

Generating functionals method of N.N.Bogolyubov and multiple production physics

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Abstract

The generating functionals (GF) method in Bogolyubov's formulation and its application for particle physics is considered. Effectiveness of the method is illustrated by two examples. So, GF method can be used as the technical trick solving the infinite sequence of algebraic equations. We will consider the example, where GF allows express the multiplicity distributions (topological cross sections) through the particles correlation functions (inclusive cross sections) to 'predict' so called the Koba-Nielsen-Olesen scaling. We will use the GF to define validity of the thermal description of the multiple production phenomena also. It will be seen that this will lead to the 'correlations relaxation condition' of N.N.Bogolyubov. This will allow to offer the experimentally measurable criteria of applicability of thermodynamical description of multiple production processes. In results we will find the closed form of perturbation theory applicable for kinetic phase of nonequilibrium processes. It is shown a way as the approach may be adapted to the definite external conditions.

1 Introduction

It is hard to imagine modern particles physics without such fundamental notions as, for instance, the phase transitions, topological defects, taken from statistical physics. This extremely fruitful connection among two branches of physics based on the euclidean postulate [1]: the formulae of particle physics are coincide with corresponding formulae of statistical physics if the transformation $t \rightarrow it$ is applied. But this coincidence exist iff the media is equilibrium only, since the time order of physical process becomes lost after the transition to imaginary time it . So, the particles static properties only can be considered by euclidean field theories.

The euclidean postulate does not 'work' for arbitrary element of S -matrix and, by this reason, there is not, at first glance, general connection between particles and statistical physics. Our aim is demonstrate this connection considering the multiple production example, staying in the real-time theory frame.

The multiple production is a typical dissipative process of the incident kinetic energies transition into the energies (masses) of produced particles. This is the nonequilibrium process and the fluctuations, generally speaking, may be high in it. Experimental data confirms this general expectation at the mean multiplicities region, when $n \sim \bar{n}$ [2].

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Considering multiple production we would like to note firstly that the mean multiplicity \bar{n} of hadrons for modern accelerator energies (~ 10 Tev) is large $\bar{n}(s) \simeq 100$. So, it is practically impossible to describe the system with $N = 3\bar{n} - 10 \simeq 300$ degrees of freedom using ordinary methods.

Secondly, it is natural to assume that the entropy \mathcal{S} tends to maximum with rising multiplicity n and reach the maximum at $n \sim n_{max} \sim \sqrt{s}$, since the dissipation take place in the vacuum (presumably with zero energy density)². But the experiment shows that at high energies $n \sim \bar{n}(s) \sim \ln^2 s \ll n_{max}(s)$ are essential. This means that there is not total dissipation of incident energy in considered thermalization process [3]. Absence of thermalization may be a consequence of hidden conservation laws [4].

We would like to adopt following fundamental principle of nonequilibrium statistics introduced by N.N.Bogolyubov [5]. It is natural to assume that the system evaluate to the equilibrium in such a way that the ‘nonequilibrium’ fluctuations in it should tend to zero. In the frame of Bogolyubov’s principle the quantitative measure of ‘nonequilibrium’ fluctuation is the mean value of correlation functions and, therefore, this quantities should tend to zero when the media tends to equilibrium.

In our interpretation the Bogolyubov’s correlations relaxation principle means following. So, for nonequilibrium state presence ‘nonequilibrium’ fluctuations in the form of the macroscopic flow of, for instance, energy ε is natural. Then the mean value of m -point correlation functions K_m can not be small as the consequence of macroscopic flow. But in vicinity of equilibrium the macroscopic flows should relax and, accordingly, the *mean* value of correlation functions should be small, $K_m \approx 0$. To characterize the equilibrium one may consider also the particles, charge, spin, etc. densities macroscopic flows and theirs relaxation.

We would like to show in result that the correlations relaxation principle leads to the quantitative connection with real time thermodynamics of Schwinger-Keldysh type³ [6]. Just for this purpose the generating functionals (GF) method of Bogolyubov will be used since it allows to find the quantitative connections, where the euclidean postulate does not applicable.

We will use more natural for particles physics microcanonical formalism. In this formalism the thermodynamical ‘rough’ variables are introduced as the Lagrange multipliers of corresponding conservation laws. Theirs physical meaning are defined by corresponding equations of state. So, if the fluctuations in vicinity of solutions of corresponding equations are Gaussian then one can use this variables for description of the system. Corresponding condition is the Bogolyubov’s correlations relaxation condition.

