On the Spatial Impedance Control of Gough-Stewart Platforms

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Abstract

This paper looks at the control of mechanical impedance of the so-called Gough-Stewart class of parallel platforms. Two methods of compliance control are presented. One is based on global potential energy functions that have previously been applied to controlling serial manipulators and electrodynamically levitated platforms. The second uses the exponential map to associate finite displacements of the platform from equilibrium with screw displacements. Compliant wrenches are then proportional to the screw displacements. Control of spatial damping is addressed as well, justifying the classification as impedance control. Control algorithms and simulation results are given.

1 Introduction

1.1 Spatial Impedance Control

Impedance control is one of the two main distinguishable active compliant motion control paradigms. The other paradigm is hybrid position/force control, which from its outset has had a more Euclidean-geometric foundation than impedance control. Early stiffness and impedance control methods were analytic, using generalized coordinates, velocities and forces. It was not obvious that the methods were not geometric because they used "Cartesian coordinates". Using hybrid control, given Euclidean models of objects in the environment one can intuitively choose directions of desired translations or rotations, and directions of desired forces or torques. By contrast it is far more difficult to choose a desired stiffness (e.g., roll-pitch stiffness) using stiffness or impedance control, particularly when the desired equilibrium orientation has nonzero angular coordinates with respect to the reference frame. To the authors' knowledge the only other configuration representation used in impedance control is that reported by Hollis et al. [1]. They use three Cartesian coordinates to represent translation and a reduced set of Euler parameters (unitary quaternions) to represent orientation. This representation was only quasi-geometric because relative rotation was represented as an algebraic difference of the reduced Euler parameters, which is not a geometric entity.

Spatio-geometric compliance control methods using geometric potential functions were first developed for spatial, serial manipulators [2, 3]. Siciliano and Villani [4] have looked at spatial impedance control of serial manipulators using reduced Euler parameters to represent orientation. This is similar to the approach of [1] except that they properly compute the difference between two quaternions using the natural quaternion product, and not the algebraic difference. The rotational compliance energy function used is equivalent to that of [2]. They also look at control of damping and inertia.

1.2 Parallel Mechanisms

Parallel mechanisms have first been introduced by Gough and Whitehall in tire-testing equipment [5]. Later, Stewart [6] proposed to use a parallel mechanism as a motion base for a flight simulator. The rationale for the use of this type of mechanism was its high stiffness and dexterity required to impart large accelerations to a heavy load — the cockpit of an aircraft — with six degrees of freedom. This architecture, which is referred to as the 'Gough-Stewart platform', soon gained in popularity and can now be found in virtually all modern flight simulators. Parallel mechanisms have also been used in a number of other applications [7].

The stiffness properties of parallel mechanisms have been studied in [8]. Because of their inherent stiffness, parallel mechanisms are attractive for tasks requiring the control of forces. Hybrid control has been applied to parallel mechanisms [9, 10, 11]. Impedance control
techniques have also been used, in particular in haptic interfaces [12, 13, 14, 15]. Adelstein et al. [16] provide an interesting historical perspective about haptic interfaces in general and parallel joystick mechanisms in particular. However, the authors are not aware of any literature specifically addressing the impedance control of parallel mechanisms.

2 Spatial Compliance

2.1 Stiffness Parameters

We use the stiffness parametrization of [17]. The platform is assumed to be acted upon by a conservative force field. We are interested in the behavior near equilibrium. In general let \( w = [f^t, m^t] \) denote a wrench in ray coordinates. Let \( \delta T = [\delta p^t, \delta \theta^t] \) denote a small, finite (twist-)displacement of the platform from equilibrium, in axis coordinates. For small displacements we have approximately

\[
w \approx K \delta T = \begin{bmatrix} K_t & K_c \\ K_c^t & K_o \end{bmatrix} \delta T
\]

where \( K \) is a symmetric stiffness matrix. In general matrix \( K_t \) can be asymmetric, but there always exists a \textit{center of stiffness}, which is unique if \( \text{tr}(K_t) \) is not an eigenvalue of \( K_c \). At this point matrix \( K_c \) is symmetric.

