Landing a VTOL Unmanned Aerial Vehicle on a moving platform using optical flow

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Abstract—This paper presents a nonlinear controller for a Vertical Take-off and Landing (VTOL) Unmanned Aerial Vehicle (UAV) that exploits a measurement of the average optical flow to enable hover and landing control on a moving platform, such as, for example, the deck of a sea going vessel. The VTOL vehicle is assumed to be equipped with a minimum sensor suite (a camera and an IMU), manoeuvring over a textured flat target plane. Two different tasks are considered in this paper: the first concerns stabilizing the vehicle relative to the moving platform, that is maintaining a constant offset from a moving reference. The second concerns regulation of automatic vertical landing onto a moving platform. Rigorous analysis of system stability is provided and simulations are presented. Experimental results are provided for a quadrotor UAV to demonstrate the performance of the proposed control strategy.

Keywords: Optical Flow, Automatic landing, Unmanned Aerial Vehicle, Nonlinear control

I. INTRODUCTION

Recent advances in technology and a long list of potential applications have led to a growing interest in aerial robotic vehicles [1]. Unmanned aerial vehicles are an ideal solution for many indoor and outdoor applications that presently jeopardize human or material safety, such as for example; monitoring traffic congestion, regular inspection of infrastructure such as bridges, dam walls and power cables, investigation of hazardous environments, etc. An important capability for a subset of potential applications, particularly those associated with maritime scenarios, is the ability to autonomously land the vehicle on a moving platform such as the deck of a sea going vessel, or indeed any landing pad attached to a moving vehicle. The associated capability of stabilizing the motion of the UAV with respect to a dynamic moving environment is itself of significance in a wide range of more general applications. Autonomous landing of UAV on moving platforms has been investigated using a model of the vertical motion of landing platform [2], [3], a tether-guide [4] or known target motion [5], [6]. The main idea of prior work is based on obtaining a prediction of the motion of the moving landing pad to provide a feed-forward compensation during the landing manoeuvre. This approach has the advantage that, should a reliable predictive model of the motion of the landing pad be determined, the resulting performance of the landing manoeuvre is of high quality. The approach suffers from the disadvantage that in many applications of interest it is difficult to determine a reliable predictive model of the motion of the landing pad either because the motion of the landing pad is primarily stochastic and no predictive model is valid, or due to the limited amount of data available to the UAV during the landing manoeuvre. In such situations the vehicle control algorithm must fall back on feedback control strategies. An approach that stems from the insight into the behaviour of flying insects and animals uses visual flow [7] as feedback for aerial vehicles in the control of motion in dynamic environments. Since optical flow provides relative velocity and proximity information with respect to the local environment, it is an ideal cue that can be used to perform landing control strategies [9], [7], as well as obstacle avoidance [10], [11], [12], terrain following [13], [14], [15], visual servo control [16], or even in both localization and control [17]. It is rare that moving obstacles are considered in prior literature using optical flow, however it is well known that insects show great capabilities in achieving landing tasks on moving objects such as, for example, a bee landing on a flower. Moreover, the full vehicle dynamics analysis is rarely discussed in the majority of work on the analysis of insect flight behaviour, since the flight regime of insects is highly damped due to their high drag to mass ratios. The control strategies that have been observed in the various biological studies do not necessarily generalise to high-inertia, low-drag aerial vehicles.

In this paper, an optical flow based control law for hovering flight and landing manoeuvre on a moving platform is proposed using only IMU and optical flow measurements. The image information considered is the average optical flow obtained from a textured target plane, using additional information provided by an embedded IMU for deotation of the flow. A non-linear PI-type controller is designed for hovering flight while another nonlinear controller, exploiting the vertical optical flow (similar to the inverse of the well-known time-to-contact), is proposed for vertical landing. It is necessary to assume bounded dynamics of the moving platform, however, no predictive model of the platform is required to obtain the desired closed-loop performance. To prove global stability and convergence of the closed-loop system, Lyapunov analysis is used both for the stabilisation of the hovering flight relative to a static plane and for the vertical landing relative to a horizontal plane moving with unknown (bounded) dynamics in the vertical direction. In
practice, the stabilisation and vertical landing also works with lateral motion. Experimental results are obtained on a quad-rotor UAV capable of quasi-stationary flight developed at CEA (French Atomic Energy Commission). A ‘high gain’ controller is used to stabilise the orientation dynamics of the vehicle, an approach classically known in aeronautics as guidance and control (or hierarchical control) [18], and the stabilisation and landing control is developed for the resulting reduced translational dynamics of the vehicle. The proposed closed-loop control schemes demonstrate efficiency and performance for the hovering flight and vertical landing manoeuvre. The material presented in this present paper is an extension of the prior work [19]. It incorporates the ground effect, considers the situation of target is moving, contains detailed proof of the system stability and incorporates additional simulations and experiments.

The body of the paper consists of six sections followed by a conclusion. Section II presents the fundamental equations of motion for an X4-flyer UAV. In Section III, fundamental equations of optical flow are presented. Sections IV and V present the proposed control strategies for hovering and vertical landing manoeuvre respectively. Section VI describes simulations results and Section VII describes the experimental results obtained on the quad-rotor vehicle.

Fig. 1: The quadrotor UAV developed in Centre d’Energie Atomique, and used for the experimental results in the paper.

II. UAV DYNAMIC MODEL AND TIME SCALE SEPARATION

The VTOL UAV is represented by a rigid body of mass $m$ and of tensor of inertia $I$ along with external forces due to gravity and forces and torques provided by rotors. To describe the motion of the UAV, two reference frames are introduced: an inertial reference frame $I$ associated with the vector basis $[e_1, e_2, e_3]$ and a body-fixed frame $B$ attached to the UAV at the center of mass and associated with the vector basis $[e_1^b, e_2^b, e_3^b]$. The position and the linear velocity of the UAV in $I$ are respectively denoted $\xi = (x, y, z)^T$ and $v = (\dot{x}, \dot{y}, \dot{z})^T$. The orientation of the UAV is given by the orientation matrix $R \in SO(3)$ from $B$ to $I$. Finally, let $\Omega = \Omega_1, \Omega_2, \Omega_3)^T$ be the angular velocity of the UAV defined in $B$.

A translational force $F$ and a control torque $\Gamma$ are applied to the UAV. The translational force $F$ combines thrust, lift, drag and gravity components. For a miniature VTOL UAV in quasi-stationary flight one can reasonably assume that the aerodynamic forces are always in direction $e_3^b$, since the thrust force predominates over other components [20]. The gravitational force can be separated from other forces and the dynamics of the VTOL UAV can be written as:

\[ \dot{\xi} = v \]
\[ m\dot{v} = -TR\dot{e}_3 + mge_3 + \Delta \]
\[ \epsilon\dot{R} = R\dot{\Omega}_x, \quad \dot{\Omega} = \epsilon\Omega \]
\[ \epsilon\dot{\Omega}_x = -\bar{\Omega} \times \bar{\Omega} + \bar{\Gamma}, \quad \bar{\Gamma} = \epsilon^2\Gamma \]

In the above notation, $g$ is the acceleration due to gravity, and $T$ a scalar input termed the thrust or heave, applied in direction $e_3^b = Re_3$, the third-axis unit vector. The term $\Delta$ represents constant (or slowly time varying unmodeled) forces. The matrix $\Omega_x$ denotes the skew-symmetric matrix associated to the vector product $\Omega_x x := \Omega \times x$ for any $x$.