Formally, the generating functions method presents the integral transformation to new variables. One can choose them as the ‘rough’ thermodynamical variables. To describe the far from equilibrium system we will introduce the ‘local equilibrium hypothesis’. In its frame the preequilibrium state consist from equilibrium domains. In this case new variables should depend on the coordinates of domain and, in result, we are forced to use

²This consideration lie in the basis of earliest Fermi-Landau ‘statistical’ model of hadrons multiple production.

³Last one includes the nonequilibrium thermodynamics also.

the generating functionals (GF) formalism.

We will consider two example to illustrate effectiveness of the GF method. In Sec.2 we will consider the transformation (multiplicity $n \rightarrow$ activity z) to show the origin of the Koba-Nielsen-Olesen scaling (KNO-scaling)⁴.

In Sec.3 we will investigate a possibility of temperature description of the multiple production processes. We will consider for this purpose the transformation (particles energies $\varepsilon \rightarrow$ temperature $1/\beta$) to find the S -matrix interpretation of thermodynamics. It will be shown that this interpretation would be rightful iff the correlations are relax.

In Sec.4 we will use this interpretation to formulate the perturbation theory in the case when β and z are local coordinates of temperature (x, t) [7]. One can use this closed form of perturbation theory for description of nonequilibrium media (in kinetic phase) and for description of the multiple production process as well.

2 KNO-scaling

We would like start from note that the generating functions method allows connect inclusive spectra f_k [8] and exclusive cross sections $\sigma_n(s)$. One can use for this purpose the normalization condition:

$$\bar{f}_k \sigma_{tot} \equiv \int d\omega_k(q) f_k(q_1, q_2, \dots, q_k) = \sum_{n=k} \frac{n!}{(n-k)!} \sigma_n, \quad \bar{f}_k \equiv 0 \quad k > n_{max}, \quad (2.1)$$

where, as usual,

$$d\omega_k(q) = \prod_{i=1}^k d^3 q_i / (2\pi)^3 2\varepsilon(q_i), \quad \varepsilon(q) = \sqrt{q^2 + m^2}$$

, is the Lorentz-covariant element of phase space.

Eq.(2.1) can be considered as the set of coupled equations for σ_n . One may multiply both sides of (2.1) on $(z-1)^k/k!$ and sum over k to solve them. We will see that this is equivalent of introduction of ‘big partition function’ $\Xi(z)$, where z is the ‘activity’: the chemical potential $\mu \sim \ln z$.

We will find in result of summation over k that

$$\Xi(z) \equiv \sum_k \frac{(z-1)^k}{k!} \bar{f}_k = \sum_n z^n \frac{\sigma_n}{\sigma_{tot}}. \quad (2.2)$$

Then, assuming that $\Xi(z)$ is known,

$$\sigma_n = \sigma_{tot} \frac{1}{2\pi i} \oint_C \frac{dz}{z^{n+1}} \Xi(z), \quad (2.3)$$

where the closed contour C includes point $z = 0$. Here $\Xi(z)$ is defined by left hand side of (2.2) and is the generating function of σ_n .

⁴In privet discussion with one of the authors (A.S.) at summer of 1973 Z.Koba noted that the main reason of investigation leading to the KNO-scaling was just the GF method of N.N.Bogolyubov.

The coefficients C_m in decomposition:

$$\ln \Xi(z) = \sum_m \frac{(z-1)^m}{m!} C_m. \quad (2.4)$$

are the (binomial) correlators. Indeed,

$$C_1 = \bar{f}_1 = \bar{n}, \quad C_2 = \bar{f}_2 - \{\bar{f}_1\}^2, \quad C_3 = \bar{f}_3 - 3\bar{f}_2 \{\bar{f}_1\}^2 + 2\{\bar{f}_1\}^3 \quad (2.5)$$

and so on. If $C_m = 0$, $m > 1$, then σ_n is described by Poisson formulae:

$$\sigma_n = \sigma_{tot} e^{-\bar{n}} \frac{(\bar{n})^n}{n!}. \quad (2.6)$$

It corresponds to the case of absence of correlations.

