Modeling complex heterogeneous objects with non-manifold heterogeneous cells

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Received 21 April 2005; accepted 24 November 2005

Abstract

This paper presents a new approach to model complex heterogeneous objects with simultaneous geometry intricacies as well as complex material distributions. Different from most of the existing approaches, which utilize manifold B-Rep and the assembly representations, the proposed scheme takes advantage of the non-manifold cellular representations to model the geometries of the heterogeneous objects. With the aid of the cell adjacency information and attribute based reasoning, complex, smooth and versatile material distributions can be defined upon the intricate geometries. Compared with other similar approaches, the proposed scheme (1) generates heterogeneous object models with higher data consistencies and lower redundancies; (2) naturally avoids unnecessary/repetitive computations and thus improves computation efficiencies; (3) represents versatile material variations/distributions using different heterogeneous feature tree (HFT) structures. The detailed representation, associated algorithms and a prototype software package are presented. Example heterogeneous objects modeled with the proposed approach are provided.

Keywords: Non-manifold; Cellular model; Boolean operation; Heterogeneous material; Attribute based reasoning

1. Introduction

Heterogeneous objects have caught considerable research interests in CAD, CAE and CAM communities. Recent studies have shown that heterogeneous objects prevail over homogeneous ones in certain aspects in terms of mechanical, thermal, electrical or combinations of these properties. Using heterogeneous components, multiple properties can be obtained in a single object; material incompatibilities and stress concentrations can be avoided with gradual compositional or structural variations. Due to these excellent performances and unique features, heterogeneous objects have gained great popularities in recent years. Extensive applications in mechanical [1–3], electrical [4,5], chemical [6], thermal [7–12], optical [13] and bio-medical [14–19] fields have been reported.

Heterogeneous object modeling has been extensively studied in the past few years. These works range from heterogeneous object representations [20–26], object construction tools development (e.g. extrusion, lofting, patterns and Boolean operations etc.) [25–29] to material distribution modeling, optimizations and finite element analysis [8,28,30,31].

Recently, modeling complex heterogeneous objects with simultaneous intricate geometries as well as complex material distributions has attracted enormous attentions. Many of the existing approaches tend to utilize manifold B-Rep and the part-assembly representation to model the geometries of the complex objects, and then define different material distributions for each manifold component/part of the assembly. Although such methods are intuitive in concept and easy to implement, however, strong data redundancy and low data consistency are also introduced (as will be elaborated shortly).

In addition, with such approaches, the material distributions are usually independently defined for each component/part, and the overall material distributions/properties (e.g. smooth material variations throughout the complex geometries) are seldom considered in a unified context.

This paper is motivated to provide an alternative approach to tackle the above problems. Non-manifold cellular representations [32] and the heterogeneous features tree representations [25] are jointly utilized to model complex heterogeneous objects with simultaneous geometry intricacies as well as complex material distributions.
2. Related work and motivations

Literature reviews on existing heterogeneous object modeling schemes can be found in many thesis (e.g. [33–38]) and separate research papers (e.g. [8,22,24–30,39–46]). In this section, we discuss these existing efforts in the context of complex heterogeneous object modeling, and particular emphasis is put on their complex representational capacities in both geometries and material distributions.

In general, the complex heterogeneous objects to be modeled can be classified into three types:

(I) Objects with complex geometries but simple material distributions: e.g. multiple material objects [20,27] or objects with 1D material gradation [23] etc.

(II) Objects with compound (2D or 3D dependent) material distributions, but simple, regular geometries: e.g. heterogeneous extruded, revolved, lofted objects [25,29].

(III) Objects with intricate geometries as well as complex material distributions [28,33,45,47,48].

2.1. Modeling type (I) and (II) heterogeneous objects

Kumar and Dutta [20] proposed \( r_m \)-classes (sets of \( r_m \)-sets) to represent multi-material objects and defined corresponding Boolean operators on \( r_m \)-sets and \( r_m \)-classes. Sun and Hu [27] proposed reasoning Boolean operations and the complex union operator in heterogeneous object design. ‘\( M_A \) dominant’ and ‘\( M_B \) dominant’ material modeling strategies are proposed. Qian and Dutta [43] also introduced priority tags to define the material compositions for different regions in their constructive operations. These approaches model the complex heterogeneous objects through Boolean operations; however, the complexity refers to complex geometries only. The material distributions modeled are usually simple (e.g. multi-material objects). In general, sharp material changes occur inside the complex objects and abrupt property variations are usually inevitable. In practical applications, smooth material transition, however, is one of the most important requirements and attractive properties of heterogeneous objects. Smooth material transitions help to resolve the material incompatibility problem and eliminate the crack, fracture or load concentration problems in many applications [17,30,49,50]. As can be seen, these approaches fail to assure such smooth material variations throughout the complex geometries.

Siu and Tan [23] introduced heterogeneous insertion and immersion operators in their source based scheme, and functionally graded material distributions can be defined over the complex geometries; however, the modeled material distributions are usually 1D graded and it is also difficult to model compound material distributions which are 2D or 3D dependent on the points’ locations. Such simplicities in material distributions though make the heterogeneous object modeling straight-forward and intuitive, however, the overall performances of the heterogeneous components are generally degraded.

In general, all these approaches are targeted for modeling type (I) heterogeneous objects, i.e. the objects with complex geometries but simple material distributions.