The positive parameter $0 < \epsilon < 1$ is introduced for timescale separation between the translation and orientation dynamics. It means that the orientation dynamics of the VTOL UAV are compensated with separate high gain control loop ($\Gamma = \bar{\Gamma}/\epsilon^2$). For this hierarchical control, the time-scale separation between the translational dynamics (slow time-scale) and the orientation dynamics (fast time-scale) can be used to design position and orientation controllers under simplifying assumptions. Although reduced-order subsystems can hence be considered for control design, the stability must be analyzed by considering the complete closed-loop system [18]. In this paper, however, we will focus on the control design for the translational dynamics.

Thus, the full vectorial term $TRe_3$ will be considered as control input for these dynamics. We will assign its desired value $u \equiv (TRe_3)^d = T^dRe_3$. Assuming that actuator dynamics can be neglected, the value $T^d$ is considered to be instantaneously reached by $T$. For the orientation dynamics of (3)-(4), a high gain controller is used to ensure that the orientation $R$ of the UAV converges to the desired orientation $R^d$. The resulting control problem is simplified to

\[ \dot{\xi} = v, \quad m\dot{v} = -u + mge_3 + \Delta \]

Thus, we consider only control of the translational dynamics (5) with a direct control input $u$. This common approach is used in practice and may be justified theoretically using singular perturbation theory [21].
III. OPTICAL FLOW EQUATIONS

In this section image plane kinematics and spherical optical flow are derived. The camera is assumed to be attached to the center of mass of the vehicle so that the camera frame coincides with the body-fixed frame.

A. Kinematics of an image point under spherical projection

We compute optical flow in spherical coordinates in order to exploit the passivity-like property discussed in [22]. The main advantage is that, in spherical coordinates, the optical flow is expressed in a simple form. Moreover, it is shown in [23] that optical flow equations can be numerically computed from an image plane to a spherical retina. A Jacobian matrix that optical flow equations can be numerically computed from an image plane to a spherical retina. A Jacobian matrix

\[ \begin{bmatrix} 23 \end{bmatrix} \]

yields

\[ \begin{bmatrix} \end{bmatrix} \]

flow are derived. The camera is assumed to be attached to the center of mass of the vehicle so that the camera frame coincides with the body-fixed frame.

B. Average optical flow

Measuring the optical flow is a key aspect of the practical implementation of the control algorithms proposed in the sequel. The optical flow \( \hat{p} \) can be computed using a range of algorithms (correlation-based techniques, features-based approaches, differential techniques, etc) [24]. Note that due to the rotational ego-motion of the camera, (8) involves the angular velocity as well as the linear velocity [8]. For the control problem we define an inertial average optical flow from the integral of all observed optical flow corrected for rotational angular velocity. By integrating optical flow over an aperture, in this case a solid angle on the sphere, we obtain information on the scaled velocity of the vehicle.

We assume that the target is a textured plane moving with a pure translational velocity \( V_t \) (no rotational velocity). Thus, for any target points \( P \) on the plane, \( V_P = V_t \). We also assume that the normal direction \( \eta \) is known and the available data are \( \hat{p} \), \( R \) and \( \Omega \) where \( R \) and \( \Omega \) are estimated from the IMU data [25]. The average optical flow is obtained by integrating the observed optical flow over a solid angle \( \mathcal{W}^2 \) of the sphere around the pole normal to the target plane (Fig. 3). The average of the optical flow on the solid angle \( \mathcal{W}^2 \) is given by (see the appendix for more details):

\[ \phi = \int_{\mathcal{W}^2} \hat{p} \, dp = -\pi (\sin \theta_0)^2 \Omega \times R^T \eta - \frac{Q(V - V_t)}{d} \]  

where the parameter \( \theta_0 \) and the matrix \( Q \) depend on the size of the solid angle \( \mathcal{W}^2 \). It can be verified that \( Q = R^T (R_t \Lambda R_t^T) R \) is a symmetric positive definite matrix. The matrix \( \Lambda \) is a constant diagonal matrix depending on parameters of the solid angle \( \mathcal{W}^2 \) and \( R_t \) represents the orientation matrix of the target plane with respect to the inertial frame. For instance, if \( \mathcal{W}^2 \) is the hemisphere centered at \( \eta \), corresponding to the visual image of the infinite target plane, it can be shown that [16]

\[ \Lambda = \frac{\pi}{4} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \]
From (9) it is straightforward to obtain a measurement of the inertial average optical flow corrected for rotational angular velocity

\[ w = -(R_t A^{-1} R_t^T) R(\phi + \pi (\sin \theta_0)^2 \Omega \times R^T \eta) \]  
(11)

By expressing it with respect to the rigid body motion, it yields:

\[ w = \frac{v - v_t}{d} + \text{noise} \]  
(12)

where \( v_t = RV_t \) is the translational velocity of the target plane expressed in the inertial frame. Note that, for theoretical analysis provided in Sections IV and V, the noise of equation (12) is ignored. Its effects on the convergence is considered in Section VI.

**Remark 3.1:** In the particular situation where the target plane is stationary \((v_t = 0)\), (12) becomes

\[ w = \frac{v}{d} + \text{noise} \]  
(13)

**IV. STABILIZATION OF THE HOVERING FLIGHT OVER A TEXTURED TARGET**

In this section a control design ensuring hovering flight over a textured flat plane is proposed. The control problem considered is the stabilization of the linear velocity about zero despite unmodeled constant (or slowly time varying) dynamics. In particular, the velocity of the target will be assumed to be constant \((v_t \equiv 0)\). A PI-type non-linear controller depending only on the measurable variable \( w = (v - v_t)/d = \tilde{v}/d \) (12) is proposed for the translational dynamics (5). The result is stated in the following theorem.

**Theorem 4.1:** Assume that \( \eta \) is known and invariant and \( \Delta \) is a constant. Consider the dynamics (5) and assume that the control input \( u \) is chosen as

\[ u = k_P w + k_I \int_0^t w \, d\tau + mg e_3 \kappa, k_P, k_I > 0 \]  
(14)

Then, for any initial conditions \( d_0 = d(0) > 0 \), the linear velocity error \( \tilde{v} \) converges asymptotically to zero. More precisely:

1. \( \dot{d} = -\langle \tilde{v}, \eta \rangle \) converges to 0 and \( d(t) > 0 \), \( \forall t \geq 0 \).
2. the horizontal velocity \( \tilde{v}^\parallel = \pi_\eta \tilde{v} \) converges to zero.

