Interference Cancellation and Joint Decoding for Collision Resolution in Slotted ALOHA

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Abstract—We present a novel decoding scheme for slotted ALOHA which is based on concepts from physical-layer network coding (PNC) and multi-user detection (MUD). In addition to recovering individual user packets from a packet collision as it is usually done with MUD, the receiver applies PNC to decode packet combinations that can be used to retrieve the original packets using information available from other slots. We evaluate the novel scheme and compare it with another scheme based on PNC that has been proposed recently and show that both attain important gains compared to basic successive interference cancellation. This suggests that combining PNC and MUD can lead to significant gains with respect to previously proposed methods on either one or the other.

I. INTRODUCTION

In a random multiple-access scheme, various users access the same channel with no or very limited coordination which may result in packet collisions. In contrast to demand assignment multiple-access (DAMA) schemes, in which collisions are avoided by pre-allocating the radio resources to the users, in random multiple-access protocols, such as e.g. ALOHA, packet collisions cannot be avoided due to the lack of coordination. Such uncoordinated multiple-access protocols are preferable over scheduled access in various scenarios including

• systems with long inherent feedback delay, e.g. satellite networks
• networks with a large user population, e.g. wireless sensor networks or RF-IDs tags
• systems with sporadic user activity patterns, as occurs frequently in machine-type communication.

A small but efficient degree of coordination is introduced by slotted ALOHA, in which the MAC frame is divided into slots and all users transmit packets of the same length and aligned to the slots. In such a system, packets from different users may collide within a slot, although packets do not fall between two adjacent slots. Methods leveraging on the power imbalance among the colliding signals [1] or the transmission of multiple replicas of the same packet [2] have been proposed to mitigate the severe loss in throughput caused by collisions. Recently, several approaches based on multiple packet transmission and iterative interference cancellation [3]–[5] have been proposed that yield dramatic performance improvements with respect to previous solutions. The method proposed in [4] applies concepts from graph codes such as belief propagation on a packet level in order to efficiently resolve collisions. For an overview of these developments and a novel scheme for coded slotted ALOHA, we refer to [4], [6].

Another method that has been proposed recently to extract information from colliding packets, is physical layer network coding [7], [8]. In [9]–[11] the denoise-and-forward approach with channel decoding [8] has been applied in the context of random multiple access systems. An information theoretical analysis of the performance of physical layer network coding has been presented in [12]. In [13], the application and practical implementation of PNC and MUD in the multiple access channel of a WLAN for the case of two colliding signals has been presented.

In this paper, we consider a coded slotted ALOHA system and examine several options for decoding more than one packet per slot in case of a collision. In addition to the well-known successive interference cancellation (SIC), we consider a novel scheme that comprises an additional decoding step based on network coding and compare the two with a recent scheme which employs a joint decoder [14] for all collided packets [15]. The advantage of the newly proposed scheme with respect to the one in [15] is its lower complexity, although this is achieved with a certain loss in terms of decoding probability.

In contrast to [4], we focus on the decoding within a single slot based on the received signal, i.e. on the physical-layer aspects of coded slotted ALOHA. Our approach is complementary to those that apply graph-based methods on a packet level.

II. SYSTEM MODEL

We assume that a MAC frame is divided into slots of equal length and packets of $K$ users arrive synchronously. The channel is modeled as block fading, i.e. the fading coefficients $h_k$ are constant during one slot, but independent for each user and each slot. In addition, each user applies the same channel code of length $N$ and rate $R$. This property will turn out to be useful for the network-decoding step and for joint decoding. We denote a codeword of user $k \in \{1, 2, \ldots, K\}$ by $c_k \in F_2^{1 \times N}$, where $F_2$ denotes the binary Galois field. The message of user $k$ is denoted accordingly by $u_k \in F_2^{1 \times RN}$ and we can write for the encoding function $c_k = u_k G$ with a common generator
matrix \( G \in \mathbb{F}^{RN \times N} \). The codeword symbols \( c_{k,n} \in \mathbb{F}_2 \) are mapped to BPSK symbols \( x_{k,n} = \mu(c_{k,n}) \in \{-1, 1\} \), and the mapping function \( \mu \) for bit vectors is defined element-wise as

\[
\mu(0) = -1, \quad \mu(1) = 1.
\]

We chose this modulation for ease of exposition, noting that the extension to higher-order QAM is relatively straightforward and does not change the main principles. The received signal for a collision of \( K \) packets can then be written as

\[
y_n = \sum_{k=1}^{K} h_k x_{k,n} + w_n, \quad w_n \sim \mathcal{N}(0, 1)
\]

where the fading coefficient follows a Rayleigh distribution with \( h_k = |h_k^{(c)}| \) for \( h_k^{(c)} \sim \mathcal{CN}(0, \sqrt{\text{SNR}}) \). The fading coefficients are known at the receiver but not at the transmitters.