To facilitate modeling compound material variations, Samanta and Koc [29] proposed using B-spline curves, B-spline surfaces and B-spline volumes to represent 1D, 2D and 3D-dependent material variations. Martin and Cohen [39] suggested using trivariate splines to represent and specify volumetric attributes. Natekar et al. [51] utilized NURBS volume to represent the geometry as well as the field definitions in their constructive solid analysis. These approaches use B-spline basis functions as weighting functions to blend the material compositions defined at the control points and are essentially able to represent 3D dependent material distributions. However, one of the limitations of these approaches is that not all complex material distributions can be represented in such explicit, parametric form. To intuitively convey complex material distributions and surmount these limitations, Kou and Tan [25] proposed a hierarchical representation based on a heterogeneous feature tree (HFT) structure, which can be regarded as a direct extension to the source-based scheme [23]. The HFT maintains the material variation dependencies with one or more hierarchies and the material composition of a feature in a higher level is dependent on (or determined by) the material composition of its child features. By recursively evaluating the material variation dependencies, the material composition of an arbitrary point inside the object can be dynamically evaluated at runtime. Within each hierarchy, reverse distance based material blending or other user-defined functions can be used to model the desired material distributions. Biswas et al. [30] generalized the intuitive material modeling strategies and provided a natural space parameterization scheme using the distance (from the material features) as the single parameter; they proposed both explicitly defined functions as well as implicitly constrained material functions to model continuously varying material properties.

These methods focused on modeling heterogeneous objects with complex (possibly 2D or 3D dependent) material variations, however, the geometries of these objects are usually of simple forms, for example extruded, swept, lofted solid or other well established geometric models within the ’existing solid modeling framework’ [30]. They fall in the category of type (II) heterogeneous objects, which have compound, smooth material gradations but simple, regular shapes.
Note that each type (II) object allows for one and only one type of material distribution (represented with either explicit or implicit functions). For complex heterogeneous objects, there may coexist two or more various types of material distributions simultaneously in different regions of the same object [24,47] (e.g. hybrid homogeneous, 1D, 2D or 3D FGM distributions in various portions of the same part). It is difficult, and sometimes impossible to model such diverse material distributions with a single material representation.

2.2. Modeling type (III) heterogeneous objects

As can be seen clearly from Section 2.1, most of type (I) and type (II) heterogeneous object modeling schemes avoid simultaneous complexity in geometry and material distributions, therefore representable heterogeneous objects and potential applications are limited. To further enhance the functionalities of heterogeneous objects and promote their applications, effective schemes for modeling type (III) heterogeneous objects are expected. Type (III) objects represent more general types of heterogeneous components and form the overwhelming majority of heterogeneous objects in nature: for example, human bones or bamboos have complex shapes as well as highly heterogeneous material distributions [49].

To model type (III) heterogeneous objects, a natural idea might be using hybrid representations described in Section 2.1, with which complex heterogeneous objects are generally represented by a collection of heterogeneous components and each component, however, could have potentially complex material distributions.

Chen and Feng [48] proposed to divide complex objects into different parts according to the material compositions and microstructures. Adzhiev et al. [47] proposed a hybrid cellular-functional model for complex heterogeneous objects and the objects are generally composed of mixed-dimensional cells. The attributes (e.g. material compositions) within a cell can be represented with a function of the point coordinates. Cheng and Lin [45] modeled the bone structure by first decomposing the geometries of the bone into several portions, and then each portion is associated with a defined material distribution. Kumar et al. [21] proposed a rigorous mathematical framework in which the geometry of a heterogeneous object is represented by an r-set composed of a finite set of disjoint decompositions. Each decomposed set (termed as a ‘compact’) is represented with an ‘n-manifold’; and the material distribution of each compact is represented with an ‘attribute manifold’. The whole object model is treated as ‘a fiber bundle’ [21]. Liu et al. [28], in their feature based design scheme, also presented an example of decomposing the overall geometry of an object with two spherical surfaces, different material modeling strategies are then applied to these decomposed parts.

In addition to these approaches, which directly decompose the existing (overall) geometries into separate interconnected components, there are also a few constructive approaches, which generate complex decompositions from simple heterogeneous primitives. For example, Shin [33] proposed a general constructive representation, which allows the generation of more complex decompositions through Boolean operations. A complex heterogeneous object ‘unioined’ from two heterogeneous primitives P and Q are modeled with a three-part-decomposition, namely, $P \oplus Q, Q\cap P$ and $P \cap Q$, where $\oplus$ and $\cap$ denote the regularized Boolean difference and intersection operators. In the overlapping region (i.e. $P \cap Q$) of the input objects, constant material blending (from the primitive’s material compositions) is applied while in other regions (i.e. $P \oplus Q$ and $Q \cap P$), distance based material blending functions are used.

2.3. Problems and challenges

The approaches discussed in Section 2.2 seem to have offered satisfactory solutions to modeling type (III) complex heterogeneous objects, however, there are a few less obvious but important problems leaving unresolved.

2.3.1. Modeling spatial decompositions

Although it is now widely recognized that spatial decomposition based representations are well suited for representing type (III) heterogeneous objects; however, from the implementation point of view, constructing consistent and coherent spatial decompositions is far more from a trivial task. As Kumar et al. [21] put it, ‘identification of appropriate decompositions of irregular/complex geometry for defining and evaluating attribute functions is a topic of ongoing research’.

Chen and Feng [48] and Zhu [35] proposed to decompose complex heterogeneous objects ‘using current CAD graphic software’, however, how these decompositions are generated and how they are represented remain implicit. Shin [33] defined ‘effective material function domains’ to model ‘multiple material mappings’ within a single hp-set (which is equivalent to the concept of an r_m-set) and avoided explicit spatial decompositions. However, such representations are not general enough to representing 2D/3D decompositions because modeling such implicit decompositions for arbitrary 2D/3D domains remains challenging.

