Some bounds for M(t)/M(t)/S queue with catastrophes

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ABSTRACT

We consider the $M(t)/M(t)/S$ queue with catastrophes. The bounds of the rate of convergence to the limit regime and the estimates of the limit probabilities are obtained. We also study the bounds for the mean of the queue and consider an example.

Keywords

Nonstationary queues, Markovian model with catastrophes, weak ergodicity, bounds

1. INTRODUCTION

The simplest queueing models with catastrophes have been studied some years ago, see the motivation and first results in [1, 5]. Here we consider the essentially general situation of nonstationary Markovian queue $M(t)/M(t)/S$ with catastrophes and obtain some interesting bounds and approximations.

Our approach is based on the method introduced by Gnedenko and Makarov (see [3]) and successively worked out by one of the authors in [6] and [7].

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Let $X = X(t)$, $t \geq 0$ be a queue-length process for $M(t)/M(t)/S$ queue with catastrophes with arrival, service, and catastrophe intensities $\lambda(t)$, $\mu(t)$ and $\xi(t)$, respectively.

Let $p_{ij}(s,t) = Pr\{X(t) = j \,|X(s) = i\}$ for $i,j \geq 0$, $0 \leq s \leq t$ be the transition probability functions of the process $X = X(t)$ and $p_i(t) = Pr\{X(t) = i\}$ for $i \in E_S$, $t > 0$ be the state probabilities.

The probabilistic dynamics of the process is represented by the forward Kolmogorov system of differential equations:

$$\begin{cases} \frac{dp_0}{dt} = -\left(\lambda(t) + \xi(t)\right) p_0 + \mu(t)p_1 + \xi(t), \\ \frac{dp_k}{dt} = \lambda(t)p_{k-1} - \left(\lambda(t) + k\mu(t) + \xi(t)\right) p_k + \\ \qquad (k+1)\,\mu(t)p_{k+1}, \quad 1 \leq k < S \\ \frac{dp_k}{dt} = \lambda(t)p_{k-1} - \left(\lambda(t) + S\mu(t) + \xi(t)\right) p_k + \\ \qquad S\mu(t)p_{k+1}, k \geq S. \end{cases} \tag{1.1}$$

We denote by $\mathbf{p}(t) = (p_0(t), p_1(t), \dots)^T$, $t > 0$ the column vector of state probabilities and by $\mathbf{A}(t) = \{a_{ij}(t),\ t \geq 0\}$ the matrix related to (1.1) where

$$a_{ij}(t) = \begin{cases} \lambda_{i-1}(t), & if\ j = i-1 \\ \mu_{i+1}(t), & \text{if } j = i+1 \\ -\left(\lambda_i(t) + \mu_i(t) + \xi(t)\right), & \text{if } j = i \\ 0, & \text{if otherwise}, \end{cases} \tag{1.2}$$

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and $\lambda_i(t) = \lambda(t)$, $\mu_i(t) = \mu(t)\min(i, S)$.

Then we can rewrite the system (1.1) in the form

$$\frac{d\mathbf{p}}{dt} = \mathbf{A}(t)\,\mathbf{p} + \mathbf{g}(t), \quad t \geq 0, \qquad (1.3)$$

as a differential equation in the space of sequences l_1, where $\mathbf{g}(t) = (\xi(t), 0, 0, \dots)^T$.

Let $\Omega = \{\mathbf{x} : \mathbf{x} \geq 0, \|\mathbf{x}\|_1 = 1\}$.

Throughout the whole paper we assume that $\lambda(t)$, $\mu(t)$ and $\xi(t)$ are locally integrable for $t \geq 0$. Moreover, we suppose (only for simplicity) that these functions are bounded, namely

$$\lambda(t) + \mu(t) + \xi(t) \leq L < \infty, \qquad (1.4)$$

for almost all $t \geq 0$.

Then

$$\|A(t)\|_1 = \sup_j \sum_i |a_{ij}(t)| \leq 2SL, \qquad (1.5)$$

for almost all $t \geq 0$.