Figure 1. \( K \)-user multiple-access channel with block fading. All users apply the same channel code.

### III. Decoding Methods

The receiver tries to recover as many user packets as possible per slot, based on the received signal \( y = [y_1, y_2, \ldots, y_N] \). In the following, we focus only on decoding per slot, noting that this does not exclude interference cancellation techniques with packets decoded in other slots but is complementary.

#### A. Separate Decoding

The simplest approach is to decode each packet separately, considering all other packets as interference. As for all other schemes to follow, we utilize the channel state information of all other users and the known transmit alphabet, i.e., the BPSK constellation\(^1\). With this, we can write for the log-likelihood value (L-value) of user \( k \) and bit position \( n \),

\[
L_{k,n} \triangleq \ln \frac{P[x_k,n = 1 \mid y_n]}{P[x_k,n = 0 \mid y_n]} = \ln \frac{P[x_k,n = 1 \mid y_n]}{P[x_k,n = -1 \mid y_n]},
\]

Since the received symbol \( y_n \) depends on all symbols, we need to marginalize over all other users’ symbols. For this, we define the sets \( X_k^n = \{ \mathbf{x} = [\mathbf{x}^{(c)}] : \mathbf{d} \in \mathbb{F}_2^R, \mathbf{d} = b \} \) for \( b \in \mathbb{F}_2 \), with cardinality \( |X_k^n| = 2^{K-1} \). We can think of the variable \( \mathbf{d} \) as the vector of the coded bits of all users at the same position, i.e., \( \mathbf{d}_n = [c_{1,n}, c_{2,n}, \ldots, c_{K,n}]^T \). We obtain for the L-values

\[
L_{k,n} = \ln \frac{\sum_{\mathbf{x} \in X_k^n} P[\mathbf{x} \mid y_n]}{\sum_{\mathbf{x} \in X_k^n} P[\mathbf{x} \mid y_n]} = \ln \frac{\sum_{\mathbf{x} \in X_k^n} P(y_n \mid \mathbf{x})}{\sum_{\mathbf{x} \in X_k^n} P(y_n \mid \mathbf{x})} = \ln \frac{\sum_{\mathbf{x} \in X_k^n} \exp \left( - (y_n - h^T \mathbf{x})^2 \right)}{\sum_{\mathbf{x} \in X_k^n} \exp \left( - (y_n - h^T \mathbf{x})^2 \right)} = \text{jacln} \left\{ - (y_n - h^T \mathbf{x})^2 \right\}
\]

where \( \text{jacln} \{ x \} \triangleq \ln \sum_{i=1}^{n} \exp(x_i) \) denotes the Jacobian logarithm, which can be computed recursively and for which computationally efficient approximations exist\(^1\). These L-values are input to a soft-input decoder, which typically is a Viterbi, a turbo or an LDPC decoder.

#### B. Successive Interference Cancellation with Seek & Decode (SIC+S&D)

A straightforward and well-known extension of basic single-user decoding is SIC: if a packet \( u_k \) is successfully decoded, its corresponding codeword \( c_{k} \) and symbol sequence \( x_{k,}\) are known and can be subtracted from the received signal \( y_n \), creating a multiple-access channel with \( K - 1 \) users. This process can be repeated until decoding of all remaining packets fails. To avoid unnecessary computations, we exploit the knowledge of the instantaneous SNRs and order the users accordingly: let \( \pi \) be a permutation of \( \{1, 2, \ldots, K\} \) such that

\[
h_{\pi(1)} \geq h_{\pi(2)} \geq \cdots \geq h_{\pi(K)}
\]

then decoding starts with user \( \pi(1) \). Apart from reducing computational complexity, this ordering is also useful to reduce the probability of undetected errors. To detect correct decoding of a packet, in general an additional error detection code, e.g. a CRC (cyclic redundancy check) code, has to be introduced into each message \( u_k \). Since there is a non-zero probability that an erroneous decoding is not detected, the number of decoding attempts with low probability of success should be kept to a minimum.

