The Metaphysical Significance of the Ugly Duckling Theorem

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Abstract

According to Satosi Watanabe’s “theorem of the ugly duckling”, the number of (possible) predicates satisfied by any two different particulars is a constant, which does not depend on the choice of the two particulars. If the number of (possible) predicates satisfied by two particulars is their number of properties in common, and the degree of resemblance between two particulars is a function of their number of properties in common, then it follows that the degree of resemblance between any two different particulars is a constant, which does not depend on the choice of the two particulars either. Avoiding this absurd conclusion requires questioning assumptions involving infinity in the proof or interpretation of the theorem, adopting a sparse conception of properties according to which not every (possible) predicate corresponds to a property, or denying that degree of resemblance is a function of number of properties in common. After arguing against the first two options, this paper argues for a version of the third which analyses degrees of resemblance in terms of degrees of naturalness of common properties. In the course of doing so, it presents a novel account of natural properties.
Introduction

“...the point of philosophy,” according to Bertrand Russell, “is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it” (Russell, 1918, 514). The argument discussed in this paper, which proceeds from nearly provable or platitudinous premises about predicates and properties to the barely believable conclusion that every two different particulars resemble each other to the same degree, does not fall far short of Russell’s goal. The paper attempts to assess the metaphysical significance of the argument by considering which premise should be rejected, and whether any surrogate premise can capture its platitudinous aspects, without entailing a barely believable conclusion.

The first premise, which Satosi Watanabe dubs “the theorem of the ugly duckling” (Watanabe, 1969, 376), is:

(1) The number of (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is a fixed constant, which equals the number of (possible) predicates satisfied by only the first, as well as the number of (possible) predicates satisfied by only the second.

As Watanabe explains “The reader will soon understand the reason for referring to the story of Hans Christian Anderson, because this theorem, combined with the foregoing interpretation [or premises two and three below], would lead to the conclusion that an ugly duckling and a swan are just as similar to each other as are two (different) swans.” (Watanabe, 1969, 376). The rationale for the first premises may be divided into a provable
part, that the number of elements in a complete boolean lattice with \( n \) atoms is \( 2^n \), and a very plausible part, that the (possible) predicates ordered by entailment are a complete boolean lattice. In the first section, this rationale is explained in detail.

The second premise follows from an abundant conception of properties, according to which a particular has a property if and only if it satisfies a (possible) predicate corresponding to that property, and so the number of properties is the number of (possible) predicates. It is:

(2) The number of (possible) predicates satisfied by two particulars is the number of properties they have in common, the number of (possible) predicates satisfied by only the first is the number of properties the first has not in common with the second, and the number of (possible) predicates satisfied by only the second is the number of properties the second has not in common with the first.

Snow, for example, has the property of being white, according to abundant conceptions of properties, if and only if snow satisfies the corresponding predicate ‘is white’. Likewise, two peas in a pod have the properties of greenness, roundness and yuckiness in common, according to abundant conceptions of properties, if and only if they satisfy the corresponding predicates ‘is green’, ‘is round’, and ‘is yucky’. And cheese has the property of edibility not in common with chalk if and only if cheese but not chalk satisfies the predicate ‘is edible’.

Proponents of sparse conceptions of properties, according to which the number of properties is less than the number of (possible) predicates, will find this the obvious premise to reject. But I will argue in the second section that many sparse conceptions
of properties which are otherwise well motivated nevertheless fail to escape the barely believable conclusion that the degree of resemblance between two different particulars is a constant, independent of the choice of the two particulars. According to David Armstrong’s influential conception, for example, there are instantiated conjunctive, but no negative, disjunctive or uninstantiate properties (Armstrong, 1978b). But Armstrong’s conception turns out to entail that every particular has at most one property, and so the degree of resemblance between different particulars is zero, regardless of the choice of the two particulars.

The third premise is supported by the analysis of resemblance as having properties in common, which suggests that the more properties particulars have in common, the more they resemble each other, and the more properties particulars have not in common with each other, the less they resemble each other. It is:

(3) The degree of resemblance between two particulars is a function of the number of properties they have in common, the number of properties the first has not in common with the second, and the number of properties the second has not in common with the first.

It’s natural to suggest, for example, that two peas in a pod resemble each other to a high degree because they have many properties in common and few properties not in common. Likewise, it’s natural to suggest that the degree of resemblance between a raven and a writing desk is low because a raven and a writing desk have few properties in common and many properties not in common.

For illustrative purposes, I will focus on the suggestion that the degree of resemblance
between particulars is their proportion of properties in common or, in other words, their number of properties in common, divided by their number of properties in total (the sum of their number of properties in common and number of properties not in common). This is convenient because the degree of resemblance between particulars which have all of their properties in common is one, whereas the degree of resemblance between particulars which have none of their properties in common is zero. But nothing important depends on this choice of illustration.

The first premise and the second in combination entail that the number of properties in common and the number of properties not in common between two different particulars is a constant, which in combination with the third premise entails that degree of resemblance between two particulars is a function of a constant, and so:

(4) The degree of resemblance between two particulars which do not satisfy all the same (possible) predicates, or which do not have all of their properties in common, or which differ from each other, is a fixed constant, which does not depend on the choice of the two particulars.

This conclusion is barely believable. According to it, a raven resembles a writing desk, for example, to the same degree as a raven resembles a magpie, and a cygnet resembles a duckling to the same degree as two different ducklings resemble each other.

Through Nelson Goodman’s later work, especially *Seven Strictures on Similarity*, Watanabe’s argument has become extremely familiar. As Goodman writes “Shall we say that two things $a$ and $b$ are more alike than two others $c$ and $d$ if and only if $a$ and $b$ have more properties in common than do $c$ and $d$? If that has a more scientific
sound and seems safer, it is unfortunately no better; for any two things have exactly as many properties in common as any other two. If there are just three things in the universe, then any two of them belong together in exactly two classes and have exactly two properties in common: the property of belonging to the class consisting of the two things, and the property of belonging to the class consisting of all three things. If the universe is larger, the number of properties will be larger but will still be the same for every two elements. Where the number of things in the universe is \( n \), each two things have in common exactly \( 2^{n-2} \) properties out of the total \( 2^n - 1 \) properties; each thing has \( 2^{n-2} \) properties that the other does not, and there are \( 2^{n-2} - 1 \) properties that neither has. If the universe is infinite, all these figures become infinite and equal” (Goodman, 1972, 443-444). Despite its familiarity, I believe the argument is worth revisiting in detail for two reasons.

Firstly, although it seems clear that the conclusion of the argument should be avoided by rejecting either the second or the third premise, it is not at all clear which premise ought to be rejected. Ideally, a motivation for rejecting one premise rather than the other ought to include a surrogate for the rejected premise, which captures what is intuitive about the original premise, but which escapes the barely believable conclusion. However, I will argue below that many of the most obvious and popular surrogates, while succeeding in capturing what is intuitive about the original premises, in fact fail to escape the barely believable conclusion. I am most optimistic about the prospects for an intuitive revision of the third premise. But no solution which I know of is completely satisfactory.
Secondly, contemporary discussions of the problem almost invariably focus on the case in which the number of (possible) predicates or properties is infinite. David Lewis, for example, writes “Because properties are so abundant, they are undiscriminating. Any two things share infinitely many properties, and fail to share infinitely many others. That is so whether the two things are perfect duplicates or utterly dissimilar. Thus properties do nothing to capture facts of resemblance” ((Lewis, 1983, 346); see also (Lewis, 1986, 59-60)). In a similar vein, Graham Priest writes, about similarity between worlds, “Presumably, how similar two worlds are depends on what holds in each of these. But how can one define similarity in terms of these things? One certainly cannot define it in terms of the number of propositions over which the worlds differ. For if there are any differences at all, there will be an infinite number” (Priest, 2008, 97).

Since the number of possible predicates is infinite, the focus on the infinite case is unsurprising. But the focus on the infinite case also makes it hard to avoid the impression that infinity is the source of the problem, which in turn suggests that clear thinking about infinity may be the route to a solution. I will argue below that there are two controversial assumptions relating to infinity which could be rejected in order to resist the conclusion of the argument; namely, the axiom of choice and the completeness of the lattice of (possible) predicates under entailment or, in other words, the thesis that every collection of (possible) predicates, whether finite or infinite, has a disjunction and conjunction. But because the argument goes through even in the finite case without either of these controversial assumptions, I will also argue that controversies concerning infinity are a distraction, and do not undermine the force of the problem.
In the first section, I will explain the rationale behind the first premise, with particular attention to the case in which the number of (possible) predicates is infinite, and to assumptions which may be resisted in order to resist the conclusion of the argument. In the second section, I will consider rejecting the second premise by denying that there is a property corresponding to every (possible) predicate, and so adopting a sparse conception of properties. In the third section, I will consider rejecting the third premise by retaining an abundant conception of properties, but denying that degrees of resemblance are a function of common and uncommon properties. The fourth section concludes.

1 The First Premise

This section explains the rationale behind the first premise, according to which the number of (possible) predicates satisfied by two particulars which do not satisfy all the same predicates is a fixed constant equal to the number of (possible) predicates satisfied by only the first. The rationale for the first premise may be divided into a provable part and a plausible part. The provable part is that the number of elements in a complete boolean lattice (or, in other words, a complete complemented distributive lattice with a minimum and maximum element) with \( n \) atoms is \( 2^n \). The plausible part is that the lattice of (possible) predicates ordered under entailment is a complete boolean lattice. So the section first clarifies the nature of (possible) predicates, then explains why the set of predicates is a complete boolean lattice under the relation of entailment, and finally explains the proof that a complete boolean lattice with \( n \) atoms has \( 2^n \) elements.

A predicate is a sentence with a name removed. The predicate ‘is white’, for example,
A (named) particular satisfies a predicate if and only if replacing the gap in the predicate by a name of the particular results in a true sentence. Snow satisfies ‘is white’, for example, because the sentence ‘snow is white’ is true. According to abundant conceptions of properties, there is a property corresponding to every predicate: corresponding to the predicate ‘is white’, for example, is the property of being white.

There are some properties that do not correspond to any actual predicate. As David Armstrong writes “It is clearly possible, and we believe it to be the case, that particulars have certain properties and relations which never fall under human notice” (Armstrong, 1978a, 21). Nevertheless, if these properties were to fall under human notice, we could introduce predicates to talk about them, so corresponding to every property is a possible predicate. The need to consider possible predicates is a complicating factor at some points in the argument, because of their great number, but a simplifying factor at others, because of their completeness. When possible, I may omit to mention them.