In [2] it is shown that the Cauchy problem for linear differential equations in Banach spaces with bounded and integrable operator functions has unique solutions for arbitrary initial conditions. This means that, under our assumptions, the existence and uniqueness of the solution do not pose any problem. Moreover, the Cauchy problem formed by (1.3) with the initial condition $\mathbf{p}(0)$ has the unique solution

$$\mathbf{p}(t) = U(t)\mathbf{p}(0) + \int_0^t U(t,\tau)\mathbf{g}(\tau)\, d\tau, \qquad (1.6)$$

where $U(t, s)$ is the Cauchy operator of equation (1.3), see [2].

Moreover, if $\mathbf{p}(s) \in \Omega$ then $\mathbf{p}(t) \in \Omega$ for any $t \geq s$.

2. ERGODICITY

THEOREM 1. *Let*

$$\int_0^\infty \xi(t)\, dt = \infty. \qquad (2.7)$$

Then $X(t)$ is weakly ergodic in uniform operator topology. Moreover, the following bound holds

$$\|\mathbf{p}^*(t) - \mathbf{p}^{**}(t)\| \leq 2e^{-\int_0^t \xi(\tau)\, d\tau}, \qquad (2.8)$$

for any initial conditions $\mathbf{p}^*(0), \mathbf{p}^{**}(\mathbf{0})$.

Proof. We use the notion of the logarithmic norm of operator function and the related estimates, see [4] and [8]. We have the equality

$$\gamma\left(A(t)\right)_1 = \sup_i \left(a_{ii}(t) + \sum_{j \neq i} a_{ji}(t) \right) = -\xi(t). \qquad (2.9)$$

Hence we obtain the following (sharp!) bound:

$$\|U(t,s)\| \leq e^{-\int_s^t \xi(\tau)\, d\tau}, \qquad (2.10)$$

for any $0 \leq s \leq t$, and our claim.

REMARK 1. *Weak ergodicity of a Markov chain means that the state probability distribution (say, $\mathbf{p}(t)$) does not depend on the initial "condition" $\mathbf{p}(0)$ as $t \to \infty$. Hence, in the general case we can consider any $\mathbf{p}^*(t)$ as the limit regime of state probability distribution. However, if all intensities ($\lambda(t)$, $\mu(t)$ and $\xi(t)$) are 1-periodic, then there exists 1-periodic limit regime, say $\boldsymbol{\pi}(t) = (\pi_0(t), \pi_1(t), \dots)^T$ which is the unique 1-periodic solution (in Ω) of the differential equation (1.3).*

Consider now the family of truncated processes $X_n(t)$, $n \geq S$ on the state space $E_n = \{0, 1, \dots, n\}$ with the same intensities for $k \leq n$ and intensity matrices $A_n(t)$.

THEOREM 2. *Let $X(0) = X_n(0) = 0$. Then*

$$\|\mathbf{p}(t) - \mathbf{p}_n(t)\| \leq \frac{SLe^{Lt}}{2^{n-2}} \qquad (2.11)$$

for any $t \geq 0$ and any $n \geq S$.

Proof. We will identify here vectors $(x_1, \dots, x_n, 0, 0, \dots)^T$ and $(x_1, \dots, x_n)^T$. Consider the forward Kolmogorov system (1.3) for original process in the following form

$$\frac{d\mathbf{p}}{dt} = A_n(t)\mathbf{p} + \mathbf{g}(t) + (A(t) - A_n(t))\,\mathbf{p}, \qquad (2.12)$$

and the respective system

$$\frac{d\mathbf{p_n}}{dt} = A_n(t)\mathbf{p_n} + \mathbf{g}(t) \qquad (2.13)$$

for the truncated process.

We have

$$\mathbf{p}_n(t) = U_n(t)\mathbf{p}(0) + \int_0^t U_n(t,\tau)\mathbf{g}(\tau)\, d\tau, \qquad (2.14)$$

for $\mathbf{p}(0) = \mathbf{p}_n(0)$, and

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$$\mathbf{p}(t) = U_n\,(t)\,\mathbf{p}(0) + \int\limits_0^t U_n(t,\tau)\mathbf{g}(\tau)\,d\tau + \int\limits_0^t U_n\,(t,\tau)\,(A(\tau) - A_n(\tau))\,\mathbf{p}(\tau)\,d\tau. \quad (2.15)$$

Then

$$\|\mathbf{p}(t) - \mathbf{p}_n(t)\| = \left\| \int\limits_0^t U_n\,(t,\tau)\,(A(\tau) - A_n(\tau))\,\mathbf{p}(\tau)\,d\tau \right\|, \quad (2.16)$$

Further we have

$$\|U_n\,(t,\tau)\| \le 1, \quad (2.17)$$

for any $n,\ t \ge 0,\ \tau \le t$.

