Logical analysis of the Bohr Complementarity Principle in Afshar's experiment under the NAFL interpretation

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Abstract

The NAFL (non-Aristotelian finitary logic) interpretation of quantum mechanics requires that no 'physical' reality can be ascribed to the wave nature of a photon. The NAFL theory QM, formalizing quantum mechanics, treats the superposed state (S) of a single photon taking two or more different paths at the same time as a logical contradiction that is formally unprovable in QM. Nevertheless, in a nonclassical NAFL model for QM in which the law of noncontradiction fails, S has a meaningful metamathematical interpretation that the classical path information for the photon is not available (i.e., it has not been measured or axiomatically asserted). It is argued that even the existence of an interference pattern does not logically amount to a proof of the wave nature (self-interference) of a single photon. This fact, when coupled with the temporal nature of NAFL truth, implies the logical validity of the retroactive assertion of the path information in Afshar's experiment; consequently, the Bohr Complementarity Principle holds, despite the co-existence of the interference pattern. NAFL supports, but not demands, a metalogical reality for the particle nature of the photon even for those times when the semantics of QM requires the state S.

1 Introduction

Shahriar S. Afshar [1] has recently performed a variant of Young's two-slit experiment, in which he claims to have demonstrated the falsification of the celebrated Bohr Complementarity Principle, and thereby, the Copenhagen Interpretation of quantum mechanics. In Afshar's experiment, sketched in Fig. 1, a source emits photons towards a screen with two slits marked U and L. A suitably located converging lens downstream of the screen focuses the photon streams into sharp images of the two slits U and L, marked U′ and L′ respectively, at location σ_2 (for simplicity, we do not distinguish between the image plane and the focal plane in Fig. 1). Afshar notes that these images serve to provide sharp

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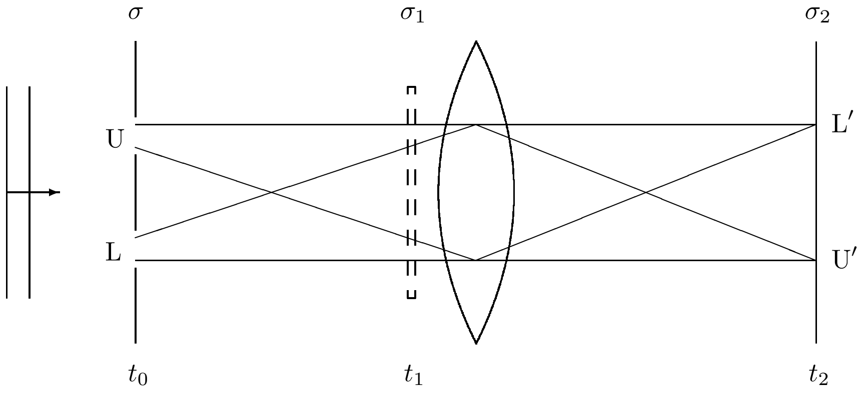

Figure 1: Sketch of Afshar's experiment

which-way information for the photons (thereby confirming their particle status), a claim which has been challenged by Kastner [2]. A wire grid is placed at location σ_1 in Fig. 1, such that the (thin) wires occur at precisely the theoretically predicted minima of the interference pattern due to the superposed states of the photons. Afshar observes that the wire grid effectively performs a nondestructive confirmation of the superposition; when both slits are open, no distortion or reduction in intensity of the images at σ_2 is observed as compared to the case when the wire grid is removed (the expected distortion and reduction in intensity do occur when one of the slits is closed). This seemingly rules out the classical particle state of the photons betweeen the locations σ and σ_1 when both slits are open, and confirms their wave nature, since blocking of photons by the wire grid does not occur to the classically expected extent. Afshar concludes that both the sharp particle and wave natures of the photon are exhibited in a single experiment, which therefore falsifies the Bohr Complementarity Principle.

It is the purpose of this paper to analyze the status of the Complementarity Principle in Afshar's experiment, particularly in the light of the non-Aristotelian finitary logic (NAFL) recently proposed by the author [3, 4, 5]. We will not concern ourselves with the nuts and bolts of Afshar's experiment and attempt to evaluate his claims from the point of view of experimental physics. Instead, we consider the experiment from a purely logical angle, in the case when Afshar's claims are granted as experimentally sound. It will be seen that the NAFL interpretation of the quantum superposition principle does indeed uphold Complementarity, despite the co-existence of the interference pattern *and* the path information for the photons. The essential argument is that the superposed state does not imply any 'reality' for the wave nature of the photon. The interference pattern merely reflects a nonclassical probability distribution for particle-like photons, which occurs because of the lack of path information. Hence the NAFL interpretation does not bar a retroactive assertion of the path information when it does become available to the observer. Kastner's analysis [2] is also critically examined and her claim that Cramer's Transactional

Interpretation of quantum mechanics [6] rescues the Complementarity Principle in Afshar's experiment is disputed.

2 The NAFL interpretation of quantum superposition

At the outset, we hasten to note that the NAFL interpretation is still nascent and incomplete in the sense that a lot of work remains to be done in demonstrating how real analysis can be done in NAFL [7], and ultimately, how all of quantum mechancis can be formalized in this logic. What has been accomplished at this stage is a completely new and logical interpretation of some of the "weird" phenomena of quantum mechanics, in particular, superposition and entanglement [3, 4, 5]. In this section, we will confine ourselves to a brief exposition of the NAFL interpretation of quantum superposition, and refer the reader to the original references for further details. The reader who is already familiar with these details may skip to Sec. 3. Our purpose herein is to provide just enough information on NAFL so as to enable an appreciation of the delicate and subtle logical issues involved in the interpretation of the Bohr Complementarity Principle in Afshar's experiment, which is discussed in Sec. 3.

The language, well-formed formulae and rules of inference of NAFL theories [3] are formulated in exactly the same manner as in classical first-order predicate logic with equality (FOPL), where we shall assume, for convenience, that natural deduction is used; however, there are key differences and restrictions imposed by the requirements of the Main Postulate of NAFL, which is explained in this section. In NAFL, truths for *formal propositions* can exist *only* with respect to axiomatic theories. There are no absolute truths in just the *language* of an NAFL theory, unlike classical/intuitionistic/constructive logics. There do exist absolute (metamathematical, Platonic) truths in NAFL, but these are truths *about* axiomatic theories and their models. As in FOPL, an NAFL theory T is defined to be consistent if and only if T has a model, and a proposition P is undecidable in T if and only if neither P nor its negation $\neg P$ is provable in T.

2.1 The Main Postulate of NAFL

If a proposition P is provable/refutable in a consistent NAFL theory T, then P is true/false with respect to T (henceforth abbreviated as 'true/false in T'); *i.e.*, a model for T will assign P to be true/false. If P is undecidable in a consistent NAFL theory T, then the Main Postulate [4] provides the appropriate truth definition as follows: P is true/false in T if and only if P is provable/refutable in an *interpretation* T* of T. Here T* is an axiomatic NAFL theory that, like T, temporarily resides in the human mind and acts as a 'truth-maker' for (a model of) T. The theorems of T* are precisely those propositions that are assigned 'true' in the NAFL model of T, which, unlike its classical counterpart, is not 'pre-existing' and is instantaneously generated by T*. Note that for a given consistent theory T, T* could vary in time according to the free will of the human mind that interprets T; for example, T* could be T+P or T+$\neg P$ or just T itself at different times for a given human mind, or in the context of quantum mechanics, for a given observer. Further, T* could vary from one observer to another at any given time; each observer determines T* by his or her own free will. The essence of the Main Postulate is that P is true/false in T if and only if it has been *axiomatically declared* as true/false by virtue of its provability/refutability in T*. In the absence of any such axiomatic declarations, *i.e.*, if P is undecidable in T* (*e.g.* take T*=T), then P is 'neither true nor false' in T and Proposition 1 shows that consistency of T requires the laws of the excluded middle and noncontradiction to fail in a nonclassical model for T in which $P \& \neg P$ is the case.