Kumar and Dutta [20] and Sun and Hu [27] proposed using regularized Boolean operations to generate the geometries of the decomposed parts and then assembly these parts into an assembly representation. Liu et al. [28] also utilized the assembly representation to model the spatial decompositions with the commercial Solidworks system. Although representing the spatial decomposition with the part-assembly model is intuitive in concept and easy to implement, however, strong data redundancy and low data consistency are also introduced in the assembly representation, which in turn, may ‘propagate numerical errors’ [33] and result in server robustness problems. To better understand such problems, Fig. 1 shows a 2D example illustrating the detailed assembly generation process. Fig. 1(a) shows two primitives P and Q which undergo three Boolean operations. To model the decomposition shown in Fig. 1(f), $P \oplus Q, Q \cap P$ and $P \cap Q$ need to be computed first. As is known that in regularized Boolean operations, typical
processes include boundary intersection/subdivision, boundary classification and boundary update (merging, removal etc.) [53]. Therefore, the boundary entities of primitive \( P \) and \( Q \) are intersected and subdivided first, as shown in Fig. 1(b). The boundaries of \( P \cap Q \) are then classified according to their containment statuses as in, on or out of primitive \( Q(P) \) [54], as shown in the legend of Fig. 1. All these boundaries of \( P \) and \( Q \) become candidate boundaries of the output object. Depending on specific Boolean operators, some of them are discarded while others are remained and merged as the output boundaries [53], as illustrated in Fig. 1(c)-(e).

Note that the boundaries rendered with the dotted lines are referenced two times in the assembly shown in Fig. 1(f). For example, the blue dotted lines are both the boundary edges of \( P \cap Q \) and \( P \setminus Q \). Therefore, they are stored and maintained in separate parts of the assembly (Fig. 1(f)), although they carry exactly the same geometric information. Such strong data redundancies are apt to result in serious data inconsistencies and make the assembly representation and relevant model manipulations vulnerable to computation errors. Also note that in both the regularized Boolean intersection \((P \cap \ast Q)\) and difference \((P \setminus Q \ast )\), the same boundary intersections of \( P \) and \( Q \) (Fig. 1(b)) need to be computed independently, even though such computations are exactly the same. In addition, in the boundary update (merging, removal) stage, some discarded boundary entities in Boolean intersection are just the needed (kept) entities in Boolean difference. These repetitive boundary-intersection computations, unnecessary entity removals and recreations, in general, will degrade the overall computation efficiencies. As stated by Qian and Dutta [43], such operations ‘lead to unnecessary operations and over-segmented 3D regions.’

Actually, all these problems come from over relying on the regularized Boolean operations and the manifold boundary representations, which in essence, are based on material homogeneity premises. Relevant studies have shown that manifold solids ‘can only represent one closed volume minus its internal structures’ [55] and manifold based systems ‘have a hard time modeling contact relationship between solids’ [56]. Conversely, non-manifold cellular models are said to be good solutions to represent objects with interior boundaries [56,57], and can ‘surmount the limitations of traditional solid models’ [55].

2.3.2. Unified material modeling and smooth material transitions

In addition to the aforementioned spatial decomposition problem, unified material modeling strategy is also an important problem which deserves particular attentions. Independent material modeling for each separate decomposition (in a divide and conquer manner) fails to consider the complex heterogeneous object as a whole, and the overall material distributions can hardly be considered in a unified context. For example, type (I) heterogeneous object modeling approaches cannot assure smooth material transitions between the different portions of the complex geometries, as described earlier.

A desired approach should provide unified material modeling strategies to model the local, as well as the global/overall material distributions in a systematic manner. To achieve such goals, Cheng and Lin [45] modified/replaced the material compositions around sharp material interfaces locally. Smooth material variations between different portions of the object are achieved; however, such material transition regions were not explicitly represented by separate decompositions, but treated as an attached part of an existing decomposition. Conceptually, it is unreasonable to maintain generically diverse (e.g. partially homogeneous and partially functionally graded) point sets in a same decomposition. Different from Cheng’s approach, Liu et al. [28] explicitly represents the smooth material transition regions with separate decompositions, however, assembly representations are used to model the complex geometries, which also suffer the robustness problems described earlier.
2.4. Motivations and overall solutions

As discussed above, most of type (I) and type (II) heterogeneous object modeling schemes avoid simultaneous complexity in geometries and material distributions; representable heterogeneous objects and potential applications are limited. Each of the existing type (III) object modeling approaches also has certain limitations.

In geometric representations, many existing approaches over rely on the manifold B-Rep and the part-assembly representations. Strong data redundancy and low data consistency problems form the major challenges that prevent such representations from wide applications. In material modeling, there lack effective and unified material modeling schemes to model the overall material distributions in a systematic manner.

This paper attempts to provide an alternative approach to tackle these aforementioned problems. Non-manifold cellular representations are utilized to represent complex geometries; non-regular Boolean operations are applied to construct the complex geometries with shared internal boundaries (co-boundaries). Based on the proposed scheme, the complex geometries of heterogeneous objects can be generated with higher data consistencies and lower redundancies; unnecessary/repetitive computations can be avoided and thus computation efficiencies can be improved. Complex and versatile material distributions are modeled with different but inter-related homogeneous feature tree structures. Cell adjacency information and attribute based reasoning are utilized to assure the smooth material variations throughout the complex heterogeneous object. The proposed mathematical model, detailed methodologies and associated algorithms are presented as follows.

3. A general representation for complex heterogeneous objects

3.1. Mathematical model

A heterogeneous cellular representation (HC-Rep) is proposed to model complex heterogeneous objects. A heterogeneous object $O$ is generally represented by quasi-disjoint [58] heterogeneous cells (or simply cells), which can be mathematically formulated with Eqs. (1)–(4):

$$ O = (G, M) = \{H^0, H^1, \ldots, H^n\}, $$

$$ H^i = (G^i, T^i) $$

$$ G = \bigcup_{i=0}^n G^i $$

$$ \text{Dim}(G^0) = \text{Dim}(G^1) = \cdots = \text{Dim}(G^n) = \text{Dim}(G) $$

$$ \text{Dim}(G^i \cap G^j) \leq \text{Dim}(G) - 1, \quad \forall i \neq j, \ 0 \leq i \leq n, \ 0 \leq j \leq n $$

where $\bigcup$ and $\cap$ denote the set theoretic union and intersection operator, $\text{Dim}(\cdot)$ denotes the object’s dimension, $G$ and $M$ denote the geometry and material representation, $T$ indicates the heterogeneous feature tree (HFT) structure, which is used to represent heterogeneous material distributions [25]. As the HFT representation underlies the material representations in this paper, to make this paper self-contained, a brief introduction on the HFT representation is provided in Appendix.