**Proof:**

**Proof of item (1):** Recall the dynamics of the vehicle (5) and consider the component \( \tilde{v}^\perp = \langle \tilde{v}, \eta \rangle \) in direction \( \eta \). One obtains:

\[ m \ddot{\tilde{v}}^\perp = -k_P \frac{\tilde{v}^\perp}{d} - k_I \int_0^t \frac{\dot{\tilde{v}}^\perp}{d} \, d\tau + \langle \Delta, \eta \rangle \]  
(15)

Note that \( \dot{\tilde{v}}^\perp = -\dot{d} \). Equation (15) can also be written as follows:

\[ m \ddot{d} = -k_P \frac{\dot{d}}{d} - k_I \int_0^t \frac{\dot{d}}{d} \, d\tau - \langle \Delta, \eta \rangle \]  
(16)

\[ = -k_P \frac{\dot{d}}{d} - k_I \ln \left( \frac{d}{d_\infty} \right) \]  
(17)

where \( d_\infty = d_0 e^{-\langle \Delta, \eta \rangle/k_I} \). Note that the control law is well defined and smooth for \( d > 0 \). For any initial conditions such that \( d_0 = d(0) > 0 \), define the Lyapunov function candidate \( \mathcal{L}_\eta \) by

\[ \mathcal{L}_\eta = \frac{m}{2d_\infty} d^2 + k_I \left[ \frac{d}{d_\infty} \left( \ln \left( \frac{d}{d_\infty} \right) - 1 \right) + 1 \right] \geq 0 \]  
(18)

Since function \( u \mapsto (u \ln u - 1) + 1 \geq 0, \forall u > 0 \), it is straightforward to verify that \( \mathcal{L}_\eta \geq 0 \). Differentiating \( \mathcal{L}_\eta \) and recalling equation (17), one obtains

\[ \dot{\mathcal{L}}_\eta = -k_P \frac{\dot{d}^2}{d_\infty} \]  
(19)

This implies that \( \mathcal{L}_\eta < \mathcal{L}_\eta(0) \) as long as \( d(t) > 0 \). Two different cases may occur depending on the initial value of \( \mathcal{L}_\eta \); \( \mathcal{L}_\eta(0) < k_I \) and \( \mathcal{L}_\eta(0) \geq k_I \). From the expression of the Lyapunov function (18), the first case \((\mathcal{L}_\eta(0) < k_I)\) implies that there exists \( \varepsilon > 0 \) such that \( d(t) > \varepsilon > 0, \forall t \). Consequently, \( d \) remains strictly positive and equation (17) is well defined for all time. Application of LaSalle’s principle shows that the invariant set is contained in the set defined by \( \mathcal{L}_\eta = 0 \). This implies that \( d \equiv 0 \) in the invariant set. Recalling (17), it is straightforward to show that \( d \) converges asymptotically to \( d_\infty \).

For the second situation \((\mathcal{L}_\eta(0) \geq k_I)\), we have to show that \( d \neq 0 \) for all time. Assume that there exists a first time \( t_1 \) such that \( d(t_1) < 0 \) and \( d(t_1) < d_\infty \). If we show that there exists a second time \( t_2 > t_1 \) such that \( d(t_2) = 0 \) and \( d(t_2) > 0 \) then, \( \mathcal{L}_\eta(t_2) < k_I \) and conditions of the first case are verified, and the result follows. We proceed using a proof by contradiction. Assume that for all time \( t > t_1 \), \( d(t) < 0 \). This implies \( d(t) < d(t_1) < d_\infty \), \( \forall t > t_1 \). Thus, recalling equation (17), it follows that there exists \( \varepsilon > 0 \) such that \( d(t) > \varepsilon > 0, \forall t > t_1 \). As a consequence, there exists a time \( T > t_1 \) such that \( d \) converges to 0 \((d > 0)\) when \( t \) tends to \( T \). Recalling equation (17), it yields:

\[ \dot{d} = -k_P \frac{\dot{d}}{m} \geq 0, \forall t > t_1 \]  
(20)

Integrating this equation, it follows:

\[ d - d(t_1) > -k_P m \ln \left( \frac{d}{d(t_1)} \right), \forall t > t_1 \]  
(21)

Since \( d \) converges to 0, \( \dot{d} \) converges to \(+\infty\). This contradicts the fact that \( d < 0, \forall t > t_1 \). It follows that \( d(t) > 0, \forall t \geq 0 \) and consequently \( d(t) \) converges to \( d_\infty \).

**Proof of item (2):** Let \( \tilde{v}^\parallel \) be the planar velocity \( \pi_\eta \tilde{v} \in \mathcal{I} \). Consider the component perpendicular to \( \eta \) of the control law (14),

\[ u = \pi_\eta u = k_P \frac{\tilde{v}^\parallel}{d} + k_I \int_0^t \frac{\dot{\tilde{v}}^\parallel}{d} \, d\tau + mg \pi_\eta e_3 \]  
(22)

Recall the dynamics of the vehicle (5) and consider the component perpendicular to \( \eta \). Substituting the control law (22) into (5), one obtains

\[ m \ddot{\tilde{v}}^\parallel = -k_P \frac{\dot{\tilde{v}}^\parallel}{d} - k_I \int_0^t \frac{\dot{\tilde{v}}^\parallel}{d} \, d\tau + \Delta^\parallel \]  
(23)

where \( \Delta^\parallel = \pi_\eta \Delta \). Let \( \delta_1 \) be the following variable:

\[ \delta_1 = \int_0^t \frac{\tilde{v}^\parallel}{d} \, d\tau - \frac{\Delta^\parallel}{k_I} \]  
(24)
Differentiating $\delta_1$, it yields
\[
\dot{\delta}_1 = \frac{\ddot{v}}{d}\tag{25}
\]
Consider the following Lyapunov function candidate:
\[
\mathcal{L}_{\pi_\alpha} = k_t \frac{||\delta_1||^2}{2} + m \frac{||\delta_2||^2}{2}
\]
where $\delta_2 = \frac{\ddot{v}}{\sqrt{d}}$. Differentiating $\mathcal{L}_{\pi_\alpha}$ and recalling equation (23), one obtains
\[
\dot{\mathcal{L}}_{\pi_\alpha} = -||\delta_2||^2 \left(\frac{kp + m\dot{d}/2}{d}\right)
\]
Using the fact that $(d, \dot{d})$ converges to $(d_\infty, 0)$, one can insure that there exists a time $T$ and $\varepsilon > 0$ such that
\[
(kp + m\dot{d}/2) > \varepsilon > 0, \forall t > T
\]
Therefore, $\dot{\mathcal{L}}_{\pi_\alpha} < -\varepsilon ||\delta_2||^2$, $\forall t > T$. Moreover, it is straightforward to verify that $\mathcal{L}_{\pi_\alpha}$ remains bounded in $[0, T]$ by noticing that
\[
\dot{\mathcal{L}}_{\pi_\alpha} < \frac{d_{\max}}{d_{\min}} \mathcal{L}_{\pi_\alpha}, \forall t \in [0, T]
\]
This implies that $\mathcal{L}_{\pi_\alpha}(t) < \mathcal{L}_{\pi_\alpha}(T), \forall t > T$. To show that $\delta_2$ converges to 0, we need to show that $\mathcal{L}_{\pi_\alpha}$ is uniformly continuous. Then, application of Barbalat’s Lemma (see [21]) will conclude the proof. To this purpose it is sufficient to show that $\mathcal{L}_{\pi_\alpha}$ is bounded. Since $d$ and $\dot{d}$ are bounded, it remains to show that $\delta_2$ and $\dot{\delta}_2$ are bounded to satisfy the condition. $\delta_1$ and $\delta_2$ are bounded since $\mathcal{L}_{\pi_\alpha}$ is bounded. Moreover, it is straightforward to show that $\dot{\delta}_2$ is bounded using its expression:
\[
\dot{\delta}_2 = -\frac{k_P}{md} \delta_2 - \frac{k_t}{m\sqrt{d}} \delta_1 - \frac{1}{2} \frac{d}{d} \dot{\delta}_2
\]
Thus, $\dot{\mathcal{L}}_{\pi_\alpha}$ is uniformly continuous, hence $\delta_2$ converges to 0.