Let point \( p_c \) be any convenient, otherwise fixed, point on the platform. Let \( R_c \) be a rotation matrix representing the orientation of the platform. Let \( H_c \) be the homogeneous matrix defined by \( p_c \) and \( R_c \) representing the configuration of the platform. Let parameter \( d_{cs} \) be the desired displacement of the center of stiffness from \( p_c \), so that

\[
H_{cs} = \begin{bmatrix} R_{cs} & p_{cs} \\ 0^t & 1 \end{bmatrix} = \begin{bmatrix} R_t & p_t + R_c d_{cs} \\ 0^t & 1 \end{bmatrix}
\]

is a homogeneous matrix representing the configuration of the platform, which we can identify with a frame with origin at point \( p_{cs} \).

Let \( H_c \) be a homogeneous matrix representing the virtual equilibrium configuration of the platform, so that \( H_{cs} = H_c \) in equilibrium. The term \textit{virtual equilibrium} is used because it corresponds to an actual equilibrium only if no net wrench is exerted on the platform by its environment. We then have that

\[
H_{cs} \approx H_c \begin{bmatrix} \delta \theta & \delta p \\ 0^t & 0 \end{bmatrix}
\]

where in general \( \tilde{v} \) is the \textit{cross-product matrix} associated with vector \( v \), the matrix \( \tilde{v} \) such that \( \tilde{v}w = v \times w \) for any vector \( w \). We also use the notation \( (v)^- = \tilde{v} \).

Matrices \( K_t, K_c \) and \( K_o \) are determined by their respective sets of orthonormal principal axes and principal stiffnesses. Matrix \( K_t = R_t \Gamma_t R_t^t \) is the translational stiffness matrix. Columns of orthonormal matrix \( R_t = [e_1t, e_2t, e_3t] \) are the principal axes of translational stiffness. Matrix \( \Gamma_t = \text{diag}(\gamma_{1t}, \gamma_{2t}, \gamma_{3t}) \) is a matrix of \textit{principal translational stiffnesses}. A displacement along any one of the principal axes results in a translational force along the same axis.

Matrix \( K_c = R_c \Gamma_c R_c^t \) is the coupling stiffness matrix. Columns of orthonormal matrix \( R_c = [e_1c, e_2c, e_3c] \) are the \textit{principal coupling axes} of stiffness. Matrix \( \Gamma_c = \text{diag}(\gamma_{1c}, \gamma_{2c}, \gamma_{3c}) \) is a matrix of \textit{principal coupling stiffnesses}. A displacement along any one of the principal axes results in a torque about the same axis. A displacement about any one of the principal axes results in a translational force along the same axis.

Matrix \( K_o = R_o \Gamma_o R_o^t \) is the rotational stiffness matrix. Columns of orthonormal matrix \( R_o = [e_{10}, e_{20}, e_{30}] \) are the \textit{principal axes} of rotational stiffness. Matrix \( \Gamma_o = \text{diag}(\gamma_{10}, \gamma_{20}, \gamma_{30}) \) is a matrix of \textit{principal rotational stiffnesses}. A rotation about any one of the principal axes results in a torque about the same axis.

2.2 Potential Function Approach

The first family of compliances considered is based on globally defined potential functions. For analytical purposes it is useful to define co-stiffness matrices. Let matrix \( G_t = R_t \Lambda_t R_t^t = \frac{1}{2} \text{tr}(K_t)I - K_t \) be the translational co-stiffness matrix. Matrix \( I \) is the identity matrix. Matrix \( \Lambda_t \) is a diagonal matrix of \textit{principal translational co-stiffnesses}. We then have \( K_t = \text{tr}(G_t)I - G_t \). Similarly, let matrix \( G_c = R_c \Lambda_c R_c^t = \frac{1}{2} \text{tr}(K_c)I - K_c \) be the coupling co-stiffness matrix. Let matrix \( G_o = R_o \Lambda_o R_o^t = \frac{1}{2} \text{tr}(K_o)I - K_o \) be the rotational co-stiffness matrix.

