A linear algorithm for embedding of cycles in crossed cubes with edge-pancyclic *

Chang-Hsiung Tsai† and Chia-Jui Lai
Department of Computer Science and Information Engineering
National Dong Hwa University
Shoufeng, Hualien, Taiwan 97401, R.O.C.
E-mail: chtsai@mail.ndhu.edu.tw; lai@ms01.dahan.edu.tw

Abstract
In this paper, we consider the problem of embedding a cycle containing number of nodes from 4 to $2^n$ through a prescribed edge in an $n$-dimensional crossed cube $CQ_n$. The main contribution of this paper is providing a systematic $O(l)$ algorithm to find a cycle of length $l$ containing $(u, v)$ in $CQ_n$ for any $(u, v) \in E(CQ_n)$ and any integer $l$ with $4 \leq l \leq 2^n$.

Keywords: Interconnection network; Cycles embedding; Edge-pancyclic; Embedding algorithm; Crossed cubes;

1 Introduction

A cycle structure is a fundamental network for multiprocessor systems and suitable for developing simple algorithms with low communication cost. Many efficient algorithms were designed with respect to cycles for solving a variety of algebraic problems, graph problems, and some parallel applications, such as those in image and signal processing [2, 16]. To carry out a cycle-structure algorithm on a multiprocessor computer or a distributed system, the processes of the parallel algorithm need to be mapped to the nodes of the interconnection network in the system such that any

---

*This work was supported in part by the National Science Council of the Republic of China under Contract NSC 98-2115-M-259-002-MY2 and 100-2115-M-008.
†Corresponding authors: C.-H. Tsai (chtai@mail.ndhu.edu.tw);
two adjacent processes in the cycle are mapped to two adjacent nodes of the network. Besides, to execute a parallel program efficiently, the size of the allocated cycle must accord to the problem size. Thus, many researchers focus their studies on how to embed cycles of different sizes into an interconnection network. In distributed systems, each node and each edge may be assigned with distinct resource and distinct bandwidth, respectively. For this purpose, it is meaningful to study the problem of how to embed cycles into a network such that these cycles pass through a special node/edge.

The crossed cube proposed by Efe [6] is one of the most notable variations of hypercube, but some properties of the former are superior to those of the latter. For example, the diameter of the crossed cube is almost the half of that of the hypercube. With regard to cycles embedding of crossed cubes, many interesting results have received considerable attention [1, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20].

Finding cycles of arbitrary length passing through any given edge in crossed cubes has attracted special attention from researchers. In this paper, we consider the problem of embedding a cycle of arbitrary numbers of nodes that passes through a given edge on the crossed cube. A concept, cycle pattern, is in use to construct an efficient algorithm for embedding a desired cycle with arbitrary given edge into the crossed cube. Furthermore, the algorithm is linear with respect to numbers of nodes in the desired cycle that would be found.

The rest of this paper is organized as follows. Section 2 introduces definitions and reflected edge label sequence that will be used throughout this paper. In Section 3, we propose cycle pattern concept and determine how to construct a cycle pattern of any length. Finally, we propose a linear time algorithm to construct the desired cycle. Conclusions are given in the final section.
2 Preliminaries

A topology of an interconnection network is conveniently represented by an undirected simple graph $G = (V, E)$, where $V(G)$ and $E(G)$ is the vertex set and the edge set of $G$, respectively. Throughout this paper, vertex and node, edge and link, graph and network are used interchangeably. For graph terminology and notation not defined here we refer the reader to [16]. A walk in a graph is a finite sequence $ω : λ_0, e_1, λ_1, e_2, λ_2, . . . , λ_{k−1}, e_k, λ_k$ whose terms are alternately vertices and edges such that, for $1 ≤ i ≤ k$, the edge $e_i$ has ends $λ_{i−1}$ and $λ_i$, thus each edge $e_i$ is immediately preceded and succeeded by the two vertices with which it is incident. In particular, a walk $ω$ is called a path if all internal vertices, $λ_i$ for $1 ≤ i ≤ k − 1$, of the walk $ω$ are distinct. The first vertex $λ_0$ of $ω$ is called its start vertex, and the vertex $λ_k$ is called a last vertex. Both of them are called end-vertices of the path $ω$. For simplicity, the path $ω$ is also denoted by $λ_0, λ_1, . . . , λ_k$. If $λ_0 = λ_k$, then $ω$ is called a cycle. A cycle of length $l$ is called an $l$-cycle.