A predicate entails another predicate if and only if necessarily any particular which satisfies the former also satisfies the latter. The predicate ‘is white’ entails the predicate ‘is coloured’, for example, because it’s necessary that any particular which satisfies ‘is white’ also satisfies ‘is coloured’. Entailment between predicates is a reflexive and transitive relation: since necessarily any particular which satisfies ‘is white’ satisfies ‘is white’, for example, ‘is white’ entails ‘is white’. And since ‘is scarlet’ entails ‘is red’ and ‘is red’ entails ‘is coloured’, ‘is scarlet’ entails ‘is coloured’.

It’s also convenient to stipulate that the relation of entailment between predicates is
antisymmetric. In other words, if a predicate entails a second predicate and the second
predicate entails the first, then they are the same predicate. Since ‘is white’ entails ‘is
not unwhite’ and ‘is not unwhite’ entails ‘is white’, for example, ‘is white’ and ‘is not
unwhite’ are the same predicate. This stipulation is convenient because it ensures that
there is only one (possible) predicate corresponding to each property, so if there is a
(possible) predicate corresponding to every property, then there is exactly one (possible)
predicate corresponding to every property (Armstrong, 1978a, 7).

One clarification. Consider the purported predicate ‘is not satisfied by itself’. If an
abundant conception of properties is correct, then a predicate satisfies ‘is not satisfied
by itself’ if and only if it satisfies the corresponding property of not being satisfied by
itself or, in other words, if and only if it does not satisfy itself. So ‘is not satisfied by
itself’ satisfies itself if and only if it does not satisfy itself, which is a contradiction.
This is a serious problem for abundant conceptions of properties (see Field (2004) for
discussion), but not the problem under discussion in this paper (for my own part, I
think it is serious problem in any case, so a general solution should not require the
rejection of the abundant conception of properties). In order to avoid it, I consider
below only predicates applying to individual particulars, and not predicates applying to
other predicates, properties, or sets.

A relation which is reflexive, antisymmetric and transitive is called a partial ordering
relation. The ordered pair \(\langle A, \leq \rangle\) of a set \(A\) and a partial ordering relation \(\leq\) between
elements of \(A\) is called a partially ordered set (Gratzer, 2011, 1). Since entailment be-
tween predicates is a reflexive, antisymmetric and transitive relation, entailment between
predicates is a partial ordering relation, and predicates are a partially ordered set under the relation of entailment. (Since I am considering only predicates applying to individual particulars, and not predicates applying to other predicates, properties, or sets, there is nothing paradoxical involved in the assumption that there is a set of all predicates. As well as the relation of entailment between predicates, ‘≤’ below may stand for the standard ordering relation between real numbers: context should disambiguate. As usual I will say that \( a < b \) if and only if \( a \leq b \) but it is not the case that \( b \leq a \).)

An element \( a \in A \) is the conjunction \( b \land c \) of two elements \( b, c \in A \) of a partially ordered set \( \langle A, \leq \rangle \) if and only if (i) \( a \leq b \) and \( a \leq c \) and (ii) for all \( d \in A \) if \( d \leq b \) and \( d \leq c \), then \( d \leq a \). In general, an element \( a \in A \) is the conjunction \( \bigwedge B \) of the elements in a subset \( B \subseteq A \) if and only if (i) for all \( b \in B \), \( a \leq b \) and (ii) for all \( c \in A \) if \( c \leq b \) for all \( b \in B \), then \( c \leq a \) (Gratzer, 2011, 5). The predicate ‘is red and square’ is the conjunction of ‘is red’ and ‘is square’, for example, because ‘is red and square’ entails ‘is red’ and entails ‘is square’, and because every predicate which entails ‘is red’ and entails ‘is square’ entails ‘is red and square’.

An element \( a \in A \) is the disjunction \( b \lor c \) of two elements \( b, c \in A \) of a partially ordered set \( \langle A, \leq \rangle \) if and only if (i) \( b \leq a \) and \( c \leq a \) and (ii) for all \( d \in A \) if \( b \leq d \) and \( c \leq d \), then \( a \leq d \). In general an element \( a \in A \) is the disjunction \( \bigvee B \) of the elements in a subset \( B \subseteq A \) if and only if (i) for all \( b \in B \), \( b \leq a \) and (ii) for all \( c \in A \) if \( b \leq c \) for all \( b \in B \), then \( a \leq c \) (Gratzer, 2011, 5). The predicate ‘is red’, for example, is the disjunction of ‘is scarlet’, ‘is crimson’, ‘is maroon’, ... and so on, because ‘is scarlet’, ‘is crimson’, ‘is maroon’, ... and so on all entail ‘is red’, and every predicate which entails
A lattice is a partially ordered set \( \langle A, \leq \rangle \) in which every pair of elements \( a, b \in A \) has a conjunction \( a \land b \) and a disjunction \( a \lor b \). Equivalently, a lattice is a partially ordered set \( \langle A, \leq \rangle \) in which every finite nonempty subset of elements \( B \subseteq A \) has a conjunction \( \bigwedge B \) and a disjunction \( \bigvee B \) (Gratzer, 2011, 9). Since every pair and so every finite nonempty subset of (possible) predicates has a conjunction which may be formed with ‘and’ and a disjunction which may be formed with ‘or’, the partial ordering of (possible) predicates under the relation of entailment is a lattice.

A lattice \( \langle A, \leq \rangle \) is complete if and only if every subset of elements \( B \subseteq A \) has a conjunction \( \bigwedge B \) and a disjunction \( \bigvee B \) (Gratzer, 2011, 50). Every finite lattice is complete, since every nonempty subset of elements of a finite lattice is a finite nonempty subset (and the conjunction and disjunction of the empty subset of elements are \( \bigvee A \) and \( \bigwedge A \) respectively, which exist because \( A \) is finite), so every nonempty subset of elements has a conjunction and disjunction. So if there were only a finite number of predicates, then the partial ordering of predicates under the relation of entailment would be a complete lattice.

But there is an infinite number of predicates. The predicates ‘is one year old’, ‘is two years old’, ‘is three years old’, ... and so on, for example, are countably infinite. And though some infinite subsets of the set of predicates do have a disjunction and conjunction, it’s controversial whether every infinite subset does, because it’s controversial whether the disjunction or conjunction of an infinite set of predicates can be formed by joining every predicate in the set with the words ‘or’ or ‘and’. Although the disjunction
of ‘is one year old’, ‘is three years old’, ‘is five years old’, ... and so on, for example, is ‘is an odd number of years old’, it’s controversial whether this disjunction can be properly expressed as ‘is one year old or is three years old or is five years old ...’ and so on. So it’s controversial whether the lattice of predicates under the relation of entailment is complete.

Nevertheless, although the existence of disjunctions and conjunctions of infinite subsets of predicates is not obviously guaranteed by the possibility of using the words ‘or’ or ‘and’ to join all the predicates in the subset, and so whether every infinite subset of predicates has an actual predicate as its disjunction and conjunction is not obvious, it still seems plausible that every infinite subset of predicates has a possible predicate as its disjunction and conjunction, because it’s plausible that for every infinite subset of predicates, there is both a possible predicate which is satisfied by all and only the particulars which satisfy one of the predicates in the set and a possible predicate which is satisfied by all and only the particulars which satisfy all of the predicates in the set. So it’s plausible that the lattice of possible predicates under the relation of entailment is complete.

An element \( a \in A \) is the maximum \( \top \) of a poset \( \langle A, \leq \rangle \) if and only if \( b \leq a \) for all \( b \in A \) (Gratzer, 2011, 5). The predicate ‘exists’ or ‘is white or not white’, for example, is the maximum element of the set of predicates under the relation of entailment, because necessarily, if a particular satisfies any predicate, then it satisfies ‘exists’ and ‘is white or not white’ or, in other words, every predicate entails ‘exists’ and ‘is white or not white’. (In a complete or finite lattice, there must be a maximum \( \top \) since \( \bigvee A \) exists
and \( a \leq \bigvee A \) for all \( a \in A \).

Likewise, an element \( a \in A \) is the minimum \( \bot \) of a poset \( \langle A, \leq \rangle \) if and only if \( a \leq b \) for all \( b \in A \) (Gratzer, 2011, 5). The predicate ‘does not exist’ or ‘is white and not white’, for example, is the minimum element of the set of predicates under entailment, because necessarily, if a particular satisfies ‘does not exist’ or ‘is white and not white’, then it satisfies every predicate (albeit vacuously so, because necessarily no particular satisfies ‘does not exist’ or ‘is white and not white’) or, in other words, ‘does not exist’ or ‘is white and not white’ entail every predicate. (In a complete or finite lattice, there must be a minimum \( \bot \) since \( \bigwedge A \) exists and \( \bigwedge A \leq a \) for all \( a \in A \).)

An element \( a \in A \) is the negation \( \neg b \) of an element \( b \in A \) if and only if \( a \lor b = \top \) and \( a \land b = \bot \) (Gratzer, 2011, 97). The predicate ‘is not white’, for example, is the negation of the predicate ‘is white’ and vice versa, because ‘is white or not white’ is equivalent to ‘exists’ or ‘is white or not white’ and ‘is white and not white’ is equivalent to ‘does not exist’ or ‘is white and not white’. Likewise, the predicate ‘is abstract’ is the negation of the predicate ‘is concrete’ and vice versa, because ‘is abstract or concrete’ is equivalent to ‘exists’ or ‘is white or not white’ and ‘is abstract and concrete’ is equivalent to ‘does not exist’ or ‘is white and not white’.

A lattice \( \langle A, \leq \rangle \) is complemented if and only if it has a minimum \( \bot \), a maximum \( \top \) and every \( a \in A \) has a negation \( \neg a \) (Gratzer, 2011, 98). The set of (possible) predicates under the relation of entailment is a complemented lattice, since its minimum element is ‘does not exist’ or ‘is white and not white’, its maximum element is ‘exists’ or ‘is white or not white’ and since every (possible) predicate has a negation, which may be formed
using the word ‘not’.

