CAN THE INTERACTION BETWEEN BABY UNIVERSES GENERATE A BIG UNIVERSE?

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ABSTRACT

We explore a simple toy model of interacting universes to establish that a small baby universe could become large ($\gg$ Planck length) if a third quantization mechanism is taken into account.

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In quantum cosmology the Wheeler–de Witt (WDW) equation describes the dynamics of a single universe as a whole. Many solutions of the WDW equation that can be interpreted as wave functions of the universe are known in the literature but all seems unsuccessful in describing our universe; as in the classical framework, also in the quantum theory a cosmological constant causing the inflation of the Planck-scale baby universe solution of the WDW equation is required to explain the main properties of our universe (flatness, oldness...): in minisuperspace models this fact can be easily understood observing how the presence of a cosmological term modifies the harmonic potential in the WDW equation and allows the tunnelling from "nothing" to a de Sitter universe (see, for instance, [1]). In this paper we will establish how a solution describing a universe of size $\gg l_{Pl}$ becomes natural if the interaction with a quantum baby universes spacetime foam is taken into account. In the following we will discuss a simple toy model and hope that this paper will produce further discussions on this argument.

As we have mentioned, the WDW equation ($H\psi = 0$) describes the dynamics of a single universe [2,3] and it is essentially a zero energy Schrödinger equation in the superspace. Generally we can assume that a complete set of solutions do exist: we label this set by an index k. To discuss interactions between universes, we assume the existence of a field theory on superspace whose free field equation is given by the WDW equation. Therefore we form linear combinations of the creation and destruction operators of universes $c^\dagger$ and c:

$$\begin{aligned} \Psi(h) &= \sum_k \psi_k(h) c_k, \\ \Psi^\dagger(h) &= \sum_k \psi_k^\dagger(h) c_k^\dagger, \end{aligned} \tag{1}$$

where h represents the three-dimensional metric of the manifold mapping the superspace. Ψ and $\Psi^\dagger$ are field operators in the abstract occupation number Hilbert space and the operators c_k, $c_k^\dagger$ satisfy the boson commutation relations (the universes are bosons). The kinetic term of the field theory action takes the form [4,5]:

$$S = -\frac{1}{2} \int Dh \Psi^\dagger H \Psi. \tag{2}$$

Then it is not difficult to convince itself that the interactions between universes modify the WDW equation (tree level) with a potential term [4]:

$$H\Psi = -\frac{dV[\Psi]}{d\Psi}. \tag{3}$$

The idea is that an observer in a given universe could interpret the potential term as an effective term in the WDW equation of the universe where he lives. In this way it is possible that the wave function modified by the interactions could describe a universe similar to ours in a natural way without the presence of a cosmological constant. To construct our model we suppose to work in the minisuperspace, i.e. we consider only spatially homogeneus and isotropic closed universes; in this case the WDW equation reduces to a one-dimensional zero energy Schrödinger equation in the scale factor variable a. We suppose also that the matter contribution to the WDW equation separates from the gravitational contribution: this is not a very strange requirement because the conformal scalar field or the Yang-Mills radiation field satisfy this condition [6-8]. With these assumptions the WDW equation separates in the gravitational ($H_g(a)$) and matter ($H_M(\phi)$) parts and the wave function can be written (in the following we put the Planck length $l_{Pl} = 1$):

$$\psi_k(a, \phi) = \chi_k(a)\Phi(\phi, k) \tag{4}$$

where Φ represents the matter part of the wave function and χ_k is the k-th harmonic oscillator wave function ($H_g\chi_k = \epsilon_k\chi_k$); the energy ϵ_k is related to the energy density of the matter field ϕ. Then (4) represents a quantum closed spherically symmetric universe and for large values of the energy, i.e. for large quantum numbers ($k \approx 10^{120}$), (4) describes our universe [2,8]: as in the classical theory our universe appears very strange! In fact a typical solution (4) has small quantum numbers, i.e. dimensions of order of the Planck length. We suppose now the existence of a foam of baby universes (4) and a two body interaction $v(a, a')$ between universes in the minisuperspace. The gravitational part of the single universe quantum equation in the Hartree–Fock [9] approximation can be written:

$$\big(H_g + V_H\big)\tilde{\chi}_k = \tilde{\epsilon}_k\tilde{\chi}_k \tag{5}$$

where V_H is the Hartree–Fock potential

$$V_H = \int da' \sum_l \tilde{\chi}_l^2(a')v(a, a'). \tag{6}$$

In particular if we consider $N - 1$ universes in the foam ground state ($k = 0$) and one universe in an excited state with a small quantum number, the one-body equation for the excited universe becomes ($v(a, a') = -gv(a)\delta(a - a')$, where g is a positive coupling constant $O(1)$):

$$\frac{1}{2}\left[-\frac{d^2}{da^2} + a^2 - 2g(N-1)v(a)\tilde{\chi}_0^2\right]\tilde{\chi}_k = \tilde{\epsilon}_k\tilde{\chi}_k. \tag{7}$$

Choosing a potential $v(a) \approx \exp(a^2)$, (5) can be cast in the form of a Schrödinger equation for a harmonic oscillator. This potential can really arise when two universes are involved [6], being $\exp(-S)$ the generic process amplitude; the energy levels are:

$$\tilde{E}_k \approx \tilde{\epsilon}_k + g(N-1) = (k + 1/2). \tag{8}$$

In absence of interactions, $\tilde{\epsilon}_k = \epsilon_k$, then $0 < \tilde{\epsilon}_k \ll g(N-1)$ and we have:

$$k \approx g(N-1). \tag{9}$$

Therefore, supposing a sufficiently large number of interacting universes, we see that the solutions have necessarily large quantum numbers and (4) can actually describe our (classical) universe without introducing other fields or the cosmological constant: the third quantization mechanism depends only on the gravitational part of the WDW equation. In the toy model described above, the main result (8) is strictly dependent on the potential $v(a)$ chosen. This problem can be overcome by assuming a condensate of baby universes in the ground state (the foam) and calculating the vertex between an excited universe and the foam in the presence of an external potential. The condensate provides the "energy reservoir" to "increase" the excited universe; moreover, the presence of the condensate enhances the baby-large universe interactions with respect to the large-large universe interactions. With this procedure, a wide class of potentials produces analogous results to the Hartree–Fock theory discussed previously.

We do not expect the model discussed above to really explain the properties of our universe, but we hope that this paper will stimulate work in this direction.

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References.

[1] A. Vilenkin, *Phys. Rev.* **D37**, 888 (1988).

[2] B. DeWitt, *Phys. Rev.* **D160**, 1113 (1967).

[3] J.A. Wheeler, in *Battelle Rencontres* eds. C. de Witt and J.A. Wheeler, (Benjamin, 1968).

[4] S.B. Giddings and A. Strominger, *Nucl. Phys.* **B321**, 481.

[5] K. Ghoroku, *Class. Quantum Grav.* **8**, 447-452 (1991).

[6] J.B. Hartle and S.W. Hawking, *Phys. Rev.* **D28**, 2960 (1983).

[7] O. Bertolami and J.M. Mourão, *Class. Quantum Grav.* **8**, 1271 (1991).

[8] M. Cavaglià and V. de Alfaro, *Mod. Phys. Lett.* **A9**, 569 (1994).

[9] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (Mc Graw-Hill Inc., 1971).