The potential energy function consists of three terms: translational, rotational, and coupling potential energies. The \textit{translational potential energy}, \( U_t \), is a quadratic function of the displacement of the center of stiffness frame from the virtual equilibrium frame,

\[
\Delta p = p_{cs} - p_c = p_t + R_c d_{cs} - p_c
\]

The translational potential energy is defined to be

\[
U_t = -\frac{1}{2} \text{tr}(\Delta \tilde{p} G_t \Delta \tilde{p}) = \frac{1}{2} \Delta p^t K_t \Delta p
\]
The associated compliance force is
\[ f^0_{cl} = 2 \text{as}(G_c \delta p) \quad (6) \]
where \( \text{as}(M) \) denotes the antisymmetric (skew-symmetric) part of \( M \). This is equivalent to the familiar expression
\[ f^0_{cl} = K_1 \delta p \quad (7) \]

The rotational (orientational) potential energy, \( U_o \), is a function of the relative orientation of the center of stiffness frame with respect to the virtual equilibrium frame. Specifically,
\[ U_o = -\text{tr}(R_c R_c^T G_o) \quad (8) \]
The associated compliance moment is
\[ m^0_{cl} = 2 \text{as}(R_c R_c^T G_o) \quad (9) \]

The coupling potential energy, \( U_c \), is a function of both the relative displacement and orientation of the center of stiffness frame with respect to the virtual equilibrium frame:
\[ U_c = -\text{tr}(R_c R_c^T G_c \delta \tilde{p}) \quad (10) \]
The associated compliance wrench is
\[ f^0_c = 2 \text{as}(R_c R_c^T G_c) \quad \text{and} \quad m^0_c = 2 \text{as}(R_c R_c^T G_c \delta \tilde{p}) \quad (11) \]

The total potential energy is the sum of the translational, rotational and coupling potential energies, \( U = U_t + U_o + U_c \). This function is bounded below if the translational stiffness matrix is positive-definite. The desired compliant wrench with respect to the platform frame in coordinates of the base frame is then
\[ \begin{bmatrix} f^0_{cl} \\ m^0_{cl} \end{bmatrix} = \begin{bmatrix} 1 \\ (R_c d_{cs})^{-1} \end{bmatrix} \begin{bmatrix} f^0_{cl} + f^0_{c} \\ m^0_{cl} + m^0_{c} \end{bmatrix} \quad (12) \]
This wrench can be determined from (4)–(12) given \( H_r \) and all compliance parameters.

2.3 Exponential Map Approach

The same stiffness can be generated using a very different approach. The set of twists \( se(3) \) maps to the set of rigid body transformations \( SE(3) \) via the exponential map. This map is locally diffeomorphic, defining a set of exponential coordinates on \( SE(3) \) [17]. Again let \( H_v \) be the virtual equilibrium configuration of the platform, coinciding with \( H_{cs} \) in equilibrium.