A lattice \( \langle A, \leq \rangle \) is distributive if and only if for all elements \( a, b, c \in A \), \( a \land (b \lor c) = (a \land b) \lor (a \land c) \) and \( a \lor (b \land c) = (a \lor b) \land (a \lor c) \) (Gratzer, 2011, 14-15). Equivalently, a lattice \( \langle A, \leq \rangle \) is distributive if and only if for all finite nonempty subsets \( B \subseteq A \) and elements \( a \in A \), \( a \land \bigvee B = \bigvee \{a \land b \mid b \in B\} \) and \( a \lor \bigwedge B = \bigwedge \{a \lor b \mid b \in B\} \). Predicates under the relation of entailment are a distributive lattice. The predicate ‘is red or both square and large’, for example, is equivalent to ‘is both either red or square and either red or large’, and the predicate ‘is both red and either square or large’ is equivalent to ‘is either both red and square or both red and large’.

A lattice is boolean if and only if it is distributive and complemented, and it has a minimum and maximum element (Gratzer, 2011, 15). So the lattice of predicates under the relation of entailment is boolean, because predicates under the relation of entailment are distributive and complemented (since every element has a negation), and it has a minimum and maximum element (‘does not exist’ or ‘is white and not white’ and ‘exists’ or ‘is white or not white’). That completes the explanation of the plausible part of the rationale for the first premise. The next paragraph begins the explanation of the provable part.

In a complete boolean lattice \( \langle A, \leq \rangle \), like the lattice of predicates under the relation of entailment, every (finite or infinite) subset satisfies distributivity. In other words, for any \( B \subseteq A \) and elements \( a \in A \), \( a \land \bigvee B = \bigvee \{a \land b \mid b \in B\} \) and \( a \lor \bigwedge B = \bigwedge \{a \lor b \mid b \in B\} \). To prove this first note that \( \bigvee B \) exists in a complete lattice, and so (i) for all \( a \land b \in \{a \land b \mid b \in B\} \), \( a \land b \leq a \land \bigvee B \). Then suppose \( c \leq a \land b \) for all \( a \land b \in \{a \land b \mid b \in B\} \).
For all $b \in B$, $b = b \lor (a \land \neg a)$ and so it follows from distributivity that $b = b \lor (a \land \neg a) = (b \land a) \lor (b \land \neg a) \leq c \lor \neg a$ and so $\lor B \leq c \lor \neg a$. But also $a \land \lor B \leq a \land (c \lor \neg a)$ and so from distributivity $a \land \lor B \leq a \land (c \lor \neg a) = (a \land c) \lor (a \land \neg a) = a \land c \leq c$. So (ii) for all $c \in A$ if $c \leq a \land b$ for all $a \land b \in \{a \land b \mid b \in B\}$ then $a \land \lor B \leq c$. The proof that $a \lor \land B = \land \{a \lor b \mid b \in B\}$ is similar (Gratzer, 2011, 154).

An element $a \in A$ of a lattice $\langle A, \leq \rangle$ is an atom if and only if $\bot < a$ and for all $b \in A$ if $b < a$ then $b = \bot$ (Gratzer, 2011, 101). In other words, an element is an atom if and only if no element is smaller, except the minimum. In the lattice of predicates which apply to a die in virtue of the number it lands, for example, there are six atoms: ‘lands one’, ‘lands two’, ‘lands three’, ‘lands four’, ‘lands five’ and ‘lands six’. (Note that an “atom” in this context is not a syntactically simple predicate: ‘lands odd’, for example, is not an atom, because it is strictly entailed by ‘lands one’, whereas ‘lands on an even number and lands on a prime number’ is an atom, because it is strictly entailed by ‘does not land’ but not by any other predicate.)

If $\langle A, \leq \rangle$ is a finite lattice, then every element $a \in A$ must be greater than or equal to some atom $b \in A$, since otherwise for all $c \in A$ such that $\bot < c < a$ there must be some $d \in A$ such that $\bot < d < c$, and $A$ would have an infinite number of elements. But if $A$ is not finite, then it’s possible that $\langle A, \leq \rangle$ has no atoms. Take, for example, the countably many logically independent predicates ‘lands heads on the first toss’, ‘lands heads on the second toss’, ‘lands heads on the third toss’, ... and so on. The conjunctions, disjunctions and negations of finite subsets of these predicates, which may be formed using the words ‘and’, ‘or’ and ‘not’ finitely many times, ordered under the relation of entailment form a
boolean lattice. But for each consistent predicate in the lattice, a consistent predicate which entails it may be formed by conjoining one of the simple predicates from which it is not formed. The predicate 'lands heads on the first toss or does not land heads on the second toss', for example, is not an atom, because it is entailed by 'lands heads on the first toss or does not land heads on the second toss and lands heads on the third toss'. So the lattice has no atoms.

But if \( \langle A, \leq \rangle \) is a complete boolean lattice, then every element \( a \in A \) must be greater than or equal to some atom \( b \in A \). Take again, for example, the countably many logically independent predicates 'lands heads on the first toss', 'lands heads on the second toss', 'lands heads on the third toss', ... and so on. The conjunctions, disjunctions and negations of arbitrary (finite or infinite) subsets of these predicates, which may be formed using the words 'and', 'or' and 'not' any arbitrary (finite or infinite) number of times, ordered under the relation of entailment form a complete boolean lattice. Every consistent predicate in the lattice is entailed by at least one atom, which may be formed by conjoining predicates with which it is consistent until there are no more. The predicate 'lands heads on the first toss', for example, is entailed by the atom formed by the infinitely long conjunction 'lands heads on the first toss and lands heads on the second toss and lands heads on the third toss...' and so on.

The formal proof of this claim depends on the axiom of choice, according to which for each set there is a function which chooses exactly one element from each of its subsets or, in other words, according to which for every set \( A \) there is a function \( f : \mathcal{P}(A) \rightarrow A \) from the subsets to the elements of \( A \) such that for each \( B \in \mathcal{P}(A), f(B) \in B \). Let
be a complete boolean lattice and \( f : \mathcal{P}(A) \mapsto A \) be such a function. Then for each element \( a \in A \) consider the set \( B \subseteq A \) such that both \( a \in B \) and for every subset \( C \subseteq B \) if there is some \( b \in A \) such that \( \bot < b < \bigwedge C \), then \( f \{ c \in A \mid \bot < c < \bigwedge C \} \in B \).

Since \( a \in B \), \( \bigwedge B \leq a \). And since \( B \subseteq B \), if there is some \( b \in A \) such that \( \bot < b < \bigwedge B \), then \( f \{ c \in A \mid \bot < c < \bigwedge B \} \in B \) and so \( \bigwedge B \leq f \{ c \in A \mid \bot < c < \bigwedge B \} \). But this contradicts the fact that \( f \{ c \in A \mid \bot < c < \bigwedge B \} < \bigwedge B \). So there is no \( b \in A \) such that \( \bot < b < \bigwedge B \). So \( \bigwedge B \) is an atom less than or equal to \( a \).

So far it has been proved that every element in a complete boolean lattice is greater than or equal to some atom. The next step in the proof is to use this fact to show that each predicate corresponds to a subset of atoms, and vice versa, so that the number of elements in a complete boolean lattice with \( n \) atoms is \( 2^n \). The rest of the section explains this part of the proof.

Every element \( a \in A \) in a complete boolean lattice \( \langle A, \leq \rangle \) is equivalent to the disjunction \( \bigvee \{ b \in B \mid b \leq a \} \) of a subset of the atoms \( B \subseteq A \) since (i) for all \( b \in \{ b \in B \mid b \leq a \} \), \( b \leq a \) and (ii) for all \( c \in A \) if \( b \leq c \) for all \( b \in \{ b \in B \mid b \leq a \} \), then \( a \leq c \). To prove (ii) suppose the contrary, that for some \( c \in A \), \( b \leq c \) for all \( b \in \{ b \in B \mid b \leq a \} \), but \( a \not\leq c \). Since \( a \not\leq c \), \( a \) is consistent with \( \neg c \) and so \( \bot < a \land \neg c \). Then since \( A \) is a complete boolean lattice, there must be an atom \( d \in B \) such that \( d \leq a \land \neg c \). But since \( d \in B \) and \( d \leq a \), \( d \in \{ b \in B \mid b \leq a \} \) and so according to the supposition \( d \leq c \). Yet, also \( d \leq \neg c \). So \( d = \bot \), which contradicts the fact that, since \( d \) is an atom, \( \bot \leq d \) (Davey and Priestly, 2002, 114).

Moreover, for every subset \( C \subseteq B \) of the atoms \( B \subseteq A \) in a complete boolean lattice,
\(C = \{b \in B \mid b \leq \bigvee C\}\). \(C \subseteq \{b \in B \mid b \leq \bigvee C\}\) since for all \(b \in C, b \in B\) and \(b \leq \bigvee C\).

To prove \(\{b \in B \mid b \leq \bigvee C\} \subseteq C\), suppose \(b \in B\) and \(b \leq \bigvee C\) (if no \(b \in B\) is such that \(b \leq \bigvee C\), then \(\bigvee C = \bot\) and so \(C = \{b \in B \mid b \leq \bot\}\)). Since \(b\) is an atom, either for all \(c \in C\), \(b \land c = \bot\) or for some \(c \in C\), \(b \land c = b\). But if for all \(c \in C\), \(b \land c = \bot\), then since every (finite or infinite) subset in a complete boolean lattice satisfies distributivity
\(b = b \land \bigvee C = \bigvee \{b \land c \mid c \in C\} = \bot\), contradicting that \(b\) is an atom. So if \(b \in B\) and \(b \leq \bigvee C\), then for some \(c \in C\), \(b \land c = b\) and so, since \(b\) and \(c\) are atoms, \(b = c\) (Davey and Priestly, 2002, 114-115).

It follows that in a complete boolean lattice \(\langle A, \leq \rangle\) the function \(\eta : a \mapsto \{b \in B \mid b \leq a\}\) is a one to one mapping from \(A\) to the set of subsets \(\mathcal{P}(B)\) of the atoms \(B \subseteq A\) (with \(\eta^{-1}(C) = \bigvee C\) for all \(C \in \mathcal{P}(B)\) as its inverse). Moreover, it follows that \(c \leq d\) if and only if \(\eta(c) \subseteq \eta(d)\) for all \(c, d \in A\). For if \(c \leq d\), then \(c = \bigvee \eta(c) \leq d = \bigvee \eta(d)\), so for all \(b \in \{b \in B \mid b \leq c\}\), \(b \in \{b \in B \mid b \leq d\}\), so \(\eta(c) = \{b \in B \mid b \leq c\} \subseteq \eta(d) = \{b \in B \mid b \leq d\}\).

And if \(\eta(c) \subseteq \eta(d)\), then \(\{b \in B \mid b \leq c\} \subseteq \{b \in B \mid b \leq d\}\), so \(c = \bigvee \eta(c) \leq d = \bigvee \eta(d)\) and \(c \leq d\) (Davey and Priestly, 2002, 115).