Finally, using the fact that $\ddot{v} = \ddot{v}^\perp + \ddot{v}^\parallel$, it follows that $\ddot{v}$ converges to zero.

**V. LANDING CONTROL ON A MOVING TEXTURED TARGET**

In this section we consider the landing manoeuvre of the aerial robot on a horizontal plane moving vertically. The primary goal is to address the question of the vertical landing on a moving platform (target) with unknown dynamics. The most important application concerns landing on a deck of a ship in high seas and tough weather [2], [4], [5], [6]. A common model of the vertical motion $z_G$ of the platform as the motion of the ship involved by the wave seas is [2]:
\[
z_G = \sum_{i=1}^n a_i \cos (\omega_i t + \phi_i)
\]
where $a_i$, $\omega_i$, $\phi_i$ are unknown constants. The classical approach estimates the parameters of motion and uses these to add a feed-forward compensation term in the control input. In this paper, we consider a more general vertical motion $z_G$ of the platform with respect to the inertial frame $T$. We assume that $z_G$ is a smooth function of class $C^2$ ($z_G$ and $\dot{z}_G$ are continuous functions of time $t$) such that $\ddot{z}_G$ is bounded by a known value.

We assume that the target plane belongs to the plane $x$-$y$ of the inertial frame so that $d \equiv h$ is the height of the vehicle with respect to the moving platform. The vertical velocity of the target plane is $\dot{z}_G e_3$. Consequently, from (12) with $v^*_t = \dot{z}_G e_3$, it is straightforward to verify that
\[
w^* = \langle w, e_3 \rangle = \frac{w^* - \dot{z}_G e_3}{h}
\]

hence,
\[
w = \frac{-h}{h} w^*
\]

as the desired average optical flow. Note that the vertical component of the inertial average optical flow acts analogously to optical flow divergence. It is straightforward to show that if $w \equiv w_0$ one has $v_x, v_y = (0, 0)$ and $v_z = h\omega^* \exp(-\omega^* t)$. Consequently, $h(t) = h_0 \exp\left(-\omega^* t\right)$ insuring a smooth vertical landing. In practice, it is impossible to exactly track $\omega^*$ and it is necessary to implement a feedback system.

We propose to use the previous control law (14) for the $x$-$y$ dynamics to stabilise the vehicle over the landing pad. We still need to provide a control scheme for the remaining degree of freedom ($h \equiv |z - z_G|$). In particular, we fix a desired set point, $\omega^*$, for the flow divergence (the flow in the normal direction to the target plane, equal to the inverse of the time-to-contact) and design a control law that regulates $(h/h + \omega^*)$ around $0$. The controller is a direct application of the controller proposed in [26], along with a complete and more rigorous proof of the exponential convergence and stability of $(h, h)$ to $(0, 0)$ despite unknown dynamics and unknown terms. Consider the dynamics
\[
m \dot{v} = -b(t) u + m g e_3 + \Delta(t)
\]
where $b(t)$ is a slowly time varying parameter that models the ground effect ($b \geq 1$). An approximate model for $b(t)$ can be found in [27], [20]
\[
\frac{1}{b(t)} = 1 - \left(\frac{D_0}{h(t) + l_0}\right)^2
\]
where $l_0$ and $D_0$ can be identified on a physical system. Note that $l_0 > D_0$ so that $b > 1$ when $h = 0$ (see Figure 4). Note also that $\max(b(t)) = b_{\max}$ is obtained when $h = 0$.

**Theorem 5.1:** Consider the dynamics of the vertical component of (30) and assume that the vertical component $u_z$ of the thrust vector $u$ is the control input. Choose $u_z$ as
\[
u_z = mk(w_z - \omega^*) + mg
\]
Assume that $z_G$ is at least $C^2$, assume that $z_G$, $\Delta(t)$ are bounded and uniformly continuous, and assume that $b(t) \geq 1$. Choose the control gain $k$ such that
\[
k > \frac{\Delta_z |\max + m |\dot{z}_G| |\max + mg |b_{\max} - 1|}{m \omega^*}
\]
Then, for all initial conditions such that \( h_0 > 0 \) (\( h_0 \equiv |z(0) - z_G(0)| \)):

1) the third component of the differential equation (30) along with (32) is smooth and non-singular. This implies that the solution \((h(t), \dot{h}(t))\) is well defined for all time \( t \geq 0 \).

2) \( h(t) > 0 \) remains positive and \((h, \dot{h})\) converge exponentially to zero.

3) the control law (32) is bounded for all time and \( \dot{h} \to 0 \).

Proof: In the first step, we prove that the third component of the differential equation (30) along with (32) is smooth and non-singular while \( h(t) > 0 \). This implies that there exists a time \( T_{\text{max}} > 0 \) such that the solution \((h(t), \dot{h}(t))\) exists and is well defined on \( t \in [0, T_{\text{max}}) \). In a second step, we prove item (2) while showing that \( T_{\text{max}} = \infty \) and finally we will prove item (3).

Proof of item (1) for \( t \in [0, T_{\text{max}}) \):

Firstly, recall that the dynamics of the considered system are decoupled and recall the dynamics of the third component of (30)

\[
m\dot{v}_z = -b(t)u_z + mg + \Delta_z(t)
\]

It follows that the height dynamics can be written:

\[
m\dot{h} = mkb(t)(w_z - \omega^*) - \Delta_z + (b(t) - 1)mg + m\ddot{z}_G
\]

\[
= -mkb(t)\left(\frac{\dot{h}}{h} + \alpha(t)\right)
\]

(36)

where,

\[
\alpha(t) = \left(\omega^* + \frac{\Delta_z}{mkb(t)} - (b(t) - 1)\frac{g}{kb(t)} - \ddot{z}_G\right)
\]

(37)

Recalling condition (33), it is straightforward to show that \( \alpha(t) \) is a positive and bounded function \( \alpha(t) > 0, \forall t \geq 0 \).

