More Graviton Physics

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The interactions of gravitons with spin-1 matter are calculated in parallel with the well known photon case. It is shown that graviton scattering amplitudes can be factorized into a product of familiar electromagnetic forms, and cross sections for various reactions are straightforwardly evaluated using helicity methods. Universality relations are identified. Extrapolation to zero mass yields scattering amplitudes for photon-graviton and graviton-graviton scattering.

1. INTRODUCTION

The calculation of photon interactions with matter is part of any introductory (or advanced) course on quantum mechanics. Indeed the evaluation of the Compton scattering cross section is a standard exercise in relativistic quantum mechanics, since gauge invariance together with the masslessness of the photon allow the results to be presented in terms of relatively simple analytic forms [1].

Naively, one might expect a similar analysis to be applicable to the interactions of gravitons since, like photons, gravitons are massless and subject to a gauge invariance. Also, just as virtual photon exchange leads to a detailed understanding of electromagnetic interactions between charged systems, a careful treatment of virtual graviton exchange allows an understanding not just of Newtonian gravity, but also of spin-dependent phenomena—geodetic precession and Lense-Thirring frame dragging—associated with general relativity which have recently been verified by gravity probe B [2]. However, despite these parallels, examination of quantum mechanics texts reveals that (with one exception [3]) the case of graviton interactions is *not* discussed in any detail. There are at least three reasons for this situation:

i) the graviton is a spin-two particle, as opposed to the spin-one photon, so that the interaction forms are more complex, involving symmetric and traceless second rank tensors rather than simple Lorentz four-vectors;

ii) there exist fewer experimental results with which to confront the theoretical calculations. **added text** Fundamental questions beyond the detection of quanta of gravitational fields have been exposed in [4];

iii) in order to guarantee gauge invariance one must include, in many processes, the contribution from a graviton pole term, involving a *triple-graviton coupling.* This vertex is a sixth rank tensor and contains a multitude of kinematic forms.

added text: Hundred years after the classical theory of general relativity and Einstein[1] argument for a quantization of gravity [5], we are still looking after experimental signature of quantum gravity effects. This paper present and extend recent works where elementary quantum gravity processes display new and very distinctive behaviour.

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[1] "*Gleichwohl müßten die Atome zufolge der inneratomischen Elektronenbewegung nicht nur elektromagnetische, sondern auch Gravitationsenergie ausstrahlen, wenn auch in winzigem Betrage. Da dies in Wahrheit in der Natur nicht zutreffen dürfte, so scheint es, daß die Quantentheorie nicht nur die Maxwellsche Elektrodynamik, sondern auch die neue Gravitationstheorie wird modifizieren müssen*" [5].

Recently, however, using powerful (string-based) techniques, which simplify conventional quantum field theory calculations, it has been demonstrated that the scattering of gravitons from an elementary target of arbitrary spin factorizes [6], a feature that had been noted ten years previously by Choi et al. based on gauge theory arguments [7]. This factorization property, which is sometime concisely described by the phrase "gravity is the square of a gauge theory", permits a relatively elementary evaluation of various graviton amplitudes and opens the possibility of studying gravitational processes in physics coursework. In a previous paper published in this journal [8] it was shown explicitly how, for both spin-0 and spin-$\frac{1}{2}$ targets, the use of factorization enables elementary calculation of both the graviton photoproduction,

$$\gamma + S \rightarrow g + S,$$

and gravitational Compton scattering,

$$g + S \rightarrow g + S,$$

reactions in terms of elementary photon reactions. This simplification means that graviton interactions can now be discussed in a basic quantum mechanics course and opens the possibility of treating interesting cosmological applications.

In the present paper we extend the work begun in [8] to the case of a spin-1 target and demonstrate and explain the origin of various universalities, *i.e.,* results which are independent of target spin. In addition, by taking the limit of vanishing target mass we show how both graviton-photon and graviton-graviton scattering may be determined using elementary methods.

In section 2 then, we generate the electromagnetic interactions of a spin one system. In section 3 we calculate the ordinary Compton scattering cross section for a spin-1 target and compare with the analogous spin-0 and spin-$\frac{1}{2}$ forms. In section 4 we examine graviton photoproduction and gravitational Compton scattering for a spin-1 target and again compare with the analogous spin-0 and spin-$\frac{1}{2}$ results. In section 5 we study the massless limit and show how both photon-graviton and graviton-graviton scattering can be evaluated, resolving a subtlety which arises in the derivation. Finally, our results are summarized in a brief concluding section 6. Two appendices contain formalism and calculational details.

2. SPIN ONE INTERACTIONS: A LIGHTNING REVIEW

We begin by generating the photon and graviton interactions of a spin-1 system. We generate the photon interaction by writing down the free Lagrangian for a scalar field ϕ

$$\mathcal{L}_0^{S=0} = \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi \tag{1}$$

and using the minimal substitution [9]

$$i\partial_\mu \longrightarrow iD_\mu \equiv i\partial_\mu - eA_\mu$$

This procedure leads to the familiar interaction Lagrangian

$$\mathcal{L}_{int}^{S=0} = -iA_\mu\phi^\dagger \overleftrightarrow{\partial}^\mu\phi + e^2A_\mu A^\nu\eta_{\mu\nu}\phi^\dagger\phi \tag{2}$$

where e is the particle charge and A_μ is the photon field and implies the one- and two-photon vertices

$$\begin{aligned} <p_f|V_{em}^{(1)\mu}|p_i> &= ie(p_f+p_i)^\mu \\ <p_f|V_{em}^{(2)\mu\nu}|p_i> &= ie^2\eta^{\mu\nu} \end{aligned} \tag{3}$$

The corresponding charged massive spin-1 Lagrangian has the Proca form [10]

$$\mathcal{L}_0^{S=1} = -\frac{1}{2}\,B_{\mu\nu}^\dagger B^{\mu\nu} + m^2\,B_\mu^\dagger B^\mu\,, \tag{4}$$

where B^μ is a spin one field subject to the constraint $\partial_\mu B^\mu = 0$ and $B^{\mu\nu}$ is the antisymmetric tensor

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu\,. \tag{5}$$

The minimal substitution then leads to the interaction Lagrangian

$$\mathcal{L}_{int}^{S=1} = i\,e\,A^\mu B^{\nu\dagger}\left(\eta_{\nu\alpha}\overleftrightarrow{\partial}_\mu - \eta_{\alpha\mu}\overleftrightarrow{\partial}_\nu\right)B^\alpha - e^2 A^\mu A^\nu(\eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\mu\alpha}\eta_{\nu\beta})B^{\alpha\dagger}B^\beta\,, \tag{6}$$

and the one, two photon vertices

$$\begin{aligned}\left\langle p_f,\epsilon_B\left|V_{em}^{(1)\mu}\right|p_i,\epsilon_A\right\rangle_{S=1} &= -i\,e\,\epsilon^*_{B\beta}\left((p_f+p_i)^\mu\eta^{\alpha\beta} - \eta^{\beta\mu}p_f^\alpha - \eta^{\alpha\mu}p_i^\beta\right)\epsilon_{A\alpha}\,,\\ \left\langle p_f,\epsilon_B\left|V_{em}^{(2)\mu\nu}\right|p_i,\epsilon_A\right\rangle_{S=1} &= i\,e^2\,\epsilon^*_{B\beta}\left(2\eta^{\alpha\beta}\eta^{\mu\nu} - \eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\nu}\eta^{\beta\mu}\right)\epsilon_{A\alpha}\,.\end{aligned} \tag{7}$$

However, Eq. (7) is *not* the correct result for a fundamental spin-1 particle such as the charged W-boson. Because the W arises in a gauge theory, the field tensor is not given by Eq. (5) but rather is generated from the charged—$\sqrt{\frac{1}{2}}(x \pm iy)$—component of

$$\boldsymbol{B}_{\mu\nu} = D_\mu\boldsymbol{B}_\nu - D_\nu\boldsymbol{B}_\mu - g_{ga}\boldsymbol{B}_\mu\times\boldsymbol{B}_\nu \tag{8}$$

where g_{ga} is the gauge coupling. This modification implies the existence of an additional $W^\pm\gamma$ interaction, leading to an "extra" contribution to the single photon vertex

$$\langle p_f,\epsilon_B\left|\delta V_{em}^{(1)\mu}\right|p_i,\epsilon_A\rangle_{S=1} = i\,e\,\epsilon^*_{B\beta}\left(\eta^{\alpha\mu}(p_i-p_f)^\beta - \eta^{\beta\mu}(p_i-p_f)^\alpha)\right)\epsilon_{A\alpha}\,. \tag{9}$$

The significance of this term can be seen by using the mass-shell Proca constraints $p_i\cdot\epsilon_A = p_f\cdot\epsilon_B = 0$ to write the total on-shell single photon vertex as

$$\begin{aligned}\left\langle p_f,\epsilon_B\left|(V_{em}+\delta V_{em})^\mu\right|p_i,\epsilon_A\right\rangle_{S=1} &= -i\,e\,\epsilon^*_{B\beta}\left((p_f+p_i)^\mu\eta^{\alpha\beta}\;2\eta^{\beta\mu}(p_i-p_f)^\alpha\right)\epsilon_{A\alpha}\\ &+\;-2\eta^{\alpha\mu}(p_i-p_f)^\beta\,,\end{aligned} \tag{10}$$

wherein, comparing with Eq. 9 we observe that the coefficient of the term $-\eta^{\alpha\mu}(p_i-p_f)^\beta + \eta^{\beta\mu}(p_i-p_f)^\alpha$ has been modified from unity to two. Since the rest frame spin operator can be identified via[2]

$$B_i^\dagger B_j - B_j^\dagger B_i = -i\,\epsilon_{ijk}\langle f\left|S_k\right|i\rangle\,, \tag{12}$$

the corresponding piece of the nonrelativistic interaction Lagrangian becomes

$$\mathcal{L}_{\rm int} = -g\,\frac{e}{2m}\langle f\left|\boldsymbol{S}\right|i\rangle\cdot\boldsymbol{\nabla}\times\boldsymbol{A}\,, \tag{13}$$

where g is the gyromagnetic ratio and we have included a factor $2m$ which accounts for the normalization condition of the spin one field. Thus the "extra" interaction required by a gauge theory changes the g-factor from its Belinfante value of unity [11] to its universal value of two, as originally proposed by Weinberg [12] and more recently buttressed by a number of additional arguments [13]. Henceforth in this manuscript then we shall assume the g-factor of the spin-1 system to have its "natural" value $g=2$, since it is in this case that the high-energy properties of the scattering are well controlled and the factorization properties of gravitational amplitudes are valid [14].

3. COMPTON SCATTERING

The vertices given in the previous section can now be used to evaluate the ordinary Compton scattering amplitude,

$$\gamma + S \to \gamma + S,$$

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[2] Equivalently, one can use the relativistic identity

$$\epsilon^*_{B\mu}q\cdot\epsilon_A - \epsilon_{A\mu}q\cdot\epsilon^*_B = \frac{1}{1-\frac{q^2}{m^2}}\left(\frac{i}{m}\epsilon_{\mu\beta\gamma\delta}p_i^\beta q^\gamma S^\delta - \frac{1}{2m}(p_f+p_i)_\mu\epsilon^*_B\cdot q\epsilon_A\cdot q\right) \tag{11}$$

where $S^\delta = \frac{i}{2m}\epsilon^{\delta\sigma\tau\zeta}\epsilon^*_{B\sigma}\epsilon_{A\tau}(p_f+p_i)_\zeta$ is the spin four-vector.