Then

$$\|\mathbf{p}(t) - \mathbf{p}_n(t)\| \le \int\limits_0^t \|(A(\tau) - A_n(\tau))\,\mathbf{p}(\tau)\|\,d\tau. \quad (2.18)$$

The first rows of matrix $(A(t) - A_n(t))$ with numbers $0,\dots,n-1$ are zeros. Hence

$$(A - A_n)\,\mathbf{p}(\tau) = (0,\dots,0,-\lambda p_n + S\mu p_{n+1}, \lambda p_n - (\lambda + S\mu + \xi)\,p_{n+1} + S\mu p_{n+2},\dots)^T \quad (2.19)$$

and

$$\|(A(\tau) - A_n(\tau))\,\mathbf{p}(\tau)\| \le \|A(\tau)\|_1 \sum_{k\ge n} |p_k(\tau)| \le 2SL \sum_{k\ge n} p_k(\tau). \quad (2.20)$$

On the other hand, putting $u(t) = \sum\limits_{k\ge 0} 2^k p_k(t)$, we obtain

$$\frac{du(t)}{dt} = \xi(t) + \sum_{k\ge 0} 2^k\,(\lambda_k(t) - \mu_k(t)/2 - \xi(t))\,p_k \le \xi(t) + Lu(t) \le L + Lu(t), \quad (2.21)$$

and $u(0) = 1$ by assumption $X(0) = X_n(0) = 0$. Then

$$u(t) \le 2e^{Lt} - 1 \le 2e^{Lt}, \quad (2.22)$$

and

$$2^n \sum_{k\ge n} p_k(t) \le \sum_{k\ge n} 2^k p_k(t) \le u(t) \le 2e^{Lt}. \quad (2.23)$$

Now (2.16), (2.18), (2.20) and (2.23) imply (2.11).

Remark 2. *We can obtain the essential bounds for the limit characteristics (limit 1-periodic sojourn probabilities, and the limiting mean) of the considered process. Namely, let intensities ($\lambda(t)$, $\mu(t)$ and $\xi(t)$) be 1-periodic and $\int\limits_0^1 \xi(t)\,dt > 0$. Then there exists 1-periodic limit regime $\boldsymbol{\pi}(t) = (\pi_0(t), \pi_1(t), \dots)^T$, and Theorems 1, 2 give us the method of computing of limit 1-periodic sojourn probabilities $J_k(t)$ (this is probability that the length of the queue at the moment t does not exceed k) by the following way.*

Let ε be an arbitrary positive number.

1. Put $\mathbf{p}^*(0) = \boldsymbol{\pi}(0)$ (unknown!), and $\mathbf{p}^{**}(0) = \mathbf{e}_0$. Then we have in (2.8)

$$\|\boldsymbol{\pi}(t) - \mathbf{p}^{**}(t)\| \le 2e^{-\int\limits_0^t \xi(\tau)\,d\tau}. \quad (2.24)$$

Choose integer m such that $6e^{-\int\limits_0^t \xi(\tau)\,d\tau} \le \varepsilon$ for any $t \ge m$.

2. Find $n \ge S$ such that $\frac{3SLe^{L(m+1)}}{2^{n-2}} \le \varepsilon$.

3. Solve the Cauchy problem for the truncated Kolmogorov system (2.13) by any numerical approach (for, instance, using MAPLE) with initial condition $\mathbf{e}_0$ on the interval $[m; m{+}1]$ (with error $\varepsilon/3$), then this solution gives us the limit 1-periodic regime $\boldsymbol{\pi}(t) = (\pi_0(t), \pi_1(t), \dots)^T$ with the error ε.

4. Finally, the limiting behaviour of the sojourn probabilities $J_k(t) = \Pr\{X(t) \le k\}$ can be computed as $\sum_{i=0}^k \pi_i(t)$ with the same error ε.

