts (a), (d), (g), (h), and (i) of Figs. 1 and 2 agree with parts (a), (b), (d), (e), and (f) of Figs. 2 and 3 of Ref. [7]. The new results are parts (e) and (f) of Figs. 1 and 2, which are used to determine the functions in parts (b) and (c), the latter showing the correct longitudinal polarization function for the eighth gluon. In Ref. [7], this function was not computed correctly, as the effect from the fluctuations of the condensate on the polarization tensor of the gluons was not taken into account.

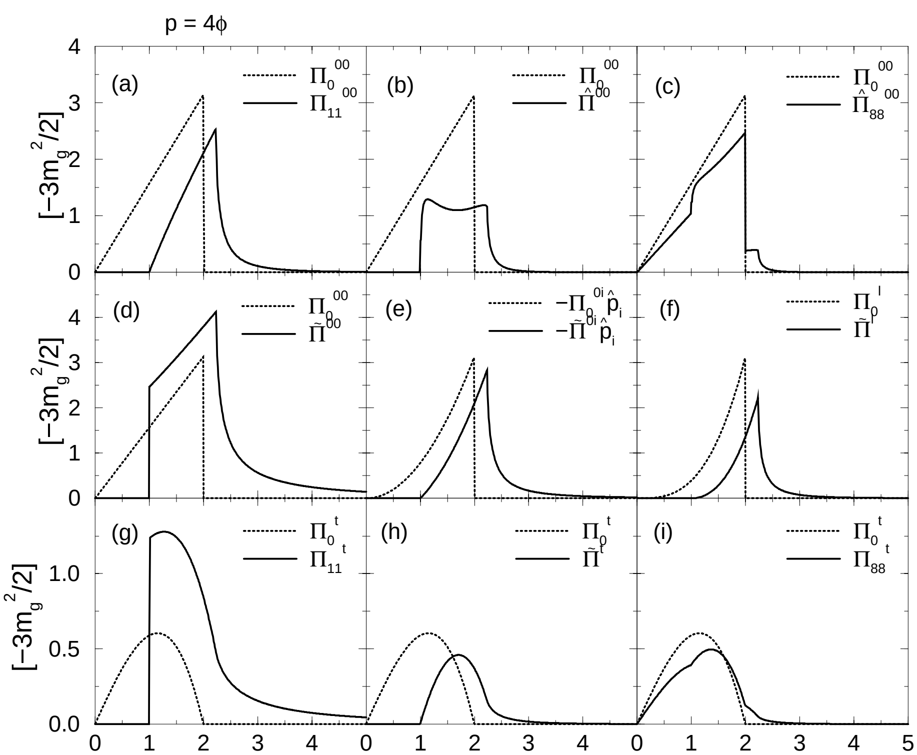

FIG. 1. Imaginary parts of the polarization tensors in a two-flavor color superconductor (solid lines) as a function of gluon energy p_0 for fixed gluon momentum $p = 4\,\phi$. (a) $\mathrm{Im}\,\Pi_{11}^{00}$, (b) $\mathrm{Im}\,\hat{\Pi}^{00}$, (c) $\mathrm{Im}\,\hat{\Pi}_{88}^{00}$, (d) $\mathrm{Im}\,\tilde{\Pi}^{00}$, (e) $-\mathrm{Im}\,\tilde{\Pi}^{0i}\,\hat{p}_i$, (f) $\mathrm{Im}\,\tilde{\Pi}^{\ell}$, (g) $\mathrm{Im}\,\Pi_{11}^t$, (h) $\mathrm{Im}\,\tilde{\Pi}^t$, (i) $\mathrm{Im}\,\Pi_{88}^t$. The corresponding results in the HDL limit, *i.e.* for $\phi = 0$, are shown as dotted lines.

The singularity around a gluon energy somewhat smaller than $p_0 = 2\,\phi$ visible in Figs. 2 (b) and (c) seems peculiar. It turns out that it arises due to a zero in the denominator of $\hat{\Pi}^{00}$ in Eq. (60), *i.e.*, when $P_\mu\,\tilde{\Pi}^{\mu\nu}(P)\,P_\nu = 0$. As discussed above, this condition defines the dispersion branch of the Nambu-Goldstone excitations [16]. Therefore, the singularity is tied to the existence of the Nambu-Goldstone excitations of the diquark condensate.