An $n$-dimensional crossed cube, denoted as $CQ_n$, was first proposed by Efe [6]. It is derived by “crossing” some edges in hypercube. With exactly same hardware cost as hypercube, it has been shown that such a simple variation gains important benefits such as greatly reduced diameter. To define crossed cubes, a relation $R = \{(00, 00), (10, 10), (01, 11), (11, 01)\}$ is introduced. Subsequently, a crossed cube of dimension $n$ is an undirected graph consisting of $2^n$ vertices labeled from 0 to $2^n − 1$ and defined recursively as following:

**Definition 1** [6] The crossed cube $CQ_1$ is a complete graph with two vertices labeled by 0 and 1, respectively. For $n ≥ 2$, an $n$-dimensional crossed cube $CQ_n$ consists of two $(n − 1)$-dimensional sub-crossed cubes, $CQ^0_{n−1}$ and $CQ^1_{n−1}$, and a perfect matching between the vertices of $CQ^0_{n−1}$ and $CQ^1_{n−1}$ according to the following rule:
Let $V(CQ_{n-1}^0) = \{0u_{n-2}u_{n-3} \cdots u_0 : u_i = 0 \text{ or } 1\}$ and $V(CQ_{n-1}^1) = \{1v_{n-2}v_{n-3} \cdots v_0 : v_i = 0 \text{ or } 1\}$. The vertex $u = 0u_{n-2}u_{n-3} \cdots u_0 \in V(CQ_{n-1}^0)$ and the vertex $v = 1v_{n-2}v_{n-3} \cdots v_0 \in V(CQ_{n-1}^1)$ are adjacent in $CQ_n$ if and only if

1. $u_{n-2} = v_{n-2}$ if $n$ is even, and
2. $(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in R$, for $0 \leq i < \lfloor \frac{n-1}{2} \rfloor$.

An edge $(u, v) \in E(CQ_n)$ is labeled by $j$ if $u_j \neq v_j$ and $u_i = v_i$ for $j + 1 \leq i \leq n - 1$, i.e., $v$ is the $j$-th dimensional neighbor (abbreviated as $j$-neighbor) of $u$. Therefore, the edge $(u, v)$ labeled $j$ might be denoted by $u[j]v$ or $v[j]u$. It is observed that each vertex $u$ in $CQ_n$ has $n$ neighbors in $CQ_n$; $u$ has exactly one $j$-neighbor for $0 \leq j \leq n - 1$. As a consequence, there are $2^{n-1}$ edges labeled by $j$, $0 \leq j \leq n - 1$, in $CQ_n$. Figure 1 is an illustration of $CQ_3$ and $CQ_4$, respectively.

A path in $CQ_n$ might be specified by the source vertex and a sequence of labels detailing the edges to be traversed, for example, the path in $CQ_3$ detailed as having the source vertex 000 and then following the edges labeled 1,2,1 (also denoted [1,2,1]) is actually the path 000, 010, 110, 100, also denoted 000[1,2,1]100. Therefore, the sequence $L = [d_1, d_2, \ldots, d_m]$ is called an edge label.
sequence in $CQ_n$ if two adjacent labels are not identical where $d_i \in Z_n$, $Z_n = \{0, 1, \ldots, n - 1\}$, for $1 \leq i \leq m$.