3. BEHAVIOUR OF THE MEAN

We shall study the following mean values

$$E_{\mathbf{p}(0)}(t) = E_{\mathbf{p}(0)}\left\{X(t)\right\} = E\left\{X(t)\,|\mathbf{p}(0)\right\}, \quad (3.25)$$

and particularly

$$E_k(t) = E\left\{X(t)\,|X(0) = k\right\}. \quad (3.26)$$

Consider firstly the following simple bounds.

Theorem 3. *Let (2.7) be satisfied. Then*

$$e^{-\int\limits_0^t \xi(\tau)\,d\tau} E_{\mathbf{p}(0)}(0) + \int\limits_0^t (\lambda(\tau) - S\mu(\tau))\,e^{-\int\limits_\tau^t \xi(s)\,ds}\,d\tau \le E_{\mathbf{p}(0)}(t) \le e^{-\int\limits_0^t \xi(\tau)\,d\tau} E_{\mathbf{p}(0)}(0) + \int\limits_0^t (\lambda(\tau))\,e^{-\int\limits_\tau^t \xi(s)\,ds}\,d\tau \quad (3.27)$$

for any $t \ge 0$ and any $\mathbf{p}(0)$.

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Proof. We have from (1.1):

$$\frac{dE_{\mathbf{p}(0)}(t)}{dt} = \sum_{k\geq 0}\left(\lambda_k(t) - \mu_k(t) - k\xi(t)\right)p_k(t). \tag{3.28}$$

Hence

$$\frac{dE_{\mathbf{p}(0)}(t)}{dt} \leq \lambda(t) - \xi(t)E_{\mathbf{p}(0)}(t), \tag{3.29}$$

and

$$\frac{dE_{\mathbf{p}(0)}(t)}{dt} \geq \lambda(t) - S\mu(t) - \xi(t)E_{\mathbf{p}(0)}(t). \tag{3.30}$$

Finally (3.29) and (3.30) imply (3.27).

Definition 1. *Markov chain $X(t)$ has the* limiting mean $\varphi(t)$ *if*

$$\lim_{t\to\infty}\left(\varphi(t) - E_k(t)\right) = 0 \tag{3.31}$$

for any k.

Definition 2. *The* double mean *of $X(t)$ is defined by*

$$\mathsf{E} = \lim_{t\to\infty}\frac{1}{t}\int_0^t E_k(u)\,du, \tag{3.32}$$

provided that the limit exists and does not depend on k.

Theorem 4. *Let arrival, service, and catastrophe rates $\lambda(t)$, $\mu(t)$, $\xi(t)$ be 1-periodic. Let $\int\limits_0^1 \xi(t)\,dt > 0$. Then X(t) has the 1-periodic limiting mean $\varphi(t)$ and double mean E.*

Proof. Firstly note that there exists $\delta > 1$ such that

$$\int\limits_0^1 \left(\xi(t) - (\delta-1)\lambda(t)\right)\,dt > 0. \tag{3.33}$$

Consider the matrix

$$D = diag\left(1, \delta, \delta^2, \dots\right), \tag{3.34}$$

and the space of sequences $\mathcal{B}$ such that

$$\|\mathbf{x}\|_{\mathcal{B}} = \sum_{i=0}^{\infty}\delta^i|x_i| < \infty,$$

and the forward Kolmogorov system (1.3) as an equation in the space $\mathcal{B}$. Now we can estimate the logarithmic norm $\gamma\left(A(t)\right)$ in $\mathcal{B}$:

$$\gamma\left(A\right)_{\mathcal{B}} = \sup_{i\geq 0}\big(\delta\lambda_i(t) - (\lambda_i(t) + \mu_i(t) + \xi(t)) + \delta^{-1}\mu_i(t)\big) = -\left(\xi(t) - (\delta-1)\lambda(t)\right). \tag{3.35}$$

Then

$$\|U(t,s)\|_{\mathcal{B}} \leq e^{-\int\limits_s^t(\xi(\tau)-(\delta-1)\lambda(\tau))\,d\tau}, \tag{3.36}$$

for any $0 \leq s \leq t$.