Given configurations \( H_{cs}, H_v \in SE(3) \) there exists a twist
\[ T^0_{rel} = \begin{bmatrix} v^0_{rel} \\ \omega^0_{rel} \end{bmatrix} \in se(3) \quad (13) \]
such that \( H_{cs} = \exp(T^0_{rel}) H_v \), where
\[ \exp(T^0_{rel}) = \exp \left( \begin{bmatrix} \omega^0_{rel} \\ 0 \end{bmatrix} \right) \quad (14) \]
The latter exponential is the usual matrix exponential. In other words given any \( H_v \) and \( H_{cs} \) there exists a locally unique \( T^0_{rel} \) with \( \exp(T^0_{rel}) = H_{cs} H_v^{-1} \). Because the inverse exponential map needs to be computed for control purposes we consider its calculation in detail. Let \( \alpha = ||\omega^0_{rel}|| \) be the angle of rotation and \( \hat{\omega} \) be a unit vector in the direction of the axis of rotation, so that \( \omega^0_{rel} = \alpha \hat{\omega} \). The exponential map can always be written in the form
\[ \exp \left( \begin{bmatrix} \omega^0_{rel} \\ 0 \end{bmatrix} \right) = \begin{bmatrix} R_{rel} & S_{rel} v^0_{rel} \\ 0 & 1 \end{bmatrix} \quad (15) \]
where
\[ R_{rel} = \cos(\alpha) I + (1 - \cos(\alpha)) \hat{\omega} \hat{\omega}^T + \frac{1 - \cos(\alpha)}{\alpha} (\hat{\omega})^T \quad (16) \]
\[ S_{rel} = \frac{\sin(\alpha)}{\alpha} I + \frac{1 - \sin(\alpha)}{\alpha} \hat{\omega} \hat{\omega}^T + \frac{1 - \cos(\alpha)}{\alpha} (\hat{\omega})^T \quad (17) \]
(c.f. eqns. (7)-(9) of [17]). Let
\[ H_{rel} = H_{cs} H_v^{-1} = \begin{bmatrix} R_r R_c^T & p_r + R_r d_{cs} - R_c R_v p_v \\ 0^T & 1 \end{bmatrix} \quad (18) \]
The angle of rotation is
\[ \alpha = \cos^{-1}(0.5(\text{tr}(R_{rel}) - 1)) \quad (19) \]
If \( \alpha = 0 \) then
\[ \omega^0_{rel} = 0 \quad \text{and} \quad v^0_{rel} = p_{rel} \quad (20) \]
If \( \alpha \neq 0 \) then
\[ \hat{\omega} = \frac{1}{\sin(\alpha)} \text{as}(R_{rel}) \quad (21) \]
\[ S_{rel} = \frac{1}{\alpha} (I - R_{rel}) (\hat{\omega})^T + \hat{\omega} \hat{\omega}^T \quad (22) \]
\[ v^0_{rel} = S_{rel}^{-1} p_{rel} \quad (23) \]
The compliant wrench is defined to be
\[ \begin{bmatrix} f_{\text{cd}}^0 \\ m_{\text{cd}}^0 \end{bmatrix} = \begin{bmatrix} K_t & K_c \\ K_c & K_0 \end{bmatrix} \begin{bmatrix} v_{\text{rel}}^0 \\ \omega_{\text{rel}} \end{bmatrix} \] (24)

The desired compliant wrench with respect to the platform frame in coordinates of the base frame is then
\[ \begin{bmatrix} f_{\text{cd}}^0 \\ m_{\text{cd}}^0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ (R_{\text{cd}}d_{\text{cd}})^{-1} & I \end{bmatrix} \begin{bmatrix} f_{\text{cd}}^0 \\ m_{\text{cd}}^0 \end{bmatrix} \] (25)

This wrench can be determined from (18)–(25) given \( H_r \) and all compliance parameters.

3 Spatial Damping

Damping acts to retard the motion of the platform. Analogous to the center of stiffness, in general there exists a center of damping at which translation and rotation are maximally decoupled. Let \( d_{\text{cd}} \) be the desired displacement of the center of damping frame from the end-effector frame, so that
\[ H_{\text{cd}} = \begin{bmatrix} R_{\text{cd}} & p_{\text{cd}} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} R_{c} & p_{c} + R_{c}d_{\text{cd}} \\ 0^T & 1 \end{bmatrix} \] (26)

Parameter \( d_{\text{cd}} \) is used to vary the location of the center of damping with respect to the fixed end-effector frame.

Analogous to matrices \( K_t, K_c \), and \( K_0 \), let \( B_t \) be the translational damping matrix, let \( B_0 \) be the rotational damping matrix, and let \( B_{\text{cd}} \) be the coupling damping matrix. Matrix \( B_t = R_t\tilde{\Gamma}_tR_t^T \), where \( R_t \) determines the principal axes of translational damping and \( \tilde{\Gamma}_t \) determines the principal translational damping coefficients. Matrix \( B_0 = R_0\tilde{\Gamma}_0R_0^T \), where \( R_0 \) determines the principal axes of rotational damping and \( \tilde{\Gamma}_0 \) determines the principal rotational damping coefficients. Matrix \( B_{\text{cd}} = R_{\text{cd}}\tilde{\Gamma}_{\text{cd}}R_{\text{cd}}^T \), where \( R_{\text{cd}} \) determines the principal coupling axes of damping and \( \tilde{\Gamma}_{\text{cd}} \) determines the principal coupling damping coefficients.