Proposition 1. *Let P be undecidable in a consistent* NAFL *theory* T*. Then $P \vee \neg P$ and $\neg(P \& \neg P)$ are not theorems of* T*. There must exist a nonclassical model $\mathcal{M}$ for* T *in which $P \& \neg P$ is the case.*

For a proof of Proposition 1, see [3] or Appendix A of [5]; this proof also seriously questions the logical/philosophical basis for the law of noncontradiction in both classical and intuitionistic logics. The interpretation of $P \& \neg P$ in the nonclassical model will be explained in Sec. 2.2. Proposition 1 is a metatheorem, *i.e.*, it is a theorem *about* axiomatic theories. The concepts in Proposition 1, namely, consistency, undecidability (provability) and the existence of a nonclassical model for a theory and hence, quantum superposition and entanglement, are strictly metamathematical (*i.e.*, pertaining to semantics or model theory) and not formalizable in the syntax of NAFL theories. An NAFL theory T is either consistent or inconsistent, and a proposition P is either provable or refutable or undecidable in T, *i.e.*, the law of the excluded middle applies to these metamathematical truths. Note that the existence of a nonclassical model does not make T inconsistent or paraconsistent, because T does not *prove* $P \& \neg P$. However, one could assert that the model theory for T requires the framework of a paraconsistent logic, so that the nonclassical models can be analyzed. NAFL is the only logic that correctly embodies the philosophy of formalism [4]; NAFL truths for formal propositions are axiomatic, mental constructs with strictly no Platonic world required.

2.2 Quantum superposition justified in NAFL

The nonclassical model $\mathcal{M}$ of Proposition 1 is a superposition of two or more classical models for T, in at least one of which P is true and $\neg P$ in another. Here '(non-)classical' is used strictly with respect to the status of P. In $\mathcal{M}$, 'P' ('$\neg P$') denotes that '$\neg P$' ('P') is not provable in T*, or in other words, $\mathcal{M}$ expresses that neither P nor $\neg P$ has been axiomatically declared as (classically) true with respect to T; thus P, $\neg P$, and hence $P \& \neg P$, are indeed (nonclassically) true *in our world*, according to their interpretation in $\mathcal{M}$. Note also that P and $\neg P$ are *classically* 'neither true nor false' in $\mathcal{M}$, where 'true' and 'false' have the meanings given in the Main Postulate. The quantum superposition principle is justified by identifying 'axiomatic declarations' of truth/falsity of P in T (via its provability/refutability in T* as defined in the Main Postulate) with 'measurement' in the real world. NAFL is more in tune with the Copenhagen interpretation of quantum mechanics than the many-worlds interpretation (MWI). Nevertheless, the *information content* in $\mathcal{M}$ is that of two or more classical models (or 'worlds'), and MWI is at least partially vindicated in this sense.

2.3 Example: Schrödinger's cat

Consider the situation wherein the cat is put into the box at time $t = t_0$ and has a probability 0.5 of being in the 'alive' state at $t = t_2$, when a 'measurement' is made of its state. Let P be the proposition that 'The cat is alive', with $\neg P$ denoting 'The cat is dead'; obviously, P is undecidable in a suitable formalization QM of quantum mechanics, which may be taken to include axioms describing this experiment. For $t_0 < t < t_2$, the observer makes no measurements, and in tune with the identification noted in Sec. 2.2, makes no axiomatic declarations regarding P in the interpretation QM* (say, let QM*=QM for this time period). In the resulting nonclassical model $\mathcal{M}$ of QM, the superposed state $P\&\neg P$ is the case; this means that the cat has not been declared (measured) to be either alive or dead, which is certainly true in the real world. At $t = t_2$, if P ($\neg P$) is observed, then the observer takes, say, QM*=QM+V ($\neg V$), where V ($\neg V$) is defined as 'The cat is alive (dead) at $t = t_2$'. Note that QM*=QM+V ($\neg V$) will prove P ($\neg P$) at $t = t_2$, *i.e.*, when the observer measures the cat to be alive (dead) in the real world, he makes the appropriate axiomatic declarations in his mind, thus setting up QM* as defined. It should be emphasized that an NAFL theory only 'sees' the observer's axiomatic declarations and does not care whether the real world exists. The observer sees the real world and the proposed identification of his measurements with his axiomatic declarations is only an informal convention that is outside the purview of NAFL. The observer could also use his free will to make his axiomatic declarations irrespective of (and possibly in contradiction to) what he measures in the real world; of course, if P is not about the real world, then he has no other choice. NAFL correctly handles the temporal nature of truth via the time-dependence of QM*. If P is observed (and axiomatically asserted via QM*=QM+V) at $t = t_2$, then the proposition U that "The cat was alive for $t_0 < t < t_2$" can be formalized for $t \geq t_2$ and proven in the NAFL theory QM*=QM+V; U does not conflict temporally or in meaning with the superposed state $P\&\neg P$, which applies for $t_0 < t < t_2$. We will return to the Schrödinger cat example in Sec. 4, in order to further elaborate upon the validity of this retroactive assertion of U.

2.4 Theory syntax and proof syntax

An NAFL theory T requires two levels of syntax, namely the 'theory syntax' and the 'proof syntax'. The theory syntax consists of precisely those propositions that are legitimate, *i.e.*, whose truth in T satisfies the Main Postulate; obviously, the axioms and theorems of T are required to be in the theory syntax. Further, one can only add as axioms to T those propositions that are in its theory syntax. In particular, neither $P \& \neg P$ nor its negation $P \vee \neg P$ is in the theory syntax when P is undecidable in T. The proof syntax, however, is classical because NAFL has the same rules of inference as FOPL; thus $\neg(P \& \neg P)$ is a valid deduction in the proof syntax and may be used to prove theorems of T. For example, if one is able to deduce $A \Rightarrow P \& \neg P$ in the proof syntax of T where P is undecidable in T and A is in the theory syntax, then one has proved $\neg A$ in T despite the fact that $\neg(P \& \neg P)$ is not a theorem (in fact not even a legitimate proposition) of T. This is justified as follows: $\neg(P \& \neg P)$ may be needed to prove theorems of T, but it does not follow in NAFL that the theorems of T imply $\neg(P \& \neg P)$ if P is undecidable in T. Let A and B be undecidable propositions in the theory syntax of T. Then $A \Rightarrow B$ (equivalently, $\neg A \vee B$) is in the theory syntax of T if and only if $A \Rightarrow B$ is *not* (classically) deducible in the proof syntax of T. It is easy to check that if $A \Rightarrow B$ is deducible in the proof syntax of T, then its (illegal) presence in the theory syntax would force it to be a theorem of T, which is not permitted by the Main Postulate. For, in a nonclassical model $\mathcal{M}$ for T in which both A and B are in the superposed state, $A \& \neg B$ must be nonclassically true, but theoremhood of $A \Rightarrow B$ will prevent the existence of $\mathcal{M}$. If one replaces B by A in this result, one obtains the previous conclusion that $\neg(A \& \neg A)$ is not in the theory syntax. For example, take T_0 to be the null set of axioms. Then nothing is provable in T_0, *i.e.*, every legitimate proposition of T_0 is undecidable in T_0. In particular, the proposition $(A \& (A \Rightarrow B)) \Rightarrow B$, which is deducible in the proof syntax of T_0 (via the *modus ponens* inference rule), is not in the theory syntax; however, if $A \Rightarrow B$ is not deducible in the proof syntax of T_0, then it is in the theory syntax. Note also that $\neg\neg A \Leftrightarrow A$ is not in the theory syntax of T_0; nevertheless, the 'equivalence' between $\neg\neg A$ and A holds in the sense that one can be replaced by the other in every model of T_0, and hence in all NAFL theories. Indeed, in a nonclassical model for T_0, this equivalence holds in a nonclassical sense and must be expressed by a different notation [4].