In other words, there is a one to one correspondence between the predicates and the subsets of the atoms the predicates are disjunctions of, and a predicate entails another if and only if it corresponds to a subset of atoms included in the subset of atoms corresponding to the predicate it entails. In the lattice of predicates which apply to a die in virtue of the number it lands, for example, ‘lands odd’ is the disjunction of ‘lands one’, ‘lands two’ and ‘lands three’, and ‘lands odd’ entails ‘lands’ because ‘lands’ is the disjunction of all the atoms, which include ‘lands one’, ‘lands two’ and ‘lands three’. 
And the same is true in the lattice of (possible) predicates, except there are not simple expressions for the atoms of which all predicates are disjunctions.

Let \( n \) be the number of atoms in a complete boolean lattice \( \langle A, \leq \rangle \). Then the number of elements in \( A \) is the number of subsets \( |\mathcal{P}(B)| = 2^n \) of the atoms \( B \subseteq A \). The number of predicates which apply to a die in virtue of the number it lands, for example, is \( 2^6 \), since there is a predicate which applies for each combination of the six atoms ‘lands one’, ‘lands two’, ‘lands three’, ‘lands four’, ‘lands five’ and ‘lands six’. Likewise, the number of predicates which apply to people in virtue of their age in years, for example, is \( 2^{\aleph_0} \), since there is a predicate which applies in virtue of each combination of the \( \aleph_0 \) atoms ‘is zero years old’, ‘is one year old’, ‘is two years old’, ‘is three years old’ ... and so on.

(One clarification. According to class nominalism, there is a property corresponding to every set of (possible) particulars, so if there is a (possible) predicate corresponding to every property, then the number of (possible) predicates is also \( 2^n \), where \( n \) is the number of (possible) particulars. While this seems to be a much quicker route to the conclusion that the number of (possible) predicates is \( 2^n \) for some \( n \) and the corollaries below, class nominalism itself needs to be justified by a result like that just proved. As Goodman, who employs this simplification in his presentation, concedes: “Of course as a nominalist, I take all talk of properties as slang for a more careful formulation in terms of predicates” (Goodman, 1972, 443). Assuming class nominalism in this way does simplify the presentation, but it obscures the justification.)

The number of predicates a particular satisfies is \( 2^{n-1} \), since because the atoms are disjoint and complete each particular must satisfy exactly one atomic predicate, but may
satisfy the disjunction of that atomic predicate with any combination of the remaining \( n - 1 \) atomic predicates. In the case of the die, for example, the number of predicates satisfied by a die satisfying ‘lands one’ is \( 2^5 \), since it satisfies a predicate corresponding to each combination of the five remaining atomic predicates ‘lands two’, ‘lands three’, ‘lands four’, ‘lands five’ and ‘lands six’. Likewise, in the case of the predicates which apply to people in virtue of their age in years, the number of predicates satisfied by a person satisfying ‘is one year old’ is \( 2^{\aleph_0 - 1} \), since it satisfies a predicate corresponding to each combination of the remaining \( \aleph_0 - 1 \) predicates ‘is zero years old’, ‘is two years old’, ‘is three years old’ ... and so on.

The number of predicates satisfied by two particulars which do not satisfy all the same predicates is \( 2^{n-2} \), since the two particulars satisfy in common the disjunctions of exactly two atomic predicates with any combination of the remaining \( n - 2 \) atomic predicates (Watanabe, 1969, 377). In the case of the die, for example, the number of predicates satisfied by a die satisfying ‘lands one’ and a die satisfying ‘lands two’ is \( 2^4 \), corresponding to the sixteen combinations of ‘lands three’, ‘lands four’, ‘lands five’ and ‘lands six’. And in the case of the predicates which apply to people in virtue of their age in years, the number of predicates satisfied by a person satisfying ‘is one year old’ and a person satisfying ‘is two years old’ is \( 2^{\aleph_0 - 2} \), since it satisfies a predicate corresponding to each combination of the remaining \( \aleph_0 - 2 \) atomic predicates ‘is zero years old’, ‘is three years old’ ... and so on.

Likewise, the number of predicates which are satisfied by only the first of two particulars which do not satisfy all the same predicates is also \( 2^{n-2} \), since the first particular
satisfies not in common with the second the disjunctions of the atomic predicate it satisfies with any combination of the remaining \( n - 1 \) atomic predicates, except for the atomic predicate satisfied by the second particular (Watanabe, 1969, 377). In the case of the die, for example, the number of predicates satisfied by a die satisfying ‘lands one’ and not satisfied by a die satisfying ‘lands two’ is still \( 2^4 \), again corresponding to the sixteen combinations of ‘lands three’, ‘lands four’, ‘lands five’ and ‘lands six’. And in the case of the predicates which apply to people in virtue of their age in years, the number of predicates satisfied by a person satisfying ‘is one year old’ and not by a person satisfying ‘is two years old’ is still \( 2^{\aleph_0 - 2} \), since the first but not the second satisfies a predicate corresponding to each combination of the remaining \( \aleph_0 - 2 \) atomic predicates ‘is zero years old’, ‘is three years old’ ... and so on.

So if the set of all (possible) predicates ordered under the relation of entailment is a complete boolean lattice, then the total number of predicates is \( 2^n \), where \( n \) is the number of atomic (possible) predicates. The total number of predicates satisfied by any particular is \( 2^{n-1} \). The number of (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is \( 2^{n-2} \), as is the number of (possible) predicates satisfied by only one. So, as the first premise of the argument states, the number of (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is a fixed constant, which equals the number of (possible) predicates satisfied by only the first and which equals the number of possible predicates satisfied by only the second (Watanabe, 1969, 376-377).

If the number of predicates satisfied by two particulars is the number of properties
they have in common, the number of predicates satisfied by only the first is the number of properties the first has not in common with the second, and the number of predicates satisfied by only the second is the number of properties the second has not in common with the first, then the number of properties in common between two different particulars is also a fixed constant, which equals the number of properties the first has not in common with the second, and the number of properties the second has not in common with the first.

So if the degree of resemblance between two particulars is their number of properties in common divided by their number of properties in total (or, in other words, their number of properties they have in common, divided by the sum of their number of properties in common, the number of properties the first has not in common with the second, and the number of properties the second has not in common with the first), then it follows that the degree of resemblance between two particulars is \( \frac{2^n - 2}{2^n - 2 + 2^{n-2} + 2^n - 2} \), which is \( \frac{1}{3} \) if \( n \) is finite and undefined otherwise. This is something so paradoxical that no one will believe it.

2 The Second Premise

Sparse conceptions of properties deny that there is a property corresponding to every (possible) predicate, and so deny that the number of properties is the number of (possible) predicates. So it’s natural for a proponent of the sparse conception to resist the conclusion of the argument by denying its second premise. The number of properties instantiated by two particulars which do not satisfy all the same (possible) predicates,
according to the sparse conception, may be any number less than or equal to $2^{n-2}$, the number of properties instantiated by only the first may be a second number less than or equal to $2^{n-2}$, and the number of properties satisfied by only the second may be a third number less than or equal to $2^{n-2}$. So even if the degree of resemblance between two particulars is a function of their number of properties in common and their number of properties not in common, it doesn’t follow, according to sparse conceptions of properties, that the degree of resemblance between different particulars is a fixed constant, which does not depend on the choice of the two particulars.

As Gonzalez Rodriguez-Pereyra, for example, writes “... what Watanabe proved is not a problem ... for it is essential to his proof that the properties in question (or “predicates” to use his terminology) are the members of the smallest complete Boolean lattice of a given set of properties ... Thus if the properties of being red and being square are among the given sparse properties, their Boolean lattice will contain properties like being red and square, being red or not being square, being neither red nor square, etc. In general the lattice will contain negative, disjunctive, and conjunctive properties. But these are not sparse or natural ...” (Rodriguez-Pereya, 2002, 66-67). Nevertheless, I will argue in this section that despite Rodriguez-Pereyra’s optimism, many conceptions of sparse properties cannot escape the absurd conclusion that the degree of resemblance between any two different particulars is a fixed constant, without also rejecting the premise that the degree of resemblance between two particulars is a function of their number of properties in common and their number of properties not in common.

Some conceptions of sparse properties cannot escape the absurd conclusion that the
degree of resemblance between any two different particulars is a fixed constant because they are not sparse enough. According to the principle of instantiation, for example, there is a property corresponding to a (possible) predicate only if some particular instantiates that (possible) predicate (Armstrong, 1978a, 113). According to the principle of instantiation, there is no property corresponding to the predicate ‘moves faster than the speed of light’, for example, because nothing moves faster than the speed of light. This suggests a sparse conception of properties according to which there is a property corresponding to a (possible) predicate if and only if some particular satisfies that predicate.

Just as every (possible) predicate corresponds to a disjunction of the $n$ atoms, every uninstantiated (possible) predicate corresponds to a disjunction of the $r$ uninstantiated atoms. So if there is a property corresponding to a (possible) predicate if and only if some particular satisfies that predicate, then the number of properties is not $2^n$, the number of (possible) predicates, but $2^n - 2^r$, the number of (possible) predicates minus the number of uninstantiated (possible) predicates. Nevertheless, the number of properties a particular satisfies is still $2^{n-1}$, since none of the (possible) predicates it satisfies are uninstantiated, the number of properties two different particulars have in common still $2^{n-2}$, since none of the (possible) predicates satisfied by two particulars are uninstantiated, and the number of properties instantiated by only the first still $2^{n-2}$, since none of the (possible) predicates satisfied by two particulars are uninstantiated. So if the degree of resemblance between two particulars is their number of properties in common divided by their number of properties not in total, the degree of resemblance
between two different particulars is still \(\frac{2^{n-2}}{2^{n-2} + 2^{n-2} + 2^{n-2}}\) or \(\frac{1}{3}\) if defined.

The principle of instantiation is sometimes combined with a principle of coinstantiation, according to which distinct properties correspond to (possible) predicates only if distinct particulars instantiate those (possible) predicates (Swoyer, 1996, 244). According to the principle of coinstantiation, distinct properties don’t correspond to the predicates ‘is a creature with a heart’ and ‘is a creature with a kidney’, for example, because no particular satisfies ‘is a creature with a heart’ but not ‘is a creature with a kidney’ and vice versa. This suggests a sparse conception of properties according to which there are distinct properties corresponding to (possible) predicates if and only if distinct particulars satisfy those (possible) predicates, and so according to which the number of properties is the number of (possible) predicates satisfied by distinct particulars.