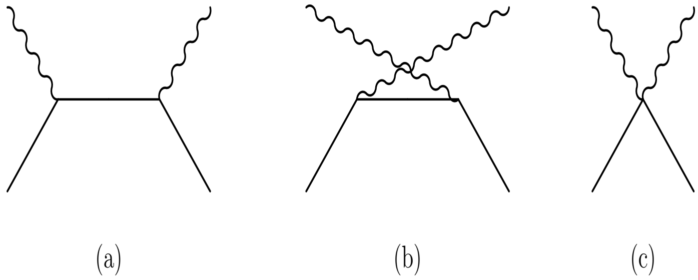

FIG. 1: Diagrams relevant to Compton scattering.

for a spin-1 system having charge e and mass m by summing the contributions of the three diagrams shown in Figure 1, yielding

$$\begin{aligned}\text{Amp}^{\text{Comp}}_{S=1} &= 2e^2\Bigg\{\epsilon_A\cdot\epsilon^*_B\left[\frac{\epsilon_i\cdot p_i\epsilon^*_f\cdot p_f}{p_i\cdot k_i}-\frac{\epsilon_i\cdot p_f\epsilon^*_f\cdot p_i}{p_i\cdot k_f}-\epsilon_i\cdot\epsilon^*_f\right]\\ &- \left[\epsilon_A\cdot[\epsilon^*_f,k_f]\cdot\epsilon^*_B\left(\frac{\epsilon_i\cdot p_i}{p_i\cdot k_i}-\frac{\epsilon_i\cdot p_f}{p_i\cdot k_f}\right)-\epsilon_A\cdot[\epsilon_i,k_i]\cdot\epsilon^*_B\left(\frac{\epsilon_f\cdot p_f}{p_i\cdot k_i}-\frac{\epsilon^*_f\cdot p_i}{p_i\cdot k_f}\right)\right]\\ &- \left[\frac{1}{p_i\cdot k_i}\epsilon_A\cdot[\epsilon_i,k_i]\cdot[\epsilon^*_f,k_f]\cdot\epsilon^*_B-\frac{1}{p_i\cdot k_f}\epsilon_A\cdot[\epsilon^*_f,k_f]\cdot[\epsilon_i,k_i]\epsilon^*_B\right]\Bigg\} \end{aligned} \tag{14}$$

with the momentum conservation condition $p_i + k_i = p_f + k_f$. It is then straightforward to verify that Eq. (14) satisfies the gauge invariance strictures

$$\epsilon^{*\mu}_f k^\nu_i \text{Amp}^{\text{Comp}}_{\mu\nu,S=1} = k^\mu_f\epsilon^\nu_i\text{Amp}^{\text{Comp}}_{\mu\nu,S=1} = 0\,. \tag{15}$$

In order to pave the way for the transition to gravity in the next section, it is useful to utilize the helicity formalism [15], wherein one evaluates the matrix elements of the Compton amplitude between initial and final spin-1 and photon states having definite helicity, where helicity is defined as the projection of the particle spin along the momentum direction. We work initially in the center of mass frame and, for a photon incident with four-momentum $k^\mu_i = p_{\text{CM}}(1,\hat{z})$, we choose the polarization vectors

$$\epsilon^{\lambda_i}_i = -\frac{\lambda_i}{\sqrt{2}}(\hat{x}+i\lambda_i\hat{y}), \qquad \lambda_i = \pm\,, \tag{16}$$

while for an outgoing photon with $k^\mu_f = p_{\text{CM}}(1,\cos\theta_{\text{CM}}\hat{z}+\sin\theta_{\text{CM}}\hat{x})$ we use polarizations

$$\epsilon^{\lambda_f}_f = -\frac{\lambda_f}{\sqrt{2}}\Big(\cos\theta_{\text{CM}}\hat{x}+i\lambda_f\hat{y}-\sin\theta_{\text{CM}}\hat{z}\Big), \quad \lambda_f = \pm\,. \tag{17}$$

We can define corresponding helicity states for the spin-1 system. In this case the initial and final four-momenta are $p^\mu_i = (E_{\text{CM}}, -p_{\text{CM}}\hat{z})$ and $p^\mu_f = (E_{\text{CM}}, -p_{\text{CM}}(\cos\theta_{\text{CM}}\hat{z}+\sin\theta_{\text{CM}}\hat{x}))$ and there exist two transverse polarization four-vectors

$$\begin{aligned}\epsilon^{\pm\mu}_A &= \left(0,\frac{\pm\hat{x}-i\hat{y}}{\sqrt{2}}\right),\\ \epsilon^{\pm\mu}_B &= \left(0,\frac{\pm\cos\theta_{\text{CM}}\hat{x}+i\hat{y}\mp\sin\theta_{\text{CM}}\hat{z}}{\sqrt{2}}\right),\end{aligned} \tag{18}$$

in addition to the longitudinal mode with polarization four-vectors

$$\begin{aligned}\epsilon^{0\mu}_A &= \frac{1}{m}\big(p_{\text{CM}},-E_{\text{CM}}\hat{z}\big)\,,\\ \epsilon^{0\mu}_B &= \frac{1}{m}\big(p_{\text{CM}},-E_{\text{CM}}(\cos\theta_{\text{CM}}\hat{z}+\sin\theta_{\text{CM}}\hat{x})\big)\,,\end{aligned} \tag{19}$$

In terms of the usual invariant kinematic (Mandelstam) variables

$$s = \big(p_i + k_i\big)^2, \quad t = \big(k_i - k_f\big)^2, \quad u = \big(p_i - k_f\big)^2,$$

we identify

$$\begin{aligned} p_{\rm CM} &= \frac{s - m^2}{2\sqrt{s}}, \qquad E_{\rm CM} = \frac{s + m^2}{2\sqrt{s}}, \\ \cos\frac{1}{2}\theta_{\rm CM} &= \frac{\big((s-m^2)^2 + st\big)^{\frac{1}{2}}}{s - m^2} = \frac{\big(m^4 - su\big)^{\frac{1}{2}}}{s - m^2}, \quad \sin\frac{1}{2}\theta_{\rm CM} = \frac{\big(-st\big)^{\frac{1}{2}}}{s-m^2}. \end{aligned} \tag{20}$$

The invariant cross-section for unpolarized Compton scattering is then given by

$$\frac{d\sigma^{\rm Comp}_{S=1}}{dt} = \frac{1}{16\pi(s-m^2)^2}\,\frac{1}{3}\sum_{a,b=-,0,+}\frac{1}{2}\sum_{c,d=-,+}\Big|B^1(ab;cd)\Big|^2. \tag{21}$$

where

$$B^1(ab;cd) = \big\langle p_f, b; k_f, d\,\big|{\rm Amp}^{\rm Comp}_{S=1}\big|\,p_i, a; k_i, c\,\big\rangle, \tag{22}$$

is the Compton amplitude for scattering of a photon with four-momentum k_i, helicity a from a spin-1 target having four-momentum p_i, helicity c to a photon with four-momentum k_f, helicity d and target with four-momentum p_f, helicity b. The helicity amplitudes can now be calculated straightforwardly. There exist $3^2 \times 2^2 = 36$ such amplitudes but, since helicity reverses under spatial inversion, parity invariance of the electromagnetic interaction requires that[3]

$$\big|B^1(ab;cd)\big| = \big|B^1(-a-b;-c-d)\big|.$$

Also, since helicity is unchanged under time reversal, but initial and final states are interchanged, T-invariance of the electromagnetic interaction requires that

$$\big|B^1(ab;cd)\big| = \big|B^1(ba;dc)\big|.$$

Consequently there exist only twelve *independent* helicity amplitudes. Using Eq. (14) we calculate the various helicity amplitudes in the center of mass frame and then write these results in terms of invariants using Eq. (20), yielding the forms quoted in Appendix A.

Substitution of these helicity amplitudes Eq. (86) into Eq. (21) then yields the invariant cross-section for unpolarized Compton scattering from a charged spin-1 target

$$\frac{d\sigma^{\rm Comp}_{S=1}}{dt} = \frac{e^4}{12\pi(s-m^2)^4(u-m^2)^2}\Big[(m^4 - su + t^2)\big(3(m^4-su)+t^2\big) + t^2(t-m^2)(t-3m^2)\Big], \tag{23}$$

which can be compared with the corresponding results for unpolarized Compton scattering from charged spin-0 and spin-$\frac{1}{2}$ targets found in ref. [8].

Often such results are written in the *laboratory* frame, wherein the target is at rest, by use of the relations

$$\begin{aligned} s - m^2 &= 2m\omega_i, \quad u - m^2 = -2m\omega_f, \\ m^4 - su &= 4m^2\omega_i\omega_f\cos^2\frac{\theta_L}{2}, \quad m^2 t = -4m^2\omega_i\omega_f\sin^2\frac{\theta_L}{2}, \end{aligned} \tag{24}$$

and

$$\frac{dt}{d\Omega} = \frac{d}{2\pi d\cos\theta_L}\left(-\frac{2\omega_i^2(1-\cos\theta_L)}{1+\frac{\omega_i}{m}(1-\cos\theta_L)}\right) = \frac{\omega_f^2}{\pi}. \tag{25}$$

[3] Note that we require only that the magnitudes of the helicity amplitudes related by parity and/or time reversal be the same. There could exist unobservable phases.

Introducing the fine structure constant $\alpha = e^2/4\pi$, we find then

$$\frac{d\sigma^{\rm Comp}_{{\rm lab},S=1}}{d\Omega} = \frac{\alpha^2}{m^2}\frac{\omega_f^4}{\omega_i^4}\left[\left(\cos^4\frac{\theta_L}{2}+\sin^4\frac{\theta_L}{2}\right)\left(1+2\frac{\omega_i}{m}\sin^2\frac{\theta_L}{2}\right)^2 + \frac{16\omega_i^2}{3m^2}\sin^4\frac{\theta_L}{2}\left(1+2\frac{\omega_i}{m}\sin^2\frac{\theta_L}{2}\right)+\frac{32\omega_i^4}{3m^4}\sin^8\frac{\theta_L}{2}\right], \tag{26}$$

Comparing with the corresponding forms found in [8], We observe that the nonrelativistic laboratory cross-section has an identical form for *any* spin

$$\left.\frac{d\sigma^{\rm Comp}_{{\rm lab},S}}{d\Omega}\right|^{NR} = \frac{\alpha^2}{m^2}\left[\left(\cos^4\frac{\theta_L}{2}+\sin^4\frac{\theta_L}{2}\right)\left(1+\mathcal{O}\Big(\frac{\omega_i}{m}\Big)\right)\right], \tag{27}$$

which follows from the universal form of the Compton amplitude for scattering from a spin-S target in the low-energy ($\omega \ll m$) limit,

$$\left\langle S,M_f;\epsilon_f\left|{\rm Amp}^{\rm Comp}_S\right|S,M_i;\epsilon_i\right\rangle_{\omega\ll m} = 2e^2\,\epsilon_f^*\cdot\epsilon_i\,\delta_{M_i,M_f}+\dots, \tag{28}$$

and obtains in an effective field theory approach to Compton scattering [16].[4]

4. GRAVITATIONAL INTERACTIONS

In the previous section we discussed the treatment the familiar electromagnetic interaction, using Compton scattering on a spin-1 target as an example. In this section we show how the gravitational interaction can be evaluated via methods parallel to those used in the electromagnetic case. An important difference is that while in the electromagnetic case we have the simple interaction Lagrangian

$$\mathcal{L}_{int} = -eA_\mu J^\mu\,, \tag{30}$$

where J^μ is the electromagnetic current matrix element, for gravity we have

$$\mathcal{L}_{int} = -\frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}\,. \tag{31}$$

Here the field tensor $h_{\mu\nu}$ is defined in terms of the metric via

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,, \tag{32}$$

where κ is given in terms of the Cavendish constant G by $\kappa^2 = 32\pi G$. The Einstein-Hilbert action is

$$\mathcal{S}_{\rm Einstein-Hilbert} = \int d^4x\,\sqrt{-g}\,\frac{2}{\kappa^2}\,\mathcal{R}\,, \tag{33}$$

where

$$\sqrt{-g} = \sqrt{-{\rm det}g} = \exp\frac{1}{2}{\rm trlog}g = 1+\frac{1}{2}\eta^{\mu\nu}h_{\mu\nu}+\dots, \tag{34}$$

is the square root of the determinant of the metric and $\mathcal{R} := R^\lambda{}_{\mu\lambda\nu}g^{\mu\nu}$ is the Ricci scalar curvature obtained by contracting the Riemann tensor $R^\mu{}_{\nu\rho\sigma}$ with the metric tensor. The energy-momentum tensor is defined in terms of the matter Lagrangian via

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{L}_{\rm mat}}{\delta g^{\mu\nu}}\,. \tag{35}$$

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[4] That this seagull contribution dominates the non relativistic cross-section is clear from the feature that

$$\text{Amp}_{\rm Born} \sim 2e^2\frac{\epsilon_f^*\cdot p\epsilon_i\cdot p}{p\cdot k} \sim \frac{\omega}{m}\times\text{Amp}_{\rm seagull} = 2e^2\epsilon_f^*\cdot\epsilon_i. \tag{29}$$

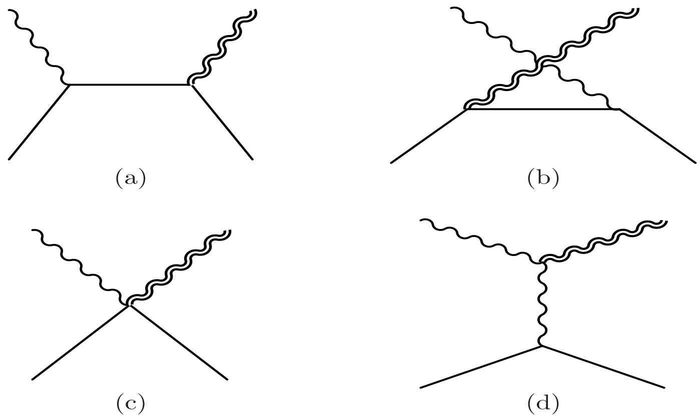

FIG. 2: Diagrams relevant to graviton photoproduction.