The dynamics (36) are well defined as long as \( h(t) > 0 \), hence there exists a first time \( T_{\text{max}} \), possibly infinite, such that \((h, \dot{h})\) is well defined on \( [0, T_{\text{max}}) \).

Proof of item (2) for \( t \in [0, T_{\text{max}}) \):

Define the following virtual state on \([0, T_{\text{max}})\):

\[
\zeta(t) = h(t) \exp\left(\int_0^t \frac{\dot{h}(\tau)}{kb(\tau)} d\tau\right)
\]

(38)

Differentiating \( \zeta \) and recalling equations (36) yields

\[
\dot{\zeta} = -\alpha \zeta
\]

(39)

Since \( \zeta_0 = h_0 \), it follows that on \([0, T_{\text{max}})\):

\[
h_0 \exp(-\alpha_{\text{max}}t) < \zeta(t) < h_0 \exp(-\alpha_{\text{min}}t)
\]

It remains to show that \( \int_0^t \frac{h(\tau)}{kb(\tau)} d\tau \) is bounded on \([0, T_{\text{max}})\).

To do this we must prove that \( \int_0^t \frac{h(\tau)}{kb(\tau)} d\tau \) is bounded by studying the evolution of \((\dot{h}, \ddot{h})\).

Proof that the sign of \( \dot{h}(t) \) does not change more than once and \( |h(t)| \) is bounded. Two situations may occur:

- \( \dot{h}(0) \geq 0 \): to show that there exists a time \( T \) such that \( \dot{h}(T) < 0 \), assume the converse; that is, \( \dot{h} \geq 0 \) for all time \( t \). Thus, from (36) where \( \alpha(t) > 0 \) and, by exploiting (38) where \( b \geq 1 \),

\[
\zeta(t) = h(t) \exp\left(\frac{\dot{h}(t) - h_0}{k}\right) \geq h_0 \exp\left(-\frac{h_0}{k}\right)
\]

Since \( \dot{\zeta} < -\alpha_{\text{min}} \zeta \), it follows that \( \zeta \) is exponentially decreasing. Therefore, there exists a time \( T \) such that \( \zeta(T) < h_0 \exp(-h_0/k) \). This contradicts the assumption.

- \( \dot{h}(0) < 0 \): to show that \( \dot{h} < 0 \), \( \forall t \in [0, T_{\text{max}}) \), assume the converse; that is, there exists \( T \) such that \( \dot{h}(T) = 0 \) and \( \dot{h}(t) < 0 \), \( \forall t < T \). Since \( \dot{h} \) is continuous and recalling (36), it follows that there exists \( \delta > 0 \) and \( \epsilon > 0 \) such that \( \dot{h}(t) < -\epsilon, \forall t \in [T - \delta, T] \). Recalling (38) one has that

\[
\zeta(t) = h(t) \exp\left(\int_0^{T-\delta} \frac{\dot{h}(\tau)}{kb(\tau)} d\tau + \int_{T-\delta}^t \frac{\dot{h}(\tau)}{kb(\tau)} d\tau\right)
\]

Using the fact that \( \dot{h}(t) < -\epsilon, \forall t \in [T - \delta, T] \), it follows that

\[
\zeta(T) < h(T) \exp\left(\int_0^{T-\delta} \frac{\dot{h}(\tau)}{kb(\tau)} d\tau\right), \forall t \in [T - \delta, T]
\]

Moreover, since \( b(t) \geq 1, \forall t \in [0, T_{\text{max}}) \),

\[
\zeta(T) \geq h(t) \exp\left(\int_0^{T-\delta} \frac{\dot{h}(\tau)}{kb(\tau)} d\tau + \int_{T-\delta}^t \frac{\dot{h}(\tau)}{k} d\tau\right)
\]

\[
\geq h(t) \exp\left(\int_0^{T-\delta} \frac{\dot{h}(\tau)}{kb(\tau)} d\tau\right) \exp\left(\frac{\dot{h}(t)}{k}\right)
\]

Using the fact that \( \dot{h}(T) = 0 \), one obtains

\[
\zeta(T) \geq h(T) \exp\left(\int_0^{T-\delta} \frac{\dot{h}(\tau)}{kb(\tau)} d\tau\right)
\]

This proves the contradiction.
To show that $\dot{h}$ is lower bounded, let $\mathcal{J}$ be the following storage function:

$$\mathcal{J} = \frac{1}{2} h^2$$

(40)

Differentiating $\mathcal{J}$ and recalling equations (36) yields

$$\dot{\mathcal{J}} = -kb(t) \frac{\dot{h}}{h} \left( \dot{h} + \alpha h \right)$$

(41)

It follows that $\mathcal{J}$ is negative as long as $|\dot{h}| > \alpha h$. Since there exists a time $T$ such that $h < 0$, $\forall t > T$, it follows that $h > 0$ is upper bounded. Consequently, $\dot{h}$ is bounded.

Due to the variation of $b(t)$, boundedness of $\dot{h}$ is not sufficient to conclude that $\int_0^t \frac{\dot{h}(\tau)}{kb(\tau)} d\tau$ is bounded. Therefore, it is necessary to study the evolution of $\dot{h}$.

**Proof that the sign of $\dot{h}$ does not change more than once.**

In the following we assume without loss of generality that $\dot{h}_0 < 0$, therefore $\dot{h}(t) < 0$ for all time.

- $\dot{h}(0) \leq 0$: to show that there exists a time $T$ such that $\dot{h}(T) > 0$, assume the converse; that is, $\dot{h} \leq 0$ for all time $t$. Since $\dot{h}$ is negative and decreasing, it follows that $h$ is strictly monotonically decreasing and cannot have a positive limit. Consequently, (36), $\dot{h}(T) > 0$ and the contradiction follows.

- $\dot{h}(0) > 0$: to show that $\dot{h}(t) \geq 0$ for all time, assume the converse; that is, there exists $T$ and $\delta > 0$ such that $\dot{h}(T - \delta) = 0$ and $\dot{h} < 0$, $\forall t \in (T - \delta, T)$. This implies that $(\dot{h}/h)(T - \delta) = -\alpha$ and $\dot{h}/h > -\alpha$, $\forall t \in (T - \delta, T]$. Using the fact that $\dot{h}/h = -\alpha$ at time $(T - \delta)$ and $|\dot{h}| < \alpha$, $\forall t \in (T - \delta, T]$ while $h$ is negative and decreasing and $h$ is positive and decreasing, the contradiction follows.