Just as every (possible) predicate corresponds to a disjunction of the \(n\) atoms, every (possible) predicate satisfied by distinct particulars correspond to a disjunction of the \(n - r\) instantiated atoms. So if distinct properties correspond to (possible) predicates if and only if distinct particulars satisfy those predicates, then the number of properties is \(2^{n-r}\) (if we also accept the principle of instantiation then the number of properties is \(2^{n-r} - 1\), since no property corresponds to the predicate ‘does not exist’ or ‘is white and not white’, with which all the unsatisfied predicates are coextensive). The number of properties a particular satisfies is \(2^{n-r-1}\), the number of properties two different particulars have in common is \(2^{n-r-2}\) and the number of properties instantiated by only the first is \(2^{n-r-1} - 2^{n-r-2}\) or \(2^{n-r-2}\). So if the degree of resemblance between two particulars is their number of properties in common divided by their number of
properties not in common, the degree of resemblance between two different particulars is \[ \frac{2^n - r - 2}{2^n - r - 2 + 2^n - r - 2 + 2^n - r - 2} \] or still \( \frac{1}{3} \) if defined.

Whereas the sparse conceptions of properties just mentioned did not escape the barely believable conclusion because they were not sparse enough, other conceptions of sparse properties cannot escape the barely believable conclusion because they are too sparse. To illustrate this thesis, I will begin with various toy examples which, while philosophically motivated, do not correspond to positions in the literature. I will then turn to the sparse conception of properties favoured by David Armstrong, which is one of the most influential positions in the literature. I will argue that Armstrong’s conception does not escape the problems of the toy examples, before turning to the question of whether any sparse conception of properties can escape the barely believable conclusion that the degree of resemblance between any two different particulars is a fixed constant, which does not depend on the choice of the two particulars.

To begin with, consider a sparse conception of properties according to which there is a property corresponding to every atomic (possible) predicate, so the number of properties is \( n \), the number of atomic predicates. (We could also consider a sparser conception according to which there is a property corresponding to every instantiated or coinstantiated atomic (possible) predicate and so the number of properties is \( n - r \), but for the reasons above, this would make no difference to the discussion.) Take, for example, the lattice of predicates which apply to a particular in virtue of its colour. In this lattice, the atomic predicates are the predicates which express determinate shades of colour, since every predicate which applies to a particular in virtue of its colour is entailed by the predicate
which expresses its determinate shade of colour, and so no consistent predicate in the lattice entails a predicate which expresses a determinate shade of colour except itself (the example illustrates that this suggestion is roughly similar to Rodriguez-Pereyra’s conception of the sparse properties as “lowest determinate properties” (Rodriguez-Pereya, 2002, 66-67)).

The conception of properties as corresponding to atomic predicates captures many of the desirable features of sparse conceptions which deny that there are disjunctive, conjunctive and negative properties, as Rodriguez-Pereyra suggests that they should in the quote above. Firstly, the atoms are not disjunctions of any (possible) predicates except ‘does not exist’ or ‘is white and not white’ and themselves (if they were then they would be strictly entailed by their disjuncts, and so fail to be atoms). In the lattice of predicates which apply to a particular in virtue of its colour, for example, every predicate is a disjunction of the predicates which express a determinate shade of colour, but the predicates which express a determinate shade of colour are not disjunctions of any consistent predicates except themselves.

Secondly, since the atoms are all inconsistent with each other no atom is the conjunction of any of the other atoms (although each atom is the conjunction of the \(2^n-1\) predicates it entails). In the lattice of predicates which apply to a particular in virtue of its colour, for example, no predicate which expresses a determinate shade of colour is a conjunction of any other predicates which express determinate shades of colour (although ‘is scarlet’, for example, is the conjunction of, for example, ‘is scarlet or crimson’, ‘is scarlet or maroon’, ‘is scarlet or crimson or maroon’, ‘is red’, ‘is red or purple’, ‘is
coloured’ and all the other predicates which apply to a particular in virtue of it’s being scarlet).

Thirdly, no atom is the negation of any of the other atoms, although each atom is the negation of the disjunction of the \( n - 1 \) other atoms (unless \( n = 2 \)). In the lattice of predicates which apply to a particular in virtue of its colour, for example, although ‘is scarlet’ is the negation of ‘is not scarlet’, ‘is not scarlet’ is not an atom which expresses a determinate shade of colour, but merely the disjunction of all the atoms except ‘is scarlet’ which express determinate shades of colour. (In the lattice of predicates which apply to a coin in virtue of which way it lands, on the other hand, the atom ‘lands heads’ is the negation of the atom ‘lands tales’ and vice versa.)

Fourthly, the conception of properties as corresponding to atomic predicates captures the sense in which ‘exists’ or ‘is self-identical’ does not correspond to a property, since ‘exists’ or ‘is self-identical’ is not an atomic predicate (except in the lattice in which ‘exists’ or ‘is self-identical’ and ‘does not exist’ or ‘is not self-identical’ are the only two predicates, and so ‘exists’ or ‘is self-identical’ is the only atom). One motivation for denying that there is a property corresponding to ‘exists’ or ‘is self-identical’ is empiricism: Armstrong suggests that although it is a priori that everything exists or is self-identical, it is not a priori that there is a property such as existence or self-identity that everything has (Armstrong, 1978b, 11). But another motivation is that conceptions of properties according to which a property corresponds to ‘exists’ or ‘self-identical’ are ill-suited to feature in an analysis of resemblance, because the mere fact that some particulars exist or are self-identical is no reason to conclude that they are alike.
Fifthly and finally, the conception of properties as corresponding to atomic predicates captures the sense in which the sparse properties almost, as David Lewis writes, “comprise a minimal basis for characterising the world completely” (Lewis, 1983, 12). The atoms comprise a basis for characterising the world completely because every (possible) predicate is a disjunction of the atoms. In the lattice of (possible) predicates which apply to a particular in virtue of its colour, for example, every predicate is equivalent to a disjunction of the (possible) predicates which express determinate shades of colour. The basis is almost minimal because each atom is the negation of the disjunction of all the other atoms, so exactly one atom, but no atom in particular, is redundant. In the lattice of (possible) predicates which apply to particular in virtue of its colour, for example, one of the predicates which express a determinate shade of colour is redundant, because it can be defined as the negation of the disjunction of all of the other predicates which express a determinate shade of colour.

So there is a case to be made that a particular instantiates a sparse property if and only if it satisfies a corresponding atomic (possible) predicate, because this is a conception of sparse properties according to which there are no negative, disjunctive or conjunctive properties, there are no properties which it is a priori that everything has, and there is just one more than enough properties for all the (possible) predicates which apply to particulars to apply in virtue of the properties the particulars satisfy. It would be extremely elegant if in addition to capturing these desirable aspects of a sparse conception of properties, the conceptions of properties as corresponding to atomic (possible) predicates also escaped the absurd conclusion that the degree of resemblance
between two different particulars is a fixed constant, which does not depend on the choice of the two particulars.

But although replacing the premise that there is a property corresponding to every (possible) predicate with the thesis that there is a property corresponding to every atomic (possible) predicate is in many ways well motivated, it fails to avoid the absurd conclusion that the degree of resemblance between any two different particulars is a fixed constant, which does not depend on the choice of the two particulars. The atomic (possible) predicates are all inconsistent with each other, so no particular satisfies more than one. And the disjunction of the atomic (possible) predicates is tautologous, so every particular satisfies at least one. So the number of atomic (possible) predicates a particular satisfies is exactly one and the number of atomic (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is zero. And if the number of atomic (possible) predicates satisfied by two particulars which do not satisfy all the same (possible) predicates is their number of properties in common, then the number of properties in common between two particulars which do not satisfy all the same properties is also zero. So if their degree of resemblance is their number of properties in common divided by their number of properties not in common, then their degree of resemblance is zero, which is a fixed constant independent of the two particulars.

In the lattice of predicates which apply to a particular in virtue of its colour, for example, every particular satisfies exactly one atomic (possible) predicate, which corresponds to its determinate shade of colour and so, if the properties correspond to the atomic
(possible) predicates, each particular instantiates exactly one property – the property of being that determinate shade of colour. No two particulars which differ in respect of colour possess the same determinate shade of colour, and so no two particulars which differ in respect of colour have their single determinate shade of colour in common. So if the degree of resemblance between two different particulars in respect of colour is their number of colour properties in common divided by their number of colour properties not in common, their degree of resemblance is zero, irrespective of their colour. This is absurd, since just in respect of colour, red particulars, for example, resemble orange particulars more than they resemble green particulars.

So the sparse conception of properties according to which there is a property corresponding to every atomic (possible) predicate is obviously too sparse to be suitable to feature in the analysis of resemblance. But there are less sparse conceptions of properties which share some of its virtues. Let the dimension of an element $a = \bigvee \{b \in B \mid b \leq a\}$ in a lattice $\langle A, \leq \rangle$ be the number $|\{b \in B \mid b \leq a\}|$ of the atoms $B \subseteq A$ it is a disjunction of (Watanabe, 1969, 327). Then the sparse conception of properties according to which there is a property corresponding to each (possible) predicate of a certain finite dimension $r$ has many of the advantages of the conception of sparse properties as corresponding to the atoms, without being quite so unaccommodatingly sparse.

Firstly, although each (possible) predicate of finite dimension $r$ is the disjunction of $r$ atoms, the (possible) predicates of finite dimension $r$ are never disjunctions of each other, since if one of the predicates had a second other than itself as a disjunct, it must be the disjunction of at least one more atom. Secondly, although each (possible) predicate
of dimension $r$ is the conjunction of $n - r$ negated atoms, the (possible) predicates of
dimension $r$ are never conjunctions of each other, since if one of the predicates had a
second other than itself as a conjunct, then it must have at least one less atom. And
thirdly, although every (possible) predicate of dimension $r$ has as its negation a (possible)
predicate of dimension $n - r$, no (possible) predicate of dimension $r$ is the negation of
another (possible) predicate of dimension $r$ (unless $n - r = r$). So the conception
according to which properties correspond to atomic predicates still captures a sense in
which there are no disjunctive, conjunctive and negative properties.