The various gravitational vertices can then be found and are given in Appendix B.

We work in harmonic (de Donder) gauge which satisfies, in lowest order,

$$\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h\,, \tag{36}$$

with

$$h = \mathrm{tr} h_{\mu\nu}\,, \tag{37}$$

and in which the graviton propagator has the form

$$D_{\alpha\beta;\gamma\delta}(q) = \frac{i}{q^2+i\epsilon}\frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta}+\eta_{\alpha\delta}\eta_{\beta\gamma}-\eta_{\alpha\beta}\eta_{\gamma\delta})\,. \tag{38}$$

Then just as the (massless) photon is described in terms of a spin-1 polarization vector ϵ_μ which can have projection (helicity) either plus- or minus-1 along the momentum direction, the (massless) graviton is a spin-2 particle which can have the projection (helicity) either plus- or minus-2 along the momentum direction. Since $h_{\mu\nu}$ is a symmetric tensor, it can be described in terms of a direct product of unit spin polarization vectors—

$$\begin{aligned} \text{helicity} &= +2: \quad h^{(2)}_{\mu\nu} = \epsilon^+_\mu\epsilon^+_\nu\,, \\ \text{helicity} &= -2: \quad h^{(-2)}_{\mu\nu} = \epsilon^-_\mu\epsilon^-_\nu\,, \end{aligned} \tag{39}$$

and just as in electromagnetism, there is a gauge condition—in this case Eq. (36)—which must be satisfied. Note that the helicity states given in Eq. (39) are consistent with the gauge requirement since

$$\eta^{\mu\nu}\epsilon^+_\mu\epsilon^+_\nu = \eta^{\mu\nu}\epsilon^-_\mu\epsilon^-_\nu = 0, \quad \text{and} \quad k^\mu\epsilon^\pm_\mu = 0\,. \tag{40}$$

With this background under our belt, we can now examine specific graviton reactions.

4.1. Graviton Photoproduction

We first use the above results to discuss the problem of graviton photoproduction on a spin-1 target—$\gamma+S \to g+S$—for which the relevant four diagrams are shown in Figure 2. The electromagnetic and gravitational vertices needed for the Born terms and photon pole diagrams—Figures 2a, 2b, and 2d—are given in Appendix B. For the photon pole diagram we require the graviton-photon coupling, Eq. (90)[5]

[5] Note that this form agrees with the previously derived form for the massive graviton-spin-1 energy-momentum tensor—Eq. (87)—in the $m \to 0$ limit.

The individual contributions from the four diagrams in Figure 2 are quoted in Appendix C and have a rather complex form. However, when combined, we find a *much* simpler result. The full graviton photoproduction amplitude is found to be proportional to the already calculated Compton amplitude for spin-1—Eq. (14)—times a universal kinematic factor, *i.e.*,

$$\langle p_f; k_f, \epsilon_f \epsilon_f \big| T \big| p_i; k_i, \epsilon_i \rangle = \sum_{i=1}^{4} \text{Amp}\,(\text{Fig.4}(i)) = H \times \Big(\epsilon^*_{f\alpha} \epsilon_{i\beta} T^{\alpha\beta}_{\text{Compton}}(S=1) \Big)\,, \tag{41}$$

where

$$H = \frac{\kappa}{2e} \frac{p_f \cdot F_f \cdot p_i}{k_i \cdot k_f} = \frac{\kappa}{2e} \frac{\epsilon^*_f \cdot p_f \, k_f \cdot p_i - \epsilon^*_f \cdot p_i \, k_f \cdot p_f}{k_i \cdot k_f}\,, \tag{42}$$

and $\epsilon^*_{f\alpha}\epsilon_{i\beta}T^{\alpha\beta}_{\text{Compton}}(S)$ is the spin-1 Compton scattering amplitude calculated in the previous section. The gravitational and electromagnetic gauge invariance of Eq. (41) is obvious, since it follows directly from the gauge invariance already shown for the Compton amplitude together with the explicit gauge invariance of the factor H. The validity of Eq. (41) allows the straightforward calculation of the cross-section by helicity methods since the graviton photoproduction helicity amplitudes are given simply by

$$C^1(ab; cd) = H \times B^1(ab; cd)\,, \tag{43}$$

where $B^1(ab; cd)$ are the Compton helicity amplitudes found in the previous section. We can then evaluate the invariant photoproduction cross-section using

$$\frac{d\sigma^{\text{photo}}_{S=1}}{dt} = \frac{1}{16\pi \big(s - m^2\big)^2} \frac{1}{3} \sum_{a=-,0,+} \frac{1}{2} \sum_{c=-,+} \big| C^1(ab; cd) \big|^2\,, \tag{44}$$

yielding

$$\begin{aligned} \frac{d\sigma^{\text{photo}}_{S=1}}{dt} &= -\frac{e^2\kappa^2(m^4 - su)}{96\pi t \big(s - m^2\big)^4 \big(u - m^2\big)^2} \Big[(m^4 - su + t^2)\big(3(m^4 - su) + t^2\big) \\ &+ \; t^2(t - m^2)(t - 3m^2) \Big]\,. \end{aligned} \tag{45}$$

Since

$$|H| = \frac{\kappa}{e} \left(\frac{m^4 - su}{-2t} \right)^{\frac{1}{2}}\,, \tag{46}$$

the laboratory value of the factor H is

$$|H_{lab}|^2 = \frac{\kappa^2 m^2}{2e^2} \frac{\cos^2 \frac{1}{2}\theta_L}{\sin^2 \frac{1}{2}\theta_L}\,, \tag{47}$$

and the corresponding laboratory cross-section is

$$\begin{aligned} \frac{d\sigma^{\text{photo}}_{\text{lab},S=1}}{d\Omega} &= \big| H_{lab} \big|^2 \frac{d\sigma^{\text{Comp}}_{\text{lab},S=1}}{dt} \\ &= G\alpha \cos^2 \frac{\theta_L}{2} \left(\frac{\omega_f}{\omega_i} \right)^4 \Bigg[\left(\text{ctn}^2 \frac{\theta_L}{2} \cos^2 \frac{\theta_L}{2} + \sin^2 \frac{\theta_L}{2} \right) \left(1 + 2\frac{\omega_i}{m} \sin^2 \frac{\theta_L}{2} \right)^2 \\ &+ \frac{16\omega_i^2}{3m^2} \sin^2 \frac{\theta_L}{2} \left(1 + 2\frac{\omega_i}{m} \sin^2 \frac{\theta_L}{2} \right) + \frac{32\omega_i^4}{3m^4} \sin^6 \frac{\theta_L}{2} \Bigg]\,. \end{aligned} \tag{48}$$

Comparing Eq. (48) with the spin-0 and spin-$\frac{1}{2}$ cross sections found in [8], we see that, just as in Compton scattering, the low-energy laboratory cross-section has a universal form, which is valid for a target of arbitrary spin

$$\frac{d\sigma^{\text{photo}}_{\text{lab},S}}{d\Omega} = G\alpha \cos^2 \frac{\theta_L}{2} \left(\text{ctn}^2 \frac{\theta_L}{2} \cos^2 \frac{\theta_L}{2} + \sin^2 \frac{\theta_L}{2} \right) \left(1 + \mathcal{O}\Big(\frac{\omega_i}{m} \Big) \right)\,. \tag{49}$$

In this case the universality can be understood from the feature that at low-energy the leading contribution to the graviton photoproduction amplitude comes *not* from the seagull, as in Compton scattering, but rather from the photon pole term,

$$\mathrm{Amp}_{\gamma-\mathrm{pole}} \underset{\omega\ll m}{\longrightarrow} \kappa\frac{\epsilon_f^*\cdot\epsilon_i\,\epsilon_f^*\cdot k_i}{2k_f\cdot k_i}\times k_i^\mu\left\langle p_f;S,M_f\middle|J_\mu\middle|p_i;S,M_i\right\rangle . \tag{50}$$

The leading piece of the electromagnetic current has the universal low-energy structure

$$\left\langle p_f;S,M_f\left|J_\mu\right|p_i;S,M_i\right\rangle = \frac{e}{2m}\left(p_f+p_i\right)_\mu\delta_{M_f,M_i}\left(1+\mathcal{O}\Big(\frac{p_f-p_i}{m}\Big)\right), \tag{51}$$

where we have divided by the factor $2m$ to account for the normalization of the target particle. Since $k_i\cdot(p_f+p_i)\underset{\omega\to 0}{\longrightarrow} 2m\omega$, we find the universal low-energy amplitude

$$\mathrm{Amp}^{NR}_{\gamma-pole} = \kappa\, e\,\omega\,\frac{\epsilon_f^*\cdot\epsilon_i\,\epsilon_f^*\cdot k_i}{2k_f\cdot k_i}\,, \tag{52}$$

whereby the helicity amplitudes have the form

$$\mathrm{Amp}^{NR}_{\gamma-pole} = \frac{\kappa\, e}{2\sqrt{2}}\begin{cases}\frac{1}{2}\sin\theta_L\left(\frac{1+\cos\theta_L}{1-\cos\theta_L}\right) = \frac{\cos\frac{\theta_L}{2}}{\sin\frac{\theta_L}{2}}\cos^2\frac{\theta_L}{2} & ++=--\,,\\ \frac{1}{2}\sin\theta_L\left(\frac{1-\cos\theta_L}{1-\cos\theta_L}\right) = \frac{\cos\frac{\theta_L}{2}}{\sin\frac{\theta}{2}}\sin^2\frac{\theta_L}{2} & +-=-+\,.\end{cases} \tag{53}$$

Squaring and averaging, summing over initial, final spins we find then

$$\frac{d\sigma^{\mathrm{photo}}_{\mathrm{lab},S}}{d\Omega} \underset{\omega\ll m}{\longrightarrow} G\,\alpha\cos^2\frac{\theta_L}{2}\left[\left(\mathrm{ctn}^2\frac{\theta_L}{2}\cos^2\frac{\theta_L}{2}+\sin^2\frac{\theta_L}{2}\right)\left(1+\mathcal{O}\left(\frac{\omega_i}{m}\right)\right)\right]. \tag{54}$$

as determined above—*cf.* Eq. (49).

Comparing the individual contributions from Appendix C with the simple forms above, the power of factorization is obvious and, as we shall see in the next section, permits the straightforward evaluation of even more complex reactions such as gravitational Compton scattering.

4.2. Gravitational Compton Scattering

In the previous section we observed the power of factorization in the context of graviton photoproduction on a spin-1 target in that we only needed to calculate the simpler Compton scattering process rather than to consider the full gravitational interaction description. In this section we consider an even more challenging example, that of gravitational Compton scattering—$g+S\to g+S$—from a spin-1 target, for which there exist the four diagrams shown in Figure 3.