A walk, $\omega(L, u) = \lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_m$, in $CQ_n$ can be generated with respect to a given edge label sequence $L$ and a given vertex $u$ as follows: $\lambda_0 = u$, and $\lambda_j$ is the $d_j$-neighbor of $\lambda_{j-1}$ in $CQ_n$ where $1 \leq j \leq m$, i.e., $\lambda_{j-1}[d_j] \lambda_j$. Thus, this walk $\omega(L, u)$ is also represented as $\lambda_0[L] \lambda_m$.

Hereafter, we are interested in a special edge label sequence called reflected edge label sequence generated by a systematic method. A reflected edge label sequence of length $2^k - 1$ is generated from a permutation with $k$ elements over $Z_n$. Let $\pi_k = \langle d_1, d_2, \ldots, d_k \rangle$, $1 \leq k \leq n$, be a permutation over $Z_n$ with $k$ elements, and let $\pi_k(i) = \langle d_1, d_2, \ldots, d_i \rangle$. The reflected edge label sequence, $RL_{\pi_k}$ defined by $\pi_k$, be generated recursively as follows:

$$
\begin{align*}
RL_{\pi_k}(1) &= d_1 \\
RL_{\pi_k}(i) &= RL_{\pi_k}(i-1), d_i, RL_{\pi_k}(i-1), 2 \leq i \leq k; \text{ and} \\
RL_{\pi_k} &= RL_{\pi_k}(k)
\end{align*}
$$

**Lemma 1** [20] For any vertex $u$ in $CQ_n$ and any $\pi_n$ permutation over $Z_n$ with $n$ elements, the walk $\omega(RL_{\pi_n}, u)$ corresponds to a Hamiltonian path of $CQ_n$ that start from $u$.

Subsequently, we obtain immediately the following corollary.

**Corollary 1** For any vertex $u$ in $CQ_n$ and any permutation $\pi_k$ over $Z_n$ with $k$ elements, the walk $\omega(RL_{\pi_k}, u)$ corresponds to a path of length $2^k - 1$ that start from $u$ in $CQ_n$.

3 Cycle patterns

This section is devoted to auxiliary results that will be used later to verify certain steps of our algorithm. Let $L$ be an edge label sequence of $CQ_n$ with $l$ elements. We called $L$ an $l$-cycle pattern, $l$-CP for short, of $CQ_n$ if the walk $\omega(L, u)$ forms an $l$-cycle for every vertex $u$. 
Particularly, appending $d_k$ to the end of the sequence $RL_{\pi_k}$, we obtain an edge label sequence $[RL_{\pi_k}, d_k]$ where $\pi_k = (d_1, d_2, \ldots, d_k)$. For convenience, we use $C_{\pi_k}$ to denote the special sequence $[RL_{\pi_k}, d_k]$ in the following; thus, $C_{\pi_k}$ contains $2^k$ edge labels for any permutation, $\pi_k$, with $k$ elements. We are interested in the special permutation, $\pi_k$ for $2 \leq k \leq n$, with $k$ elements over $\mathbb{Z}_n$ satisfies that $C_{\pi_k}$ is a $2^k$-CP of $CQ_n$. Consequently, the following lemma gives the necessary and sufficient conditions for determining $\pi_k$ such that $C_{\pi_k}$ is a $2^k$-CP of $CQ_n$.

**Lemma 2** [17] For $n \geq 3$ and $2 \leq k \leq n$, let $\pi_k = (d_1, d_2, \ldots, d_k)$ be a permutation over on $\mathbb{Z}_n$. Then, $C_{\pi_k}$ is a $2^k$-CP of $CQ_n$ if and only if

1. $\min\{d_{k-1}, d_k\}$ is odd, or
2. $|d_{k-1} - d_k| = 1$.

The following lemma proposes how to generate 5-, 6-, and 7-CPs in $CQ_n$.