Using the fact that there exists a time $T \in [0, T_{\max})$ from which $\dot{h} < 0$ is bounded and $\dot{h} \geq 0$, $\forall t \in [T, T_{\max})$, it is straightforward to verify that $\int_0^t \frac{\dot{h}(\tau)}{kb(\tau)} d\tau$ remains bounded on $[T, T_{\max})$. Therefore, since $\zeta$ is exponentially decreasing, one can ensure that $h$ remains positive and exponentially decreasing on $[T, T_{\max})$.

Now, we prove that $T_{\max} = \infty$ and thus that $\zeta$ is well defined on $[0, \infty)$. Assume that $T_{\max} \neq \infty$, it means that there exists a positive number $\delta$ such that $h(t) > 0$ by continuity and such that $\int_0^t \frac{\dot{h}(\tau)}{kb(\tau)} d\tau$ is unbounded on $[T_{\max}, T_{\max} + \delta]$. This contradicts the above discussion. It follows that $h$ converges exponentially to zero. Moreover, using (40) and (41) with direct application of the Input-to-State-Stable (ISS) argument, it follows that $\dot{h}$ is exponentially stable.

Proof of item (3) for $t \in [0, \infty)$:

Now, we prove that the controller (32) is bounded by proving that $\dot{h} \to 0$. Analogously to the proof of Barbatal’s Lemma, we proceed by contradiction. Assume that $\dot{h}$ does not converge to zero. Since $\int_0^t \frac{\dot{h}(\tau)}{kb(\tau)} d\tau$ and $b(t)$ are bounded and there exists a time $T$ such that $\dot{h} \geq 0$, $\forall t > T$, it follows that there exists $\epsilon > 0$ and two sequences $(T_n)_{n \geq 1} \in \mathbb{R}_+$ and $(\delta_n)_{n \geq 1} \in \mathbb{R}_+$ such that

(i) $T_n \to +\infty$ as $n \to +\infty$,

(ii) $\frac{\dot{h}}{h}(T_n - \delta_n) = \epsilon/2$ and $\frac{\dot{h}}{h}(T_n) = \epsilon$.

(iii) $\epsilon/2 \leq \frac{\dot{h}}{h}(t) \leq \epsilon$, $\forall t \in [T_n - \delta_n, T_n]$.

We need to show that $(\delta_n)_{n \geq 1}$ is lower bounded by a strictly positive number. Using the fact that $\dot{h}/kb \leq \epsilon$, $\forall t \in [T_n - \delta_n, T_n]$ and recalling (36), one has

$$\left( \frac{\dot{h}}{h} + \alpha \right) \geq -\epsilon$$

Integrating this inequality within $[T_n - \delta_n, T_n]$ one obtains

$$\ln \left( \frac{h(t)}{h(T_n - \delta_n)} \right) \geq -\int_{T_n - \delta_n}^{T_n} (\alpha + \epsilon) \, d\tau$$

Given that

$$-\int_{T_n - \delta_n}^{T_n} (\alpha + \epsilon) \, d\tau \geq -\int_{T_n - \delta_n}^{T_n} (\alpha_{\text{max}} + \epsilon) \, d\tau,$$

one has

$$\ln \left( \frac{h(t)}{h(T_n - \delta_n)} \right) \geq - (\alpha_{\text{max}} + \epsilon) \delta_n$$

Hence, $h(t) \geq h(T_n - \delta_n) \exp(-(\alpha_{\text{max}} + \epsilon) \delta_n)$, $\forall t \in [T_n - \delta_n, T_n]$ and therefore, using the fact that $h < 0$ and increasing ($\dot{h} \geq 0$),

$$\frac{\dot{h}}{h} \geq \frac{\dot{h}}{h(T_n - \delta_n)} \exp \left( (\alpha_{\text{max}} + \epsilon) \delta_n \right)$$

(42)

Using $(\dot{h}/kb)(T_n - \delta_n) = \epsilon/2$ and $(\dot{h}/kb)(T_n) = \epsilon$, one also has

$$\frac{\dot{h}}{h}(T_n) = -\epsilon - \alpha(T_n)$$

$$\frac{\dot{h}}{h}(T_n - \delta_n) = -\epsilon/2 - \alpha(T_n - \delta_n)$$

Recalling inequality (42), it follows that

$$\exp \left( (\alpha_{\text{max}} + \epsilon) \delta_n \right) \geq \frac{\epsilon + \alpha(T_n)}{\epsilon/2 + \alpha(T_n - \delta_n)}$$

Now, we need the uniform continuity of $\alpha(t)$. To show this, we first need to show that $1/b(t)$ is itself uniformly continuous (see (37)). The result is straightforward to show using (31) and the fact that $(\dot{h}, h)$ converge to 0 ($b/b^2$ is bounded, then $1/b$ is uniformly continuous). Using assumptions of the theorem and the fact that $1/b$ is uniformly continuous, it follows that $\alpha(t)$ is uniformly continuous. Thus, there exists $\gamma > 0$ such that $|T_n - t| \leq \gamma \Rightarrow |\alpha(T_n) - \alpha(t)| \leq \epsilon/4$. Considering the case $\delta_n < \gamma$, one obtains $\alpha(T_n) \geq \alpha(T_n - \delta_n) - \epsilon/4$ and, therefore

$$\exp \left( (\alpha_{\text{max}} + \epsilon) \delta_n \right) \geq \frac{3\epsilon/4 + \alpha(T_n - \delta_n)}{\epsilon/2 + \alpha(T_n - \delta_n)}$$

Using the fact that $\alpha(t)$ is bounded, it is straightforward to verify that

$$\frac{3\epsilon/4 + \alpha(t)}{\epsilon/2 + \alpha(t)} \geq \frac{3\epsilon/4 + \alpha_{\text{max}}}{\epsilon/2 + \alpha_{\text{max}}}, \forall t > 0$$
This implies that
$$
\exp ( (\alpha_{\text{max}} + \epsilon) \delta_n) \geq \frac{3\epsilon/4 + \alpha_{\text{max}}}{\epsilon/2 + \alpha_{\text{max}}} > 1, \ \forall n \geq 1
$$
Consequently, there exists $\delta > 0$ such that $\delta_n \geq \delta$ for all $n \geq 1$. The next step of the proof is a direct application of the proof of Barbalat’s Lemma. By definitions (ii)-(iii), for all $t \in [T_n - \delta_n, T_n]$ and for all $n \geq 1$ we have
$$
\frac{\dot{h}}{kb}(t) = \left| \frac{\dot{h}}{kb}(t) \right| = \left| \frac{\dot{h}}{kb}(T_n) - \left( \frac{\dot{h}}{kb}(T_n) - \frac{\dot{h}}{kb}(t) \right) \right|
\geq \epsilon - \frac{\epsilon}{2}
\geq \frac{\epsilon}{2}
$$
As a consequence, $\int_0^t (\dot{h}/kb)(\tau) \, d\tau$ converges to $+\infty$ which contradicts the hypothesis.