The contributions from the four individual diagrams can be calculated using the graviton vertices given in Appendix B and are quoted in Appendix C. Each of the four diagrams has a rather complex form. Again however, when added together the total simplifies enormously. Defining the kinematic factor

$$Y = \frac{\kappa^2}{8e^4}\frac{p_i\cdot k_i\,p_i\cdot k_f}{k_i\cdot k_f} = \frac{\kappa^4}{16e^4}\frac{(s-m^2)\,(u-m^2)}{t}\,, \tag{55}$$

the sum of the four diagrams is found to be given by

$$\begin{aligned}\left\langle p_f,\epsilon_B;k_f,\epsilon_f\epsilon_f\left|\mathrm{Amp}_{\mathrm{grav}}\right|p_i,\epsilon_A;k_i,\epsilon_i\epsilon_i\right\rangle_S &= \sum_{i=1}^{4}\mathrm{Amp}\,(\mathrm{Fig.5}(i))\\ &= Y\times]\left\langle p_f,\epsilon_B;k_i,\epsilon_f\left|\mathrm{Amp}_{\mathrm{em}}\right|p_i,\epsilon_A;k_i,\epsilon_i\right\rangle_S\times\left\langle p_f;k_i,\epsilon_f\left|\mathrm{Amp}_{\mathrm{em}}\right|p_i;k_i,\epsilon_i\right\rangle_{S=0}\,,\end{aligned} \tag{56}$$

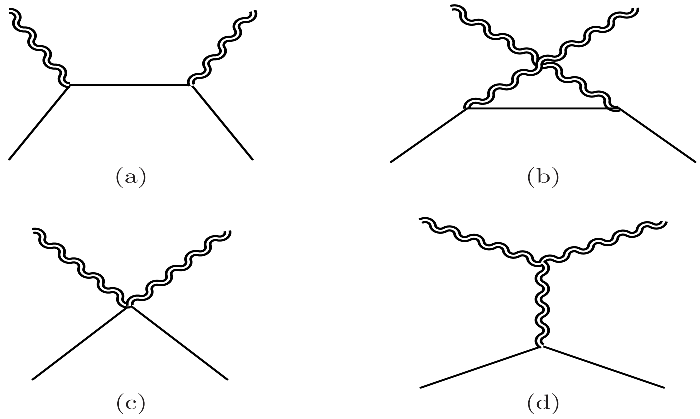

FIG. 3: Diagrams relevant for gravitational Compton scattering.

with $S=1$, where

$$\langle p_f;k_i,\epsilon_f|\mathrm{Amp}_{\rm em}|p_i;k_i,\epsilon_i\rangle_{S=0}=2e^2\left[\frac{\epsilon_i\cdot p_i\,\epsilon_f^*\cdot p_f}{p_i\cdot k_i}-\frac{\epsilon_i\cdot p_f\,\epsilon_f^*\cdot p_i}{p_i\cdot k_f}-\epsilon_f^*\cdot\epsilon_i\right], \tag{57}$$

is the Compton amplitude for a spinless target.

In [8] the identity Eq. (56) was verified for simpler cases of $S=0$ and $S=\frac{1}{2}$. This relation is a consequence of the general connections between gravity and gauge theory tree-level amplitudes derived using string-based methods as explained in [17]. Here we have demonstrated its validity for the much more complex case of spin-1 scattering. The corresponding cross-section can be calculated by helicity methods using

$$D^1(ab;cd)=Y\times B^1(ab;cd)\times A^0(cd)\,, \tag{58}$$

where $D^1(ab;cd)$ is the spin-1 helicity amplitude for gravitational Compton scattering, $B^1(ab;cd)$ is the ordinary spin-1 Compton helicity amplitude calculated in section 3, and $A^0(cd)$ are the helicity amplitudes for spin zero Compton scattering arising from Eq. (57) calculated in [8], as given in Appendix A. Using Eq. (56) the invariant cross-section for unpolarized spin-1 gravitational Compton scattering

$$\frac{d\sigma^{\rm g-Comp}_{S=1}}{dt}=\frac{1}{16\pi\big(s-m^2\big)^2}\,\frac{1}{3}\sum_{a=-,0,+}\frac{1}{2}\sum_{c=-,+}\big|D^1(ab;cd)\big|^2\,, \tag{59}$$

is found to be

$$\begin{aligned}\frac{d\sigma^{\rm g-Comp}_{S=1}}{dt}&=\frac{\kappa^4}{768\pi\big(s-m^2\big)^4\big(u-m^2\big)^2t^2}\,\big[(m^4-su)^2\big(3(m^4-su)+t^2\big)(m^4-su+t^2)\big)\\&+\;m^4t^4(3m^2-t)(m^2-t)\big]\,.\end{aligned} \tag{60}$$

and this form can be compared with the unpolarized gravitational Compton cross-sections found in [8].

The corresponding laboratory frame cross-section is

$$\begin{aligned}\frac{d\sigma^{\rm g-Comp}_{{\rm lab},S=1}}{d\Omega}&=G^2m^2\frac{\omega_f^4}{\omega_i^4}\left[\left(\mathrm{ctn}^4\frac{\theta_L}{2}\cos^4\frac{\theta_L}{2}+\sin^4\frac{\theta_L}{2}\right)\left(1+2\frac{\omega_i}{m}\sin^2\frac{\theta_L}{2}\right)^2\right.\\&+\;\frac{16}{3}\frac{\omega_i^2}{m^2}\left(\cos^6\frac{\theta_L}{2}+\sin^6\frac{\theta_L}{2}\right)\left(1+2\frac{\omega_i}{m}\sin^2\frac{\theta_L}{2}\right)\\&+\left.\;\frac{16}{3}\frac{\omega_i^4}{m^4}\sin^2\frac{\theta_L}{2}\left(\cos^4\frac{\theta_L}{2}+\sin^4\frac{\theta_L}{2}\right)\right],\end{aligned} \tag{61}$$

We observe that the low-energy laboratory cross-section has the universal form for any spin

$$\frac{d\sigma^{\rm g-Comp}_{{\rm lab},S}}{d\Omega} = G^2m^2\left[\left({\rm ctn}^4\frac{\theta_L}{2}\cos^4\frac{\theta_L}{2}+\sin^4\frac{\theta_L}{2}\right)\left(1+\mathcal{O}\Big(\frac{\omega_i}{m}\Big)\right)\right]. \tag{62}$$

It is interesting to note that the "dressing" factor for the leading (++) helicity Compton amplitude—

$$\big|Y\big|\,\big|A^{++}\big| \;=\; \frac{\kappa^2}{2e^2}\;\frac{m^4-su}{-t}\;\;\stackrel{\rm lab}{\longrightarrow}\;\;\frac{\kappa^2m^2}{2e^2}\;\frac{\cos^2\frac{\theta_L}{2}}{\sin^2\frac{\theta_L}{2}}\,, \tag{63}$$

—is simply the square of the photoproduction dressing factor H, as might intuitively be expected since now *both* photons must be dressed in going from the Compton to the gravitational Compton cross-section.[6] In this case the universality of the nonrelativistic cross-section follows from the leading contribution arising from the graviton pole term

$${\rm Amp}_{g-pole}\;\underset{\omega\ll m}{\longrightarrow}\;\frac{\kappa}{4k_f\cdot k_i}\;\left(\epsilon_f^*\cdot\epsilon_i\right)^2\;\left(k_f^\mu k_f^\nu+k_i^\mu k_i^\nu\right)\;\frac{\kappa}{2}\;\left\langle p_f;S,M_f\left|T_{\mu\nu}\right|p_i;S,M_i\right\rangle. \tag{65}$$

Here the matrix element of the energy-momentum tensor has the universal low-energy structure

$$\frac{\kappa}{2}\left\langle p_f;S,M_f\left|T_{\mu\nu}\right|p_i;S,M_i\right\rangle=\frac{\kappa}{4m}\left(p_{f\mu}p_{i\nu}+p_{f\nu}p_{i\mu}\right)\delta_{M_f,M_i}\left(1+\mathcal{O}\Big(\frac{p_f-p_i}{m}\Big)\right), \tag{66}$$

where we have divided by the factor $2m$ to account for the normalization of the target particle. We find then the universal form for the leading graviton pole amplitude

$${\rm Amp}_{g-pole}\;\underset{\rm non-rel}{\longrightarrow}\;\frac{\kappa^2}{8m\,k_f\cdot k_i}\;\left(\epsilon_f^*\cdot\epsilon_i\right)^2\left(p_i\cdot k_f\,p_f\cdot k_f+p_i\cdot k_i\,p_f\cdot k_i\right)\delta_{M_f,M_i}\,. \tag{67}$$

Since $p\cdot k\underset{\omega\ll m}{\longrightarrow}m\omega$ the helicity amplitudes become

$${\rm Amp}^{\rm NR}_{g-pole}=4\pi Gm\begin{cases}\frac{\left(1+\cos\theta_L\right)^2}{2\left(1-\cos\theta_L\right)}=\frac{\cos^4\frac{\theta_L}{2}}{\sin^2\frac{\theta_L}{2}} & ++=--\,,\\ \frac{\left(1-\cos\theta_L\right)^2}{2\left(1-\cos\theta_L\right)}=\frac{\sin^4\frac{\theta_L}{2}}{\sin^2\frac{\theta_L}{2}} & +-=-+\,.\end{cases} \tag{68}$$

Squaring and averaging,summing over initial,final spins we find

$$\frac{d\sigma^{\rm g-Comp}_{{\rm lab},S}}{d\Omega}\;\underset{\omega\ll m}{\longrightarrow}\;G^2\,m^2\left[{\rm ctn}^4\frac{\theta_L}{2}\cos^4\frac{\theta_L}{2}+\sin^4\frac{\theta_L}{2}\right], \tag{69}$$

as found in Eq. (62) above.

5. GRAVITON-PHOTON SCATTERING

In the previous sections we have generalized the results of [8] to the case of a massive spin-1 target. Here we show how these spin-1 results can be used to calculate the cross-section for photon-graviton scattering. In the Compton scattering calculation we assumed that the spin-1 target had charge e. However, the photon couplings to the graviton are identical to those of a graviton coupled to a charged spin-1 system in the massless limit, and one might assume then that, since the results of the gravitational Compton scattering are independent of charge, the graviton-photon cross-section can be calculated by simply taking the $m\to 0$ limit of the graviton-spin-1 cross-section. Of course,

---

[6] In the case of $+-$ helicity the "dressing" factor is

$$\big|Y\big|\,\big|A^{+-}\big|=\frac{\kappa^2}{2e^2}\;m^2\,. \tag{64}$$

so that the non-leading contributions will have different dressing factors.

the laboratory cross-section no longer makes sense since the photon cannot be brought to rest, but the invariant cross-section is well defined

$$\frac{d\sigma^{\rm g-Comp}_{S=1}}{dt} \underset{m\to 0}{\longrightarrow} \frac{4\pi\, G^2\left(3s^2u^2-4t^2su+t^4\right)}{3s^2t^2}=\frac{4\pi G^2(s^4+u^4+s^2u^2)}{3s^2t^2}\,, \tag{70}$$

and it might be (naively) assumed that Eq. (70) is the graviton-photon scattering cross-section. However, this is *not* the case and the resolution of this problem involves some interesting physics.