**Lemma 3** Let $n \geq 3$ and $2 \leq d \leq n - 1$. Then,

1. $[1, 0, d, 0, d]$ is a 5-CP,
2. $[1, d, 0, 1, 0, d]$ is a 6-CP, and
3. $[1, 0, 1, d, 0, 1, d]$ is a 7-CP.

**Proof.** Let $n \geq 3$ and $2 \leq d \leq n - 1$. For any vertex $u$ in $CQ_n$, $u$ is denoted by $u_{n-1}u_{n-2}\ldots u_d\ldots u_1u_0$ where $u_i = 0, 1$ and $0 \leq i \leq n - 1$. Let $S = \{00, 01, 10, 11\}$ and $f_R : S \rightarrow S$ be a bijection function defined by that for $x \in S$, $(x, f_R(x))$ belongs to the relation $R$. It is observed that $f_R(f_R(x)) = x$ for $x \in S$. 

6
The path $\omega([1, 0, d, 0, d], u)$ is denoted by $u[1, 0][d][w][0][z][d]v$. Let $x_1 = u_1 \oplus u_0$ and $x_0 = u_0$ where $\oplus$ is an exclusive or operation. Without loss of generality, we may assume that $d$ is an even integer. By Table 1, we have that $f_R(x_1 x_0) = u_1 u_0$ (see Table 1), $u = v$. Hence $u[1, 0, d, 0, d]v$ is a 5-cycle for any vertex $u$ in $CQ_n$. Therefore, $[1, 0, d, 0, d]$ is a 5-CP in $CQ_n$. Using similar methods, (2) and (3) can be verified.

Consequently, the following lemma provides an approach to establish a longer cycle pattern by combining two small cycle patterns.

**Lemma 4** For $n \geq 3$, let $L_1 = [d_1, d_2, \ldots, d_{k-1}, d_k]$ and $L_2 = [d'_1, d'_2, \ldots, d'_{l-1}, d'_l]$ be a $k$-CP and an $l$-CP in $CQ_n$, respectively where $d_k = d'_l$. Then, the edge label sequence $[d_1, d_2, \ldots, d_{k-1}, g, d'_1, d'_2, \ldots, d'_{l-1}, g]$ is a $(k + l)$-CP if $[d_k, g, d_k, g]$ is a 4-CP and $g > \max_{1 \leq i \leq k, 1 \leq j \leq l} \{d_i, d'_j\}$.

**Proof.** Let $u$ be any vertex of $CQ_n$. Assume that $d_k = d'_l = d$. Hence $u[d_1, d_2, \ldots, d_{k-1}]v[d]u$ is a $k$-cycle in $CQ_n$. Let $g > \max_{1 \leq i \leq k, 1 \leq j \leq l} \{d_i, d'_j\}$ and $w$ be the $g$-th neighbor of $v$ in $CQ_n$. Since $[d'_1, d'_2, \ldots, d'_{l-1}, d]$ is an $l$-CP of $CQ_n$, $w[d'_1, d'_2, \ldots, d'_{l-1}]z[d]w$ is an $l$-cycle of $CQ_n$. It is observed that the $g$-th bit position of each vertex in the path $u[d_1, d_2, \ldots, d_{k-1}]v$ is equal to $u_g$.
the $g$-th bit position of each vertex in the path $w[d'_1, d'_2, \ldots, d'_{l-1}]z$ is equal to $u_g$. Thus, these two paths are disjoint. Since $C_{(d,g)}$ is a 4-CP of $CQ_n$, $u[d]v[g]w[d]z[g]u$ is a 4-cycle. Therefore, $u[d_1, d_2, \ldots, d_{k-1}]v[g]w[d'_1, d'_2, \ldots, d'_{l-1}]z[g]u$ forms a $(k + l)$-cycle. \hfill \Box

For $2 \leq j \leq m$, let $\tau(m,j) = (m-j, m-j+1, \ldots, m-1)$ be a permutation over $Z_n$. By Lemma 2, $C_{\tau(m,j)}$ is a $2^j$-CP in $CQ_n$. Since $C_{\tau(m-1,j)} = [RL_{\tau(m-1,j)}, m-2]$ and by Lemma 4, one can combine $C_{\tau(m-1,j)}$ and $C_{\tau(m-1,k)}$ to a $(2^j + 2^k)$-CP such as $[RL_{\tau(m-1,j)}, m-1, RL_{\tau(m-1,k)}, m-1]$, for any $2 \leq j, k \leq m - 1$.