Eq. (1) formulates a complex heterogeneous object with a set of heterogeneous cells $H^i (0 \leq i \leq n)$, and each cell has its own boundary and HFT representations [25]. Eq. (2) shows that the geometries of all these cells form a space decomposition or a partition [53] of the complex object. Eqs. (3) and (4) impose two extra constraints on this cellular representation: Eq. (3) requires that all cells are dimensionally homogeneous and have the same dimensionality as the complex object. The reason for adding such a constraint is that the emphasis of the proposed representation is to model complex heterogeneous objects through cellular decompositions, dangling geometric features with mixed dimensionalities are not necessary in this context. Eq. (4) requires that the geometry intersections of any two cells are either empty set $\emptyset$ or of lower dimension entities only. Eqs. (3) and (4) together form an alternative description for quasi-disjoint decompositions and this description is slightly different from other similar definitions, e.g. the work by Ferrucci [58]. The major difference can be reflected from the definition of cell adjacency. We define that two cells $G^i$ and $G^j$ are adjacent, if and only if the following condition is met:

$$ \forall i \neq j, \ 0 \leq i \leq n, \ 0 \leq j \leq n $$

Fig. 2 illustrates two examples of quasi-disjoint cellular decompositions. In Fig. 2 (a), object $O = \{A, B, C, D, E\}$ consists of five 2D cells; in Fig. 2 (b), object $O = \{F, G, H, I\}$ is decomposed into four 3D cells. According to the adjacency definition, cell pairs $\{A, B\}$, $\{B, C\}$, $\{B, D\}$, $\{B, E\}$, $\{F, G\}$, $\{G, H\}$, $\{G, I\}$ are mutually adjacent. Note that cells $\{A, D\}$ and $\{F, I\}$ are regarded as disjoint even though $\{A, D\}$ shares a common vertex and $\{F, I\}$ shares common edges because of violation of Eq. (5).

Assume the material variations within a single heterogeneous cell are smooth (which can be assured by applying distance based material blending [30] in the HFT representation [25]), another condition can be imposed to ensure smooth material variations (material continuum) throughout the complex heterogeneous object:

$$ M(P, T^i) = M(P, T^j), \quad \forall P \in (G^i \cap G^j), $$

$$ P \neq \emptyset, \ i \neq j, \ 0 \leq i \leq n, \ 0 \leq j \leq n $$

where $M(P, T)$ denotes the material composition of point $P$ evaluated from a given heterogeneous feature tree $T$. Note that this condition is expected to be satisfied in most applications, unless the designer intentionally wants to model objects with sharp material interfaces/transitions.
3.2. Properties of the heterogeneous cellular representation

The following properties of the proposed heterogeneous cellular representation make it suitable for modeling complex heterogeneous objects:

1. Complex objects are modeled with cellular decompositions. This allows objects with intricate geometries to be represented. Note that there are no special restrictions on the decompositions, as long as they altogether form a partition of the final complex object. As will be seen shortly that diverse decompositions are possible for the same complex geometries. This flexibility introduces extra freedoms for representing complex objects with versatile decomposition layouts.

2. The proposed scheme utilizes non-manifold boundary representation instead of the part-assembly representation to describe the objects’ geometries. In non-manifold representation, the topological elements (e.g. edges, faces) are distinguished from their reference uses [59,60]. For example, a face use is one of the two uses (sides) of a face, and each use is oriented with respect of the face geometry [60]. Any two boundary uses that share the same boundary element(s) are generally termed as co-boundaries (e.g. boundary use of a3 and b1, see Fig. 2(e)). All the element uses that form the boundary of the same cell are termed as partner boundary elements (e.g. a1–a4 and b1–b4 in Fig. 2(e)). In an assembly representation, cell A and B in Fig. 2(a) may be represented with separate boundary representations, and the sharing information between a3 and b1 is lost. While in non-manifold cellular decompositions, the boundaries of each cell are not explicitly evaluated into two-manifold ones; instead, they are represented with boundary reference uses. Two adjacent cells may share the same topological element; however, each cell keeps its own boundary uses. The boundary sharing information among the cells can be explicitly represented with a cell adjacency graph (CAG), as shown in Fig. 2(c) and (d), the nodes of the CAG are the heterogeneous cells, and two nodes are linked with an arc if they are adjacent.

3. More than one type of material distributions can be defined over the quasi-disjoint decompositions. This allows complex material distributions to be defined over the intricate geometries. Note that in a typical type (II) object, only a single material distribution is allowed. For example, in our previous paper [25], each object has one and only one HFT structure in material representation, and the material distributions that can be modeled are usually limited. With the proposed representation, multiple HFTs $T^0(1 \leq i \leq n)$ can be used in combination to model more complex material distributions.

4. These multiple HFTs ($T^0$) are not independently defined for each cell, as formulated by Eq. (6). The topology information between the cells is also utilized in the material modeling process. The complex object is considered as a whole, rather than loosely ‘assembled’ parts. If the material distributions of any two adjacent cells are distinct, i.e. there are sharp material transitions, new heterogeneous cells (with new cell boundaries and HFT representations) can be generated to offer a smooth transition in between. This makes it possible to define smooth material variations throughout the complex object (within each cell and across different cells).