3 Bohr Complementarity and the NAFL interpretation of Afshar's experiment

The Bohr Complementarity Principle easily follows from the NAFL interpretation of quantum superposition discussed in Sec. 2. As applied to Afshar's experiment, the relevant definition is as follows.

Definition 1 (Bohr Complementarity Principle (BCP)). The particle and wave nature of the photon cannot be *simultaneously* demonstrated to hold at any given spatial location and time within a given experiment.

Note carefully the location of the word "*simultaneously*" in this definition; if we were to change BCP to "...cannot be demonstrated to hold *simultaneously* at any given spatial location and time within a given experiment", then such a definition would arguably fail even in NAFL. The ability of NAFL to handle the temporal nature of mathematical truth and the distinctions NAFL makes between syntax and semantics are demonstrated to be indispensable for a correct logical explanation of the results in Afshar's experiment.

3.1 Quantum superposition in Afshar's experiment

Let QM be the NAFL theory formalizing quantum mechanics. We assume that axioms providing a detailed description of the single-photon version of Afshar's experiment [1] (which he has reportedly performed with the same results) have already been included in QM.

Definition 2. Let P denote the proposition "The photon passed through (only) slit U at location σ and time t_0 in Fig. 1."

Definition 3. Let the negation $\neg P$ of P denote the proposition "The photon passed through (only) slit L at location σ and time t_0 in Fig. 1."

Since P and $\neg P$ are equally probable, it follows that P is undecidable in QM. Here it is important to understand the NAFL concept of negation (see Sec. 2.2 of [4]); in particular, mutually exclusive classical possibilities (e.g. in the real world) are negations of each other and $\neg\neg P$ is equivalent to P, just as in classical logic. But unlike classical logic, $P \vee \neg P$ is *not* a theorem of QM, and unlike intuitionistic logic, $\neg(P \& \neg P)$ is also *not* a theorem of QM; as noted in Sec. 2.1, there must exist a nonclassical model $\mathcal{M}$ for QM in which $P \& \neg P$ is the case. The interpretation of $P \& \neg P$ in $\mathcal{M}$ is identical to that explained in Secs. 2.2 and 2.3; 'P' (resp. '$\neg P$') of $P \& \neg P$ has the nonclassical meaning that '$\neg P$' (resp. 'P') is not provable in the observer's interpretation QM* of QM. In keeping with the informal convention noted in Secs. 2.2 and 2.3, the observer agrees to keep his axiomatic assertions in tune with his measurements in the real world, so that the superposition $P \& \neg P$ in $\mathcal{M}$ also has an equivalent nonclassical meaning with 'P' (resp. '$\neg P$') now denoting that "The observer has not measured the photon to pass through slit L (resp. slit U) at location σ and time t_0 in Fig. 1." One can see that the NAFL interpretation of $P \& \neg P$ is meaningful in the real world because $\neg P$ is not *really* the negation of P in $\mathcal{M}$; if the observer has not measured (axiomatically declared) that the photon passed through slit L, it does not follow that he has measured (axiomatically declared) that the photon passed through slit U. Consistency demands that the observer should never be able to *prove* $P \& \neg P$ in any NAFL theory, in particular, QM or QM*; for such a proof would imply the contradiction: "The photon *really* passed through both slits". The only reality that exists, as far as the observer is concerned, is that he has not measured or axiomatically asserted either P or $\neg P$ in the real world. This same reality is accurately modelled in the nonclassical model $\mathcal{M}$ where, somewhat paradoxically, P, $\neg P$ and $P \& \neg P$ all hold, but with the nonclassical interpretations noted above; the existence of such a nonclassical model for QM is a requirement of consistency in NAFL.

NAFL requires that both $P\&\neg P$ and its negation $\neg(P\&\neg P)$ (or equivalently, $P\vee\neg P$) are not legitimate propositions in the theory syntax of QM; see Sec. 2.4. What this means is that *formally*, the status of the photon in the theory syntax of QM is indeterminate; it cannot be proven to be either a particle (in which case $P \vee \neg P$ should be a theorem) or a wave (in which case $P\&\neg P$ should be a theorem). Consequently, both classical and nonclassical models, with respect to the proposition P, exist for QM. Note that the NAFL semantics for the nonclassical model $\mathcal{M}$ of QM does not take a stand either on the status of the photon as a particle or non-particle. Nevertheless, the NAFL theory QM tacitly supports (but not requires) the 'reality' of the particle nature of the photon in a *metalogical* sense, *i.e.*, outside of both theory syntax and semantics. This will be fully explained in Sec. 3.2. Here we observe that the proof syntax of QM requires the deduction of $P \vee \neg P$; see Sec. 2.4, where it is noted that the rules of inference of NAFL theories, which determine the proof syntax, must be classical. It is possible (but not necessary) to interpret this deduction as a metalogical assertion of the particle nature of the photon in the sense that the photon 'really' had to pass through one and only one of the two slits U and L at time t_0 in Fig. 1, even though, at that instant, the observer did not have a proof that either of these paths was traversed. It is this lack of knowledge at time t_0 that NAFL semantics expresses as a requirement of logical consistency, via the nonclassical model $\mathcal{M}$, rather than any perceived reality for the particle nature of the photon. But such a perceived reality became inevitable the moment Definition 3 was formulated in QM as the negation of P. Here it should be kept in mind that the 'photon' referred to is that which must necessarily pass through at least one of the slits. The coin toss experiment, considered in Sec. 4, will further illustrate the metalogical 'reality' of $P \vee \neg P$ in NAFL.

3.2 Bohr Complementarity in Afshar's experiment

Consider again the single-photon version of Afshar's experiment in Fig. 1, with P, $\neg P$ and the theory QM as defined in Sec. 3.1. At time $t = t_0$ a photon passes through the slit(s); at $t = t_1 > t_0$, the said photon passes the wire grid in the "interference plane" at location σ_1, and subsequently passes through the converging lens; at $t = t_2 > t_1$, the photon ends up at one of the locations marked U$'$ or L$'$ (say, U$'$, for the sake of definiteness). Note that in the single-photon version of Afshar's experiment, the restriction is that only one photon at a time can pass through the slit(s), although several photons actually end up at U$'$ and L$'$.

Definition 4. Take the proposition Q to denote that "The photon reaches the image U$'$ of slit U at location σ_2 and time t_2 in Fig. 1".