#### C. Successive Interference Cancellation with Seek & Decode (SIC+S&D)

For a coded slotted ALOHA system, a further decoding step after SIC is possible. Assume that after the SIC procedure described above, \( K - K_1 \) packets have been correctly decoded, hence leaving \( K_1 \in \{2, \ldots, K\} \) packets for which decoding failed. In this situation, the receiver can try to decode a combined packet, which is given by the sum of two or more packets. Assume that after SIC, users \( 1, 2, \ldots, K_1 \) have not been decoded. Then the receiver can try to decode a subset of \( \{1, 2, \ldots, K_1\} \), e.g. given by \( K = \{k_1, k_2, \ldots, k_l\} \subset \{1, 2, \ldots, K_1\} \). For this subset we define the sets of constel-
For the decoding of packet combinations, we give an instructive example for the subset 

$$X^{(b)}_\ell \triangleq \{ x = \mu(d) : d \in \mathbb{F}_2^\ell \text{ with } \sum_{i=1}^{\ell} d_i = b \}, \ b \in \mathbb{F}_2$$

and obtain the corresponding L-values as

$$L^K = \ln \sum_{x \in X^{(k)}} \exp \left( - (y_n - [h_{k_1}, h_{k_2}, \ldots, h_{k_\ell}] x)^2 \right)$$

These L-values $L^K_1, L^K_2, \ldots, L^K_K$ are fed to the soft-input decoder, which, if successful, finds the corresponding codeword $\sum_{k \in K} c_k$ or message $\sum_{k \in K} u_k$. Note that the sum of messages or codewords is defined in the finite field $\mathbb{F}_2$, which is the same as the bit-wise XOR.

After successful decoding of a packet combination, a subsequent idea is to re-apply interference cancellation with the codeword $\sum_{k \in K} c_k$ or message $\sum_{k \in K} u_k$. This can be done with the relation

$$\sum_{k \in K} c_k = \sum_{k \in K} u_k G$$

For error detection, since CRC codes are also binary linear codes, the same CRC can be used. For $K_1$ undecoded packets, there exist

$$\sum_{\ell=2}^{K_1} \binom{K_1}{\ell} = 2^{K_1} - K_1 - 1$$

combinations of two and more packets, for which a decoding attempt is possible from the L-values defined by (7). With this definition, note that the subsets $X^{(b)}_\ell$ only depend on $b$ and on the number of packets $\ell$ but not on their indices $k_1, \ldots, k_\ell$. After successful decoding of a packet combination, a subsequent idea is to re-apply interference cancellation with the new packet combination. This, however, is not directly possible since the combined codeword $\sum_{k \in K} c_k$ does not correspond to any symbol sequence $x_k$ in (2) and the sum of codewords and symbol sequences are taken over different fields, namely $\mathbb{F}_2$ and $\mathbb{R}$.

However, the knowledge of a combined packet $c_K$ might still be useful for another decoding attempt: the cardinality of the sets $X^{(b)}_\ell$ can be reduced by a factor of two by introducing the additional constraint of the known combined packet. Then, the L-values can be recomputed and new decoding attempts, including $\ell = 1$ for individual packets can be undertaken. This approach brings about a slight additional complexity due to the constraint on the decoded combination. In this case, the sets $X^{(b)}_\ell$ will additionally depend on $n$ and hence have to be computed for each coded bit.

**Example 1.** For the decoding of packet combinations, we give an instructive example for the subset $K = \{1, 2\}$, which shows the connection to physical-layer network coding in the two-way relay channel. In Table I, the possible combinations of the coded bits $d_1 = \hat{c}_{1,n}, d_2 = \hat{c}_{2,n}$ are listed. From this table, we can easily find the sets

$$X^{(0)}_2 = \left\{ \begin{array}{c|c} -1 & 1 \\ \hline -1 & 1 \end{array} \right\}$$

$$X^{(1)}_2 = \left\{ \begin{array}{c|c} -1 & 1 \\ \hline 1 & -1 \end{array} \right\}$$

which corresponds to (7) in [17].