Based on the proposed heterogeneous cellular representation, complex heterogeneous objects can be effectively modeled by first generating non-manifold cellular decompositions for the complex geometries (cell constructions) and then applying different but inter-related material modeling strategies to each heterogeneous cells (material modeling).
Table 1
Constructive modeling operators and their interpretations

<table>
<thead>
<tr>
<th>Directive</th>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Op^{(G)}$</td>
<td>Union: $A \cup B$</td>
<td>{ { } } \in A or { } \in B</td>
</tr>
<tr>
<td></td>
<td>Intersection: $A \cap B$</td>
<td>{ { } } \in A, and { } \in B</td>
</tr>
<tr>
<td></td>
<td>Difference: $A/B$</td>
<td>{ { } } \in A, and { } \notin B</td>
</tr>
<tr>
<td>$Op^{(M)}$</td>
<td>Transition_in_None</td>
<td>No material transition regions</td>
</tr>
<tr>
<td></td>
<td>Transition_in_A</td>
<td>Material transition region occurs inside operand A</td>
</tr>
<tr>
<td></td>
<td>Transition_in_B</td>
<td>Material transition region occurs inside operand B</td>
</tr>
<tr>
<td></td>
<td>Transition_in_Both</td>
<td>Material transition regions occur inside both A and B</td>
</tr>
</tbody>
</table>

where $n_a$, $n_b$ and $n_c$ are the number of heterogeneous cells of the input objects $A$, $B$ and the output object $C$. $H_a^{(i)}$, $H_b^{(i)}$ and $H_c^{(i)}$ are heterogeneous cells.

In this paper, we propose to define the constructive operator $\otimes$ with two parts of operational directives: geometrical directive $Op^{(G)}$ and material variation directive $Op^{(M)}$, as described in Eq. (9):

$$\otimes = \{ Op^{(G)}, Op^{(M)} \}$$

$Op^{(G)}$ can be any traditional Boolean operators, e.g. union, intersection or difference; $Op^{(M)}$ is proposed to control different material variation layouts during the complex object construction. Typical values of $Op^{(G)}$, $Op^{(M)}$ and their interpretations are briefed in Table 1. Different material transition layouts are illustrated in Fig. 3(c) and Figs. 11–17.

For clarity reasons, in the following sub sections, 2D entities are generally utilized as examples to describe the relevant algorithms. The same algorithms can also be directly applied to construct complex 3D objects. The constructive operation $\otimes$ as described in Eq. (9) is used as an example and unless explicitly stated, $Op^{(G)}$ indicates the ‘union’ operation, $Op^{(M)}$ equals to ‘Transition_in_A’.

4. Methodologies and implementations

4.1. Problem statement

Let $A(G_a, M_a)$, $B(G_b, M_b)$ be the input heterogeneous primitives, $C(G_c, M_c)$ be the output complex heterogeneous object constructed from $A$ and $B$ ($G$ and $M$ indicate geometry and material), subject to certain constructive operations:

$$C = A \otimes B$$

(7)

where $\otimes$ denotes the constructive modeling operator.

According to Eq. (1), heterogeneous objects can be generally represented by cellular decompositions, and Eq. (7) can be then rewritten as:

$$\{ H_c^{(0)}, H_c^{(1)}, \ldots, H_c^{(n_c)} \} = \{ H_a^{(0)}, H_a^{(1)}, \ldots, H_a^{(n_a)} \} \otimes \{ H_b^{(0)}, H_b^{(1)}, \ldots, H_b^{(n_b)} \}$$

(8)

Note that in the cell construction process, one can model the non-manifold cellular decompositions either by (a) directly decomposing an existing geometry or by (b) constructing complex decompositions from simple primitives, as discussed in Section 2.3.1. Cheng and Lin [45] presented an example to generate non-manifold cellular decompositions with type (a) approach; Cavalcanti et al. [56] also generated spatial subdivisions using a set of surface patches as delimitation boundaries. As type (b) approaches are more complex and also rare in literatures, in the following sections, we present the detailed methodologies following type (b) approach. The presented cell construction and material modeling algorithms, however, are general enough to be applicable to both cases.

4.2. Algorithms and implementations

Take the following objects as an example: the input objects to the constructive operation are the objects in Fig. 3(a) and the output object is shown in Fig. 3(c).

4.2.1. Algorithm overview

To model the complex heterogeneous objects in Fig. 3(c), both the geometries and material distributions need to be properly represented. Non-manifold quasi-disjoint cellular decompositions are first constructed, as shown in Fig. 3(b).

After cell constructions, the material distributions of each cell are to be defined. To ensure smooth material variations as formulated by Eq. (6), the material distribution for each constructed cell is not independently modeled. The constructed cells are first categorized into different classes based on their material variation behaviors in the construction process: namely the connection cells (CC), transition cells (TC) and static cells (SC).

- A connection cell is the cell that simultaneously connects more than one cell and whose adjacent cells have different material distributions. Sharp material transitions occur around the boundaries of the connection cell, for example the cell ‘C1’ in Fig. 3(b).

![Fig. 3. A complex heterogeneous object construction example. (a) Operand A and B. (b) Non-manifold quasi-disjoint cells. (c) Output complex object.](image-url)
A transition cell serves as the media for different materially defined cells and offers smooth material transition regions between its adjacent cells, for example the cell ‘T1’ in Fig. 3(b).

A static cell is the cell that maintains its original material distributions before and after the constructive operation. The cells ‘S1’ and ‘S2’ in Fig. 3(b) are example static cells.

Based on such cell classifications, different material modeling strategies are then applied to each type of cells.

A schematic flow chart showing the overall algorithm is shown in Fig. 4. The figure offers an overview of the whole processes involved, and the details of each process are explained as follows.

4.2.2. Cell construction

The following procedures are carried out in the cell construction process.

Non-regular Boolean union operation [61–63] is first applied on the input object A and B. The non-regular Boolean union is different from the regularized counterpart in that the internal boundary entities are kept in the output object. This naturally preserves co-boundary elements that are usually obliterated in the regularized Boolean operations. The outputs of non-regular Boolean operation are quasi-disjoint cells, in this example \( \{O_1, O_2, O_3\} \), as shown in Fig. 5(b).

After the non-regular Boolean union, the connection cell \( O_2 \) is then identified from these cells (Fig. 5(c)). Conceptually, the connection cell corresponds to the Boolean intersection of primitive \( A \) and \( B \).