Using the fact that $\dot{h}(t)$ converges to 0, it is straightforward to verify that:
- $\dot{h}/h = -\alpha(t)$ when $t$ tends to $+\infty$. This means that the descent speed depends on $\alpha(t)$.
- $\dot{h}$ is bounded and therefore the control law (32) is bounded.

**Remark 5.2:** Note that the stability of the control law (14) used for the lateral dynamics during the landing manoeuvre can also be proved in the case where $\Delta^l$ and $b$ are constant; the proof is similar to the second part of the proof of Theorem 4.1 using the fact that $\dot{h}$ is bounded and converges to 0. The authors do not have a formal proof of stability in the case where both $\dot{h}(t)$ and $\Delta^l(t)$ vary over time or in case where the lateral dynamics of the target plane is not zero. Nevertheless, if these variables vary sufficiently slowly ($\dot{b}(t) \equiv 0$, $\Delta^l(t) \equiv 0$ and $\dot{v}_n \equiv 0$), then the robustness of Theorem 4.1 will ensure stability. Characterising the stability conditions is a difficult problem that remains open.

**VI. SIMULATIONS**

In order to evaluate the efficiency of the proposed servo control technique, Matlab simulations of the vertical landing of an idealised quadrotor (34) on static or moving platform are presented. The simulations presented consider only the vertical landing problem of the vehicle on a static and a moving platform.

The mass of the vehicle is chosen $m = 0.85$kg. It corresponds to the physical mass of the quadrotor used for experiments. The control gain is set to $k = 10$, the error $\Delta_z$ is chosen $\Delta_z = -0.3$. For the parameter $b$ defined in (31), incorporating the ground effect, we have chosen $l_0 = 0.5m$ and $D_0 = 0.15m$. The desired set point $\omega^*$ is set to $0.5s^{-1}$. Using the above values of the different parameters involved in the vertical motion (36), it is straightforward to show that condition (33) is verified. Figure 5 shows the closed-loop trajectory of the vertical motion of the vehicle. One can verify that the vertical optical flow $w_z$ remains positive for all time even if it does not reach $\omega^*$ and the height $h = -z + z_G$ converges exponentially to 0. We also notice that the height remains positive during the manoeuvre, implying that the vehicle does not collide with the platform.

Figure 6 shows the result with a moving platform. We keep the same parameters as before. The vertical motion of the platform is chosen as
$$
z_G = a_G \sin (2\pi f_G t) \quad \text{with} \quad a_G = 0.1m \quad \text{and} \quad f_G = 0.3s^{-1}
$$
It is straightforward to verify that condition (33) holds. Note that, during the simulation, $z_G$ is assumed to be unknown. This means that no feed-forward compensations is performed. Figure 6 shows the closed-loop trajectory of the vertical motion of the vehicle. Observe that the vertical optical flow remains positive for all time even if it does not reach $\omega^*$ and the height $h = -z + z_G$ converges exponentially to 0 despite the fact that the vertical motion of the platform is unknown. Figure 7 shows the same result with an additional noise on the measured optical flow. One observes that the convergence is not affected, even close to the touchdown.

**Fig. 5:** Simulation of vertical landing on a static platform using controller (32)

**Fig. 6:** Simulation of vertical landing on a stochastically moving platform using controller (32)

Figure 8 shows the result with a stochastically moving platform. The vertical motion of the platform is now the sum of $n = 7$ sinusoidal signals (see equation (28)), where parameters $a_i \in [0, 0.1]$, $\omega_i \in [1, 6]$ and $\phi_i \in [0, 2\pi]$ are chosen stochastically. The bounds of the parameters are chosen to ensure that the condition (33) is verified. During the simulation, $z_G$ is still assumed to be unknown. Once again, one observes the expected behaviour, the height $h = -z + z_G$ converges exponentially to 0 despite the unknown motion of the platform.

In Figure 9, a phase diagram for different trajectories is presented. For each trajectory, parameters are chosen stochastically and analogously to the previous simulation. $\Delta_z \in [-2, 2]$ is also chosen stochastically. As for the set point $\omega^*$ and initial conditions, they are chosen such that the figure is understandable: $h_0 \in [1, 4]$, $\dot{h}_0 \in [-4, 6]$ and $\omega^* \in [0.5, 4.5]$. 

The figure shows robustness of the approach since the expected behaviour is observed for all trajectories. One can observe that the trajectories satisfy the result of theorem 5.1, that is $h(t) > 0$ remains positive for all time and $(\hat{h}, \dot{h})$ converge to zero. One also verify that there exists a time $T \geq 0$ such that $\hat{h} < 0$, $\forall t \geq T$ (see the proof of the theorem).

VII. EXPERIMENTAL RESULTS

In this section, we present two experiments that demonstrate the performance of the proposed control scheme on a physical vehicle. The UAV used for the experimentation is the quadrotor, constructed by the CEA (Fig. 1), a vertical take off and landing vehicle ideally suited for stationary and quasi stationary flight [28].

A. Prototype description

The X4-flyer is equipped with a set of four electronic boards designed by the CEA. Each electronic board includes a micro-controller and has a particular function. The first board integrates motor controllers which regulate the rotation speed of the four propellers. The second board integrates an Inertial Measurement Unit (IMU) consisting of 3 low cost MEMS accelerometers, that give the gravity components in the body frame, 3 angular rate sensors and 2 magnetometers. On the third board, a Digital Signal Processing (DSP), running at 150 MIPS, is embedded and performs the control algorithm of the orientation dynamics and filtering computations. The final board provides a serial wireless communication between the operator’s joystick and the vehicle. An embedded camera with a view angle of 70 degrees pointing directly down, transmits video to a ground station (PC) via a wireless 2.4 GHz analogue link. A Lithium-Polymer battery provides nearly 10 minutes of flight time. The loaded weight of the prototype is about 850g. In parallel the video signal, the X4-flyer sends inertial data to the ground station at a frequency of 15Hz. The data is processed by the ground station PC and incorporated into the control algorithm. Desired orientation and desired thrust are generated on the ground station PC and sent to the drone.
A key challenge for the implementation lies in the relatively large time latency between the inertial data and visual features. For orientation dynamics, an embedded ‘high gain’ controller in the DSP running at 166Hz, independently ensures the exponential stability of the orientation towards the desired set point.

B. Experiments

The target plane used is a large board painted with random contrast textures (Fig. 10). It is held and moved manually. A Pyramidal implementation of the Lucas-Kanade [29] algorithm is used to compute the optical flow. The efficiency of the algorithm is increased by defocusing the camera to low-pass filter images. The field of view of the aperture is of 30° around the direction of observation \( \eta \). Optical flow is computed on 210 points on this aperture and a least-square estimation of motion parameters is used to obtain robust measurements of the average optical flow \( w \) [30].