Hence

$$\|\mathbf{p}^*(t) - \mathbf{p}^{**}(t)\|_{\mathcal{B}} \leq e^{-\int\limits_0^t(\xi(\tau)-(\delta-1)\lambda(\tau))\,d\tau}\|\mathbf{p}^*(0) - \mathbf{p}^{**}(0)\|_{\mathcal{B}}, \tag{3.37}$$

for any acceptable initial conditions $\mathbf{p}^*(0)$, $\mathbf{p}^{**}(0)$.

On the other hand, assumptions of Theorem imply the bound

$$\limsup_{t\to\infty}\|\mathbf{p}(t)\|_{\mathcal{B}} \leq \limsup_{t\to\infty}\left(\|U(t)\mathbf{p}(0)\|_{\mathcal{B}} + \int\limits_0^t \|U(t,\tau)\mathbf{g}(\tau)\,d\tau\|_{\mathcal{B}}\right) \leq \tag{3.38}$$

$$\limsup_{t\to\infty}\int\limits_0^t \xi(\tau)e^{-\int\limits_\tau^t(\xi(u)-(\delta-1)\lambda(u))\,du}\,d\tau = \mathsf{M} < \infty,$$

for any $\mathbf{p}(0)$.

Put

$$W = \sup_n \frac{n}{\delta^n} < \infty. \tag{3.39}$$

Then we obtain

$$\limsup_{t\to\infty}E_{\mathbf{p}(0)}(t) = \limsup_{t\to\infty}\sum_{k=0}^{\infty}kp_k(t) \leq W\limsup_{t\to\infty}\|\mathbf{p}(t)\|_{\mathcal{B}} \leq W\mathsf{M}, \tag{3.40}$$

for any acceptable $\mathbf{p}(0)$.

Put now $\mathbf{p}^*(0) = \boldsymbol{\pi}(0)$, $\phi(t) = \sum\limits_{k=0}^{\infty}k\pi_k(t)$, and $\mathbf{p}^{**}(0) = \mathbf{p}(0) = \mathbf{e}_0$. Then we have in (3.38):

$$\|\boldsymbol{\pi}(t) - \mathbf{p}(t)\|_{\mathcal{B}} \leq e^{-\int\limits_0^t(\xi(\tau)-(\delta-1)\lambda(\tau))\,d\tau}\sum_{k=0}^{\infty}k\pi_k(0) \leq \mathsf{M}e^{-\int\limits_0^t(\xi(\tau)-(\delta-1)\lambda(\tau))\,d\tau}. \tag{3.41}$$

Finally, we obtain

$$|\varphi(t) - E_0(t)| \leq W\mathsf{M}e^{-\int\limits_0^t(\xi(\tau)-(\delta-1)\lambda(\tau))\,d\tau}. \tag{3.42}$$

The right-hand side of (3.42) tends to zero as $t \to \infty$ in accordance with (3.33). One can see that $|\varphi(t) - E_k(t)| \to 0$ as $t \to \infty$ for *any* k. Hence, $\varphi(t)$ is 1-periodic limiting mean for $X(t)$. The existence of double mean follows from periodicity of $\varphi(t)$.

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Corollary 1. *Let the assumptions of Theorem 4 be fulfilled. Then the bound of the rate of convergence to the limiting mean (3.42) holds.*

Consider now the problem of approximation of the limit mean characteristics of $X(t)$. Put $\|\mathbf{x}\|_{1E} = \sum_{k=0}^{\infty} k|x_i|$, and denote by $E_{n,0}(t)$ the respective mean for truncated process.

Corollary 2. *Let the assumptions of Theorem 4 be fulfilled. Then the following bound of the error of truncation holds:*

$$|E_0(t) - E_{n,0}(t)| \leq \frac{6nSLe^{Lt}}{2^n} \qquad (3.43)$$

for any $t \geq 0$ and any $n \geq S$.

Proof. We have now

$$|E_0(t) - E_{n,0}(t)| = \|\mathbf{p}(t) - \mathbf{p}_n(t)\|_{1E}$$

and instead of (2.16)-(2.23) we obtain the following estimate (see also the reasoning of Theorem 2 [8])

$$\|U_n(t,\tau)(A(\tau) - A_n(\tau))\mathbf{p}(\tau)\|_{1E} \leq 3SL \sum_{k \geq n} kp_k(t) =$$

$$3SL \sum_{k \geq n} \frac{k}{2^k} 2^k p_k(t) \leq \frac{3SLn}{2^n} \sum_{k \geq n} 2^k p_k(t) \qquad (3.44)$$

and (3.43).