Fourthly, the conception of properties as corresponding to (possible) predicates of a
certain finite dimension $r$ captures the sense in which ‘exists’ or ‘is self-identical’ does
not correspond to a property, since ‘exists’ or ‘is a self-identical’ is the disjunction of all
$n$ atoms and so is a predicate of dimension $r$ only if $r = n$. Fifthly and finally, properties
corresponding to (possible) predicates of a certain (finite) dimension $r$ comprise a basis
for characterising the world completely because unless $r$ is zero or $n$ every (possible)
predicate of dimension greater than $r$ is a disjunction and every (possible) predicate
of dimension less than $r$ a conjunction of the (possible) predicates of dimension $r$ (but
not in general an irredundant basis). So there is some philosophical motivation for a
conception of sparse properties according to which a particular instantiates a property if
and only if it satisfies a corresponding (possible) predicate of a certain finite dimension
$r$.

But the conception of properties as corresponding to (possible) predicates of a certain
finite dimension $r$ does not escape the barely believable conclusion. The number of
predicates of finite dimension \( r \) is \( \binom{n}{r} \), the number of disjunctions of length \( r \) which can be formed from \( n \) predicates. If there were just four atomic predicates – ‘is red and square’, ‘is red and not square’, ‘is not red and square’ and ‘is not red and not square’, for example, – of dimension one, then there would be six predicates of dimension two – ‘is red’, ‘is square’, ‘is red if and only if square’, ‘is red if and only if not square’, ‘is not red’, ‘is not square’ –, four of dimension three – ‘is red or square’, ‘is square if red’, ‘is red if square’ and ‘is not red or not square’ –, and one each of dimension of zero and four – ‘is square and not square’ and ‘is square or not square’.

The number of predicates of finite dimension \( r \) satisfied by a particular is \( \binom{n-1}{r-1} \), since the disjunctions of length \( r \) which apply to it must include the single atom satisfied by the particular, but may include any \( r - 1 \) of the remaining \( n - 1 \) atoms. If ‘is red and square’, ‘is red and not square’, ‘is not red and square’ and ‘is not red and not square’, for example, were the four atomic predicates, then a red square would satisfy three dimension two predicates – ‘is red’, ‘is square’ and ‘is red if and only if square’ –, and three dimension three predicates – ‘is red or square’, ‘is square if red’, ‘is red if square’.

Likewise, the number of predicates of finite dimension \( r \) satisfied by two particulars which do not satisfy all the same predicates is \( \binom{n-2}{r-2} \), since the disjunctions of length \( r \) which apply to it must include the two atoms corresponding to the two combinations of simple predicates they satisfy, but may include any \( r - 2 \) of the remaining \( n - 2 \) atoms (Watanabe, 1969, 377). If ‘is red and square’, ‘is red and not square’, ‘is not red and square’ and ‘is not red and not square’, for example, were the four atomic predicates,
then a red square and a non-red square would jointly satisfy one dimension two predicate – ‘is square’ – and two dimension three predicates – ‘is red or square’ and ‘is square if red’.

So the number of predicates of finite dimension \( r \) which are satisfied by only the first of two particulars which do not satisfy all the same predicates is \( \binom{n-1}{r-1} - \binom{n-2}{r-2} \), or the number of predicates of finite dimension \( r \) which are satisfied by both particulars subtracted from the number of predicates of finite dimension \( r \) which are satisfied by the first, or, in other numerals, \( \binom{n-2}{r-1} \). If ‘is red and square’, ‘is red and not square’, ‘is not red and square’ and ‘is not red and not square’, for example, were the four atomic predicates, then a red square would satisfy two dimension two predicates – ‘is red’ and ‘is red if and only if square’ – and one dimension three predicate – ‘is red if square’ – not satisfied by a non-red square.

So the number of (possible) predicates of finite dimension \( r \) satisfied by two particulars which do not satisfy all the same (possible) predicates is a fixed constant, as is the number of (possible) predicates of dimension \( r \) satisfied by only the first, which equals the number of (possible) predicates of dimension \( r \) satisfied by only the second. (Though the number of (possible) predicates of dimension \( r \) satisfied by two particulars is not equal to the number of (possible) predicates of dimension \( r \) satisfied by only the first, but to the number of (possible) predicates of dimension \( r - 1 \) which are satisfied by only the first (Watanabe, 1969, 377).) So if there is a property corresponding to every (possible) predicate of a certain dimension \( r \), and the degree of resemblance between two particulars is a function of their number of properties in common and their number
of properties not in common, then the degree of resemblance between two particulars is still a constant, which depends on the number of atomic (possible) predicates $n$ and the dimension $r$, but not on the choice of the two particulars.

So whereas conceptions according to which the sparse properties correspond to the instantiated or coinstantiated (possible) predicates are not sparse enough to escape the conclusion that the degree of resemblance between two different particulars is constant, conceptions according to which the sparse properties correspond to the (possible) predicates of a certain dimension, especially the conception according to which the sparse properties correspond to the atoms or the (possible) predicates of dimension one, are too sparse to escape the conclusion that the degree of resemblance between two different particulars is constant. This suggests pursuing a conception of properties of intermediate sparseness. In particular, whereas the conception according to which the sparse properties correspond to the (possible) predicates of dimension $r \neq \frac{n}{2}$ offered a precise sense in which there were no negative, disjunctive or conjunctive properties, it suggests pursuing a conception of properties according to which there are some, but only some, negative, disjunctive or conjunctive properties.

According to one popular intermediately sparse conception of properties, there are no negative or disjunctive properties, but there are conjunctive properties. As David Armstrong, for example, writes “... it is implausible to say that in the case of disjunctive and negative “properties”, there is something identical [a resemblance] in all the objects in virtue of which the corresponding predicates apply. By contrast, if a number of particulars each have two properties, $P$ and $Q$, it is perfectly natural to say this this
constitutes a respect [of resemblance] in which they are identical” (Armstrong, 1978b, 34). But it is not obvious whether even intermediately sparse conception of properties which accept that there are conjunctive properties can escape the absurd conclusion that the degree of resemblance between different particulars is a fixed constant.

Armstrong’s motivation to introduce conjunctive properties begins with the following consideration: “If every complex universal (complex property or relation) were a complex of simple universals then it would be rather natural to speak of the simple universals as the universals. But I do not think that it can be shown that every complex universal is a complex of simple universals. In the particular case of properties, it is logically epistemically possible that all properties are conjunctive properties.” (Armstrong, 1978b, 34). In this passage, Armstrong first considers and then rejects a sparse conception of properties according to which properties are never conjunctive, since if they were, we would take their simpler conjuncts to be the properties instead.

Instead of a sparse conception according to which properties correspond to the atomic predicates, this suggests a sparse conception according to which properties correspond to the dual atomic predicates. (An element $a \in A$ of a lattice $\langle A, \leq \rangle$ is a dual atom if and only if $a < \top$ and for all $b \in A$ if $a < b$ then $b = \top$ (Gratzer, 2011, 101).) This is because just as the atoms are not disjunctions of any (possible) predicates except ‘does not exist’ or ‘is white and not white’ and themselves, the dual atoms are not conjunctions of any (possible) predicates except ‘exists’ or ‘is white or not white’ and themselves. The dimension of the dual atoms is $n - 1$. (If $a$ is a dual atom, it cannot disjoin all the atoms, since then $a = \top$ contradicting the definition. But if $b$ and $c$ are atoms $a$ must disjoin
at least one of them, since otherwise \( a < a \lor b < \top \) or \( a < a \lor c < \top \), contradicting the definition. So if \( a \) is a dual atom it fails to disjoin exactly one atom.

The number of dual atoms or predicates of dimension \( n-1 \) satisfied by a particular is \( \binom{n-1}{n-1} = n-1 \), since the disjunctions of length \( n-1 \) which apply to it must include the single atom satisfied by the particular, but may include any \( n-1-1 \) of the remaining \( n-1 \) atoms. Likewise, the number of predicates of dimension \( n-1 \) satisfied by two particulars which do not satisfy all the same predicates is \( \binom{n-2}{n-1-2} = n-2 \) and the number of predicates of dimension \( n-1 \) which are satisfied by only the first of two particulars which do not satisfy all the same predicates is \( (n-1) - (n-2) = 1 \). So if the degree of resemblance between two particulars is their number of properties in common divided by their number of properties in total, the degree of resemblance between two different particulars is \( \frac{n-1}{n-1+1+1} = \frac{n-1}{n+1} \), which is still a fixed constant which depends only on \( n \), and not on the choice of the two particulars.

Since if \( n \) is finite the conception of properties according to which there is a property corresponding to every dual atom or (possible) predicate of dimension \( n-1 \) is simply an instance of the conception according to which there is a property corresponding to every (possible) predicate of a certain dimension \( r \), it was already obvious that it would be unsuitable to feature in the analysis of degrees of resemblance. But Armstrong’s reason for rejecting the conception is very different. He writes “The following proposition seems to involve no contradiction. For all properties, \( P \), there exists a property, \( Q \), and a wholly distinct property, \( R \), such that \( P = Q \land R \). Thus, make the unlikely supposition that \( humanity \), \( animality \) and \( rationality \) are properties. Suppose further that \( humanity \) is
the conjunction of animality and rationality. Why might not animality and rationality themselves be conjunctions of further distinct properties and so *ad infinitum*?”

This suggests a conception of properties according to which there is a property corresponding to every predicate *below* a certain dimension *r*. If *r* is finite then the simple properties could be identified as those of dimension *r* − 1, and the conjunctive properties as those of dimension less than *r* − 1. But if *r* were, for example, ℵ₀, then according to this conception every property would be conjunctive, since every predicate of dimension *k* < *r* = ℵ₀ would be the conjunction of two predicates of dimensions *k* + 1 < *r* = ℵ₀. In the lattice of predicates which apply to people in virtue of their age in years, for example, every predicate of a finite dimension *k* is the conjunction of two predicates of dimension *k* + 1 – ‘is one year old’ is the conjunction of ‘is one year old or two years old’ and ‘is one year old or three years old’, ‘is one year old or two years old’ is the conjunction of ‘is one year old or two years old or three years old’ and ‘is one year old or two years old or four years old’, ... and so on.

If *n* and *r* are finite then the number of predicates of dimension less than *r* satisfied by a particular is the sum \( \sum_{i=1}^{r-1} \binom{n-1}{i-1} \) of the number of predicates of each dimension less than *r* satisfied by the particular. The number of predicates of dimension less than *r* satisfied by two particulars which do not satisfy all the same predicates is the sum \( \sum_{i=2}^{r-1} \binom{n-2}{i-2} \) of the number of predicates of each dimension less than *r* satisfied by two different particulars. And the number of predicates of dimension less *r* which are satisfied by only the first of two particulars which do not satisfy all the same predicates is the sum \( \sum_{i=1}^{r-1} \binom{n-2}{i-1} \) of the number of predicates of each dimension less than *r*
satisfied by only the first of two particulars which do not satisfy all the same predicates. So if there is a property corresponding to every (possible) predicate of less than a certain finite dimension \( r \), and the degree of resemblance between two particulars is a function of their number of properties in common and their number of properties not in common, then the degree of resemblance between two particulars is still a constant, which depends on the number of atomic (possible) predicates \( n \) and the dimension \( r \), but not on the choice of the two particulars.