![Fig. 10: Hovering flight above the landing pad](image)

Given that the divergent flow magnitude is relatively small compared to the lateral flow in the forward and backwards directions [31] and since only the divergent flow is used for landing manoeuvre, the control approach is split into two sequential phases. In the first phase the vehicle is stabilized over the landing plane. Once the velocity has stabilised to zero the landing phase is initiated. During the experiments, the yaw velocity is also separately regulated to zero. This has no effect on the proposed control scheme. The landing pad has been moved both vertically and laterally to show performances of the control algorithms. For the vertical landing, the desired set point \( w^d \) is set to \((0, 0, 0.1)^T\). This ensures a relatively rapid descent (approximatively in 10s).

Note that no measurements of the relative position \( \tilde{\xi} = \xi - \xi_G \) of the UAV with respect to the platform are available. Nevertheless, an estimation of the UAV’s relative position can be computed from the average optical flow using

\[
\frac{\tilde{\xi} - \tilde{\xi}_0}{h_0} = \int_0^t w_\gamma(\tau) d\tau = \int_0^t w \exp \left( -\int_0^\tau w_z d\delta \right) d\tau
\]

where \( \tilde{\xi} \) denotes the relative position of the UAV with respect to the platform: \( \tilde{\xi} = \xi - \xi_G \). Note that

\[
\gamma(\tau) = \exp \left( -\int_0^\tau w_z d\delta \right) = \exp \left( \int_0^\tau \frac{\dot{h}}{h} d\delta \right) = \frac{h(\tau)}{h_0}
\]

In Figures 11 and 12, the 3 components of the relative position \( (\xi - \xi_0)/h_0 \) are presented. Figure 11 shows the result using controller (14) for the stabilisation of the X4-flyer with respect to the platform (from 0s to 140s) and controller (32) for the vertical landing manoeuvre (from 140s). For the stabilisation phase, the platform is moving laterally (from 0s to 100s) and vertically (from 100s to 140s). During the landing manoeuvre \( (t \geq 140s) \) the platform is moving only vertically. Note that, during the landing phase, controller (14) is still used for the \( x-y \) dynamics. This ensures that the vehicle remains stable over the landing pad. Figure 11 shows the exponential descent of the height while the lateral position remains stable. Note that the relative position \( (y - y_G)/h_0 \) converges around −1, this is due to an initial bias of the inertial measurements in \( y \)-direction that has been compensated by the integral term in the controller (14). Note also that, contrary to what was expected, the height \( h \) is slowly oscillating during the landing phase. This implies that condition (33) is not verified for all time \( t \) and therefore, the positivity of \( \alpha(t) \) (see Section V) is not always guaranteed. This problem is mainly due to the fact that experimental constraints (large time latency, outer loop’s sampling time which is of 15Hz) prevent us from choosing a higher gain \( k \) which strictly respect the condition. The vehicle lands at time 180s. We notice that, due to the landing gear, the final position is not \( h \equiv 0 \).

Figure 12 shows the result when the landing pad is moving with high oscillations. The controller for the stabilisation of the X4-flyer with respect to the platform is used from 0s to 55s and the controller for landing is used from \( t = 55s \) while oscillations start from 30s. During landing manoeuvre, the height \( h \) is highly oscillating, which means that the gain \( k \) is not high enough to compensate the oscillations. Nevertheless, the distance with the ground remains positive which insures the non-collision with the moving target and the UAV is even able to land with a satisfactory behaviour.

These results can be watched on the video accompanying the paper or at the following url: http://www.youtube.com/watch?v=hl18Fykax8M.

VIII. CONCLUDING REMARKS

This paper presented a nonlinear controller for vertical landing of a VTOL UAV using the measurement of average optical flow on a spherical camera along with the IMU data. The originality of our approach lies in the fact that neither linear velocity nor distance with the target is reconstructed. Inertial data is used only for \( \text{derotation} \) of the flow and the proposed approach is an image based visual control algorithm. Both stabilisation and vertical landing with respect to a moving platform were considered and a rigorous analysis of the stability of the closed-loop systems was provided. Simulations provide a clear picture of the predicted response of the proposed algorithm. The experimental results indicate
some of the difficulties with obtaining high gain feedback control and show that the proposed scheme is effective even if the assumptions in the theorems don’t necessarily hold.

There are several directions in which further work is of interest. The practical limitations of real world systems limit magnitude of the feedback gain that can be applied and lead to limitations in the applicability of the approach in the presence of aggressive motion of the environment. Improving the time response of the optical sensors would already provide a major improvement in the closed-loop response and alleviate much of this difficulty. The consideration of a feedforward compensation could alleviate the dependence on high gain feedback when the environmental motion can be modelled. How to accomplish this within the image based paradigm is a challenge. Finally, although the robustness of the proposed approach indicates that small variation of orientation and error in estimation of the normal direction will not destroy the stability analysis obtained, it is of interest to consider the situation where the platform orientation is time varying and the normal of the platform is not assumed to be known.

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APPENDIX

In this appendix, we provide derivation of optical flow integration described in Eq. (9) and used in Paragraph III-B. Using the notations of Section III, consider the average of the optical flow in the direction $\eta$ over a solid angle $\mathcal{W}^2$ of $S^2$. Define $(\alpha_e, \alpha_a)$ to be the spherical coordinates of $\eta$ where $\alpha_e$ is the elevation angle and $\alpha_a$ is the azimuth angle. With these parameters, define $R_t$ to be the orientation matrix from a frame of reference with $\eta$ in the z-axis assuming no yaw rotation to the inertial frame $\mathcal{I}$

$$R_t = \begin{pmatrix} c(\alpha_e) c(\alpha_a) & -s(\alpha_a) & s(\alpha_e) c(\alpha_a) \\ c(\alpha_e) s(\alpha_a) & c(\alpha_a) & s(\alpha_e) s(\alpha_a) \\ -s(\alpha_e) & 0 & c(\alpha_e) \end{pmatrix}$$

Define $\theta_0$ as the angle associated with the apex angle $2\theta_0$ of the solid angle $\mathcal{W}^2$. Then:

$$\phi = \int_{\mathcal{W}^2} \dot{\phi} dp = -\pi (\sin \theta_0)^2 \Omega \times R^T \eta - \frac{Q(V-V_i)}{d}$$

where, $Q = R^T (R_t \Lambda R_t^T) R$ is a symmetric positive definite matrix. The matrix $\Lambda$ is a positive diagonal matrix depending on the solid angle $\mathcal{W}^2$. It can be written as

$$\Lambda = \int_{\mathcal{W}^2} \pi_q(p, R^T \eta) dq = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} (I - q q^T) \langle q, R_t^T \eta \rangle \sin \theta \, d\theta \, d\phi$$

where $q^T = (s(\theta) c(\phi), s(\theta) s(\phi), c(\theta))$. Eventually, straightforward but tedious calculations verify that:

$$\Lambda = \frac{\pi (\sin \theta_0)^4}{4} \begin{pmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

where $\lambda = \frac{(\sin \theta_0)^2}{4 - (\sin \theta_0)^2}$. 

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\footnote{for all $x \in \mathbb{R}$, $s(x) = \sin(x)$, $c(x) = \cos(x)$, $t(x) = \tan(x)$}

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