We begin by noting that in the massless limit the only nonvanishing helicity amplitudes are

$$\begin{aligned} D^1(++;++)_{m=0} &= D^1(--;--)_{m=0}=8\pi\, G\,\frac{s^2}{t}\,,\\ D^1(--;++)_{m=0} &= D^1(++;--)_{m=0}=8\pi\, G\,\frac{u^2}{t}\,,\\ D^1(00;++)_{m=0} &= D^1(00;--)_{m=0}\ =8\pi\, G\,\frac{su}{t}\,, \end{aligned} \tag{71}$$

which lead to the cross-section

$$\begin{aligned} \frac{d\sigma^{\rm g-Comp}_{S=1}}{dt} &= \frac{1}{16\pi s^2}\,\frac{1}{3}\sum_{a=+,0,-}\frac{1}{2}\sum_{c=+,-}\left|D^1(ab;cd)\right|^2\\ &= \frac{1}{16\pi s^2}\,\frac{1}{3\cdot 2}\,(8\pi G)^2\times 2\times\left[\frac{s^4}{t^2}+\frac{u^4}{t^2}+\frac{s^2u^2}{t^2}\right]\\ &= \frac{4\pi}{3}\,G^2\frac{s^4+u^4+s^2u^2}{s^2t^2}\,, \end{aligned} \tag{72}$$

in agreement with Eq. (70). However, this result reveals the problem. We know that in Coulomb gauge the photon has only *two* transverse degrees of freedom, corresponding to positive and negative helicity—there exists *no* longitudinal degree of freedom. Thus the correct photon-graviton cross-section is obtained by deleting the contribution from the $D^1(00;++)$ and $D^1(00;--)$ multipoles

$$\begin{aligned} \frac{d\sigma_{g\gamma}}{dt} &= \frac{1}{16\pi s^2}\,\frac{1}{3}\sum_{a=+,-}\frac{1}{2}\sum_{c=+,-}\left|D^1(ab;cd)\right|^2\\ &= \frac{1}{16\pi s^2}\,\frac{1}{2\cdot 2}\left(8\pi G\right)^2\times 2\times\left[\frac{s^4}{t^2}+\frac{u^4}{t^2}\right]=2\pi\, G^2\,\frac{s^4+u^4}{s^2t^2}\,, \end{aligned} \tag{73}$$

which agrees with the value calculated via conventional methods by Skobelev [18]. Alternatively, since in the center of mass frame

$$\frac{dt}{d\Omega}=\frac{\omega_{\rm CM}}{\pi}\,, \tag{74}$$

we can write the center of mass graviton-photon cross-section in the form

$$\frac{d\sigma_{\rm CM}}{d\Omega}=2\,G^2\,\omega^2_{\rm CM}\left(\frac{1+\cos^8\frac{\theta_{\rm CM}}{2}}{\sin^4\frac{\theta_{\rm CM}}{2}}\right)\,, \tag{75}$$

again in agreement with the value given by Skobelev [18].

So what has gone wrong here? Ordinarily in the massless limit of a spin-1 system, the longitudinal mode decouples because the zero helicity spin-1 polarization vector becomes

$$\epsilon^0_\mu \underset{m\to 0}{\longrightarrow} \frac{1}{m}\left(p,\left(p+\frac{m^2}{2p}+\ldots\right)\hat{z}\right)=\frac{1}{m}p_\mu+\left(0,\frac{m}{2p}\hat{z}\right)+\ldots \tag{76}$$

However, the term proportional to p_μ vanishes when contracted with a conserved current by gauge invariance while the term in $\frac{m}{2p}$ vanishes in the massless limit. However, what takes place when *two* longitudinal spin-1 particles are present is that the product of longitudinal polarization vectors is proportional to $1/m^2$, while the correction term to the four-momentum p_μ is $\mathcal{O}(m^2)$ so that the product is nonvanishing in the massless limit and this is why the multipole $D(00;++)_{m=0}=D(00;--)_{m=0}$ is nonzero. One deals with this problem by simply omitting the longitudinal degree of freedom explicitly, as done above.

5.1. Extra Credit

Before leaving this section it is interesting to note that graviton-graviton scattering can be treated in a parallel fashion. That is, the graviton-graviton scattering amplitude can be obtained by dressing the product of two massless spin-1 Compton amplitudes [19]—

$$\begin{aligned}&\left\langle p_f, \epsilon_B\epsilon_B; k_f, \epsilon_f\epsilon_f \left|\mathrm{Amp}^{\mathrm{tot}}_{grav}\right| p_i\epsilon_A\epsilon_A; k_i, \epsilon_i\epsilon_i\right\rangle_{m=0,S=2}\\ &= Y \times \langle p_f, \epsilon_B; k_f, \epsilon_f \,|\mathrm{Amp}^{\mathrm{Comp}}_{em}|\, p_i, \epsilon_A; k_i\epsilon_i\rangle_{m=0,S=1}\\ &\times\langle p_f, \epsilon_B; k_f, \epsilon_f \,|\mathrm{Amp}^{\mathrm{Comp}}_{em}|\, p_i, \epsilon_A; k_i\epsilon_i\rangle_{m=0,S=1}\,. \end{aligned} \tag{77}$$

Then for the helicity amplitudes we have

$$E^2(++;++)_{m=0} = Y\big(B^1(++;++)_{m=0}\big)^2\,, \tag{78}$$

where $E^2(++;++)$ is the graviton-graviton $++;++$ helicity amplitude while $B^1(++;++)$ is the corresponding spin-1 Compton helicity amplitude. Thus we find

$$E^2(++;++)_{m=0} = \frac{\kappa^2}{16e^4}\,\frac{su}{t} \times \left(2e^2\,\frac{s}{u}\right)^2 = 8\pi\, G\,\frac{s^3}{ut}\,, \tag{79}$$

which agrees with the result calculated via conventional methods [20].

6. THE FORWARD CROSS-SECTION

Before closing, we note some intriguing physics associated with the forward-scattering limit. In this limit, *i.e.*, $\theta_L \to 0$, in the laboratory frame, the Compton cross-section evaluated in section 3 has a universal structure independent of the spin S of the massive target

$$\lim_{\theta_L\to 0}\frac{d\sigma^{\mathrm{Comp}}_{\mathrm{lab},S}}{d\Omega} = \frac{\alpha^2}{2m^2}\,, \tag{80}$$

reproducing the well-known Thomson scattering cross-section.

For graviton photoproduction, however, the small angle limit is very different, since the forward-scattering cross-section is divergent—the small angle limit of the graviton photoproduction of section 4.1 is given by

$$\lim_{\theta_L\to 0}\frac{d\sigma^{\mathrm{photo}}_{\mathrm{lab},S}}{d\Omega} = \frac{4G\alpha}{\theta_L^2}\,, \tag{81}$$

and arises from the photon pole in figure 2(d).

The small angle limit of the gravitational Compton cross-section derived in section 4.2 is given by

$$\lim_{\theta_L\to 0}\frac{d\sigma^{\mathrm{g-Comp}}_{lab,S}}{d\Omega} = \frac{16G^2m^2}{\theta_L^4}\,. \tag{82}$$

where again the limit is independent of the spin S of the matter field. The behavior in Eqs. (82) is due to the graviton pole in figure 3(d), and is typical of the small-angle behavior of Rutherford scattering in a Coulomb potential.

7. CONCLUSION

In [8] it was demonstrated that the gravitational interaction of a charged spin-0 or spin-$\frac{1}{2}$ particle is greatly simplified by use of factorization, which asserts that the gravitational amplitudes can be written as the product of corresponding electromagnetic amplitudes multiplied by a universal kinematic factor. In the present work we demonstrated that the same simplification applies when the target particle carries spin-1. Specifically, we evaluated the graviton photoproduction and graviton Compton scattering amplitudes explicitly using direct and factorized techniques and demonstrated that they are identical. However, the factorization methods are *enormously* simpler, since they require only electromagnetic calculations and eliminate the need to employ less familiar and more cumbersome tensor quantities. As a result it is now straightforward to include graviton interactions in a quantum mechanics course in order to stimulate student interest and allowing access to various cosmological applications.

We studied the massless limit of the spin-1 system and showed how the use of factorization permits a relatively simple calculation of graviton-photon scattering, discussing a subtlety in this graviton-photon calculation having to do with the feature that the spin-1 system must change from three to two degrees of freedom when $m \to 0$ and we explained why the zero mass limit of the spin-1 gravitational Compton scattering amplitude does not correspond to that for photon scattering. The graviton-photon cross section may possess interesting implications for the attenuation of gravitational waves in the cosmos [23]. We also calculated the graviton-graviton scattering amplitude.

Finally, we discussed the main feature of the forward cross-section for each process studied in this paper. Both the Compton and the gravitational Compton scattering have the expected $1/\theta_L^4$ behavior, while graviton photoproduction has a different shape that could in principle lead to an interesting new experimental signature of a graviton scattering on matter—$\sim 1/\theta_L^2$. Again this result has potentially intriguing implications for the photoproduction of gravitons from stars [24, 25].

Appendix A: Compton Scattering Helicity Amplitudes

For a spin-0 target we have

$$A^0(cd) \equiv L(s,u)H^0(cd) \tag{83}$$

where

$$L(s,u) = \frac{2e^2}{(s-m^2)(u-m^2)}$$

and

$$\begin{aligned} H^0(++) &= m^4 - su\,, \\ H^0(+-) &= -m^2 t\,, \end{aligned} \tag{84}$$

For a spin-1 target we find

$$B^1(ab;cd) \equiv K(s,u)H^1(ab;cd) \tag{85}$$

where

$$K(s,u) = \frac{2e^2}{(s-m^2)^3(u-m^2)}$$

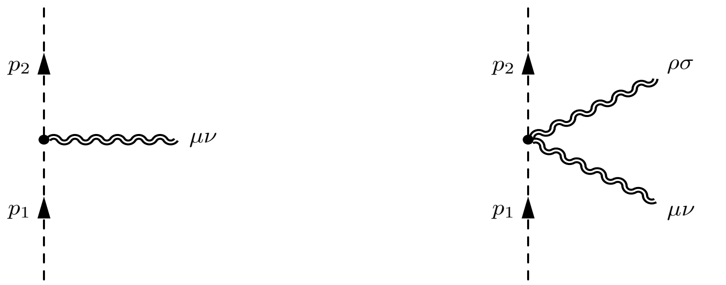

FIG. 4: (a) The one-graviton and (b) two-graviton emission vertices from a vector particle.

and

$$
\begin{aligned}
\left|H^1(++;++)\right| &= \left|H^1(--;--)\right| = ((s-m^2)^2+m^2t)^2\,,\\
\left|H^1(++;--)\right| &= \left|H^1(--;++)\right| = (m^4-su)^2\,,\\
\left|H^1(+-;+-)\right| &= \left|H^1(-+;-+)\right| = m^4t^2\,,\\
\left|H^1(+-;-+)\right| &= \left|H^1(-+;+-)\right| = s^2t^2\,,\\
\left|H^1(++;+-)\right| &= \left|H^1(--;-+)\right| = \left|H^1(++;-+)\right| = \left|H^1(--;+-)\right|\,,\\
&= m^2t(m^4-su)\,,\\
\left|H^1(+-;++)\right| &= \left|H^1(-+;--)\right| = \left|H^1(-+;++)\right| = \left|H^1(+-;--)\right|\,,\\
&= m^2t(m^4-su)\,,\\
\left|H^1(0+;++)\right| &= \left|H^1(0-;--)\right| = \left|H^1(+0;++)\right| = \left|H^1(-0;--)\right|\,,\\
&= \sqrt{2}m(tm^2+(s-m^2)^2)\sqrt{-t(m^4-su)}\,,\\
\left|H^1(0+;+-)\right| &= \left|H^1(0-;-+)\right| = \left|H^1(+0;-+)\right| = \left|H^1(-0;+-)\right|\,,\\
&= \sqrt{2}mst\sqrt{-t(m^4-su)}\,,\\
\left|H^1(0+;-+)\right| &= \left|H^1(0-;+-)\right| = \left|H^1(+0;+-)\right| = \left|H^1(-0;-+)\right|\,,\\
&= \sqrt{2}m^3t\sqrt{-t(m^4-su)}\,,\\
\left|H^1(0+;--)\right| &= \left|H^1(0-;++)\right| = \left|H^1(+0;--)\right| = \left|H^1(-0;++)\right|\,,\\
&= \sqrt{2}m\sqrt{-t(m^4-su)^3}\,,\\
\left|H^1(00;++)\right| &= \left|H^1(00;--)\right| = (2tm^2+(s-m^2)^2)(m^4-su)\,,\\
\left|H^1(00;+-)\right| &= \left|H^1(00;-+)\right| = m^2t((s-m^2)^2+2st)\,. \qquad (86)
\end{aligned}
$$

Appendix B: Gravitational Vertices

Using the formalism discussed in section 3, the needed gravitational couplings can be obtained. The spin 1-one single graviton emission vertex shown in Fig. 4(a) is

$$
\begin{aligned}
\left\langle p_f,\epsilon_B\left|V^{(1)\mu\nu}_{grav}\right|p_i,\epsilon_A\right\rangle_{S=1} &= -i\,\frac{\kappa}{2}\Big[\epsilon^*_B\cdot\epsilon_A\big(p_i^\mu p_f^\nu+p_i^\nu p_f^\mu\big)-\epsilon^*_B\cdot p_i\,\big(p_f^\mu\epsilon_A^\nu+\epsilon_A^\mu p_f^\nu\big)\\
&- \epsilon_A\cdot p_f\big(p_i^\nu\epsilon_B^{*\mu}+p_i^\mu\epsilon_B^{*\nu}\big)+\big(p_f\cdot p_i-m^2\big)\big(\epsilon_A^\mu\epsilon_B^{*\nu}+\epsilon_A^\nu\epsilon_B^{*\mu}\big)\\
&- \eta^{\mu\nu}\left[\big(p_i\cdot p_f-m^2\big)\epsilon^*_B\cdot\epsilon_A-\epsilon^*_B\cdot p_i\,\epsilon_A\cdot p_f\right]\Big]\,, \qquad (87)
\end{aligned}
$$