Let us consider more weak assumption:

$$C_m(s) = \gamma_m (C_1(s))^m, \quad (2.7)$$

where γ_m is the energy independent constant. Then

$$\ln \Xi(z, s) = \sum_{m=1} \frac{\gamma_m}{m!} \{(z-1)\bar{n}(s)\}^m. \quad (2.8)$$

To find consequences of this assumption let us find the mostly probable values of z . The equation:

$$n = z \frac{\partial}{\partial z} \ln \Xi(z, s) \quad (2.9)$$

has increasing with n solutions $\bar{z}(n, s)$ since $\Xi(z, s)$ is the increasing function of z , iff $\Xi(z, s)$ is the nonsingular at finite z function. Last condition has deep physical meaning and practically assumes that absence of first order phase transition [9].

Let us introduce new variable:

$$\lambda = (z-1)\bar{n}(s). \quad (2.10)$$

Corresponding eq.(2.9) looks as follows:

$$\frac{n}{\bar{n}(s)} = \left(1 + \frac{\lambda}{\bar{n}(s)}\right) \frac{\partial}{\partial \lambda} \ln \Xi(\lambda). \quad (2.11)$$

So, with $O(\lambda/\bar{n}(s))$ accuracy, one can assume that

$$\lambda \simeq \lambda_c(n/\bar{n}(s)). \quad (2.12)$$

are essential. It follows from this estimation that such scaling dependence is rightful at least in the neighborhood of $z = 1$, i.e. in vicinity of main contributions into σ_{tot} . This gives:

$$\bar{n}(s)\sigma_n(s) = \sigma_{tot}(s)\psi(n/\bar{n}(s)), \quad (2.13)$$

where

$$\psi(n/\bar{n}(s)) \simeq \Xi(\lambda_c(n/\bar{n}(s))) \exp\{n/\bar{n}(s)\lambda_c(n/\bar{n}(s))\} \leq O(e^{-n}) \quad (2.14)$$

is the unknown function. The asymptotic estimation follows from the fact that $\lambda_c = \lambda_c(n/\bar{n}(s))$ should be, as follows from nonsingularity of $\Xi(z)$, nondecreasing function of n .

The estimation (2.12) is rightful at least at $s \rightarrow \infty$. The range validity of n , where solution of (2.12) is acceptable depends from exact form of $\Xi(z)$. Indeed, if $\ln \Xi(z) \sim \exp\{\gamma\lambda(z)\}$, $\gamma = \text{const} > 0$, then (2.12) is rightful at all values of n and it is enough to have the condition $s \rightarrow \infty$. But if $\ln \Xi(z) \sim (1 + a\lambda(z))^\gamma$, $\gamma = \text{const} > 0$, then (2.12) is acceptable iff $n \ll \bar{n}^2(s)$.

Representation (2.13) shows that just $\bar{n}(s)$ is the natural scale of multiplicity n [10]. This representation was offered firstly as a reaction on the so called Feynman scaling for inclusive cross section:

$$f_k(q_1, q_2, \dots, q_k) \sim \prod_{i=1}^k \frac{1}{\varepsilon(q_i)}. \quad (2.15)$$

As follows from estimation (2.14), the limiting KNO prediction assumes that $\sigma_n = O(e^{-n})$. In this regime $\Xi(z, s)$ should be singular at $z = z_c(s) > 1$. The normalization condition

$$\left. \frac{\partial \Xi(z, s)}{\partial z} \right|_{z=1} = \bar{n}(s)$$

gives: $z_c(s) = 1 + \gamma/\bar{n}(s)$, where $\gamma > 0$ is the constant. Note, such behavior of big partition function $\Xi(z, s)$ is natural for stationary Markovian processes described by logistic equations [11]. In the field theory such equation describes the QCD jets [12].

It is known that at *Tevatron* energies the mean hadrons multiplicity rise with transverse momentum. The associated mean multiplicity is

$$C_1(q_{tr}) = \bar{n}(q_{tr}) = \frac{\sum_n n d\sigma_n/dq_{tr}}{\sum_n d\sigma_n/dq_{tr}}.$$

So, if

$$C_m(q_{tr}) = \gamma_m (C_1(q_{tr}))^m : f_k(q_1, q_2, \dots, q_k) \sim \prod_{i=1}^k \frac{1}{\varepsilon(q_i)} \Omega(q_{tr}),$$

then:

$$\bar{n}(q_{tr}) \frac{d\sigma_n/dq_{tr}}{\sum_n d\sigma_n/dq_{tr}} = \Psi(n/\bar{n}(q_{tr})).$$

This prediction is in good agreement with experiment [13].