Definition 5. Let the proposition R denote that "The images L$'$ and U$'$ of the slits L and U respectively are undistorted and unchanged in intensity when the wire grid is inserted at the calculated minima of the expected interference pattern at location σ_1 in Fig. 1".

Consider the following contentious inferences that are inherent in the controversy over Afshar's experiment.

Inference 1. The theory QM+Q proves P.

This, of course, implies the particle nature of the photon.

Inference 2. The theory QM+R proves $P\&\neg P$.

In words, Inference 2 means "The photon passed through both slits U and L at location σ and time t_0 in Fig. 1", which in turn implies the wave nature of the photon. Since Q and R are true, in the sense that they are observed, one may conclude that if Inferences 1 and 2 are granted, the resulting 'true' (but inconsistent) theory QM+Q+R violates BCP; see Definition 1. Kastner [2] seems to permit Inference 1, but disputes that it 'really' establishes 'which-way' information for the photon; she argues that Cramer's Transactional Interpretation [6] supports her stand. Kastner concludes that only the wave nature of the photon is unambiguously exhibited at the slits in Afshar's experiment, via Inference 2, which she allows (although neither she nor Afshar interpret the wave nature of the photon as a proposition of the form $P\&\neg P$, as NAFL requires). In what follows, we will argue that the NAFL interpretation allows Inference 1, but *not* Inference 2; this fact, when coupled with the temporal nature of mathematical truth in NAFL, means that Bohr Complementarity survives. The claim that a single photon 'really' exhibits wave nature is questionable from the NAFL point of view and will be critically examined in Sec. 3.3. However, we emphasize at the outset that the wave nature of light could well follow unambiguously in a new, as yet unknown, version of QM formalized in NAFL. The present argument applies only to our current understanding of QM and the nature of light.

The NAFL justification of BCP is as follows. For $t_0 \leq t < t_2$, the observer has the theory QM in mind with the interpretation QM*=QM. Hence by Proposition 1 (see Sec. 2.1), $P\&\neg P$ holds for the observer, in a nonclassical model $\mathcal{M}$ for QM. At $t = t_2$, upon measuring Q, the observer switches to the interpretation QM*=QM+Q and concludes P in a classical model for QM, via a proof in QM* (Inference 1). Note that P only applies retroactively, for times $t \geq t_2$; therefore it does not temporally conflict with the superposition $P\&\neg P$, which applied for $t_0 \leq t < t_2$. The second observation here is that the theory QM does not *prove* $P\&\neg P$; as noted earlier, $P\&\neg P$ is not even a legitimate proposition in the theory syntax of QM. Such a proof of $P\&\neg P$ would make QM inconsistent in NAFL, which is *not* a paraconsistent logic, as noted in Sec. 2.1. In fact, the Main Postulate of NAFL and consequently, the existence of $\mathcal{M}$, are strictly metamathematical results, *i.e.*, pertaining to semantics or model theory; these concepts are not formalizable in either the theory syntax or proof syntax of QM. The theory QM* can therefore prove P without loss of consistency. Since the deduction of P and the meta-deduction of the superposition $P\&\neg P$ were made in non-overlapping time intervals, and, in particular, were *not* made *simultaneously*, BCP survives in NAFL.

One might object that there is something unsatisfactory about this state of affairs; if we interpret P and $P\&\neg P$ as affirming the particle and the wave natures of the photon respectively, they seem to be contradictory even if asserted at non-overlapping time intervals, since, after all, they both apply to the same photon at the same spatial location and time. The answer to this objection is that in the nonclassical model $\mathcal{M}$, $P\&\neg P$ only means that the observer has not axiomatically asserted P via a proof in QM* (or measured P in the real world) and he has not axiomatically asserted $\neg P$ via a proof in QM* (or measured $\neg P$ in the real world); as noted in Sec. 3.1, $P\&\neg P$ does *not* imply that the photon 'really' exhibited wave nature and passed through both slits. But the retroactive assertion of P (via a proof in QM*), valid for $t \geq t_2$, can be taken to have the *metalogical* (see Sec. 3.1) meaning that the photon 'really' passed through only slit U at time t_0, and therefore does not conflict in meaning with the metadeduction of $P\&\neg P$ made in $\mathcal{M}$ during the time interval $t_0 \leq t < t_2$; both are indeed 'true' when appropriately interpreted. To summarize, for $t_0 \leq t < t_2$, NAFL semantics only recognizes the metamathematical truth of $P\&\neg P$ in $\mathcal{M}$. For $t \geq t_2$, NAFL semantics asserts the retroactive truth of P via a classical model for QM; this truth may be taken to hold *metalogically* for $t_0 \leq t < t_2$, *i.e.*, outside of NAFL syntax and semantics. The temporal nature of NAFL truth plays an indispensable role in removing the mystery associated with wave-particle duality. In classical logic, unlike NAFL, the retroactive assertion of P at $t = t_2$ would necessarily mean that P was *always true* in the semantics of QM, including at $t = t_0$. Therefore in the framework of classical logic, such a retroactive assertion of P will be problematic and will clash with the quantum superposition state that actually held in the semantics of QM at $t = t_0$.

At this stage the reader will have the following obvious question; does not R (Definition 5) *prove* the existence of the interference pattern at location σ_1 of Fig. 1 and consequently, the reality of the wave nature of the photon when it enters the slits U and L? In the NAFL interpretation, the answer to the second part of this question is in the negative; in Sec. 3.3 we will argue that Inference 2 is not permitted on logical grounds and therefore the observation R is irrelevant to the above-noted retroactive conclusion of P by the observer. A second question that arises is whether there are logical grounds for banning Inference 1 as well. Afshar [1] has asserted that Inference 1 follows from standard optics and has subsequently pointed out elsewhere that moving one of the slits U or L in Fig. 1 causes the corresponding image U$'$ or L$'$ to co-move with the slit. Here we do not pass judgement on the experimental validity of Afshar's claims. We only wish to point out that on purely logical grounds, NAFL does permit the *conclusion* of Inference 1, namely, the retroactive assertion of P; for reasons mentioned in Sec. 3.1 and in this subsection, NAFL does not object to the metalogical existence of the photon as a particle. Thus one could also take the interpretation QM*=QM+P at $t = t_2$ (instead of QM*=QM+Q as noted above), if it turns out that there is indeed something wrong with Inference 1.

We note that although the NAFL interpretation does permit the retroactive assertion of P, via Inference 1 or otherwise, there is no *obligation* on the part of the observer to make such an assertion (or measurement). The observer can choose to live with just the *metamathematical* conclusion that both classical and nonclassical models for QM exist; he could choose to remain agnostic (*i.e.*, take no stand) on the *metalogical* status of the photon as a particle or non-particle in the nonclassical models and scrupulously avoid making any retroactive assertions/measurements. Consequently, to facilitate such an agnostic attitude, Inference 1 must either be disallowed or if permitted, must be weakened so as not to imply any 'reality' for the path of the photon at time t_0. At time t_2, the photon behaves *as though* it originated from slit U (via the weakened Inference 1), but need not have 'really' done so, from the agnostic's point of view. Kastner [2] seems to prefer this latter approach to Inference 1; however, rather than remain agnostic, she asserts that the photon passed through both slits, as a wave. In Sec. 3.4, we criticize the limiting of Inference 1 as noted above, as well as Kastner's reasons for doing so. Cramer's Transactional Interpretation [6] is also criticized from the NAFL point of view in Sec. 3.5. An *anti-realist* approach is also possible in NAFL. Anti-realism is stronger than agnosticism, in the sense that it requires the observer to *deny* any reality for the state of the photon as particle or non-particle in the nonclassical model $\mathcal{M}$. In other words, no such reality exists in the absence of an axiomatic declaration, which, by informal convention, the observer associates with 'measurement' in the real world. The anti-realist approach would presumably require the banning of Inference 1 and other retroactive assertions of reality. Of course, Inference 2 is already illegal in NAFL, as will be explained in Sec. 3.3.