There also exista a two-graviton (seagull) vertex shown in Fig. 4(b), which are found by expanding the stress-energy

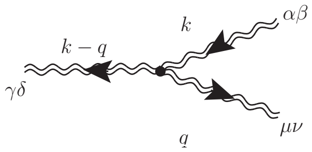

FIG. 5: The three graviton vertex

tensor to second order in $h_{\mu\nu}$.

$$
\begin{aligned}
\langle p_f,\epsilon_B;k_f\left|V^{(2)\,\mu\nu,\rho\sigma}_{grav}\right|p_i,\epsilon_A;k_i\rangle_{S=\overline{1}} &= -i\,\frac{\kappa^2}{4}\Bigg\{ \\
&+\ \left[p_{i\beta}p_{f\alpha}-\eta_{\alpha\beta}(p_i\cdot p_f-m^2)\right]\left(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma}\right) \\
&+\ \eta_{\mu\rho}\big[\eta_{\alpha\beta}\big(p_{i\nu}p_{f\sigma}+p_{i\sigma}p_{f\nu}\big)-\eta_{\alpha\nu}p_{i\beta}p_{f\sigma}-\eta_{\beta\nu}p_{i\sigma}p_{f\alpha} \\
&-\ \eta_{\beta\sigma}p_{i\nu}p_{f\alpha}-\eta_{\alpha\sigma}p_{i\beta}p_{f\nu}+(p_i\cdot p_f-m^2)\big(\eta_{\alpha\nu}\eta_{\beta\sigma}+\eta_{\alpha\sigma}\eta_{\beta\nu}\big)\big] \\
&+\ \eta_{\mu\sigma}\big[\eta_{\alpha\beta}\big(p_{i\nu}p_{f\rho}+p_{i\rho}p_{f\nu}\big)-\eta_{\alpha\nu}p_{i\beta}p_{f\rho}-\eta_{\beta\nu}p_{i\rho}p_{f\alpha} \\
&-\ \eta_{\beta\rho}p_{i\nu}p_{f\alpha}-\eta_{\alpha\rho}p_{i\beta}p_{f\nu}+(p_i\cdot p_f-m^2)\eta_{\alpha\nu}\eta_{\beta\rho}+\eta_{\alpha\rho}\eta_{\beta\nu}\big)\big] \\
&+\ \eta_{\nu\rho}\big[\eta_{\alpha\beta}\big(p_{i\mu}p_{f\sigma}+p_{i\sigma}p_{f\mu}\big)-\eta_{\alpha\mu}p_{i\beta}p_{f\sigma}-\eta_{\beta\mu}p_{i\sigma}p_{f\alpha} \\
&-\ \eta_{\beta\sigma}p_{i\mu}p_{f\alpha}-\eta_{\alpha\sigma}p_{i\beta}p_{f\mu}+(p_i\cdot p_f-m^2)\big(\eta_{\alpha\mu}\eta_{\beta\sigma}+\eta_{\alpha\sigma}\eta_{\beta\mu}\big)\big] \\
&+\ \eta_{\nu\sigma}\big[\eta_{\alpha\beta}\big(p_{i\mu}p_{f\rho}+p_{i\rho}p_{f\mu}\big)-\eta_{\alpha\mu}p_{i\beta}p_{f\rho}-\eta_{\beta\mu}p_{i\rho}p_{f\alpha} \\
&-\ \eta_{\beta\rho}p_{i\mu}p_{f\alpha}-\eta_{\alpha\rho}p_{i\beta}p_{f\mu}+(p_i\cdot p_f-m^2)\big(\eta_{\alpha\mu}\eta_{\beta\rho}+\eta_{\alpha\rho}\eta_{\beta\mu}\big)\big] \\
&-\ \eta_{\mu\nu}\big[\eta_{\alpha\beta}\big(p_{i\rho}p_{f\sigma}+p_{i\sigma}p_{f\rho}\big)-\eta_{\alpha\rho}p_{i\beta}p_{f\sigma}-\eta_{\beta\rho}p_{i\sigma}p_{f\alpha} \\
&-\ \eta_{\beta\sigma}p_{i\rho}p_{f\alpha}-\eta_{\alpha\sigma}p_{i\beta}p_{f\rho}+\big(p_i\cdot p_f-m^2\big)\big(\eta_{\alpha\rho}\eta_{\beta\sigma}+\eta_{\beta\rho}\eta_{\alpha\sigma}\big)\big] \\
&-\ \eta_{\rho\sigma}\big[\eta_{\alpha\beta}\big(p_{i\mu}p_{f\nu}+p_{i\nu}p_{f\mu}\big)-\eta_{\alpha\mu}p_{i\beta}p_{f\nu}-\eta_{\beta\mu}p_{i\nu}p_{f\alpha} \\
&-\ \eta_{\beta\nu}p_{i\mu}p_{f\alpha}-\eta_{\alpha\nu}p_{i\beta}p_{f\mu}+(p_i\cdot p_f-m^2)\big(\eta_{\alpha\mu}\eta_{\beta\nu}+\eta_{\beta\mu}\eta_{\alpha\nu}\big)\big] \\
&+\ \big(\eta_{\alpha\rho}p_{i\mu}-\eta_{\alpha\mu}p_{i\rho}\big)\big(\eta_{\beta\sigma}p_{f\nu}-\eta_{\beta\mu}p_{f\sigma}\big) \\
&+\ \big(\eta_{\alpha\sigma}p_{i\nu}-\eta_{\alpha\nu}p_{i\sigma}\big)\eta_{\beta\rho}p_{f\mu}-\eta_{\beta\mu}p_{f\rho}\big) \\
&+\ \big(\eta_{\alpha\sigma}p_{i\mu}-\eta_{\alpha\mu}p_{i\sigma}\big)\big(\eta_{\beta\rho}p_{f\nu}-\eta_{\beta\nu}p_{f\rho}\big) \\
&+\ \big(\eta_{\alpha\rho}p_{i\nu}-\eta_{\alpha\nu}p_{i\rho}\big)\big(\eta_{\beta\sigma}p_{f\mu}-\eta_{\beta\mu}p_{f\sigma}\big)\big\}\,\epsilon^{\alpha}_{A}(\epsilon^{\beta}_{B})^{*}\Bigg\}. \qquad (88)
\end{aligned}
$$

We also require the triple graviton vertex of Fig. 5, which was given in [8].

$$
\begin{aligned}
\tau^{\mu\nu}_{\alpha\beta,\gamma\delta}(k,q) &= -\frac{i\,\kappa}{2}\left[\left(I_{\alpha\beta,\gamma\delta}-\frac{1}{2}\eta_{\alpha\beta}\eta_{\gamma\delta}\right)\left[k^\mu k^\nu+(k-q)^\mu(k-q)^\nu+q^\mu q^\nu-\frac{3}{2}\eta^{\mu\nu}q^2\right]\right. \\
&+\ 2q_\lambda q_\sigma\left[I^{\lambda\sigma,}{}_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta}+I^{\lambda\sigma,}{}_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}-I^{\lambda\mu,}{}_{\alpha\beta}I^{\sigma\nu,}{}_{\gamma\delta}-I^{\sigma\nu,}{}_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta}\right] \\
&+\ \Big[q_\lambda q^\mu\big(\eta_{\alpha\beta}I^{\lambda\nu,}{}_{\gamma\delta}+\eta_{\gamma\delta}I^{\lambda\nu,}{}_{\alpha\beta}\big)+q_\lambda q^\nu\big(\eta_{\alpha\beta}I^{\lambda\mu,}{}_{\gamma\delta}+\eta_{\gamma\delta}I^{\lambda\mu,}{}_{\alpha\beta}\big) \\
&-\ q^2\big(\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta}+\eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}\big)-\eta^{\mu\nu}q^\lambda q^\sigma\big(\eta_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}+\eta_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}\big)\Big] \\
&+\ \Big[2q^\lambda\big(I^{\sigma\nu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}(k-q)^\mu+I^{\sigma\mu,}{}_{\gamma\delta}I_{\alpha\beta,\lambda\sigma}(k-q)^\nu-I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}k^\mu-I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\lambda\sigma}k^\nu\big) \\
&+\ q^2\big(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu}+I_{\alpha\beta,\sigma}{}^{\nu}I^{\sigma\mu,}{}_{\gamma\delta}\big)+\eta^{\mu\nu}q^\lambda q_\sigma\big(I_{\alpha\beta,\lambda\rho}I^{\rho\sigma,}{}_{\gamma\delta}+I_{\gamma\delta,\lambda\rho}I^{\rho\sigma,}{}_{\alpha\beta}\big)\Big] \\
&+\ \Big[\big(k^2+(k-q)^2\big)\Big(I^{\sigma\mu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\nu}+I^{\sigma\nu,}{}_{\alpha\beta}I_{\gamma\delta,\sigma}{}^{\mu}-\frac{1}{2}\eta^{\mu\nu}\big(I_{\alpha\beta,\gamma\delta}-\frac{1}{2}\eta_{\alpha\beta}\eta_{\gamma\delta}\big)\ \big) \\
&-\ \left.\big(k^2\eta_{\alpha\beta}I^{\mu\nu,}{}_{\gamma\delta}+(k-q)^2\eta_{\gamma\delta}I^{\mu\nu,}{}_{\alpha\beta}\big)\Big]\ \right]. \qquad (89)
\end{aligned}
$$

where

$$I_{\alpha\beta,\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma})$$

The photon-graviton coupling was given in [8]

$$\begin{aligned}\langle k_f, \epsilon_f \left| V^{(\gamma)\mu\nu}_{grav} \right| k_i, \epsilon_i \rangle \; &= \; i\,\frac{\kappa}{2}\,\Big[\epsilon_f^* \cdot \epsilon_i \big(k_i^\mu k_f^\nu + k_i^\nu k_f^\mu\big) - \epsilon_f^* \cdot k_i \big(k_f^\mu \epsilon_i^\nu + \epsilon_i^\mu k_f^\nu\big) \\ &- \; \epsilon_i \cdot k_f \big(k_i^\nu \epsilon_f^{*\mu} + k_i^\mu \epsilon_f^{*\nu}\big) + k_f \cdot k_i \big(\epsilon_i^\mu \epsilon_f^{*\nu} + \epsilon_i^\nu \epsilon_f^{*\mu}\big) \\ &- \; \eta^{\mu\nu}\left[k_f \cdot k_i \epsilon_f^* \cdot \epsilon_i - \epsilon_f^* \cdot k_i \epsilon_i \cdot k_f\right]\Big]. \end{aligned} \tag{90}$$

Finally, we need the seagull vertex which arises from the feature that the energy-momentum tensor depends on p_i, p_f and therefore yields a contact interaction when the minimal substitution is made, yielding the spin-1 seagull amplitude shown in Fig. 2c.

$$\begin{aligned}&\langle p_f, \epsilon_B; k_f, \epsilon_f \epsilon_f \left| T \right| p_i, \epsilon_A; k_i, \epsilon_i \rangle_{\rm seagull} = \frac{i}{2}\,\kappa\, e\,\Big[\epsilon_f^* \cdot (p_f + p_i)\,\epsilon_f^* \cdot \epsilon_i\, \epsilon_B^* \cdot \epsilon_A \\ - \; &\epsilon_B^* \cdot \epsilon_i\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot \epsilon_A - \epsilon_B^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_i\, \epsilon_f^* \cdot \epsilon_A - \epsilon_A \cdot \epsilon_i\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_B^* \\ - \; &\epsilon_A \cdot p_f\, \epsilon_f^* \cdot \epsilon_i\, \epsilon_f^* \cdot \epsilon_B^* - \epsilon_f^* \cdot \epsilon_A\, \epsilon_i \cdot (p_f + p_i)\, \epsilon_f^* \cdot \epsilon_B^* \Big]. \end{aligned} \tag{91}$$

Appendix C: Graviton Scattering Amplitudes

In this section we list the independent contributions to the various graviton scattering amplitudes which must be added in order to produce the complete and gauge invariant amplitudes quoted in the text. We leave it to the (perspicacious) reader to perform the appropriate additions and to verify the equivalence of the factorized forms shown earlier.