**Lemma 5** For $n \geq 4$, let $n > m \geq m_1 > m_2 > \cdots > m_k > 1$ where $2 \leq k \leq n - 2$. Then, the edge label sequence $[RL_{\tau(m,m_1)}, m, RL_{\tau(m-1,m_2)}, m-1, RL_{\tau(m-2,m_3)}, m-2, \ldots, RL_{\tau(m-k+3,m_k-2)}, m-k+3, RL_{\tau(m-k+2,m_k-1)}, m-k+2, RL_{\tau(m-k+2,m_k)}, m]$ is an $s$-CP in $CQ_n$, where $s = \sum_{i=1}^{k} 2^{m_i}$.

**Proof.** For $n \geq 4$, let $n > m \geq m_1 > m_2 > \cdots > m_k > 1$ where $2 \leq k \leq n - 2$ and $L = [RL_{\tau(m,m_1)}, m, RL_{\tau(m-1,m_2)}, m-1, RL_{\tau(m-2,m_3)}, m-2, \ldots, RL_{\tau(m-k+3,m_k-2)}, m-k+3, RL_{\tau(m-k+2,m_k-1)}, m-k+2, RL_{\tau(m-k+2,m_k)}]$. It is claimed that for any vertex $u \in V(CQ_n)$, (1) $u[L]v$ is a path of length $\sum_{i=1}^{k} 2^{m_i} - 1$ in $CQ_n$ and (2) $(u, v)$ is an edge of $CQ_n$ with label $m$.

It is observed that the walk $u[L]v$ is divided into $k$ subpath such as $P_1 = u^1[RL_{\tau(m,m_1)}]v^1, P_2 = u^2[RL_{\tau(m-1,m_2)}]v^2, \ldots, P_{k-1} = u^{k-1}[RL_{\tau(m-k+2,m_k-1)}]v^{k-1}$, and $P_k = u^k[RL_{\tau(m-k+2,m_k)}]v^k$ where $u^1 = u, v^k = v$, and $(v^i, u^{i+1})$ is an edge with label $m - i + 1$ for $1 \leq i \leq k - 1$. Suppose that $x$ and $y$ are two distinct vertices in the walk $u[L]v$ and $x \in V(P_i)$ and $y \in V(P_j)$.

Without loss of generality, we may assume that $i \leq j$. By Corollary 1, it is obvious that $x \neq y$ if $i = j$. Suppose that $i < j$. It is observed that $m - i + 1$ is the maximal label in the edge label sequence from vertex $x$ to $y$ and appearance once. Hence $x_{m-i+1} \neq y_{m-i+1}$. Therefore, $x \neq y$ and
Similarly, a \( (u, v) \) is an edge with label \( m \). Since \( \tau(l, j) = \langle l - j, l - j + 1, \ldots, l - 1 \rangle \) be a permutation over \( Z_n \) and by Corollary 1, the two end-vertices \( w \) and \( z \) of the path \( w[RL_{\tau(l, j)}]z \) is an edge with label \( l - 1 \). Hence \( (u^i, v^i) \) is an edge with label \( m - i \) (or \( m - k + 1 \), resp.) for \( 1 \leq i \leq k - 1 \) and \( (v^i, u^{i+1}) \) is an edge with label \( m - i + 1 \) for \( 1 \leq i \leq k - 1 \). This implies that
\[
u^1[m-1]v^1[m]u^2[m-2]v^2 \ldots u^i[m-i]v^i[m-i+1] \ldots u^{k-1}[m-k+1]v^{k-1}[m-k+2]u^k[m-k+1]v^k \text{ is a path. Since } [m-i, m-i+1, m-i, m-i+1] \text{ is a 4-CP for } 1 \leq i \leq k - 1,
\]
u^i \text{ and } v^k \text{ are adjacent by an edge with label } m - i + 1. \text{ It is implied that } (u^1, v^k), \text{ i.e. } (u, v), \text{ is an edge with label } m. \text{ Therefore, } u[L]v[m]u \text{ is a cycle of length } \sum_{i=1}^k 2^{m_i}.
\]