The ‘candidate transition cell’ (CTC) is then generated by first offsetting the connection cell \( O_2 \), and then subtracting \( O_2 \) from the offset, as illustrated in Fig. 5(d). The transition cells is derived from a regularized Boolean intersection between the CTC and the input object A (because in this case \( O_{\text{in}}^{(M)} = \text{Transition in}_A \)), see Fig. 5(e) and (f). Finally the output cellular decompositions (with transition cells) are constructed by applying a non-regular union between the original cells \( \{O_1, O_2, O_3\} \) and the derived transition cells, as shown in Fig. 5(g) and (h).

4.2.3. Cell classifications

In order to apply different material modeling strategies to different types of cells, these cells must be classified prior to the actual material modeling process. To accomplish this, we introduce unique cell attributes as the identifiers and trace the attribute changes during the constructive operations.
4.2.3.1. Attribute assignment. Denote the *ith* boundary element use for a given cell *H* as *u*(i)(H) and let AA(H, ν) be the function to assign a unique attribute value ν to all the *partner boundary element uses* of the cell *H*. We first apply the function AA (attribute assignment) to every cell of the input primitive *A* and *B*, prior to the non-regular Boolean union in the cell construction algorithm:

\[ A = \{ H_a^{(1)}, H_a^{(2)}, \ldots, H_a^{(n_a)} \} \quad B = \{ H_b^{(1)}, H_b^{(2)}, \ldots, H_b^{(n_b)} \} \]

\[ \{u^{(i)}(H_a^{(k)})|\text{Attr}(u^{(i)}(H_a^{(k)})) = \nu_i\} \]

\[ \{u^{(j)}(H_b^{(l)})|\text{Attr}(u^{(j)}(H_b^{(l)})) = \nu_j\} \]

where *n*(a,k) and *n*(b,l) are the total number of boundary element uses for the *kth*/*lth* cell of primitive *A* and primitive *B*, NULL refers to empty attribute value, Attr(·) returns the attribute value of a given boundary element use. For convenience, we call such attributes as ‘Boundary Use Attributes’ (BUA).

It should be emphasized that such attributes are assigned to the boundary element uses rather than the boundary elements themselves. This makes it possible for two different boundary element uses who share the same boundary element to have different attributes assigned to each use, for example, the boundary element use ‘a3’ and ‘b1’ in Fig. 2(e) each may have its own BUAs.

Fig. 6 illustrates this attribute assignment process with the proposed scheme. Initially, the BUAs are assigned to all the partner boundary element uses of *A* and *B*, as seen in Fig. 6(a), here *u*(A) = *p*, 1 ≤ *i* ≤ 4 (red color), and *u*(B) = *q*, 1 ≤ *j* ≤ 4 (blue color). During the non-regular Boolean union operations, the boundaries of *A* and *B* are intersected and subdivided. There are three possible cases occurred to the boundary element uses during this process, and for each case, different attribute manipulations are applied, as shown in Table 2.

4.2.3.2. Attribute propagations. After the above attribute assignment and manipulations, the attributes of all the partner boundary uses within the same cell are then propagated. The purpose of such attribute propagations is to make sure that every boundary element uses within the given cell is properly endowed with certain attribute(s) after geometric manipulations, so that further attribute-based classifications can be conducted.

The attribute propagation function \( \pi(H, \nu_0, \nu_1, \ldots, \nu_n) \) is designed to perform the following operations such that:
4.2.3.3. Attributes based cell classification. From Fig. 7, it can be
found that cell O2 is different from cell O1 and O3 in that its
boundary use has more than one BUAs, while for O1 and O3, there
is only one BUA for each cell. If a cell has \( n(n \geq 2) \) different
BUAs in its boundary use, then it is certain that the
cell has \( n \) neighboring cells with diverse BUA sources. In this
sense, if the attributes is defined to correspond to the material
distribution that a cell has, then the claim ‘A cell has \( n \) different
BUAs in its boundary use’ is equivalent to ‘A cell connects to
\( n \) cells with diverse material distributions’. This matches the
connection cell definition in this paper.

Generally, the constructed non-manifold quasi-disjoint cells
can be classified based on the following rules:

1. After BUA assignment and propagation, all the boundary
element uses of a static cell will have one unique BUA value.
2. After BUA assignment and propagation, both the connec-
tion cell and the transition cell have \( n(n \geq 2) \) BUA values in
its boundary element uses.
3. The connection cell and the transition cell have different
types of adjacent cells. For example, when \( Op^{(M)} = \)
Transition_in_A, a transition cell will have a static neighbor
cell whose BUA value is the same as primitive A’s BUA
value (for brevity, such static cells are called a ‘type A static
cells’); while a connection cell is adjacent to a ‘type B static
cell’, as shown in Fig. 5(h).
4. When \( Op^{(M)} = \) Transition_in_None, all non-static cells are
connection cells, and there will be no transition cells.

Table 3 summarizes the different cell classification cases under different \( Op^{(M)} \) directives.

4.2.4. Material modeling

Given the input operand \( A[H_{x_0}^0, H_{x_1}^1, ..., H_{x_n}^n] \) and
\( B[H_{y_0}^0, H_{y_1}^1, ..., H_{y_m}^m] \), the task of material modeling is to
represent the material distributions (or specifically the
heterogeneous feature trees \( T^{(0)} \)) for each cell \( H_c^0 \) of the output
\( C[H_{c_0}^0, H_{c_1}^1, ..., H_{c_n}^n] \).

4.2.4.1. Material modeling for static cells. The static cells
maintain the original primitive material distributions in
heterogeneous constructive operations. A proxy heterogeneous
feature tree (PHFT) structure is proposed to model a static
cell’s material distribution. The PHFT points to an existing
HFT in the modeling space, and the heterogeneous feature that
owns the existing HFT is called a proxy heterogeneous feature;
the cell that uses the PHFT is called a client.