Graviton Photoproduction: spin-1

For the case of graviton photoproduction, we find the four contributions, *cf.* Fig. 2,

$$\begin{aligned}\text{Fig.2(a)}: \; \text{Amp}_a(S=1) = \frac{\kappa e}{p_i \cdot k_i}\, &\Big[\epsilon_i \cdot p_i \left[\epsilon_B^* \cdot \epsilon_A\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot p_f - \epsilon_B^* \cdot k_f\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot \epsilon_A \right. \\ - \; &\left. \epsilon_A \cdot p_f\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot \epsilon_B^* + p_f \cdot k_f\, \epsilon_f^* \cdot \epsilon_A\, \epsilon_f^* \cdot \epsilon_B^*\right] \\ + \; &\epsilon_A \cdot \epsilon_i \left[\epsilon_B^* \cdot k_i\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot p_f - \epsilon_B^* \cdot k_f\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot k_i - p_f \cdot k_i\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot \epsilon_B^* \right. \\ + \; &\left. p_f \cdot k_f\, \epsilon_f^* \cdot k_i\, \epsilon_f^* \cdot \epsilon_B^*\right] \\ - \; &\epsilon_A \cdot k_i\, \left[\epsilon_B^* \cdot \epsilon_i\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot p_f - \epsilon_B^* \cdot k_f\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot \epsilon_i - \epsilon_i \cdot p_f\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot \epsilon_B^* \right. \\ + \; &\left. p_f \cdot k_f\, \epsilon_f^* \cdot \epsilon_i\, \epsilon_f^* \cdot \epsilon_B^*\right] \\ - \; &\epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_i\, \epsilon_f^* \cdot p_f p_i \cdot k_i \Big]. \end{aligned} \tag{92}$$

$$\begin{aligned}\text{Fig.2(b)}: \; \text{Amp}_b(S=1) = -\frac{\kappa e}{p_i \cdot k_f}\, &\Big[\epsilon_i \cdot p_f \left[\epsilon_A \cdot \epsilon_B^*\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot p_i - \epsilon_B^* \cdot p_i\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_A \right. \\ + \; &\left. \epsilon_A \cdot k_f\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_B^* - p_i \cdot k_f\, \epsilon_f^* \cdot \epsilon_A\, \epsilon_f^* \cdot \epsilon_B^*\right] \\ + \; &\epsilon_B^* \cdot k_i \left[\epsilon_A \cdot \epsilon_i\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot p_i - \epsilon_i \cdot p_i\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_A + \epsilon_A \cdot k_f\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_i \right. \\ - \; &\left. p_i \cdot k_f\, \epsilon_f^* \cdot \epsilon_A\, \epsilon_f^* \cdot \epsilon_i\right] \\ + \; &\epsilon_i \cdot \epsilon_B^* \left[\epsilon_A \cdot k_i\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot p_i - p_i \cdot k_i\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot \epsilon_A + \epsilon_A \cdot k_f\, \epsilon_f^* \cdot p_i\, \epsilon_f^* \cdot k_i \right. \\ - \; &\left. p_i \cdot k_f\, \epsilon_f^* \cdot \epsilon_A\, \epsilon_f^* \cdot k_i\right] \\ - \; &\epsilon_A \cdot \epsilon_f^*\, \epsilon_f^* \cdot p_i\, \epsilon_B^* \cdot \epsilon_i\, p_i \cdot k_f \Big]. \end{aligned} \tag{93}$$

$$\text{Fig.2(c)}: \ \text{Amp}_c(S=1) = \kappa\, e \left[\epsilon_f^* \cdot \epsilon_i (\epsilon_B^* \cdot \epsilon_A\, \epsilon_f^* \cdot (p_f + p_i) - \epsilon_A \cdot p_f\, \epsilon_B^* \cdot \epsilon_f^* - \epsilon_B^* \cdot p_i\, \epsilon_A \cdot \epsilon_f^*) \right.$$
$$\left. - \ \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_i\, \epsilon_f^* \cdot p_i - \epsilon_A \cdot \epsilon_f^*\, \epsilon_B^* \cdot \epsilon_i\, \epsilon_f^* \cdot p_f + \epsilon_f^* \cdot \epsilon_A\, \epsilon_f^* \cdot \epsilon_B^*\, \epsilon_i \cdot (p_f + p_i) \right], \tag{94}$$

and finally, the photon pole contribution

$$\begin{aligned}
\text{Fig.2(d)}: \quad & \text{Amp}_\text{d}(S=1) = -\frac{e\,\kappa}{2k_f \cdot k_i} \\
\times \ & \Big[\epsilon_B^* \cdot \epsilon_A \left[\epsilon_f^* \cdot (p_f + p_i)(k_f \cdot k_i \epsilon_f^* \cdot \epsilon_i - \epsilon_f^* \cdot k_i\, \epsilon_i \cdot k_f)\right. \\
+ \ & \left.\epsilon_f^* \cdot k_i (\epsilon_f^* \cdot \epsilon_i\, k_i \cdot (p_i + p_f) - \epsilon_f^* \cdot k_i \epsilon_i \cdot (p_f + p_i))\right] \\
- \ & 2\epsilon_B^* \cdot p_i \left[\epsilon_f^* \cdot \epsilon_A\, (k_f \cdot k_i\, \epsilon_f^* \cdot \epsilon_i - \epsilon_f^* \cdot k_i\, \epsilon_i \cdot k_f)\right. \\
+ \ & \left.\epsilon_f^* \cdot k_i\, (\epsilon_f^* \cdot \epsilon_i\, \epsilon_A \cdot k_i - \epsilon_f^* \cdot k_i\, \epsilon_i \cdot \epsilon_A)\right] \\
- \ & 2\epsilon_A \cdot p_f \left[\epsilon_f^* \cdot \epsilon_B^* (k_f \cdot k_i\, \epsilon_f^* \cdot \epsilon_i - \epsilon_f^* \cdot k_i\, \epsilon_i \cdot k_f)\right. \\
+ \ & \left.\epsilon_f^* \cdot k_i\, (\epsilon_f^* \cdot \epsilon_i\, \epsilon_B^* \cdot k_i - \epsilon_f^* \cdot k_i\, \epsilon_i \cdot \epsilon_B^*)\right]\Big].
\end{aligned} \tag{95}$$

Gravitational Compton Scattering: spin-1

In the case of gravitational Compton scattering—Figure 3—we have the four contributions

$$\begin{aligned}
\text{Fig.3(a)}: \quad & \text{Amp}_a(S=1) = \kappa^2\, \frac{1}{2p_i \cdot k_i} \Big[(\epsilon_i \cdot p_i)^2 (\epsilon_f^* \cdot p_f)^2 \epsilon_A \cdot \epsilon_B^* \\
- \ & (\epsilon_f^* \cdot p_f)^2 \epsilon_i \cdot p_i (\epsilon_A \cdot k_i \epsilon_B^* \cdot \epsilon_i + \epsilon_A \cdot \epsilon_i\, \epsilon_B^* \cdot p_i) \\
- \ & (\epsilon_i \cdot p_i)^2 \epsilon_f^* \cdot p_f\, (\epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot p_f + \epsilon_B^* \cdot k_f\, \epsilon_A \cdot \epsilon_f^*) \\
+ \ & \epsilon_i \cdot p_i\, \epsilon_f^* \cdot p_f\, \epsilon_i \cdot p_f\, \epsilon_A \cdot k_i\, \epsilon_B^* \cdot \epsilon_f^* + \epsilon_i \cdot p_i\, \epsilon_f^* \cdot p_f\, \epsilon_f^* \cdot p_i\, \epsilon_A \cdot \epsilon_i\, \epsilon_B^* \cdot k_f \\
+ \ & (\epsilon_f^* \cdot p_f)^2 \epsilon_B^* \cdot \epsilon_i\, \epsilon_A \cdot \epsilon_i\, p_i \cdot k_i + (\epsilon_i \cdot p_i)^2 \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_f^*\, p_f \cdot k_f \\
+ \ & \epsilon_i \cdot p_i\, \epsilon_f^* \cdot p_f\, (\epsilon_A \cdot k_i\, \epsilon_B^* \cdot k_f\, \epsilon_i \cdot \epsilon_f^* + \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_i\, p_i \cdot p_f) \\
- \ & \epsilon_i \cdot p_i\, \epsilon_f^* \cdot p_i\, \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_i\, p_f \cdot k_f - \epsilon_f^* \cdot p_f\, \epsilon_i \cdot p_f\, \epsilon_A \cdot \epsilon_i\, \epsilon_B^* \cdot \epsilon_f^*\, p_i \cdot k_i \\
- \ & \epsilon_i \cdot p_i\, \epsilon_A \cdot k_i\, \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_f^* \cdot \epsilon_i\, p_f \cdot k_f - \epsilon_f^* \cdot p_f\, \epsilon_B^* \cdot k_f\, \epsilon_A \cdot \epsilon_i\, \epsilon_i \cdot \epsilon_f^*\, p_i \cdot k_i \\
+ \ & \epsilon_A \cdot \epsilon_i\, \epsilon_B^* \cdot \epsilon_f^*\, p_i \cdot k_i\, p_f \cdot k_f\, \epsilon_i \cdot \epsilon_f^* - m^2 \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_i\, \epsilon_f^* \cdot p_f\, \epsilon_i \cdot p_i\Big].
\end{aligned} \tag{96}$$

$$\begin{aligned}
\text{Fig.3(b)}: \quad & \text{Amp}_b(S=1) = -\kappa^2 \frac{1}{2p_i \cdot k_f} \Big[\big(\epsilon_f^* \cdot p_i\big)^2 \big(\epsilon_i \cdot p_f\big)^2 \epsilon_A \cdot \epsilon_B^* \\
+ \ & \big(\epsilon_i \cdot p_f\big)^2 \epsilon_f^* \cdot p_i \big(\epsilon_A \cdot k_f\, \epsilon_B^* \cdot \epsilon_f^* - \epsilon_A \cdot \epsilon_f^*\, \epsilon_B^* \cdot p_i\big) \\
+ \ & \big(\epsilon_f^* \cdot p_i\big)^2 \epsilon_i \cdot p_f \big(\epsilon_B^* \cdot k_i \epsilon_A \cdot \epsilon_i - \epsilon_B^* \cdot \epsilon_i\, \epsilon_A \cdot p_f\big) \\
- \ & \epsilon_f^* \cdot p_i\, \epsilon_i \cdot p_f\, \epsilon_f^* \cdot p_f\, \epsilon_A \cdot k_f\, \epsilon_B^* \cdot \epsilon_i - \epsilon_f^* \cdot p_i\, \epsilon_i \cdot p_f\, \epsilon_i \cdot p_i\, \epsilon_A \cdot \epsilon_f^*\, \epsilon_B^* \cdot k_i \\
- \ & \big(\epsilon_i \cdot p_f\big)^2 \epsilon_B^* \cdot \epsilon_f^*\, \epsilon_A \cdot \epsilon_f^*\, p_i \cdot k_f - \big(\epsilon_f^* \cdot p_i\big)^2 \epsilon_B^* \cdot \epsilon_i\, \epsilon_A \cdot \epsilon_i\, p_f \cdot k_i \\
+ \ & \epsilon_f^* \cdot p_i\, \epsilon_i \cdot p_f \big(\epsilon_A \cdot k_f\, \epsilon_B^* \cdot k_i\, \epsilon_i \cdot \epsilon_f^* + \epsilon_B^* \cdot \epsilon_i\, \epsilon_A \cdot \epsilon_f^*\, p_i \cdot p_f\big) \\
+ \ & \epsilon_f^* \cdot p_i\, \epsilon_i \cdot p_i\, \epsilon_B^* \cdot \epsilon_i\, \epsilon_A \cdot \epsilon_f^*\, p_f \cdot k_i + \epsilon_i \cdot p_f\, \epsilon_f^* \cdot p_f\, \epsilon_A \cdot \epsilon_f^*\, \epsilon_B^* \cdot \epsilon_i\, p_i \cdot k_f \\
- \ & \epsilon_f^* \cdot p_i\, \epsilon_A \cdot k_f\, \epsilon_B^* \cdot \epsilon_i\, \epsilon_i \cdot \epsilon_f^*\, p_f \cdot k_i - \epsilon_i \cdot p_f\, \epsilon_B^* \cdot k_i\, \epsilon_A \cdot \epsilon_f^*\, \epsilon_f^* \cdot \epsilon_i\, p_i \cdot k_f \\
+ \ & \epsilon_A \cdot \epsilon_f^*\, \epsilon_B^* \cdot \epsilon_i\, p_i \cdot k_f\, p_f \cdot k_i\, \epsilon_i \cdot \epsilon_f^* - m^2 \epsilon_B^* \cdot \epsilon_i\, \epsilon_A \cdot \epsilon_f^*\, \epsilon_i \cdot p_f\, \epsilon_f^* \cdot p_i\Big].
\end{aligned} \tag{97}$$