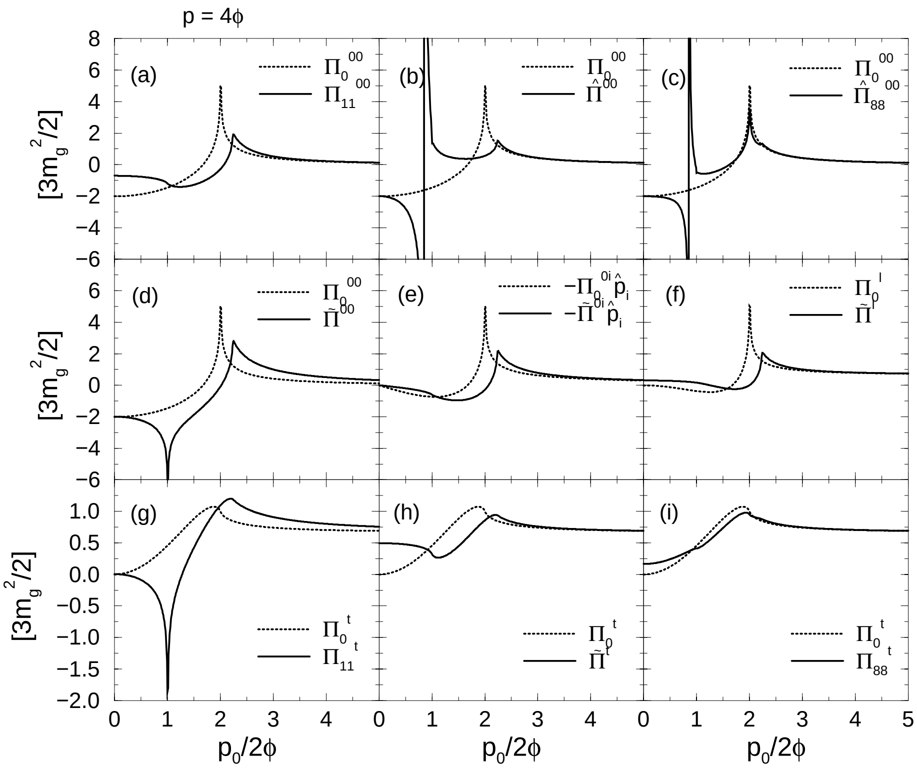

FIG. 2. The same as in Fig. 1, but for the real parts.

B. Spectral densities

Let us now determine the spectral densities for longitudinal and transverse modes, defined by (cf. Eq. (45) of Ref. [7])

$$\rho^{00}_{88}(p_0,\mathbf{p}) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\hat{\Delta}^{00}_{88}(p_0+i\eta,\mathbf{p}) \quad , \quad \rho^{t}_{88}(p_0,\mathbf{p}) \equiv \frac{1}{\pi}\,\mathrm{Im}\,\Delta^{t}_{88}(p_0+i\eta,\mathbf{p}) \tag{70}$$

The longitudinal and transverse spectral densities for gluons of color 8 are shown in Figs. 3 (c) and (d), for fixed gluon momentum $p = m_g/2$ and $m_g = 8\,\phi$. For comparison, the corresponding spectral densities for gluons of color 1 are shown in parts (a) and (b). Parts (a), (b), and (d) are identical to those of Fig. 6 of Ref. [7], part (c) is new and replaces Fig. 6 (c) of Ref. [7]. One observes a peak in the spectral density around $p_0 = m_g$. This peak corresponds to the ordinary longitudinal gluon mode (the plasmon) present in a dense (or hot) medium.

Note that the longitudinal spectral density for gluons of color 8 vanishes at an energy somewhat smaller than $p_0 = m_g/4$. The reason is the singularity of the real part of the gluon self-energy seen in Figs. 2 (b) and (c). The location of this point is where $P_\mu\,\tilde{\Pi}^{\mu\nu}(P)\,P_\nu = 0$, *i.e.*, on the dispersion branch of the Nambu-Goldstone excitations.