Such a proxy-client structure can be easily implemented
based on our previous heterogeneous feature tree structure, see
Appendix for details. Within each hierarchy of the HFT, a new
pointer is appended to each HFT node, which points an existing
(proxy) heterogeneous feature. Based on this PHFT, The
material composition evaluation for a static cell can be directly
forwarded or delegated to its proxy heterogeneous features.
If a client feature contains a valid proxy HFT pointer, the material composition for an arbitrary point inside the client’s geometry is dynamically determined by calling the proxy feature’s `EvalMaterial` procedure [25]. If the proxy feature pointer is a NULL pointer, then it degenerates to our old configurations in the HFT representation [25], existing recursive material evaluation algorithm can be applied [25].

Note that to model the material distributions for a static cell, we can also copy the proxy HFT structure instead of sharing it with the proxy feature. Sharing the HFT between the client and the proxy feature is preferred because: (1) Data redundancies are reduced; (2) Any modifications to the primitive material distributions can be immediately reflected in the output object.

4.2.4.2. Material modeling for connection cells. Since $\text{Op}^{(M)} = \text{Transition} \text{ in } A$, new transition cell(s) will be generated within the geometries of primitive $A$. In this case, the connection cell is modeled to have the same material variation as its static neighbor cell so that smooth material variations are assured. The detailed implementations include: (1) get all the static neighboring cells of the current $CC$ from the cell adjacency graph; (2) from the cell list derived in (1), get the cell $C_p$ who has the same BUA as the cells in primitive $B$, and let $C_p$ be the connection cell’s proxy.

Other cases for different $\text{Op}^{(M)}$ configurations can be easily induced as well.

4.2.4.3. Material modeling for transition cells. Material modeling for transition a cell is more complicated. As the transition cell is designed to offer smooth material gradations between two or more material distributions, directly delegating the material evaluation task to either neighboring cell will still result in sharp material transitions. To smooth the material distributions between the neighbors of a transition cell, a novel `Graft HFT (GHFT)` structure is proposed.

The idea is to generate a new HFT through ‘graft hybridization’. Fig. 8 shows an example graft HFT, which is constructed from two HFT branches. Each branch HFT is related to one of the existing material distributions defined upon a neighbor of the transition cell, and the hybridized HFT represents a smooth material gradation between these branches.

To implement the graft hybridization, an empty graft HFT is first created; then all the neighboring cells of the transition cell are identified, as shown in Fig. 9(b) and (d). For each neighbor, the shared boundary elements are then retrieved, in this example they are $\{e_1, e_2, e_3\}$ and $\{e_4, e_5, e_6\}$. All the shared elements within the same cell (e.g. $\{e_1, e_2, e_3\}$ and $\{e_4, e_5, e_6\}$) are grouped together, as shown in Fig. 9(c) and (e). The grouped heterogeneous features are then organized into HFT branches and finally hybridized into a graft tree by saving these branches as its child trees. The mesh and shaded outputs for the transition cell are shown in Fig. 9(g) and (h).

Note that the above HFT graft hybridization is carried out fully automatically without user interactions. All the necessary information can be retrieved from the cell adjacency graph, the BUA lists and the co-boundary relations.

The modeled material distributions for the complex heterogeneous object (including two static cells S1 and S2, one connection cell C1 and one transition cell T1) are shown in Fig. 3(c). Complete proxy heterogeneous feature trees, the graft heterogeneous feature tree and their relationships are depicted in Fig. 10.

As can be seen from the above cell classification and material modeling process, the topological information
(e.g. the cell adjacency and co-boundary information) is extensively utilized in material modeling, which helps to ensure material continuities throughout the complex objects.

5. Examples and applications

The proposed Heterogeneous Cellular Representation (HC-Rep) based approach has been successfully implemented in our prototype software package—CAD4D for interactive heterogeneous object design [64]. CAD4D is a stand alone CAD modeler built upon Lib4D—an object oriented C++ class library, which includes high level C++ object classes, such as CObject4D, CSurface4D, CSolid4D, CComplex4D, CCell4D etc. Both CAD4D and Lib4D are developed and maintained by the authors at Department of Mechanical Engineering, the University of Hong Kong. Commercial ACIS modeler [65] is used as the geometric modeling kernel, and OpenGL [66] is used as the rendering engine.

Some example heterogeneous objects designed by CAD4D are presented here to show the validity and power of the proposed approach.

5.1. Examples

Fig. 11 shows a 2D example constructed with the proposed approach. In this example, more than one connection cells and transition cells are generated. Fig. 12 illustrates a complex object constructed from a FGM ring and a homogeneous rectangle. The region around the transition cell in Fig. 12(b) is amplified in Fig. 12(c). As can be seen from the figures, the transition cell offers smooth material variations between its
adjacent cells, no matter what kind of material distributions these neighbors have.

Fig. 13 shows a 3D example constructed from a block and a cylinder with the constructive operator \( \{ Op^G, Op^M \} = \{ \cup, \text{Transition}_1 \} \). Fig. 14 shows a complex object modeled with successive ‘union’ (i.e. \( Op^G = \cup \)) operations. The output complex heterogeneous object is composed of nine heterogeneous cells. Fig. 15 shows a 3D example modeled with successive subtractions. The object is derived by sequentially subtracting four homogeneous/FGM objects from a homogeneous block. Fig. 16 shows an example constructed from a FGM object (with free-form surfaces) and a homogeneous sphere.

To test the model validity, some of the 3D models are fabricated with the Z Corporation’s 3D printer [67]. Fig. 17 shows the fabricated 3D objects.

As can be seen from the above figures, all these example objects are complex in both the geometry and material distributions. Smooth material variations throughout the complex objects are guaranteed at the same time. These results proved the feasibilities of the proposed approach and validated the presented theoretical representations and methodologies.