$$\text{Fig.3(c)}: \qquad \text{Amp}_c(S=1) = -\frac{\kappa^2}{4}\Big[\big(\epsilon_i\cdot\epsilon_f^*\big)^2\big(m^2-p_i\cdot p_f\big)\epsilon_A\cdot\epsilon_B^*$$
$$+\ \epsilon_A\cdot p_f\,\epsilon_B^*\cdot p_i\,\big(\epsilon_i\cdot\epsilon_f^*\big)^2+\epsilon_i\cdot p_i\,\epsilon_f^*\cdot p_f\,\big(2\epsilon_i\cdot\epsilon_f^*\,\epsilon_A\cdot\epsilon_B^*-2\epsilon_A\cdot\epsilon_2\,\epsilon_B^*\cdot\epsilon_1\big)$$
$$+\ \epsilon_i\cdot p_f\,\epsilon_f^*\cdot p_i\,\big(2\epsilon_i\cdot\epsilon_f^*\,\epsilon_A\cdot\epsilon_B^*-2\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot\epsilon_f^*\big)$$
$$+\ 2\epsilon_i\cdot p_i\,\epsilon_1\cdot p_f\,\epsilon_A\cdot\epsilon_f^*\,\epsilon_B^*\cdot\epsilon_f^*+2\epsilon_f^*\cdot p_f\,\epsilon_f^*\cdot p_i\,\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot\epsilon_i$$
$$-\ 2\epsilon_i\cdot p_i\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_A\cdot p_f\,\epsilon_B^*\cdot\epsilon_f^*-2\epsilon_f^*\cdot p_f\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_A\cdot\epsilon_i\,\epsilon_f^*\cdot p_i$$
$$-\ 2\epsilon_i\cdot p_f\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_A\cdot\epsilon_f^*\,\epsilon_B^*\cdot p_i-2\epsilon_f^*\cdot p_i\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_B^*\cdot\epsilon_i\,\epsilon_A\cdot p_f$$
$$-\ 2\big(m^2-p_f\cdot p_i\big)\epsilon_i\cdot\epsilon_f^*\big(\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot\epsilon_f^*+\epsilon_A\cdot\epsilon_f^*\,\epsilon_B^*\cdot\epsilon_i\big)\Big], \qquad (98)$$

and finally the (lengthy) graviton pole contribution is

$$3\text{(d)}: \text{Amp}_d(S=1) = -\frac{\kappa^2}{16\,k_i\cdot k_f}\Big[\epsilon_B^*\cdot\epsilon_A\Big[\big(\epsilon_i\cdot\epsilon_f^*\big)^2\Big[4k_i\cdot p_i\,p_f\cdot k_i+4k_f\cdot p_i\,k_f\cdot p_f$$
$$-\ 2\big(p_i\cdot k_i\,p_f\cdot k_f+p_f\cdot k_i\,p_i\cdot k_f\big)+6p_i\cdot p_f\,k_i\cdot k_f\Big]+4\Big[\big(\epsilon_i\cdot k_f\big)^2\epsilon_f^*\cdot p_f\,\epsilon_f^*\cdot p_i$$
$$+\ \big(\epsilon_f^*\cdot k_i\big)^2\epsilon_i\cdot p_i\,\epsilon_i\cdot p_f+\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\big(\epsilon_i\cdot p_i\,\epsilon_f^*\cdot p_f+\epsilon_i\cdot p_f\,\epsilon_f^*\cdot p_i\big)\Big]$$
$$-\ 4\epsilon_i\cdot\epsilon_f^*\Big[\epsilon_i\cdot k_f\big(\epsilon_f^*\cdot p_i\,p_f\cdot k_f+\epsilon_f^*\cdot p_f\,k_f\cdot p_i\big)+\epsilon_f^*\cdot k_i\big(\epsilon_i\cdot p_i\,p_f\cdot k_i+\epsilon_i\cdot p_f\,p_i\cdot k_i\big)\Big]$$
$$-\ 4k_i\cdot k_f\,\epsilon_i\cdot\epsilon_f^*\big(\epsilon_i\cdot p_i\,\epsilon_f^*\cdot p_f+\epsilon_i\cdot p_f\,\epsilon_f^*\cdot p_i\big)-4p_i\cdot p_f\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\Big]$$
$$-\ \big(p_i\cdot p_f\,\epsilon_B^*\cdot\epsilon_A-\epsilon_B^*\cdot p_i\,\epsilon_A\cdot p_f\big)\Big[10\big(\epsilon_i\cdot\epsilon_f^*\big)^2k_i\cdot k_f+4\epsilon_i\cdot\epsilon_f^*\,\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i$$
$$-\ 4\big(\epsilon_i\cdot\epsilon_f^*\big)^2k_i\cdot k_f-8\epsilon_i\cdot\epsilon_f^*\,\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\Big]+\big(p_i\cdot p_f-m^2\big)\Big[\big(\epsilon_i\cdot\epsilon_f^*\big)^2\big(4\epsilon_A\cdot k_i\,\epsilon_B^*\cdot k_i$$
$$+\ 4\epsilon_A\cdot k_f\,\epsilon_B^*\cdot k_f-2\big(\epsilon_A\cdot k_i\,\epsilon_B^*\cdot k_f+\epsilon_A\cdot k_f\,\epsilon_B^*\cdot k_i\big)+6\epsilon_B^*\cdot\epsilon_A\,k_i\cdot k_f\big)$$
$$+\ 4\Big[\big(\epsilon_i\cdot k_f\big)^2\epsilon_A\cdot\epsilon_f^*\,\epsilon_B^*\cdot\epsilon_f^*+\big(\epsilon_f^*\cdot k_i\big)^2\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot\epsilon_i+\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_f\big(\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot\epsilon_f^*$$
$$+\ \epsilon_A\cdot\epsilon_f^*\,\epsilon_B^*\cdot\epsilon_i\big)\Big]-4\epsilon_i\cdot\epsilon_f^*\Big[\epsilon_i\cdot k_f\big(\epsilon_A\cdot\epsilon_f^*\,\epsilon_B^*\cdot k_f+\epsilon_B^*\cdot\epsilon_f^*\,\epsilon_A\cdot k_f\big)$$
$$+\ \epsilon_f^*\cdot k_i\big(\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot k_i+\epsilon_B^*\cdot\epsilon_i\,\epsilon_A\cdot k_i\big)+k_i\cdot k_f\big(\epsilon_A\cdot\epsilon_i\,\epsilon_B^*\cdot\epsilon_f^*+\epsilon_B^*\cdot\epsilon_i\,\epsilon_A\cdot\epsilon_f^*\big)$$
$$+\ \epsilon_A\cdot\epsilon_B^*\,\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\Big]\Big]-2\epsilon_A\cdot p_f\Big[\big(\epsilon_f^*\cdot\epsilon_i\big)^2\Big[2\epsilon_B^*\cdot k_i\,p_i\cdot k_i+2\epsilon_B^*\cdot k_f\,p_i\cdot k_f$$
$$+\ 3\epsilon_B^*\cdot p_i\,k_i\cdot k_f-\big(\epsilon_B^*\cdot k_i\,p_i\cdot k_f+\epsilon_B^*\cdot k_f\,p_i\cdot k_i\big)\Big]+2\big(\epsilon_i\cdot k_f\big)^2\epsilon_B^*\cdot\epsilon_f^*\,\epsilon_f^*\cdot p_i$$
$$+\ 2\big(\epsilon_f^*\cdot k_i\big)^2\epsilon_B^*\cdot\epsilon_i\,\epsilon_i\cdot p_i+2\epsilon_i\cdot k_f\epsilon_f^*\cdot k_i\big(\epsilon_B^*\cdot\epsilon_i\,\epsilon_f^*\cdot p_i+\epsilon_i\cdot p_i\,\epsilon_B^*\cdot\epsilon_f^*\big)$$
$$-\ 2\epsilon_i\cdot\epsilon_f^*\Big[\epsilon_i\cdot k_f\big(\epsilon_B^*\cdot\epsilon_f^*\,p_i\cdot k_f+\epsilon_f^*\cdot p_i\,\epsilon_B^*\cdot k_f\big)+\epsilon_f^*\cdot k_i\big(\epsilon_B^*\cdot\epsilon_i\,p_i\cdot k_i+\epsilon_B^*\cdot k_i\,\epsilon_i\cdot p_i\big)\Big]$$
$$-\ 2k_i\cdot k_f\,\epsilon_i\cdot\epsilon_f^*\big(\epsilon_B^*\cdot\epsilon_i\,\epsilon_f^*\cdot p_i+\epsilon_B^*\cdot\epsilon_f^*\,\epsilon_i\cdot p_i\big)-2\epsilon_B^*\cdot p_i\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\Big]$$
$$-\ 2\epsilon_B^*\cdot p_i\Big[\big(\epsilon_f^*\cdot\epsilon_i\big)^2\Big[2\epsilon_A\cdot k_i\,p_f\cdot k_i+2\epsilon_A\cdot k_f\,p_f\cdot k_f+3\epsilon_A\cdot p_f\,k_i\cdot k_f$$
$$-\ \big(\epsilon_A\cdot k_i\,p_f\cdot k_f+\epsilon_A\cdot k_f\,p_f\cdot k_f\big)\Big]+2\big(\epsilon_i\cdot k_f\big)^2\epsilon_A\cdot\epsilon_f^*\,\epsilon_f^*\cdot p_f+2\big(\epsilon_f^*\cdot k_i\big)^2\epsilon_A\cdot\epsilon_i\,\epsilon_i\cdot p_f$$
$$+\ 2\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\big(\epsilon_A\cdot\epsilon_i\,\epsilon_f^*\cdot p_f+\epsilon_i\cdot p_f\,\epsilon_A\cdot\epsilon_f^*-2\epsilon_i\cdot\epsilon_f^*\Big[\epsilon_i\cdot k_f\big(\epsilon_A\cdot\epsilon_f^*\,p_f\cdot k_f$$
$$+\ \epsilon_f^*\cdot p_f\,\epsilon_A\cdot k_f\big)+\epsilon_f^*\cdot k_i\big(\epsilon_A\cdot\epsilon_i\,p_f\cdot k_i+\epsilon_A\cdot k_i\,\epsilon_i\cdot p_f\big)\Big]$$
$$-\ 2k_i\cdot k_f\epsilon_i\cdot\epsilon_f^*\big(\epsilon_A\cdot\epsilon_i\,\epsilon_f^*\cdot p_f+\epsilon_A\cdot\epsilon_f^*\,\epsilon_i\cdot p_f\big)-2\epsilon_A\cdot p_f\,\epsilon_i\cdot\epsilon_f^*\,\epsilon_i\cdot k_f\,\epsilon_f^*\cdot k_i\Big]\Big]. \qquad (99)$$

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$$H = \sqrt{m^2 + \boldsymbol{p}^2}$$

is replaced by

$$H - e\phi = \sqrt{m^2 + (\boldsymbol{p} - e\boldsymbol{A})^2}$$

Making the quantum mechanical substituations

$$H \to i\frac{\partial}{\partial t} \qquad \boldsymbol{p} \to -i\boldsymbol{\nabla}$$

we find the minimal substitution given in the text.
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