Finally, we show in Fig. 4 the dispersion relations for all excitations, defined by the roots of

$$p^2 - \mathrm{Re}\,\hat{\Pi}^{00}_{88}(p_0,\mathbf{p}) = 0 \tag{71a}$$

for longitudinal gluons (cf. Eq. (47a) of Ref. [7]), and by the roots of

$$p_0^2 - p^2 - \mathrm{Re}\,\Pi^{t}_{88}(p_0,\mathbf{p}) = 0 \tag{71b}$$

for transverse gluons (cf. Eq. (47b) of Ref. [7]). Let us mention that not all excitations found via Eqs. (71) correspond to truly stable quasiparticles, *i.e.*, the imaginary parts of the self-energies do not always vanish along the dispersion curves. Nevertheless, in that case Eqs. (71) can still be used to identify peaks in the spectral densities, which correspond to *unstable* modes (which decay with a rate proportional to the width of the peak). As long as the width of the peak (the decay rate of the quasiparticles) is small compared to its height, it makes sense to refer to these modes as quasiparticles.

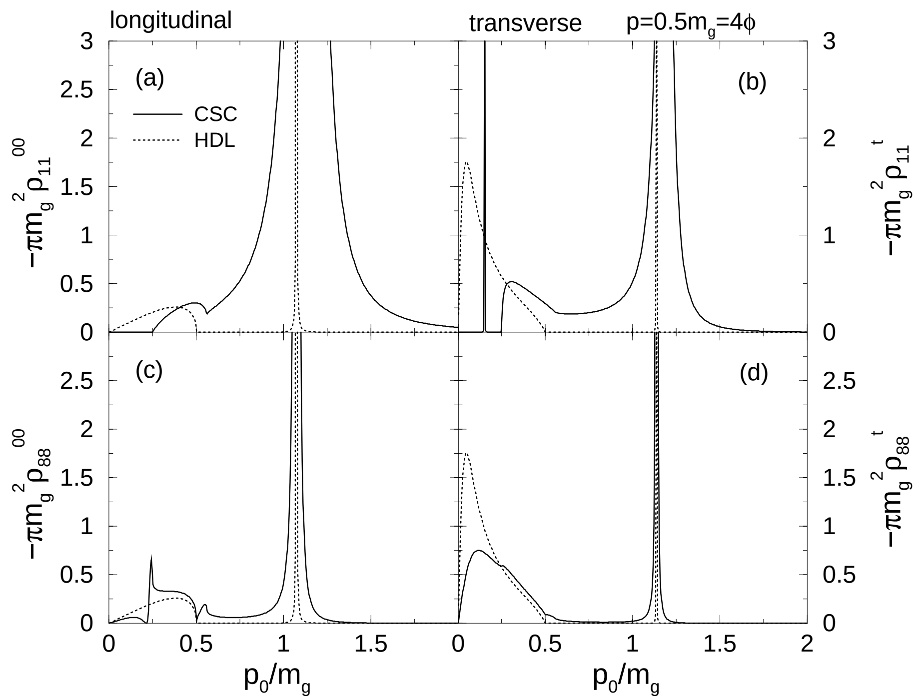

FIG. 3. The longitudinal (a), (c) and transverse (b), (d) spectral densities for gluons of color 1 (a), (b) and 8 (c), (d). The gluon momentum is $p = m_g/2$ and $m_g = 8\phi$. For comparison, the dotted lines represent the corresponding HDL spectral densities. The poles of the spectral density corresponding to stable quasiparticles are made visible by using a numerically small but nonzero imaginary part.