In order to assess the performance of this scheme, we count the number of innovative packets per slot. Innovative packets are correctly decoded packets or combinations of packets which cannot be obtained by combining other correctly decoded packets. The number of innovative packets is the same as the number of linearly independent packet combinations. After successful decoding of individual packets or combinations, we build a binary matrix $A$ whose rows $\alpha = [a_1, a_2, \ldots, a_K]$ indicate the user indices which are contained in successfully decoded combinations. For instance, if the combined packet $c_1 + c_2 + c_4$ is correctly decoded, the corresponding row is $\alpha = [1, 0, 1, 1, 0, 0]$ for $K = 6$. The number of innovative packets can then be calculated as the rank of $A$ in $\mathbb{F}_2$ arithmetic.

The original packets can be recovered from the innovative packets like in linear network coding [18], [19].

**D. Joint Decoding and Seek & Decode (JDS&D)**

From (2) we can observe that the received samples $y_n$ depend on all coded bits $c_{k,n}$ at the same bit position but are independent of bits at other positions. The optimum decoding approach is therefore to consider the vectorial symbols $d_n \triangleq [c_{1,n}, c_{2,n}, \ldots, c_{K,n}]^T$ jointly. This can be done with a joint decoder which operates on the vectors $d_n$ or on an equivalent integer representation $\bar{d}_n$ such that $\bar{d}_n = \text{bin}(d_n)$.

The notation $\text{bin}(b)$ denotes the binary representation of the non-negative integer $b$. For LDPC and for convolutional codes, such joint decoders are described in [14], [17]. The decoder input is given by the probability vector

$$\mathbf{p}_n \triangleq \begin{bmatrix} p_n(0) \\ p_n(1) \\ \vdots \\ p_n(2^K - 1) \end{bmatrix} \in \mathbb{R}^{2^K}$$
where

\[ p_n(b) \triangleq P [d = \text{bin}(b) \mid y_n] \]

\[ \propto p(y_n \mid x = \mu(\text{bin}(b))) \]  

for \( b = 0, 1, \ldots, 2^K - 1 \). Let \( \bar{x}_b = \mu(\text{bin}(b)) \), then

\[ p_n = \alpha \begin{bmatrix} \exp\left( -\left( y_n - h^T \bar{x}_0 \right)^2 \right) \\ \vdots \\ \exp\left( -\left( y_n - h^T \bar{x}_{2^K-1} \right)^2 \right) \end{bmatrix} \]

where \( \alpha \) is a scaling factor which is irrelevant for the decoding algorithm.

The decoder output is an estimate of all messages (or equivalently of all codewords),

\[ \hat{U} = \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \vdots \\ \hat{u}_K \end{bmatrix} \]

Making use of an error detecting code, the receiver checks all possible packet combinations, i.e. all \( 2^K - 1 \) non-empty subsets of \( \{1, 2, \ldots, K\} \) and builds the binary matrix \( A \in \mathbb{F}_2^{(2^K-1)\times K} \) which indicates the correct packet combinations in each row. From this matrix, the number of innovative packets is calculated as its rank.

This joint decoding approach reverses the order of the SIC+S&D method: while in SIC+S&D the packet combination is determined first and then a decoding attempt is carried out, joint decoding first tries to decode all packets jointly and then the receiver checks which combinations are correct.

E. Complexity Considerations and Possible Combined Approaches

While in the following, we focus on the achievable performance for each of the described methods, it is worth pointing out the possible combinations and performance-complexity trade-offs. For the basic separate decoding scheme, complexity could be reduced by ordering users according to their instantaneous SNR and stop decoding after the decoding of one user has failed. This will obviously cause a slight performance loss which depends mainly on the SNR differences and on the applied coding scheme, basically on the packet length. The same idea can be applied to both SIC techniques, while for SIC+S&D, a packet combination can be checked for linear independency before the decoding attempt. The complexity of SIC+S&D in the worst case is proportional to \( 2^K - 1 \) decoding attempts. The complexity of joint decoding in the case of LDPC codes is proportional to \( K \cdot 2^K \) for belief propagation with transform-based check-node processing [20], [21]. The complexity with joint decoding can therefore be lowered if the some packets can be decoded by separate decoding, thus reducing the number of collided packets. Hence, for many settings, a less complex approach might be first apply SIC and then joint decoding on the remaining packets. On the other hand, reduced-complexity decoding algorithms, which are known from non-binary LDPC codes, can also be applied to joint decoding [22].