3 Temperature description

By definition,

$$\sigma_n^{ab}(s) = \int d\omega_n(q) \delta(q_a + q_b - \sum_{i=1}^n q_i) |A_n^{ab}|^2, \quad (3.1)$$

where A_n^{ab} is the amplitude of n creation at interaction of particles a and b .

Considering Fourier transform of energy-momentum conservation δ -function one can introduce the generating function ρ_n [14]. We may find in result that σ_n is defined by equality:

$$\sigma_n(s) = \int_{-i\infty}^{+i\infty} \frac{d\beta}{2\pi} e^{\beta\sqrt{s}} \rho_n(\beta), \quad (3.2)$$

where

$$\rho_n(\beta) = \int \left\{ \prod_{i=1}^n \frac{d^3 q_i e^{-\beta \varepsilon(q_i)}}{(2\pi)^3 2\varepsilon(q_i)} \right\} |A_n^{ab}|^2. \quad (3.3)$$

The mostly probable value of β in is defined by equation of state:

$$\sqrt{s} = -\frac{\partial}{\partial \beta} \ln \rho_n(\beta). \quad (3.4)$$

Let us consider the simplest example of noninteracting particles:

$$\rho_n(\beta) = \{2\pi m K_1(\beta m) / \beta\}^n,$$

where K_1 is the Bessel function. Inserting this expression into (3.4) we can find that in the nonrelativistic case ($n \simeq n_{max}$)

$$\beta_c = \frac{3}{2} \frac{(n-1)}{(\sqrt{s} - nm)}.$$

I.e., $E_{kin} = \frac{3}{2}T$, where $E_{kin} = (\sqrt{s} - nm)$ is the kinetic energy.

It is important to note that the equation(3.4) have unique real rising with n and decreasing with s solution $\beta_c(s, n)$ [15].

The expansion of integral (3.2) near $\beta_c(s, n)$ unavoidably gives asymptotic series with zero convergence radii since $\rho_n(\beta)$ is the essentially nonlinear function of β . From physical point of view this means that, generally speaking, fluctuations in vicinity of $\beta_c(s, n)$ may be arbitrarily high and in this case $\beta_c(s, n)$ has not any physical sense. But if fluctuations are small (strictly speaking, they may be arbitrarily high, bu distribution in vicinity of $\beta_c(s, n)$ should be Gaussian), then $\rho_n(\beta)$ should coincide with partition function of n particles and $\beta_c(s, n)$ may be interpreted as the inverse temperature.

Let us define the conditions when the fluctuations are small [7]. Firstly, we should expand $\ln \rho_n(\beta + \beta_c)$ over β :

$$\ln \rho_n(\beta + \beta_c) = \ln \rho_n(\beta_c) - \sqrt{s}\beta + \frac{1}{2!}\beta^2 \frac{\partial^2}{\partial \beta_c^2} \ln \rho_n(\beta_c) - \frac{1}{3!}\beta^3 \frac{\partial^3}{\partial \beta_c^3} \ln \rho_n(\beta_c) + \dots \quad (3.5)$$

and, secondly, expend the exponent in the integral over, for instance, over $\partial^3 \ln \rho_n(\beta_c) / \partial \beta_c^3$ neglecting higher decomposition terms in (3.5). In result, k -th term of the perturbation series

$$\rho_{n,k} \sim \left\{ \frac{\partial^3 \ln \rho_n(\beta_c) / \partial \beta_c^3}{(\partial^2 \ln \rho_n(\beta_c) / \partial \beta_c^2)^{3/2}} \right\}^k \Gamma \left(\frac{3k+1}{2} \right). \quad (3.6)$$

Therefore, one should assume that

$$\partial^3 \ln \rho_n(\beta_c) / \partial \beta_c^3 \ll (\partial^2 \ln \rho_n(\beta_c) / \partial \beta_c^2)^{3/2}. \quad (3.7)$$

to neglect this term. One of possible solution of this condition is

$$\partial^3 \ln \rho_n(\beta_c) / \partial \beta_c^3 \approx 0. \quad (3.8)$$

If this condition is hold, then the fluctuations are Gaussian, but arbitrary since their value is defined by $\{\partial^2 \ln \rho_n(\beta_c) / \partial \beta_c^2\}^{1/2}$, see (3.5).