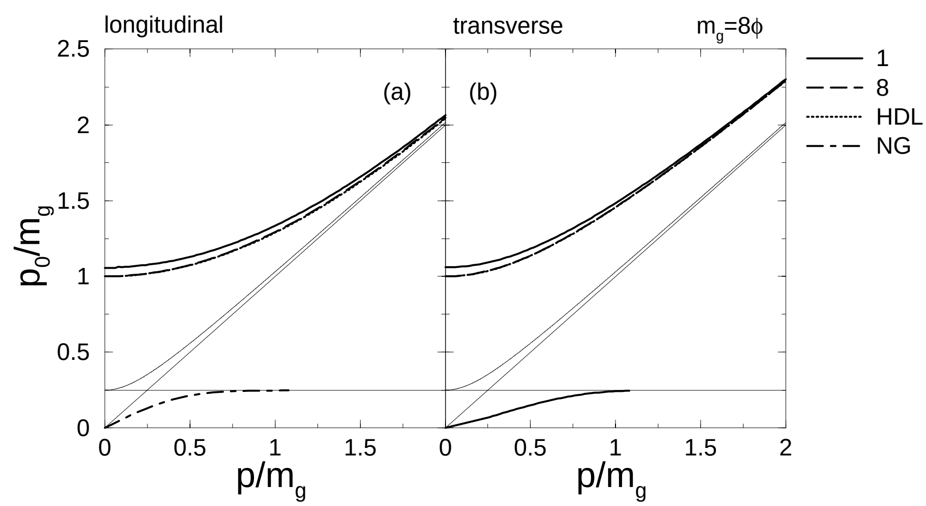

FIG. 4. Dispersion relations for (a) longitudinal and (b) transverse modes for $m_g = 8\,\phi$. The solid lines are for gluons of color 1, the dashed lines for gluons of color 8. The dotted lines correspond to the dispersion relations in the HDL limit. For both longitudinal and transverse gluons of color 8, the dispersion curves are indistinguishable from the HDL curves. The additional branch shown in (a) as dashed-dotted line is the one for the Nambu-Goldstone excitations, which appears as a zero in the longitudinal spectral density.

Fig. 4 corresponds to Fig. 5 of Ref. [7]. In fact, part (b) is identical in both figures. Fig. 4 (a) differs from Fig. 5 (a) of Ref. [7], reflecting our new and correct results for the longitudinal gluon self-energy. In Fig. 5 (a) of Ref. [7], the dispersion curve for the longitudinal gluon of color 8 was seen to diverge for small gluon momenta. In Ref. [7] it was argued that this behavior was due to neglecting the mesonic fluctuations of the diquark condensate. Indeed, properly accounting for these modes, we obtain a reasonable dispersion curve, approaching $p_0 = m_g$ as the momentum goes to zero. In Fig. 4 (a) we also show the dispersion branch for the Nambu-Goldstone excitations (dash-dotted). This is strictly speaking not given by a root of Eq. (71), but by the singularity of the real part of the longitudinal gluon self-energy. However, because this singularity involves a change of sign, a normal root-finding algorithm applied to Eq. (71) will also locate this singularity. As expected [16], the dispersion branch is linear,

$$p_0 \simeq \frac{1}{\sqrt{3}}\, p \;, \tag{72}$$

for small gluon momenta, and approaches the value $p_0 = 2\,\phi$ for $p \to \infty$.

IV. CONCLUSIONS

In cold, dense quark matter with $N_f = 2$ massless quark flavors, condensation of quark Cooper pairs spontaneously breaks the $SU(3)_c$ gauge symmetry to $SU(2)_c$. This results in five Nambu-Goldstone excitations which mix with some of the components of the gluon fields corresponding to the broken generators. We have shown how to decouple them by a particular choice of 't Hooft gauge. The unphysical degrees of freedom in the gluon propagator can be eliminated by fixing the 't Hooft gauge parameter $\lambda = 0$. In this way, we derived the propagator for transverse and longitudinal gluon modes in a two-flavor color superconductor accounting for the effect of the Nambu-Goldstone excitations.

We then proceeded to explicitly compute the spectral properties of transverse and longitudinal gluons of adjoint color 8. The spectral density of the longitudinal mode now exhibits a well-behaved plasmon branch with the correct low-momentum limit $p_0 \to m_g$. Moreover, the spectral density vanishes for gluon energies and momenta corresponding to the dispersion relation for Nambu-Goldstone excitations. We have thus amended and corrected previous results presented in Ref. [7].

Our results pose one final question: using the correct expression for the longitudinal self-energy of adjoint colors $4, \ldots, 8$, do the values of the Debye masses derived in Ref. [6] change? The answer is "no". In the limit $p_0 = 0$, $p \to 0$, application of Eqs. (120), (124), and (129) of Ref. [6] to Eq. (58) yields $\hat{\Pi}^{00}_{aa}(0) \equiv \Pi^{00}_{aa}(0)$, and the results of Ref. [6] for the Debye masses remain valid.

ACKNOWLEDGMENTS

We thank G. Carter, D. Diakonov, and R.D. Pisarski for discussions. We thank R.D. Pisarski in particular for a critical reading of the manuscript and for the suggestion to use 't Hooft gauge to decouple meson and gluon modes. D.H.R. thanks the Nuclear Theory groups at BNL and Columbia University for their hospitality during a visit where part of this work was done. He also gratefully acknowledges continuing access to the computing facilities of Columbia University's Nuclear Theory group. I.A.S. would like to thank the members of the Institut für Theoretische Physik at the Johann Wolfgang Goethe-Universität for their hospitality, where part of this work was done. The work of I.A.S. was supported by the U.S. Department of Energy Grant No. DE-FG02-87ER40328.

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