For example, suppose one will construct an 88-CP in \( CQ_7 \). Combining one \( C_{(0,1,2,3,4,5)} \), one \( C_{(1,2,3,4)} \), and one \( C_{(2,3,4)} \), an 88-CP is generated as \([RL(0,1,2,3,4,5), 6, RL(1,2,3,4), 5, RL(2,3,4), 6]\). Similarly, a \((2^m + 2^j + l_a)\)-CP can be established by combining \( C_{\tau(m,m)} \), \( C_{\tau(m-1,j)} \) and \( L_a \) satisfying \( d = m - 2 \) in Lemma 3 formed as \([RL_{\tau(m,m)}, m, RL_{\tau(m-1,j)}, m - 1, L_a, m]\) where \( j < m \).

**Lemma 6** For \( n \geq 4 \), let \( n > m \geq m_1 > m_2 > \ldots > m_k > 1 \) where \( 1 \leq k \leq n - 2 \). Then, the edge label sequence \([RL_{\tau(m,m_1)}, m, RL_{\tau(m-1,m_2)}, m - 1, RL_{\tau(m-2,m_3)}, m - 2, \ldots, RL_{\tau(m-k+2,m_{k-1})}, m - k + 2, RL_{\tau(m-k+1,m_k)}, m - k + 1, L_a, m]\) is an s-CP in \( CQ_n \), where \( s = \sum_{i=1}^k 2^{m_i} + l_a \) and \( L_a \) satisfies one of the following conditions,

1. \( L_a = [1,0,m-k,0] \) if \( l_a = 5 \).
2. \( L_a = [1,m-k,0,1,0] \) if \( l_a = 6 \).
3. \( L_a = [1,0,1,m-k,1,0] \) if \( l_a = 7 \).
Lemma 7 For \( n \geq 2 \), let \( L \) be an \( l \)-CP in \( CQ_n \). Then, for any edge \((u, v)\) with labeled \( d \) in \( CQ_n \), there exists a vertex \( z \) so the cycle \( \omega(L, z) \) passes the edge \((u, v)\) if \( d \) is appearance in \( L \).

Proof. Let \((u, v)\) be an edge labeled by \( d \) and \( L \) be an \( l \)-CP containing \( d \) in \( CQ_n \). Without loss
of generality, we may assume \( L = [d_1, d_2, \ldots, d_k, d, d_{k+2}, \ldots, d_l] \). Let \( L_1 = [d, d_k, d_{k-1}, \ldots, d_1] \) be an edge label sequence. Let \( z \) and \( u \) be two end-vertices of the path \( \omega(L_1, u) \). Clearly, \( \omega(L_1, u) \) can be represented by \( u[d]v[d_k, d_{k-1}, \ldots, d_1]z \). Since \( [d_1, d_2, \ldots, d_k, d] \) is a subsequence of \( L \) from the start, \((u, v)\) lies on the cycle \( \omega(L, z) \).

By above descriptions, we have the following theorem.

**Theorem 1** For \( n \geq 3 \), let \((u, v)\) be a \( d \)-dimensional edge of \( CQ_n \). Then, there exists an edge label sequence \( L \) of length from 4 to \( 2^n \) such that \( \omega(L, z) \) corresponds to a cycle passing through the edge \((u, v)\) in \( CQ_n \) for some \( z \in V(CQ_n) \).