In summary, there are three approaches possible in NAFL regarding the status of the photon during $t_0 \leq t < t_2$, when $\mathcal{M}$ is in force. These are, a metalogical reality for the particle state, agnosticism and anti-realism. The author believes that the first option is the most satisfactory from both philosophical and logical points of view, *given* Definition 3. With such a choice of negation in NAFL, has the observer denied any possibility that the photon can 'really' pass through both slits, thereby affirming its metalogical particle status? This is a tricky issue; the author believes that the answer is in the positive, i.e., the observer has predetermined the particle nature of the photon via Definition 3. One may attempt to justify the above choice of negation by arguing that P and $\neg P$ cannot be 'measured' together, although $P \& \neg P$ could be 'really' true in the real world (this would provide another reason for banning Inference 2, for otherwise the proposition R in Definition 5 surely amounts to a 'measurement' of $P \& \neg P$). However, such an attempt at justification fails, for the following reason. NAFL truth for undecidable propositions of a theory is purely axiomatic in nature, by the Main Postulate (see Sec. 2.1); the concept of 'measurement' cannot be formalized in the theory syntax of NAFL theories [3]. The informal convention of associating 'axiomatic declaration' with 'measurement' in the real world can, in principle, be broken in NAFL; see Sec. 2.3 as well as the final paragraph of Sec. 3.3. If the photon can 'really' (or classically) pass through both slits, there ought to be nothing to stop us from axiomatically declaring, i.e., inferring via a proof in QM*, that 'The photon passed through both slits at $t = t_0$', even if *measurement* of this event is impossible. For such a proposition to be legal in the theory syntax of QM, NAFL would require the negation of P to be modified such that it includes the case of the photon passing through both slits in disjunction with the case noted in Definition 3. *However*, as a consequence, BCP will fail by the Main Postulate of NAFL. For if both the particle and wave natures of the photon are classically possible phenomena, then the formal undecidability in QM of whether the photon exists as a particle or a wave would demand (via Proposition 1) that there must exist a nonclassical model $\mathcal{N}$ for QM in which the superposed state of the photon as a particle *and* a wave must hold. In $\mathcal{N}$, the photon will be neither a particle nor a wave, in violation of BCP. As explained in Sec. 3.3, we do not believe that this is the correct approach.

Reverting to the negation in Definition 3, the "wave nature" of the photon is a proposition of the form $P\&\neg P$. As noted earlier, neither $P\&\neg P$ nor its negation $P \vee \neg P$ (which symbolizes the particle nature of the photon) is a legitimate proposition in the theory syntax of QM. Consequently, BCP survives, because the Main Postulate and Proposition 1 apply only to formal propositions that are in the theory syntax of NAFL theories; the nonclassical model $\mathcal{N}$ noted above need not (and does not) exist. Here we have put "wave nature" in quotes because $P\&\neg P$ in the nonclassical model $\mathcal{M}$ of QM does not imply that the photon is 'really' a wave, as was argued above and in Sec. 3.1. One can also state BCP, in the context of the NAFL interpretation of Afshar's experiment, as follows:

> At any given time, the observer, via the interpretation QM*, can generate either a classical or a nonclassical model (but *not* both) for QM, with respect to the proposition P; in the classical model, $P \vee \neg P$ (and either P or $\neg P$) holds, and the photon is a particle; in the nonclassical model, $P\&\neg P$ holds and the photon may be loosely termed as a 'wave', although its true status in the real world is left ambiguous.

One can see that the above formulation of BCP is not violated in the NAFL interpretation of Afshar's experiment. The NAFL interpretation also neatly solves the problem of the mysterious "instantaneous collapse of the wavefunction", which would arise if and only if one insists that the photon 'really' exhibits wave nature. Indeed, when Q is measured at time t_2 in Fig. 1, all that happens is that the observer switches his interpretation QM* of QM as noted previously. This amounts to a switch from a state of ignorance regarding the path of the photon to one of knowledge. Obviously, there is no implication here that the photon abruptly collapsed from a wave to a particle at $t = t_2$.

3.3 Critique of Inference 2

From the point of view of logic (and in particular, the NAFL interpretation), we wish to establish that *the presence of an interference pattern, such as, that observed in Young's two-slit experiment, does* ***not prove*** *the 'reality' of the wave nature of a single photon.*

Let us first consider the single-photon version of Young's two-slit experiment. Let the wire grid at location σ_1 in Fig. 1 be replaced by an electronic screen, capable of registering and storing the arrival of single photons. The photons reach the slits one at a time, with equal probability of passing through either slit. In the standard quantum formalism, a photon is *assumed* to take all available paths to any particular spot on the screen at which it ends up, *i.e.*, the photon is assumed to pass through both slits and 'interfere with itself', in order to theoretically predict the interference fringes. Since these predictions agree with the experimental observations, the 'reality' of the wave nature of the photon is concluded. At the outset, let us grant that the interference fringes are indeed observed even in the single-photon case, even though doubts have been expressed in this regard when the rate of emission of photons is sufficiently small ([8]; see the paragraph "If quanta are to be treated as real particles, self-interference must be ruled out. . . . Nevertheless, this still means that the evidence in favour of self-interference is inconclusive."). Firstly, note that when a single photon is fired at the slits, it also ends up as a single, bright spot at a specific, unpredictable location on the screen; *it does not exhibit an interference pattern on the screen.* This fact conclusively establishes the grainy nature of the photon as it interacts with the screen. The interference pattern, which is observed to build up over time only after many photons have landed on the screen, has to be interpreted as reflecting a probability distribution, with the probability density function (PDF) proportional to some power of the intensity of the fringes. In particular, at the dark fringes of the screen, the photon has a vanishing PDF. Note that the PDF for the photon is normalized over the entire area of the σ_1 plane in which the screen is located.

Remark 1. A zero value of the probability density function (PDF) for the photon, at a point or a line located on a dark fringe (minimum) of the interference pattern on the screen in Young's two-slit experiment, does *not* imply a proof in the theory QM that a given photon cannot land at that point or line and be detected.

Remark 1 seems to follow even in the standard formalism for QM. Note that the probability (as opposed to the PDF) of the photon reaching *any* given point or line on the screen is *exactly* zero; but obviously this does not constitute a *proof* that the photon cannot be detected at that point or line. Points/lines on which the PDF vanishes, such as, the dark fringes in the two-slit experiment, are not special in this regard. An *arbitrarily* small area around such a point or line will still have a non-zero probability of recording a photon, as long as the PDF does not vanish identically in the entire area. So there does not appear to be any basis for excluding the possibility that a photon can land at the minima of the fringes, although one could expect that such an eventuality is unlikely in the real world. But one should not confuse 'statistical expectation' with 'proof'. To be sure, the observed intensity at the dark fringes is zero, and zero intensity means zero photon count rate. But remember that we are asking if a *single, given* photon can land at the dark fringes; 'count rate' and 'intensity' and even 'probability' are not really well-defined for this process. For example, a very large number of photons (N) could be fired at the slits and $N-1$ of these could conform precisely to the expected probability distribution; if the remaining photon ends up at a dark fringe, that does not constitute a violation of any law of QM. For as N is increased and no further deviations are recorded, the probability distribution will asymptotically conform to the theoretical pattern.