Let us consider now (3.8) carefully. We will find computing derivatives that this condition means following approximate equality:

$$\frac{\rho_n^{(3)}}{\rho_n} - 3 \frac{\rho_n^{(2)} \rho_n^{(1)}}{\rho_n^2} + 2 \frac{(\rho_n^{(1)})^3}{\rho_n^3} \approx 0, \quad (3.9)$$

where $\rho_n^{(k)}$ means the k -th derivative. For identical particles (see definition (3.3)),

$$\begin{aligned} \rho_n^{(k)}(\beta_c) &= n^k (-1)^k \int \left\{ \prod_{i=1}^n \varepsilon(q_i) \frac{d^3 q_i e^{-\beta \varepsilon(q_i)}}{(2\pi)^3 2\varepsilon(q_i)} \right\} |A_n^{ab}|^2 \\ &= \sigma_{tot} n^k \int \left\{ \prod_{i=1}^k \varepsilon(q_i) \frac{d^3 q_i e^{-\beta \varepsilon(q_i)}}{(2\pi)^3 2\varepsilon(q_i)} \right\} \bar{f}_k(q_1, q_2, \dots, q_k), \end{aligned} \quad (3.10)$$

where \bar{f}_k is the $(n - k) \geq 0$ -point inclusive cross section. It coincide with k -particle distribution function in the n -particle system. Therefore, l.h.s. of(3.9) is the 3-point correlator K_3 :

$$K_3 \equiv \int d\omega_3(q) \left(\langle \prod_{i=1}^3 \varepsilon(q_i) \rangle_{\beta_c} - 3 \langle \prod_{i=1}^2 \varepsilon(q_i) \rangle_{\beta_c} \langle \varepsilon(q_3) \rangle_{\beta_c} + 2 \prod_{i=1}^3 \langle \varepsilon(q_i) \rangle_{\beta_c} \right), \quad (3.11)$$

where the index means averaging with the Boltzmann factor $\exp\{-\beta_c \varepsilon(q)\}$.

In result, to have all fluctuations in vicinity of β_c Gaussian, we should have $K_m \approx 0$, $m \geq 3$. But, as follows from (3.7), the set of minimal conditions looks as follows:

$$K_m \ll K_2, \quad m \geq 3. \quad (3.12)$$

If experiment confirms this conditions then, independently from number of particles, the final state may be described by one parameter β_c with high enough accuracy β_c .

Considering β_c as physical (measurable) quantity, we are forced to assume that both the total energy of the system $\sqrt{s} = E$ and conjugate to it variable β_c may be measured with high accuracy⁵.

⁵Note, the uncertainty principle $\sim \hbar$ did not restrict ΔE and $\Delta \beta$.

4 Real-time finite temperature generating functionals

We would like to show now why and in a what conditions our S -matrix interpretation of statistics is rightful.

In modern formulations, see e.g. the textbook [16], the temperature is introduced by so called periodic Kubo-Martin-Schwinger (KMS) boundary condition [17]. Namely, in the Feynman-Kac functional integral representation of the partition function

$$\Xi(\beta) = \int D\varphi e^{-S_\beta(\varphi)}. \quad (4.1)$$

the action $S_\beta(\varphi; z)$ is defined on the Matsubara imaginary time contour $C_M: (t_i, t_i - i\beta)$, but fields should obey KMS boundary condition:

$$\varphi(t_i) = \varphi(t_i - i\beta). \quad (4.2)$$

This is natural consequence of definition: $\Xi(\beta) = \text{Spe}^{-\beta\mathbf{H}}$.

It was offered to deform Matsubara contour in a following way:

$$C_M \rightarrow C_{SK} : (t_i, t_f) + (t_f, t_i + i\beta), \quad (4.3)$$

where C_{SK} is the Mills time contour [18] and $t_f > t_i$ belongs to real axis [19]. Including the real-time parts we obtain a possibility to describe time evolution of the system

But this attempt was not successful. First of all, we have not an evident interpretation t_i and t_f [20]. Secondly, in spite of real-time parts, this formulation unable to describe the time evolution [21].

4.1 Equilibrium media

It was shown above that if σ_n is defined by (3.1) then one may introduce ρ_n using definition (3.3). The Fourier transform (3.2) connects σ_n ρ_n . On other hand, ρ_n reminds the partition function.