**Proof.** For \( n \geq 3 \), let \((u, v)\) be an edge of \( CQ_n \) labeled \( d \). We will generate an \( l \)-CP containing \( d \) for \( 4 \leq l \leq 2^n \). By Lemma 2 and Lemma 3, the desired \( l \)-CP can be easily built when \( 4 \leq l \leq 8 \). For \( 9 \leq l \leq 2^n \), \( l \) can be represented by \( l_a + r \) where \( l_a = 2^{m_1} + 2^{m_2} + \cdots + 2^{m_k} \), \( m_1 > m_2 > \cdots > m_k > 1 \) and \( r = 4, 5, 6, 7 \). Let \( \pi = (d_1, d_2, \ldots, d_i, \ldots, d_{m_1-1}, d_{m_1}) \) where \( d_i = d \) for some \( i \), \( d_{m_1} = m \geq m_1 \), and \( \min\{d_{m_1-1}, d_{m_1}\} \) is odd or \( |d_{m_1-1} - d_{m_1}| = 1 \). By Lemma 2, \( C_\pi \) is a \( 2^{m_1} \)-CP containing \( d \).

One can build a \( (l - 2^{m_1}) \)-CP denoted by \( L_1 \) by applying Lemma 2 and Lemma 5 if \( l \) is a multiple of 4, otherwise applying Lemma 6. Subsequently, the desired \( l \)-CP, \( L \), is built by combining \( C_\pi \) and \( L_1 \). Since the cycle pattern \( L \) containing \( d \) and by Lemma 7, there exists a vertex \( z \) so the cycle \( \omega(L, z) \) passes the edge \((u, v)\).

It is not difficult to verify that the time complexity of the algorithm \texttt{genCycle}(e, l)\ is \( O(l) \). Therefore, the following theorem is obtained.
Algorithm 1 genCycle(e, l)

Input: An edge e = (u, v) labeled by d in CQ_n.
Output: An l-cycle passing through the edge (u, v) in CQ_n.

1: Applying Lemma 2, 3, 5, and 6, generate an l-CP, L, containing d.
2: Let \( L = [d_1, d_2, \ldots, d_j, d, d_j+2, \ldots, d_l] \).
3: Let u and z be two end-vertices of the path \( \omega(L_1, u) \), where \( L_1 = [d, d_j, d_{j-1}, \ldots, d_1] \).
4: \( C \leftarrow \{z\} \);
5: \( v \leftarrow z \);
6: for (each label i in L) do
7: \( x \) is the i-th neighbor of v;
8: \( C \leftarrow C \cup \{x\} \);
9: \( v \leftarrow x \);
10: end for
11: return C

Theorem 2 The time complexity of algorithm genCycle(e, l) is \( O(l) \) where \( 4 \leq l \leq 2^n \) and e is an edge in CQ_n for \( n \geq 2 \).

4 Concluding remarks

The crossed cube is one of most prominent variants of hypercube. Because crossed cubes are neither edge-symmetric nor vertex-symmetric, producing an l-cycle, \( 4 \leq l \leq 2^n \), to pass any prescribed edge in a crossed cube is more intricate of a process than in a regular hypercube. In this paper, we apply a systematic edge label sequence called reflected edge label sequence to build a simple algorithm to generate an l-cycle, \( 4 \leq l \leq 2^n \), such that passes through arbitrary given edge in CQ_n. Numerous variants of hypercube, for example, Möbius cubes, Twisted cubes, and Locally Twisted cubes, have been proposed and proved that there exists a cycle of every length passing through any given edge in them. Finding an algorithm to generate a desired cycle passing through arbitrary given edge in these variants of hypercube remain yet to be solved. The methods developed in this paper should be applied to finding a cycle of every length passing through any given edge in these networks.
References