IV. Simulation Results

A. Single-User Performance

The outage probability for the single-user block Rayleigh fading channel is given by

\[ p_{out} = 1 - \exp\left( -\frac{2^{2R} - 1}{\text{SNR}} \right) \]  

This holds for the channel model (2) with \( K = 1 \) [23, Section 5.4.1] (the factor 2 before the rate is due to the real-valued channel model). For all simulations, we use an LDPC code from the WiMAX standard [24] with code rate \( R=1/2 \), word length \( N = 576 \) bits and message length \( RN = 288 \) bits. This widely known code has been chosen to facilitate comparability and for the availability of a joint decoder for LDPC codes. Note that for joint decoding, a non-binary LDPC decoder in \( \mathbb{F}_q \) with \( q = 2^K \) can be applied [25], provided the definition of the bit vector corresponds to the binary image of the Galois field elements [14], [25].

Fig. 2 shows the packet error rate (PER) for the chosen code over a block Rayleigh fading channel, in comparison with the outage probability (15). For the single-user case, the number of successfully decoded packets per slot is simply given by \( 1 - \text{PER} \). For the simulations, we assumed perfect error detection which is slight simplification while for a more realistic implementation, an additional error detecting code, e.g. a 24-bit CRC, can be included.

![Figure 2. Outage probability and packet error rate for the single-user channel with block Rayleigh fading](image)

B. Performance for Collision Resolution

Fig. 3 to Fig. 5 show the achieved number of innovative packets per slot with the described decoding techniques with 2, 4 and 8 users. We can see that for all cases, joint decoding performs best and its advantage increases with the number of users. For a high number of users, the advantage of joint decoding to all other techniques is dramatic. On the other hand, we point out that, unlike JD+S&D, the SIC+S&D scheme has
the advantage that is does not require any modification at the decoder, since only the L-value calculation is modified with respect to a standard receiver. We further note that the advantage of SIC+S&D over pure SIC decreases with the number of users. For sufficiently high SNR, all methods achieve benefits from multiple packet transmissions, which can be most clearly seen in Fig. 3 for two users. At low SNR, the average number of recovered packets per slot is nearly identical to the single-user case while for medium to high SNR, in the mean more than one packet is recovered. For all considered cases, the number of innovative packets tends to $K$, i.e. for high SNR nearly all collided packets can be decoded. This result is in line with the capacity of the multiple-access channel, which increases without bounds with the number of users.

The objective of this comparison is to compare the different approaches for decoding multiple packets, for which reason the code rate and the modulation is fixed for all cases. In general, the modulation and coding parameters should be selected according to the expected SNR.

![Figure 3. Two-user collision channel, $K = 2$.](image1)

![Figure 4. Four-user collision channel, $K = 4$.](image2)

In Fig. 6, the maximum throughput for $K \in \{2, 3, \ldots, 8\}$ users for each scheme is depicted. The curves are obtained by selecting for each SNR value the number of users, ranging from 2 to 8, with the highest average number of innovative packets. We can observe that at low SNR there is little difference between the four schemes while at medium SNR, joint decoding has a significant advantage. For (very) high SNR, all schemes are limited by the maximum number of users.

![Figure 5. Eight-user collision channel, $K = 8$.](image3)

![Figure 6. Maximum throughput for $K \in \{1, 2, \ldots, 8\}$ for all four decoding options](image4)

V. CONCLUSIONS

We proposed a new scheme based on PNC and MUD that aims at retrieving innovative packets from collisions in slotted ALOHA systems. Starting from two well-known techniques, namely separate decoding and successive interference cancellation (SIC), we have presented an additional decoding scheme which tries to decode the largest number of innovative packet combinations by simply modifying the detector at the receiver side, without making any modification to the channel decoder. We also evaluated the joint decoding of all collided packets and the subsequent detection of packet combinations, which constitutes the optimum approach, but is also characterized by
by a high complexity and requires modifications at the decoder. Simulation results show that the new scheme achieves substantial gains compared to separate decoding and to SIC.

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