Remark 3. *We can compute now the limiting 1-periodic mean and double mean by the way of the previous Section.*

4. EXAMPLE

Let $X(t)$ be a queue-length process for $M(t)/M(t)/100$ queue with catastrophes, and let $\lambda(t) = 3 + \sin 2\pi t$, $\mu(t) = 1 + \cos 2\pi t$, $\xi(t) = 2 + \sin 4\pi t$ be the respective birth, death, and catastrophe intensities.

Then $S = 100$, $L = 9$. Put $\delta = 1.2$. Then in accordance with Theorem 4 there exist 1-periodic limiting characteristics of the Markov chain (limiting regime $\boldsymbol{\pi}(t) = (\pi_0(t), \pi_1(t), \dots)^T$, the respective mean $\phi(t) = \sum_{j \geq 0} j\pi_j(t)$, the respective sojourn probabilities $J_k(t) = \sum_{j=0}^{k} j\pi_j(t)$) and double mean E. Now we obtain: $\mathsf{M} \leq 3$ in (3.39), $W \leq 2.1$ in (3.39). For $m = 6$ and $n = 108$ we have $|\mathsf{E} - 1.036| < 10^{-3}$. In figures 1 - 3 the limit 1-periodic characteristics of queue-length process (the mean $\phi(t)$ and the sojourn probabilities $J_k(t)$ for $k = 0, 10$) with error 10^{-3} are drawn.

5. ACKNOWLEDGMENTS

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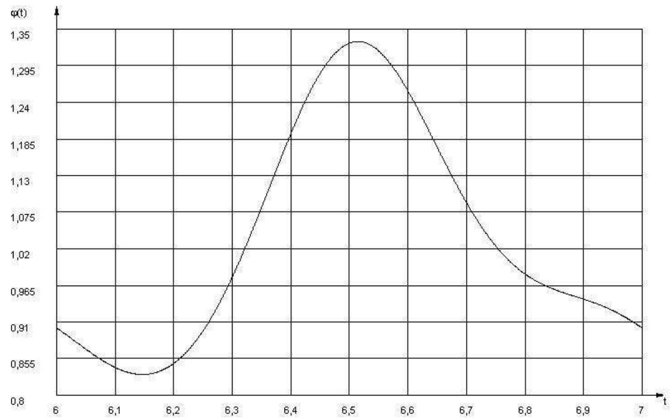
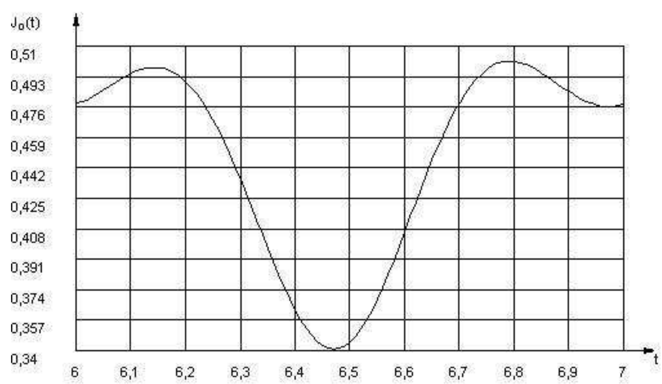


Figure 1:



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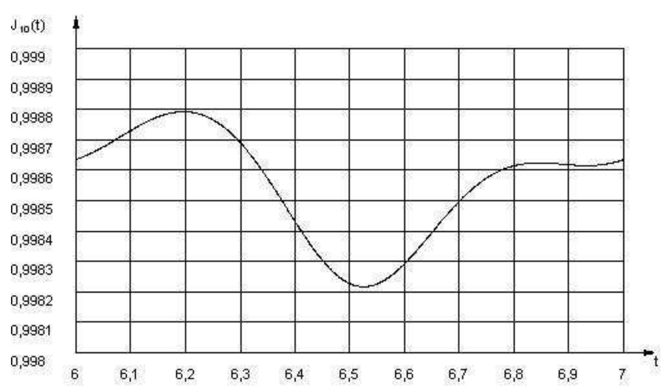

Figure 3:

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