Let \( v_{\text{cd}}^0 \) and \( \omega_{\text{cd}}^0 \) be the absolute twist of the platform. In other words, the absolute twist is the rate of change of the platform frame with respect to the "absolute" base frame, in coordinates of the base frame. The corresponding twist at the center of damping is
\[ v_{\text{cd}}^0 = v_{\text{cd}}^0 - (R_{\text{cd}}d_{\text{cd}})^{-1}\omega_{\text{cd}}^0 \quad \text{and} \quad \omega_{\text{cd}}^0 = \omega_{\text{cd}}^0 \] (27)

Vectors \( v_{\text{cd}}^0 \) and \( \omega_{\text{cd}}^0 \) are computable given \( v_{\text{cd}}^0 \) and \( \omega_{\text{cd}}^0 \). Estimation of this twist given measurements of \( p_c \) and \( R_c \) over time lies outside the scope of this work.

With respect to the end-effector frame, the wrenches at the center of damping are
\[ f_{\text{cd}}^0 = B_t v_{\text{cd}}^0 + B_{\text{cd}}\omega_{\text{cd}}^0 \] (28)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.pdf}
\caption{A Gough-Stewart platform}
\end{figure}

The desired damping wrench with respect to the platform frame is then
\[ f_{\text{dr}}^0 = f_{\text{cd}}^0 \quad \text{and} \quad m_{\text{dr}}^0 = m_{\text{cd}}^0 + (R_{\text{cd}}d_{\text{cd}})^{-1}f_{\text{cd}}^0 \] (29)

4 Control Law based on Kinesthetic Model

Figure 1 shows schematically a Gough-Stewart platform. The actuators are assumed to be prismatic, force servoactuators. The base attachment point of actuator \( i \) is denoted by \( A_i \). The joint between each actuator and the base is assumed to constrain translation of the actuator with respect to the base and to constrain rotation of that actuator about the segment connecting the attachment points, \( A_iB_i \). The platform attachment point of actuator \( i \) is denoted by \( B_i \). The joint between each actuator and the platform is assumed to constrain translation of the actuator with respect to the platform.

The configuration of the platform, \( H_r \), is assumed to be known. The frame associated with \( H_r \) is identified by \( r \) in the figure. The center of stiffness frame is identified by \( cs \). We assume that \( R_{cs} = R_r \) and \( p_{cs} = p_r + R_r d_{cs} \), where \( d_{cs} \) is a control parameter specifying the desired location of the center of stiffness with respect to the platform.

The manipulator attachment point of any actuator, \( B_i = p_r + R_r d_i \), is computable given \( H_r \). The base attachment point of each actuator, \( A_i \), is assumed to be known and stationary. Let \( u_i \) be the unit vector directed from \( A_i \) to \( B_i \), so that
\[ u_i = \frac{B_i - A_i}{\|B_i - A_i\|} \] (31)
Let $f_{ui}$ be the force exerted by actuator $i$ on the platform, so that $f_i$ is the magnitude of the force. The net force acting on the platform is then

$$f_{net}^0 = \sum_{i=1}^{6} f_{ui}$$

(32)

The net moment about $p_r$ is

$$m_{net}^0 = \sum_{i=1}^{6} (R_r d_i) \times (f_{ui})$$

(33)

We define a matrix $M$ relating actuator forces and platform wrenches:

$$M = \begin{bmatrix} u_1 & \ldots & u_6 \\ (R_r d_1) \times u_1 & \ldots & (R_r d_6) \times u_6 \end{bmatrix}$$

(34)

Let $f_{act} = [f_1 \ldots f_6]^T$ be the array of actuator forces. The actuator forces necessary to achieve the desired compliance and damping are then

$$f_{act} = -M^{-1} \begin{bmatrix} f_x^0 + f_{dir}^0 \\ m_x + m_{dir}^0 \end{bmatrix}$$

(35)

In practice it would be necessary to add other control terms to compensate for friction, gravity, eventual flexural joint compliance, etc.