To find complete analogy with statistical physics we should consider transition $m \rightarrow n$ particles with amplitude $A_{nm} = \langle out; n | in; m \rangle$. Summation over n and m is assumed. The corresponding δ -function of energy-momentum conservation law should be written in the form:

$$\delta\left(\sum_{i=1}^n q_i - \sum_{i=1}^m p_i\right) = \int d^4 P \delta(P - \sum_{i=1}^n q_i) \delta(P - \sum_{i=1}^m p_i), \quad P = (E, \vec{P}). \quad (4.4)$$

This will lead to necessity introduce independently the temperature of initial ($1/\beta_i$) and final ($1/\beta_f$) states. In particle physics we can consider the final state temperature only.

In result we get to the Fourier-Mellin transform $\rho(\beta, z) = \rho(\beta_i, z_i; \beta_f, z_f)$. Direct calculations give important factorized form:

$$\rho(\beta, z) = e^{\hat{N}(\beta, z; \phi)} \rho_0(\phi),$$

where the operator

$$\begin{aligned} \hat{N}(\beta, z; \phi) = & \int dx dx' (\hat{\phi}_+(x) D_{+-}(x-x', \beta_f, z_f) \hat{\phi}_-(x') - \\ & - \hat{\phi}_-(x) D_{-+}(x-x', \beta_i, z_i) \hat{\phi}_+(x')), \quad \hat{\phi} = \frac{\delta}{\delta \phi}, \end{aligned} \quad (4.5)$$

acts on the functional:

$$\rho_0(\phi_{\pm}) = \int D\Phi_+ D\Phi_- e^{iS(\Phi_+) - iS(\Phi_-) - iV(\Phi_+ + \phi_+) + iV(\Phi_- + \phi_-)}. \quad (4.6)$$

At the very end of calculations one should take auxiliary variables ϕ_{\pm} equal to zero.

Here D_{+-} D_{-+} are the frequency correlation functions:

$$D_{\pm\mp}(x-x', \beta) = \mp i \int d\omega(q) e^{\pm i q(x-x' + i\mp\beta)} z(q)$$

They obey the equations:

$$(\partial^2 + m^2)_x G_{+-} = (\partial^2 + m^2)_x G_{-+} = 0.$$

So, all ‘thermodynamical’ information contained in the operator $\hat{N}(\beta, z; \phi)$, but interactions are described by $\rho_0(\phi)$. One can say that the operator \hat{N} (adiabatically) maps the interacting field system on the observable states. This important property allows consider only ‘mechanical’ processes and exclude from consideration the ‘thermal’ ones.

Calculating $\rho_0(\phi)$ perturbatively one can find:

$$\rho(\beta, z) = e^{-iV(-\hat{j}_+) + iV(-\hat{j}_-)} e^{\frac{i}{2} \int dx dx' j_a(x) D_{ab}(x-x', \beta, z) j_b(x')}, \quad (4.7)$$

where D_{++} is the Feynman (causal) Green function and

$$D_{--} = (D_{++})^*$$

is the anticausal one and, as usual, $\hat{j} = \delta/\delta j$. At the very end one should take $j = 0$.

Let us assume now that our system is a subsystem of bigger system. This would lead to transformation of Boltzmann factor $\exp\{-\beta\varepsilon\}$ on corresponding statistics occupation number $\bar{N}(-\beta\varepsilon)$. This means that our interacting fields system is surrounded by black body radiation. This is mechanical model of the thermostat (heat bath of thermodynamics).

In result the matrix D_{ab} takes form (we put for simplicity $z_i = z_f = 1$):

$$\begin{aligned} iG(q; \beta) = & \begin{pmatrix} \frac{i}{q^2 - m^2 + i\epsilon} & 0 \\ 0 & -\frac{i}{q^2 - m^2 - i\epsilon} \end{pmatrix} + \\ & + 2\pi\delta(q^2 - m^2) \begin{pmatrix} \tilde{n}(\frac{\beta_f + \beta_i}{2}|q_0) & \tilde{n}(\beta_i|q_0)a_+(\beta_i) \\ \tilde{n}(\beta_f|q_0)a_-(\beta_f) & \tilde{n}(\frac{\beta_f + \beta_i}{2}|q_0) \end{pmatrix} \end{aligned} \quad (4.8)$$