Even if \( n \) or \( r \) are infinite and \( n \leq r \), then the number of predicates of dimension less than \( r \) satisfied by two different particulars which do not satisfy all the same predicates remains \( 2^{n-2} \) since if \( n \leq r \) then all the predicates are predicates of dimension less than \( r \). But if \( n \) or \( r \) are infinite and \( 0 < r < n \) then the number of predicates of dimension less than \( r \) satisfied by two different particulars which do not satisfy all the same predicates is \( n - 2 \), the number of combinations of less than length \( r \) of the \( n - 2 \) atoms which do not apply to one of the particulars. If \( n = 2^{\aleph_0} \) and \( r = \aleph_0 \), for example, then the number of predicates of dimension less than \( r = \aleph_0 \) satisfied by two different particulars which do not satisfy all of the same predicates is \( 2^{\aleph_0} - 2 \) or \( 2^{\aleph_0} \), the number of combinations of less than length \( \aleph_0 \) of the \( 2^{\aleph_0} - 2 \) atoms which do not apply to one of the particulars.

So if there is a property corresponding to every (possible) predicate of less than a certain (finite or infinite) dimension \( r \), and the degree of resemblance between two particulars is a function of their number of properties in common and their number of properties not in common, then the degree of resemblance between two particulars is still a constant, which depends on the number of atomic (possible) predicates \( n \) and
the dimension $r$, but not on the choice of the two particulars. Although it captures
the idea that it is possible for all properties to be conjunctions of other properties, the
conception of properties as corresponding to predicates of less than a certain dimension
$r$ is not equipped to escape the barely believable conclusion.

All of the sparse conceptions of properties considered so far, although philosophically
motivated in various respects, were chosen to illustrate the robustness of the conclusion
that the degree of resemblance between two particulars is a fixed constant which does not
depend on the choice of the two particulars. I now turn to consider the sparse conception
of properties favoured by David Armstrong, which is not merely a straw man, but one of
the most influential in the literature. I will begin with a simplified version of Armstrong’s
conception, which ignores the principle of instantiation. I will then discuss Armstrong’s
actual conception, which includes the principle of instantiation. I will argue that neither
conception succeeds in escaping the absurd conclusion that the degree of resemblance
between two different particular is a fixed constant, which does not depend on the choice
of the two particulars.

If we ignore the principle of instantiation, then Armstrong favours a conception of
properties which meets the following three conditions: (negation) if there is a property
corresponding to a (possible) predicate $a$, then there is no property corresponding to
its negation $\neg a$, (disjunction) if there is a property corresponding to two (possible)
predicates $a$ and $b$ and $a \neq b$, then there is no property corresponding to their disjunction
$a \lor b$, and (conjunction) if there is a property corresponding to two (possible) predicates
$a$ and $b$, then there is a property corresponding to their conjunction $a \land b$. If there is a
property corresponding to ‘is white’, for example, then there is no property corresponding
to ‘is not white’, and if there are properties corresponding to ‘is red’ and ‘is square’
then there is no property corresponding to ‘is red or square’, but there is a property
corresponding to ‘is red and square’.

This conception of properties is too sparse, because it follows that a property corre-
sponds to only one possible predicate. For suppose that there is a property corresponding
to $a$ and a property corresponding to $b$. Then according to (conjunction) there is a prop-
erty corresponding to $a \land b$. But since $a = a \lor (a \land b)$ and $b = b \lor (a \land b)$, there is a
property corresponding to $a \lor (a \land b)$ and a property corresponding to $b \lor (a \land b)$. Then
according to (disjunction) $a = a \land b$ and $b = a \lor b$, so $a = b$. So if there is a property
corresponding to $a$ and a property corresponding to $b$, then according to this conception,
$a = b$ (Bacon, 1986, 49). In other words, there is a property corresponding to only one
(possible) predicate. It follows that the only difference between particulars is in respect
of that single property, so the degree of resemblance between two different particulars is
a fixed constant, which does not depend on the choice of the two particulars.

Because Armstrong accepts the principle of instantiation, he does not accept (con-
junction) in full generality, and so it does not follow from Armstrong’s conception that
there is a property corresponding to only one (possible) predicate. Instead, Armstrong
endorses: (instantiated conjunction) if there is a property corresponding to two (pos-
sible) predicates $a$ and $b$ and some particular satisfies both $a$ and $b$, then there is a
property corresponding to their conjunction $a \land b$. If there is a property corresponding
to ‘is red’ and a property corresponding to ‘is square’, for example, and some square is
red, then there is a property corresponding to ‘is red and square’.

But Armstrong’s conception is still too sparse, since it follows that each particular instantiates only one property. For suppose that there is a property corresponding to $a$ and a property corresponding to $b$, and that some particular satisfies both $a$ and $b$. Then according to (instantiated conjunction) there is a property corresponding to $a \land b$. But since $a = a \lor (a \land b)$ and $b = b \lor (a \land b)$, there is a property corresponding to $a \lor (a \land b)$ and a property corresponding to $b \lor (a \land b)$. Then according to (disjunction) $a = a \land b$ and $b = a \land b$, so $a = b$. So if there is a property corresponding to $a$ and a property corresponding to $b$ and some particular satisfies both $a$ and $b$, then, according to Armstrong’s conception, $a = b$ (Bacon, 1986, 49). So it follows from Armstrong’s conception that only one of the (possible) predicates satisfied by a particular corresponds to a property.

So Armstrong’s conception of properties has similar consequences to the conception according to which the properties correspond to the atomic (possible) predicates. Supposing that that there are no bare particulars, or in other words that at least one of the (possible) predicates satisfied by a particular corresponds to a property, it follows from Armstrong’s conception that exactly one of the (possible) predicates satisfied by each particular corresponds to a property, or that the number of properties a particular instantiates is one. The number of particulars satisfied by two different particulars is zero, since the one property each satisfies must be different if they are different, and their number of properties not in common is two, since neither of their two properties are in common. So if their degree of resemblance is their number of properties in common
divided by their number of properties not in common, then their degree of resemblance is zero. So Armstrong’s conception of properties turns out to be too sparse to be suitable to feature in the analysis of resemblance.

In order to avoid this problem, John Bacon suggests weakening (disjunction) to (determination): if there is a property corresponding to two (possible) predicates $a$ and $b$ and a property corresponding to their disjunction $a \lor b$, then either $a \leq b$ or $b \leq a$. Supposing that there is a property corresponding to $a$ and a property corresponding to $b$, and that some particular satisfies both $a$ and $b$, then according to (determination) there is a property corresponding to $a \land b$. Then according to (conjunction) there is a property corresponding to $a \land b$. And since $a = a \lor (a \land b)$ and $b = b \lor (a \land b)$, there is a property corresponding to $a \lor (a \land b)$ and a property corresponding to $b \lor (a \land b)$. But according to (determination) it merely follows that $a \leq a \land b$ or $a \land b \leq a$ and $b \leq a \land b$ or $a \land b \leq b$, which is unexceptionable (Bacon, 1986, 49).

But even if Bacon’s conditions escape the barely believable conclusion that the degree of resemblance between two particulars is a fixed constant which does not depend on the choice of the two particulars, they do not seem well motivated on other grounds. Take, for example, the lattice of predicates in which the atoms are ‘is a husband’, ‘is a wife’, ‘is a bachelor’ and ‘is a spinster’. Intuitively, the predicates which count towards resemblance, and so which correspond to properties, in this lattice are the atoms as well as ‘is male’, ‘is married’, ‘is female’ and ‘is unmarried’, even though ‘is male’ is the disjunction of ‘is a husband’ and ‘is a bachelor’, ‘is married’ is the disjunction of ‘is a husband or a wife’, ‘is female’ is the disjunction of ‘is a wife’ and ‘is a spinster’,
and ‘is unmarried’ is the disjunction of ‘is a bachelor’ and ‘is a spinster’. (Properties do not intuitively correspond to the predicates ‘is a husband or a spinster’, ‘is a wife or a bachelor’, ‘is a non-husband’, ‘is a non-wife’, ‘is a non-bachelor’ and ‘is a non-spinster’. If they did, the barely believable conclusion would follow.)

But Bacon’s conditions are incompatible with this intuitive choice of which predicates in the lattice correspond to properties, and so which predicates in the lattice contribute towards resemblances between particulars which satisfy them. Since there are properties corresponding to ‘is a wife’, ‘is a husband’ and their disjunction ‘is married’, for example, it follows from (determination) that either ‘is a wife’ entails ‘is a husband’ or ‘is a husband’ entails ‘is a wife’, which is absurd. Likewise, in the lattice of predicates which apply to particulars in virtue of their colour, ‘is light red’ seems to correspond to a property, since the light red things resemble each other in virtue of being light red, and ‘is dark red’ seems to correspond to a property, since the dark red things resemble each other in virtue of being dark red. But their disjunction ‘is red’ also seems to correspond to a property, since the red things resemble each other in virtue of being red. But then according to (determination) either ‘is light red’ entails ‘is dark red’ or ‘is dark red’ entails ‘is light red’, which is absurd.

The underlying problem is that sparse conceptions of properties which issue a blanket rejection of properties corresponding to negations and disjunctions of (possible) predicates which do correspond to properties are too sparse, because they reject that there are properties corresponding to many predicates which in fact do contribute to resemblances between particulars. Although it’s sensible to think that ‘is a barbarian’, for example,
does not correspond to a property in the lattice of predicates which apply to people in virtue of their nationality because it’s the negation of ‘is a Greek’, it’s not sensible to think that ‘is unmarried’ does not correspond to a property in the lattice of predicates which apply to people in virtue of their marital status and gender merely because it’s the negation of ‘is married’. Likewise, although it’s sensible to think that ‘is a raven or a writing desk’ does not correspond to a property if ‘is a raven’ and ‘is a writing desk’ do, it’s not sensible to think that ‘is red’ does not correspond to a property if ‘is light red’ and ‘is dark red’ do.