At this stage one might advance the argument that a zero PDF at a point X on the screen means that the photon passed through both slits and interfered destructively with itself at X, making it impossible for the photon to reach X. This argument requires the *assumption* that the photon is 'really' a wave, from the slits all the way up to the screen, with the wavefunction 'collapsing' at the screen in order to enable the detection of the photon as a particle. But such an assumption is at best an *axiom* that is *not* provable in QM. The assertion that a photon that is fired at the slits as a particle mysteriously tranforms itself into a wave, passes through both slits, and equally mysteriously ends up as a particle at the screen, has no credibility when interpreted as a 'reality'. In any case the axiomatic nature of this assertion with respect to QM means that Remark 1 still holds. For there is no proof that the same observed probability distribution cannot be arrived at by other, less mysterious routes. In summary, if one interprets the interference pattern at the screen strictly as reflecting a probability distribution for particle-like photons, and if one views the quantum formalism as merely one possible algorithm for deriving it, one cannot infer any 'reality' for the wave nature of the photon, as demonstrated by Remark 1. Young's two-slit experiment, by itself, does *not prove* the wave nature of the photon. One can at best assert that no photon has so far been experimentally detected at the theoretical minima of the interference pattern. But that argument does not constitute a proof of the theoretical impossibility of such a detection in *all* future experiments, or for all future times in a given experiment, which would be required to establish the wave nature of the photon. Indeed, such an infinitary proposition can never be 'proven' experimentally and *must* be axiomatic with respect to QM. Of course, in the event of such an axiomatic assertion being added to QM, the wave nature of the photon (which can then be inferred) must be formulated as a classically possible phenomenon rather than as a contradiction of the form $P \& \neg P$; as observed in Sec. 3.2, this will require a modification of Definition 3, and consequently, NAFL will require the violation of BCP (Definition 1) via a nonclassical model $\mathcal{N}$ in which the photon is neither a particle nor a wave. We do not believe that this is the correct approach, not only because of the mystery associated with wavefunction collapse, but also in the light of Inference 1 in Afshar's experiment. Inference 2 must be sacrificed, as will be argued below.

Next consider the NAFL interpretation, with Definiton 3 in place. Let X be a point located on a dark fringe in Young's two-slit experiment. To 'prove' that a photon leaving the slit(s) can never reach X, the standard QM formalism proceeds as follows.

1. Assume that the photon reaches X on the electronic screen.
2. Assume P, assume $\neg P$; $P \& \neg P$ follows. The photon takes all available paths to the point X.

3. Associate a probability wave with the photon, in accordance with the standard formalism.

4. Conclude destructive interference at X. The photon has a zero probability density function at X.

5. Conclude from step 4 that step 1 is false, and the photon can never reach X on the electronic screen.

We already questioned step 5 in the preceding analysis; a vanishing PDF does not imply proof of impossibility. Even if step 5 is granted, the above proof is *not* valid in the NAFL version of QM, whose rules of inference are classical. For step 2 *asssumes* a contradiction of the the form $P\&\neg P$. Classically, *any* proposition can be inferred from a contradiction, and a proof based on such an inference has no validity. Therefore the conclusion (step 5) cannot be established as a theorem of QM based on the above 'proof', either in NAFL or in classical logic. Indeed, the wave nature of the photon is now classically impossible to prove as a theorem of QM, since it has been formulated as a contradiction. This holds in NAFL as well; a proposition of the form $P\&\neg P$ is not even legitimate in the theory syntax of the NAFL theory QM (see Sec. 2.4), and so cannot be a theorem. Further, $P\&\neg P$ cannot be added as an axiom to QM, say, at time t_1, for the purpose of obtaining an interpretation QM* that retroactively asserts the wave nature of the photon, in the same manner that Q was added at time t_2; as noted in Sec. 2.4, only propositions that are legitimate in the theory syntax of QM can be so added. *However*, it follows from Proposition 1 that consistency of QM requires the existence of a nonclassical model $\mathcal{M}$ for QM in which $P\&\neg P$ is nonclassically true. The nonclassical interpretation of $P\&\neg P$ in $\mathcal{M}$ was extensively discussed in Secs. 3.1 and 3.2. The model theory TM for $\mathcal{M}$ must be based on a paraconsistent logic (see Sec. 2.1), for TM must prove $P\&\neg P$. In such a paraconsistent logic, the classical result that *any* proposition follows from a contradiction does not hold. Therefore steps 1-4 in the above proof can be validly formulated in TM. Rather than asserting that the photon took all available paths to the dark spot X on the screen and destructively interfered with itself, we may simply conclude that the photon did not take any path to X (and indeed, is very unlikely to do so, via the proof in TM); hence the dark spot. Thus we have a setting in which the paraconsistent theory TM can, in principle, justify the standard quantum formalism in a nonclassical NAFL model $\mathcal{M}$ for QM. But note that the theorems of QM, as well as those of its interpretation QM* (which generates $\mathcal{M}$), must also be theorems of TM; the contradictions provable in TM must involve only undecidable propositions of QM*, such as, P. In particular, since infinite sets cannot exist in NAFL theories [7], including QM, it follows that TM cannot permit these either. One must develop real analysis in NAFL without infinite sets [7], so that one can justify the quantum formalism in TM. One might ask, why bother with NAFL at all? Why not just use a paraconsistent logic to begin with, and just develop the theory TM? The problem is that in such an eventuality, we do not have any logical principles by which we determine which are the contradictions that should be provable in TM. For example, there would be no particular reason for BCP to hold; without the guiding principles of NAFL, a paraconsistent logic might as well prove the contradiction that the photon is both a wave and a particle at the same time. Thus we would be reduced to using our arbitrary intuitions rather than the principles of logic in determining what should and should not be provable. One may think of TM as essentially a set of metamathematical rules that tell us how to combine various classical models of QM so as to generate the desired nonclassical model $\mathcal{M}$ that conforms with the observer's interpretation QM* (which in turn conforms with his observations/axiomatic declarations in the real world). Note that TM tries to *predict* the results of the 'measurements' made by the observer in the real world, via probabilistic reasoning. TM essentially tells us what QM* will probably look like if the observer keeps his axiomatic declarations in QM* in tune with his (statistically large number of) observations in the real world.