### 5 Simulation

This section presents simulation results using both compliance control methods. The actuated platform is modelled simply as a single rigid body acted on by a body-relative actuator wrench, $[f_x^0 m_x^0]^T$. The dynamic equations implemented in the simulation were equivalent to:

$$\frac{d}{dt} \begin{bmatrix} \dot{p}_r \\ \dot{R}_r \end{bmatrix} = \begin{bmatrix} 0 \\ -J^{-1} \ddot{\omega}_b \end{bmatrix} + \begin{bmatrix} \frac{1}{mR_r} & 0 \\ 0 & J^{-1} \end{bmatrix} \begin{bmatrix} f_a \\ m_a \end{bmatrix}$$

(36)

$$\begin{bmatrix} \dot{p}_r \\ \dot{\omega}_b \end{bmatrix} = \begin{bmatrix} -J^{-1} \omega_b \end{bmatrix} + \begin{bmatrix} 1/mR_r & 0 \\ 0 & J^{-1} \end{bmatrix} \begin{bmatrix} f_a \\ m_a \end{bmatrix}$$

(37)

where $m$ is the platform mass and $J$ is the inertia matrix of the platform with respect to the body frame. The centroid is assumed to coincide with point $p_r$. The body-relative actuator wrench is

$$\begin{bmatrix} f_a \\ m_a \end{bmatrix} = \sum_{i=1}^{6} f_i \begin{bmatrix} R^i_{x} \otimes u_i \\ d_i R^i_{x} \times u_i \end{bmatrix}$$

(38)

Figure 2: Simulation of controlled platform using global potential function method

Figure 2 depicts a wire-frame animation of a controlled platform using the first control algorithm. The platform is represented by a regular hexagonal prism. An additional segment on the top indicates orientation unambiguously. This representation is for illustration purposes only. The assumed architecture is asymmetric. Base attachment points $A_1$ and $A_2$, points $A_3$ and $A_4$, and points $A_5$ and $A_6$ are assumed to coincide, respectively. These three pairs of points form a triangle, as indicated in the figure. Platform attachment points $B_6$ and $B_1$, points $B_2$ and $B_3$, and points $B_4$ and $B_5$ are assumed to coincide, respectively, forming a second triangle. The configurations of the six linear actuators are depicted only at the equilibrium configuration of the platform.

A similar simulation was performed using the exponential map method assuming identical control parameters, dynamic model parameters and initial conditions. The results were indistinguishable from those depicted in Fig. 2. This indicates that the preceding analysis is consistent. The methods are not equivalent, so in general behavioral differences will be apparent given large displacements of the platform from equilibrium.

### 6 Conclusion

Two spatio-geometric methods for controlling the mechanical impedance of Gough-Stewart platforms were presented. Both methods require computing a desired wrench on the platform given its configuration and instantaneous motion (twist). Given the desired wrench, computation of the necessary actuator forces
is straightforward. Because the computations are unfamiliar it may appear that they are numerically difficult. Close inspection reveals this to be false. Computation of the wrench using the potential function method is particularly simple. Only addition, multiplication and copying operations are required. Wrench computations will not be a significant limiting factor in practical implementations.

The geometric methods have two advantages compared with well-known impedance control methods: First, computation of the wrench-based, Euclidean geometric control laws is more natural and intuitive than computation of the generalized-force-based, conventional control laws. A disadvantage of the new methods is that they require real-time computation of the forward kinematics if configuration of end-effector is not sensed directly. The stiffness control method of [18] requires only trivial real-time computation. The new methods will most likely be first applied to parallel manipulators with less than six DOF, for which forward kinematic computation is feasible. One possibility under investigation is the control of an active three-DOF joystick, controlling only the rotational stiffness.

Second, conventional impedance control methods are only quasi-Euclidean, using Cartesian coordinates and Euler angles to represent configuration. The associated impedance parameters like damping and stiffness matrices are not Euclidean and cannot easily be chosen directly. The parametrization of the geometric methods is Euclidean and more intuitive, making it easier to program the controlled platform to perform spatially complex tasks.

References