5.2. An application case study

Fig. 18 illustrates an application of the proposed approach to high performance cutting tools design.

Graded cutting tools have been reported ‘in high precision machining of soft components, such as plastic contact lenses, polygonal mirrors’ etc. [68]. Conventional diamond cutting tools are manufactured by joining the diamond crystal onto a metallic alloy shank with a silver solder. ‘The machining accuracy is relatively poor due to the silver solder’s lack of stiffness, which causes vibrations during machining’ [68]. In 1994, Li et al. [69] proposed ‘extremely stiff FGM diamond tools’ with graded layers of diamond/SiC between the diamond chip and SiC shank. The FGM based cutting tool has a functionally graded distribution from diamond and SiC at the cutting tip, as shown in Fig. 18(a). The shank, with the shape as shown in Fig. 18(b), is then infiltrated with molten silicon. The silicon and carbon react to form new SiC grains that bond the existing SiC grains (and the diamond tips). This union, as reported in [68], results in ‘the
formation of a strongly integrated body without metal interfaces’, with almost ‘sixfold improvement in machining precision’ and ‘a 30% extension in life’.

The above cutting tool is modeled with the proposed approach as follows. First the FGM based cutting tip is modeled, as shown in Fig. 18(a); and the material distribution of the tip is represented by a heterogeneous feature tree structure. A homogenous silicon shank is then created, as shown in Fig. 18(b). The tip (as primitive A) and the shank (as primitive B) are then joined together with the proposed constructive operator \( \{ \text{Op}^{(G)}, \text{Op}^{(M)} \} = \{ \cup, \text{Transition_in_B} \} \). The resultant high performance cutting tool is shown in Fig. 18(c).

Note that the above SiC grains generated through chemical reaction of silicon and carbon form a smooth material transition between the tool tip and the tool body, which
provide critical contributions to the performance enhancement. As can be seen that the proposed methodology can faithfully characterize the final unioned geometry as well as the smooth material transitions occurred in the fabrication process.

6. Conclusions and discussions

A new approach to model complex heterogeneous objects is proposed in this paper. Non-manifold cellular representations are utilized to model the complex geometries of the heterogeneous objects and non-regular Boolean operations are applied to construct the complex geometries with shared internal boundaries (co-boundaries). Based on the proposed scheme, the complex geometries of heterogeneous objects can be generated with higher data consistencies and lower redundancies; unnecessary/repetitive computations can be avoided and thus computation efficiencies can be improved. Complex and versatile material distributions are modeled with different but inter-related heterogeneous feature tree structures. Cell adjacency information and attribute based reasoning are utilized to assure the smooth material variations throughout the complex heterogeneous object. Detailed algorithms are presented, and example heterogeneous objects are modeled and fabricated.

The novelties of this paper include: the proposed scheme avoids over relying on the manifold B-Rep and the part-assembly representations and utilizes non-manifold cellular representations to model complex cellular decompositions. By using non-regular Boolean operations, unnecessary/repetitive computations are avoided. The generated CAD models are highly consistent in data representations and the strong redundancies are naturally eliminated. Different from other similar approaches, which independently model the material distributions for each decomposed sets, the presented approach can model the local as well as the global/overall material distributions in a unified context. Complete mathematical models as well as the detailed algorithms are presented to model general, complex heterogeneous objects.

Although the presented approach is relatively more complex than existing approaches in terms of both data structures and algorithms, however the underlying principles are intuitive and easy to understand. Two promising directions are suggested for the future work: further exploiting non-manifold boundary representations in heterogeneous object modeling and unified geometric and material modeling in a consistent framework.

Acknowledgements

The authors would like to thank the Department of Mechanical Engineering, The University of Hong Kong and the Research Grant Council of Hong Kong for supporting this project (HKU 7200/04E). The provision of a senior fellowship by the Croucher Foundation to the second author is gratefully acknowledged.

Appendix. The heterogeneous feature tree (HFT) representation [25]

Heterogeneous feature. A heterogeneous features is defined as an entity with a heterogeneous material distribution. A heterogeneous feature \( F(G, M) \) or \( F(G, T) \) is represented by its geometries \( G \) (inclusive of topologies) and material distributions \( M \). Boundary representation (B-Rep) is used to describe the geometry information, and the material information is represented by a heterogeneous feature tree (HFT) structure \( T \).
dependencies among all the constructive heterogeneous features at different hierarchies. The material composition of a feature in a higher level is dependent on (or determined by) the material composition of its child features, and the material composition evaluated from each child trees are then blended at their parent level in the material evaluation process.

The structure and key parameters of a HFT node are conceptually shown in the following figure. Each HFT node consists of three major parts: (1) the geometric data pointer; (2) the material descriptor and (3) the material variation dependency descriptor.

The geometric data pointer points to either 0D (e.g. point), 1D (e.g. EDGE), 2D (e.g. FACE) or 3D geometries (e.g. BODY) of a heterogeneous feature.

The material descriptor in each node is implemented with either a dynamic array structure or simply a $k$-dimensional vector $[r_1, r_2, \ldots, r_k]$. In the first case, the dynamic array keeps a collection of child features pointers. Otherwise, the dynamic array is of zero length, indicating that the object is made of a homogeneous material composition represented by the $k$-dimensional vector. In the extended HFT structure, a ‘proxy pointer’ (rendered in red color) is appended to model the proxy-client material variation relationship, as described in this paper.

The ‘Dependency mode’ is an enumeration data, which further defines the detailed material variation dependency types between the parent and child features. More detailed interpretations can be found in [25].

As a simple and intuitive example, a lofted heterogeneous cylinder and corresponding HFT representation are illustrated below to show the typical usage of the HFT structure.

With the HFT representation, the material modeling process roughly amounts to two sub-tasks: constructing proper HFT structures (e.g. selection of hierarchy levels, modeling material variation functions, child feature identifications/constructions etc) and the material composition evaluations, which can be accomplished with an elegant recursive approach [25].

References


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