In summary, our thesis is that the observed interference pattern at the screen in Young's two-slit experiment is a manifestation of a *particular* nonclassical NAFL model $\mathcal{M}$ for QM in which the count rate of the photon at each point of the screen is in tune with the nonclassical probability distribution derived in the theory TM of $\mathcal{M}$. There could be other nonclassical NAFL models for QM that follow different probability distributions, but these may not be relevant to our real world. The important achievement here is that we have conceptually justified the use of $P \& \neg P$ in deducing the interference pattern *without conceding any physical reality* for the wave nature of the photon. As noted earlier, $P \& \neg P$ in $\mathcal{M}$ merely reflects the observer's ignorance of the path information of the photon and is used *only* for the purpose of deriving the probability distribution; the contradictions in the paraconsistent structure $\mathcal{M}$ have no 'physical' reality. To the various bright spots on the screen, each individual photon 'really' takes only one path. The only mystery here is that this 'reality' is not revealed via a classical probability distribution when a large number of photons are fired at the screen; instead, the resulting interference pattern revealed by Nature seems to indirectly confirm, via a nonclassical probability distribution, the observer's ignorance of the path information. Why does Nature act in this manner? The answer may have something to do with the paradoxical nature of probability, which is, even classically, a problematic notion that is dependent on the information available to the observer (hence, 'conditional' probabilities). While the photon passes through only one slit at a time as a particle, the status of the other inactive slit (*i.e.*, whether it is open or closed) seems to impose some sort of conditionality on the probability distribution. Perhaps formulation of real analysis in NAFL, development of consistent postulates for QM, and then developing the paraconsistent theory TM of $\mathcal{M}$ will shed further light on this mystery. Many of the concepts of standard quantum theory, such as, the notion of probability, will not be formalizable in the NAFL theory QM and will have to be dealt with in TM, and in general, at a metamathematical level. This is understandable, since these concepts may be specific to the real world and the model $\mathcal{M}$ which applies to it. It is also possible that radically new concepts of space, time, and in particular, light, may be needed for a more satisfactory description of Nature in an NAFL theory that does away with probability altogether.

Let us now revert to Afshar's experiment. Afshar [1] argues that if a classical probability distribution had applied, the wire grid at location σ_1 in Fig. 1 should have blocked about 6.6% of the photons. Instead, the wire grid blocked fewer than 0.1% of the photons, as expected from the probability distribution calculated using the standard quantum formalism. Let us grant Afshar's argument and concede that the interference pattern does indeed exist. As argued above, this does not legitimize Inference 2. In the light of the observation of Q and Inference 1, one concludes that the photon still has a metalogical particle nature for $t_0 \leq t < t_2$, but with a nonclassical probability distribution corresponding to the interference pattern, as deducible in a nonclassical model $\mathcal{M}$ for QM. In particular, there was no 'destructive interference' at the dark fringes corresponding to the locations of the wires; the photons that ended up at the images simply missed the wires and the dark fringes.

Note that the axiomatic nature of NAFL truth allows the observer to *axiomatically declare*, via a choice of QM*=QM+P at time t_0 in Fig. 1, that a given photon passed through slit U, even though such a *measurement* was not made at $t = t_0$. Thus the observer breaks with the informal convention of keeping his axiomatic declarations in tune with his measurements, and generates a classical particle model of the photon for $t_0 \leq t < t_2$, instead of the nonclassical model $\mathcal{M}$. If at $t = t_2$, the photon is found to end up at the image U′, this would vindicate the observer's choice made through guesswork and free will. Thus in principle, NAFL allows the observer to bring the particle nature of the photon within the scope of its semantics, even though in practice, continued success on this front would require improbable guesswork on the part of the observer. Of course, consistency of QM requires $\mathcal{M}$ to exist (see Proposition 1), even if the observer does not choose it in any particular instance.

3.4 Critique of Kastner's argument

Kastner [2] has criticized the interpretation of Inference 1 as retroactively asserting the particle nature of the photon. She has essentially two reasons for her reservations, as stated below.

- According to Kastner, the photon exhibited wave nature at time t_0 in Fig. 1 in the sense that it 'really' passed through both slits, as confirmed by Inference 2 (which she supports with the caveat that the wave nature of the photon, being real, is not to be formulated as a contradiction of the form $P \& \neg P$). So, as was noted in Sec. 3.2, Kastner requires Inference 1 to be limited to the assertion that the photon was post-selected in the state of slit U, without any retroactive implication that the photon 'really' passed through only slit U (as a particle). In other words, Inference 1 does not provide 'which-way information' for the photon.

- Kastner cites Cramer's Transactional Interpretation (TI) of quantum mechanics [6] as unambiguously showing that the photon was selected in the superposed state of both slits at time t_0, and also was post-selected in the

> state of slit U at time t_2. However, there are intermediate times in Fig. 1, when the photon was between the locations σ_1 and σ_2, during which the ontological state of the photon is ambiguous according to TI (see Fig. 3 of [2]). Kastner states that at these locations, the 'offer wave' of TI shows the photon to be in a superpositon state of both slits, while the backwards-in-time 'confirmation wave' of TI shows the photon to be in a state of slit U. For this reason, Kastner asserts that it would be wrong to retroactively infer at time t_2 that the photon passed through only slit U, since such an inference would require the photon to be determinately a particle at all intermediate locations betwen σ and σ_2 in Fig. 1.

On the basis of the above criticisms, Kastner concludes that Afshar's experiment does not refute BCP (Definition 1).

We have exhaustively addressed the problems with the first of Kastner's criticisms. From the point of view of NAFL, Inference 1 is merely the observer's retroactive axiomatic declaration of the particle nature of the photon. But the concept of 'axiomatic declaration' itself cannot be formalized in the theory syntax of NAFL theories [3]. So when the observer 'post-selects' the photon as being in the 'state of slit U', he can *only* have in mind that the photon 'really' passed through slit U as a particle. Therefore, in NAFL, the classical model of QM that is generated at time t_2 via the observer's interpretation QM*=QM+Q, *must* reflect such a retroactive implication for Inference 1. In fact, the completeness theorem of first-order logic (which is a metamathematical principle in NAFL) *requires* that there *must* exist such a classical model for QM. For, as we have already pointed out, QM does not prove either $\neg P$ or Kastner's claim that the photon passed through both slits at time t_0. This latter claim, affirming the wave nature of the photon at time t_0, is at best another *metamathematical* requirement that Kastner chooses to impose. In the conflict between these two metamathematical requirements, the completeness theorem wins out in NAFL; it is a sacred principle of logic that cannot be sacrificed. On the other hand, the nonclassical NAFL model $\mathcal{M}$ of QM that affirms $P\&\neg P$ at time t_0 does not conflict with the above retroactive assertion of P, either temporally or in meaning, as we have already pointed out in Secs. 3.1 and 3.2. In particular, $\mathcal{M}$ only affirms the observer's lack of information on the path of the photon at time t_0, rather than Kastner's assertion that the photon 'really' passed through both slits.

The second of Kastner's objections is also problematic, in the sense that her stated purpose of rescuing BCP is not served by her invocation of Cramer's TI. If TI requires that the photon's ontological state is ambiguous between the locations σ_1 and σ_2 in Fig. 1, that amounts to a violation of BCP when the photon was at these locations. For we have determined, using TI, that the photon was neither a particle nor a wave when it was at these locations. So the NAFL model of QM that applies at these times would have to be a superposition of both the particle and wave states of the photon, *i.e.*, a superposition of a classical and a nonclassical model of QM. But such a superposition of models does not exist in NAFL and clearly violates BCP; see Definition 1 and also the NAFL version of

BCP as stated at the end of Sec. 3.2. This is not surprising, for TI requires us to ascribe reality to the superposed state of the photon passing through both slits; this is also Kastner's belief, as noted above. As a consequence, NAFL would *require* that Definition 3 be modified to force, via Proposition 1, the existence of a nonclassical model $\mathcal{N}$ for QM that violates BCP (as was pointed out in Sec. 3.2).