where

$$a_{\pm}(\beta) = -e^{\frac{\beta}{2}(|q_0| \pm q_0)}.$$

Following Green functions:

$$D_{ab}(x - x', \beta) = \int \frac{d^4 q}{(2\pi)^4} e^{iq(x-x')} G_{ab}(q, \beta)$$

was introduced and the occupation number

$$n_{++}(q_0) = n_{--}(q_0) = \left\{ e^{|q_0|(\beta_f + \beta_i)/2} - 1 \right\}^{-1} \equiv \tilde{n}(|q_0| \frac{\beta_i + \beta_f}{2}). \quad (4.9)$$

and

$$n_{+-}(q_0) = \Theta(q_0)(1 + \tilde{n}(q_0\beta_f)) + \Theta(-q_0)\tilde{n}(-q_0\beta_i), \quad (4.10)$$

$$n_{-+}(q_0) = \Theta(q_0)\tilde{n}(q_0\beta_i) + \Theta(-q_0)(1 + \tilde{n}(-q_0\beta_f)). \quad (4.11)$$

Assuming that $\beta_i = \beta_f = \beta_c$ it is easy to find:

$$G_{+-}(t - t') = G_{-+}(t - t' - i\beta), \quad G_{-+}(t - t') = G_{+-}(t - t' + i\beta), \quad (4.12)$$

i.e. our Green function obey KMS boundary condition.

So, representation (4.7) with Green functions (4.8) coincide identically with (4.1) calculated perturbatively, see also [19].

4.2 Nonequilibrium media

Our attempt introduce the temperature as the quantitative characteristic of *whole* system based on assumption that mean value of correlators is small. We can ‘localize’ this condition assuming that this rough description may be extended only on subdomains of the system. For definiteness the subdomains may be marked by space-time coordinate r .

It should be underlined that we divide on the subdomains not the system under consideration but the device, where external particles are measured. Noting that external flow consist from noninteracting particles (including the flow of black body radiation) the division on subdomains can not influence on the fields interaction.

In result we introduce the ‘local’ temperature $1/\beta(r)$ for r -th group of interacting particles assuming that fluctuations in vicinity of $\beta(r)$ are Gaussian. This means that the mean value of correlation in the group is small, but the correlation between groups may be high. Nevertheless last one is not important since the external particles are on the mass shell. At the same time dimension of group may be arbitrary, but large then some r_0 to have possibility to introduce the temperature as the collective variable.

We can distinguish following scales. Let L_q be the characteristic 4-scale of quantum fluctuations, L_s be the scale thermodynamical fluctuations and L be the scale of subdomain. It is natural to assume that $L_s \gg L \gg L_q$.

Corresponding generating *functional* has the form:

$$\rho_{cp}(\alpha_1, \alpha_2) = e^{\hat{N}(\phi_a^* \phi_b)} \rho_0(\phi_{\pm}).$$

One may note that the ‘localization’ gives influence on the operator only:

$$\hat{N}(\phi_a^* \phi_b) = \int dY dy \hat{\phi}_a(Y + y/2) \tilde{n}_{ab}(Y, y) \hat{\phi}_b(Y - y/2),$$

The occupation numbers $n_{ab}(Y, q)$ have same form, $\beta \rightarrow \beta(Y)$ and

$$\tilde{n}_{ij}(Y, y) = \int d\omega(q) e^{iqy} n_{ij}(Y, q)$$

We find calculating ρ_0 perturbatively that:

$$\begin{aligned} \rho_{cp}(\beta) = & \exp\{-iV(-i\hat{j}_+) + iV(-i\hat{j}_-)\} \times \\ & \exp\{i \int dY dy [j_a(Y + y/2) G_{ab}(y, (\beta(Y))) j_b(Y - y/2)]\} \end{aligned} \quad (4.13)$$

where the matrix Green function $G(q, (\beta(Y)))$ was defined in (4.8).

5 Conclusion

One more detail. Our consideration has show the uniqueness of Bogolyubov’s solution of the nonequilibrium thermodynamics problem. Indeed, without vanishing of correlations perturbation series in the β_c vicinity, being asymptotic, is divergent.

We would like to stress in conclusion that Bogolyubov’s creative works naturally unite particle and statistical physics. In result, using Bogolyubov’s mathematical basis, we have the united scientific space in which both branches of physics, thermodynamics and quantum field theory, supplement each other.

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