3.5 Critique of Cramer's TI in the delayed-choice scenario

One example of a quantum mystery arises from the well-known delayed choice experiment of Wheeler [9]. In Fig. 1, *after* a photon passes through the slit(s) in the σ plane at time t_0, the observer could either choose to insert a screen in the σ_1 plane, *or* he could choose to allow the photon to pass through the lens and reach the σ_2 plane. So according to Wheeler, this delayed choice of measurement means that the observer could post-select (*i.e., after* time t_0) each photon to pass through both slits, as a wave, or one slit only, as a particle. But this amounts to choosing one's past after the event and seems paradoxical. In the NAFL interpretation, the metalogical reality of the photons as particles means that the delayed choice does *not* influence the past; each photon always passed through one and only one slit. The interference pattern, which one sees on the screen after many photons are recorded on it, *also* exists (but is not measured) for the photons that end up at the σ_2 plane, as is spectacularly confirmed in Afshar's experiment.

Cramer's Transactional Interpretation (TI) [6], on the other hand, provides an explanation for delayed choice by positing an 'atemporal' transaction that takes place between 'offer waves' and 'confirmation waves', with the latter propogating backwards in time. From the point of view of NAFL, however, which rejects the relativistic conception of 'spacetime' (see Appendix B of [5]), such an atemporal transaction is not physically possible, and neither can anything propagate backwards in time. The temporal and axiomatic nature of NAFL truth requires absolute time, as well as Euclidean space. Cramer's aphysical approach to delayed choice really amounts to taking an anti-realist stand that the 'past' exists if and only if, and only *after*, it is clearly defined. This is confirmed by Cramer's assertion that "No offer is a transaction until it is a confirmed transaction", which corresponds to Wheeler's "No phenomenon is a phenomenon until it is an observed phenomenon". Such an anti-realist stand is possible in NAFL by associating 'measurement' or 'observation' with 'axiomatic declaration', and denying any metalogical reality outside of NAFL syntax and semantics (as was noted in Sec. 3.2). But in NAFL, the superposed state is not really a physical state of the photon passing through both slits, as is assumed by Cramer; the anti-realist stand, when imposed upon the NAFL interpretation, would further require that the photon has no determinate state in the real world when it is deemed to be in a quantum superposition of passing through both slits. Hence from the point of view of NAFL, Cramer's TI is not consistent with either realism or anti-realism. As was discussed in Sec. 3.2, positing a metalogical reality is not only compatible with NAFL, but is also philosophically a more satisfactory resolution of the paradoxes associated with Afshar's experiment. From this realist point of view, Cramer's TI, by positing waves propagating backwards in time, seems to accept that one *can* influence the past; this is denied in the NAFL interpretation as aphysical. In Sec. 3.4, we have criticized Kastner's [2] defence of BCP, using Cramer's TI, as logically problematic; the said defence upholds BCP at the slits by requiring its violation elsewhere, at least according to the NAFL interpretation.

4 The coin toss and Schrödinger cat experiments

Consider the coin toss experiment described in Sec. 2.5 of [4]. An observer tosses a fair coin and, as it lands at time t_0, covers the coin under the palm of his hand without seeing the outcome. Let P ($\neg P$) represent "The outcome is 'heads' ('tails')". Let the observer have the NAFL theory T in mind, which includes axioms describing this coin toss experiment. Further, at $t = t_2$, the observer lifts his hand and sees the outcome, say, 'heads'. For $t_0 \leq t < t_2$, the observer chooses the interpretation T*=T. Hence $P \& \neg P$ holds for the observer in a nonclassical model $\mathcal{T}$ for T, signifying that he has not measured (axiomatically declared) the outcome to be either 'heads' or 'tails' during this time interval. Let R denote the proposition that "The outcome is 'heads' for $t \geq t_2$". For $t \geq t_2$, the observer takes T*=T+R, in tune with his observation, so that P (which is provable in T* for these times) holds in a classical model for T. Let the proposition Q, formulated for $t \geq t_2$, denote "The outcome was 'heads' during $t_0 \leq t < t_2$". Our contention is that the theory T*=T+R proves Q, so that the observer has retroactively asserted at $t = t_2$ that the outcome was always 'heads' during $t_0 \leq t < t_2$.

Is the inference of Q in T+R legal? In this case, the observer can feel confident that the outcome 'heads' must be metalogically ('really') true for $t_0 \leq t < t_2$. The observer *knew* that the coin was flat under the palm of his hands during $t_0 \leq t < t_2$, but as was noted in Sec. 2.5 of [4], this fact, being provably equivalent to $P \vee \neg P$ in the proof syntax of T, *cannot* be formalized as a legal proposition in the theory syntax of T. Such an (illegal) formalization would force $P \vee \neg P$ to be a theorem of T, and prevent the existence of the nonclassical model $\mathcal{T}$ required by Proposition 1 for consistency of T in NAFL. The reason this example is interesting is that the observer *knows* that the superposed state of 'heads and tails' in $\mathcal{T}$ has no 'physical' reality, but nevertheless it correctly reflects the observer's ignorance of the outcome during $t_0 \leq t < t_2$. In Afshar's experiment, the situation is logically similar, but the observer does not have such a clear intuition for the particle nature of the photon, and consequently, for the validity of Inference 1. But nevertheless, NAFL treats both cases similarly and in a logically consistent manner.

Next reconsider the Schrödinger cat experiment described in Sec. 2.3. Once again the observer has the intuition that the retroactive assertion of U, via an inference in the theory QM*=QM+V, is true in the real world. This inference is based on the principle (deducible in the proof syntax of

QM*) that if one finds the cat to be alive at time t_2 and if one knows that the cat was alive at an earlier time t_0, then the cat was alive for $t_0 < t < t_2$. Thus at $t = t_2$, we *know* that the cat was (metalogically, 'really' and unambiguously) alive during $t_0 < t < t_2$. NAFL supports such a rock-solid, unimpeachable inference that conforms with the standard definition of 'alive'.

5 Concluding Remarks

The NAFL interpretation upholds the Bohr Complementarity Principle (BCP) in Afshar's experiment [1] by retroactively affirming the 'real' particle status of the photon, even while *semantically* the photon was in a superposed state (S) in a nonclassical model $\mathcal{M}$ of the NAFL theory QM formalizing quantum mechanics. However, such a 'reality' for the particle state, which is outside of both the syntax and the semantics of QM, can be said to be *metalogical.* In NAFL, S (or the 'wave nature' of the photon) only reflects the fact that the observer has not measured (axiomatically asserted) the true, classical path of the photon; no *physical* reality, to the effect that the photon 'really passed though both slits and interfered with itself', can be assigned to S. The interference pattern in the σ_1 plane of Fig. 1 must be interpreted in NAFL as reflecting a nonclassical probability distribution for the photons, still treated as particles, that is derivable within $\mathcal{M}$. We have no explanation yet for *why* the lack of which-way information influences the probability distribution in this manner.

The still nascent NAFL interpretation of quantum mechanics has enormous potential for future research and seems to be absolutely indispensable for a satisfactory explanation of the puzzling consequences of Afshar's experiment. The subtle formulation of the syntax and semantics of NAFL theories to combine both classical and intuitionistic principles, the ability of NAFL to handle the temporal nature of mathematical truth, and the demonstration of the need for a paraconsistent logic to handle the model theory of NAFL theories are features that make the NAFL interpretation spectacularly suitable for the purpose of explaining the mysteries of quantum mechanics. This, despite the limitations that NAFL imposes on classical infinitary reasoning [3, 4, 5, 7]. In general, NAFL can be said to provide a logical basis for many of Niels Bohr's great ideas that were spelt out in the Copenhagen Interpretation, while differing from it in significant ways.

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