So although it’s natural for a proponent of the sparse conception to resist the conclusion of the argument by denying its second premise, we have seen that sparse theories of properties which are motivated on other grounds don’t escape the conclusion that the degree of resemblance between two different particulars is a fixed constant as easily as might otherwise be expected. Moreover, even those sparse conceptions of properties which do escape the absurd conclusion are nevertheless ill-suited to feature in the analysis of resemblance, because they still reject the existence of some negative and disjunctive properties which ought to feature in the analysis of degree of resemblance. While there may be other conceptions of sparse properties which are more suited to feature in the analysis of degrees of resemblance, it is disappointing that such a conception is not obviously one which is well motivated on other grounds. In the next section, I will discuss whether a better solution to the puzzle is to retain an abundant conception of properties, according to which there is a property corresponding to every possible predicate, but to deny that the degree of resemblance between two different particulars is a function of
their number of common and uncommon properties.

3 The Third Premise

In the last section, I argued that many well-motivated ways of denying that there is a property corresponding to every (possible) predicate, and so denying the second premise of the argument, fail to avoid the absurd conclusion that the degree of resemblance between two different particulars is a constant, which does not depend on the choice of the two particulars. This suggests that a better motivated way to avoid the conclusion of the argument may be to revise the premise that the degree of resemblance between two different particulars is a function of their number of common and uncommon properties. Different properties, according to this revision, have different weights in determining degrees of resemblance: the degree of resemblance between two different particulars is a function of the weights of the properties they have in common, the weights of the properties the first has not in common with the second, and the weights of the properties the second has not in common with the first (Watanabe, 1969, 382).

More precisely, if the weights of a property or predicate is given by a function $w : A \rightarrow \mathbb{R}$ from the set of predicates $A$ to the real numbers, and if for each particular $x$ there is a function $f_x : A \rightarrow \{0, 1\}$ which takes each predicate in $A$ to zero if it is satisfied by $x$ and one if it is not satisfied by $x$, then the degree of resemblance between $x$ and $y$ can be defined as $\frac{\sum_{a \in A} w(a)f_x(a)f_y(a)}{\sum_{a \in A} w(a)f_x(a)f_y(a) + w(a)f_x(a) + w(a)f_y(a)}$. If all of the $2^n$ predicates in $A$ have the same uniform weight, then the degree of resemblance between $x$ and $y$
collapses to \( \frac{2^{n-2}}{2^{n-2} + 2^{n-2} + 2^{n-2}} \) or \( \frac{1}{3} \) if defined, but if some of the \( 2^n \) predicates in \( A \) are unevenly weighted then the degree of resemblance between \( x \) and \( y \) may be any real number between zero and one (if finite and defined), and will depend on the choice of the two particulars \( x \) and \( y \). So in this case, the degree of resemblance between two particulars is not a fixed constant, and does depend on the choice of the two particulars.

(The sum of an infinite collection of real numbers is the smallest number which is greater than the sum of all the finite subsets. In other words, if \( A \) is infinite and \( g : A \to \mathbb{R} \) is a function from \( A \) to the real numbers, then the sum \( \sum_{a \in A} g(a) \) can be defined as the number \( r \) such that (i) for every finite subset \( B \subseteq A \), \( \sum_{b \in B} f(b) \leq r \) and (ii) for every number \( s \) if for every finite subset \( B \subseteq A \), \( \sum_{b \in B} f(b) \leq s \), then \( r \leq s \). Note that this definition does not require the elements in \( A \) to be summed in a particular order. It agrees with the usual definition if \( A \) is countably infinite and \( g(a) \geq 0 \) for all \( a \in A \), but has the advantage of applying even if \( A \) is uncountably infinite. Since the sum of an infinite collection of real numbers may be infinite or undefined, the resulting degree of resemblance may also turn out to be infinite or undefined.)

Revising the premise that the degree of resemblance between two different particulars is a function of their number of common and uncommon properties to the thesis that the degree of resemblance between two different particulars is a weighted function of their common and uncommon properties must be at least as good a way to avoid the barely believable conclusion as adopting a given sparse conception of properties, because if the weights of the predicates are given by the function \( w : A \to \{0,1\} \) which takes each predicate in \( A \) to one if it corresponds to a property according to the sparse conception
but to zero if it does not, then the sum of the weights of the predicates satisfied by two
particulars which don’t satisfy all the same predicates will be equal to their number of
properties in common according to the sparse conception, and the sum of the weights of
the predicates satisfied by just one will be equal to the number of properties instantiated
by just one according to the sparse conception, so the revision to the premise will give
the same number (if defined) as the degree of resemblance between two particulars as
the given sparse conception of properties did.

Nevertheless, just as it was unsatisfying to be told that the conclusion should be
avoided by adopting a sparse conception of properties and denying that there is a prop-
erty corresponding to every predicate, without being given any further information about
which sparse conception of properties to adopt and which predicates do not correspond
to any property, it is equally as unsatisfying to be told that the conclusion should be
avoided by revising the third premise to the thesis that the degree of resemblance be-
tween two different particulars is a weighted function of their common and uncommon
properties, without being given any further information about the nature of the weights
and what the weights of the various predicates are. One important question, which
I shall not try to answer, is whether the weights should be interpreted as subjective
degrees of importance (as Nelson Goodman (1972) argues) or objective degrees of
naturalness (as David Lewis (1983) argues). Nevertheless, I will argue in this section
that even without resolving this question, there is more to be said about the nature of
weights than can be said about which predicates correspond to properties according to
an appropriate sparse conception.
One natural proposal, for example, is to define the weighting function \( w : A \rightarrow \mathbb{R} \) from the set of predicates \( A \) to the real numbers as a function such that for all \( a \in A \), \( w(a) = 1 - p(a) \), where \( p : A \rightarrow \mathbb{R} \) is a function from the set of predicates \( A \) to the real numbers which meets the following three conditions: (non-negativity) for all \( a \in A \), \( 0 \leq p(a) \), (normalisation) \( p(\top) = 1 \) and (finite additivity) for all \( a, b \in A \) such that \( a \land b = \bot \), \( p(a \lor b) = p(a) + p(b) \) (or, equivalently, for every finite nonempty subset \( B \subseteq A \) such that \( a \land b = \bot \) for all \( a, b \in B \), \( p(\lor B) = \sum_{b \in B} p(b) \)). The weightiness of a property, according to this idea, is the opposite of it’s peculiarity, where the minimum degree of peculiarity is zero, a predicate like ‘exists’ or ‘is white or not white’ which applies to anything no matter what it is like has the maximum degree of peculiarity, and the peculiarity of a disjunction such as ‘is red or green’ is the sum of the peculiarity of the disjuncts ‘is red’ and ‘is green’, when they are inconsistent.

This characterisation of the weighting function has a number of the desirable features of sparse conceptions of properties which deny there are properties corresponding to negative, disjunctive and conjunctive predicates, while avoiding some of the features which made such conceptions so ill-equipped to feature in the analysis of degrees of resemblance. Firstly, it follows from (normalisation) that \( p(a \lor \neg a) = 1 \) since for all \( a \in A \), \( a \lor \neg a = \top \) and it follows from (additivity) that \( p(a \lor \neg a) = p(a) + p(\neg a) \) since for all \( a \in A \), \( a \land \neg a = \bot \), so it follows from (normalisation) and (finite additivity) that \( p(\neg a) = 1 - p(a) \). This captures the idea that if ‘is a raven’ has a high weight in determining the degrees of resemblance of birds to each other, ‘is a non-raven’ will have a correspondingly low weight, while accommodating the idea that ‘is left-handed’ and ‘is
right-handed’ may have an equal weight of half even if one is the negation of the other.

Secondly, this characterisation captures the desired asymmetry between conjunctive and disjunctive properties which sparse conceptions which maintained that properties exist corresponding to conjunctive but not to disjunctive predicates were unable to. For any pair of predicates $a, b \in A$ such that $a \leq b$ or $a$ entails $b$, it follows that $a \land \neg b = \bot$ and so according to (finite additivity) $p(a \lor \neg b) = p(a) + p(\neg b) = p(a) + 1 - p(b)$ and so $p(b) + p(a \lor \neg b) = p(a) + 1$. But since $p(a \lor \neg b) \leq 1$, it follows that $p(a) \leq p(b)$. And since for all $a, b \in A$, $a \land b \leq a \leq a \lor b$ it follows that for all $a, b \in A$, $p(a \land b) \leq p(a) \leq p(a \lor b)$ or, in other words, that the weight of a conjunction is greater than or equal to the weight of the conjuncts, whereas the weight of a disjunction is less than or equal to the weight of the disjuncts.

Despite these advantages, this proposal has a number of counterintuitive consequences. Firstly, if $p(a) = 0$ and $p(b) = 0$ then $p(a \lor b) = 0$. For since the peculiarity of a conjunction is less than or equal to the peculiarity of a conjunct, $0 \leq p(a \land b) \leq p(a) = 0$, $0 \leq p(a \land \neg b) \leq p(a) = 0$ and $0 \leq p(\neg a \land b) \leq p(b) = 0$. And since according to (finite additivity) $p((a \land b) \lor (a \land \neg b) \lor (\neg a \land b)) = p(a \land b) + p(a \land \neg b) + p(\neg a \land b)$, it follows that $p(a \lor b) = 0 + 0 + 0 = 0$. But if ‘is red’ and ‘is green’, for example, are not at all peculiar or perfectly natural (as Lewis would say), it shouldn’t follow that their disjunction ‘is red or green’ is not at all peculiar or perfectly natural, since there is a wider diversity between the things which are red or green than between the things which are red or than between the things which are green.

Secondly, if $p(a) = p(b)$, $p(c) = p(d)$, $a \land c = \bot$, and $b \land d = \bot$, then $p(a \lor c) = p(b \lor d)$,
since \( p(a \lor c) = p(a) + p(c) = p(b) + p(d) = p(b \lor d) \). But if ‘is red’ and ‘is yellow’ are peculiar or natural to the same degree, and ‘is orange’ and ‘is purple’ are peculiar or natural to the same degree, it shouldn’t follow that ‘is red or orange’ is peculiar or natural to the same degree as ‘is yellow or purple’. Rather, ‘is red or orange’ should have a higher weight in determining degree of resemblance and so a lower degree of peculiarity than ‘is yellow or purple’, since red and orange particulars are similar with respect to colour whereas yellow and purple particulars are not.

Thirdly, suppose (finite additivity) is strengthened to (general additivity), according to which for every subset, finite or infinite, \( B \subseteq A \) such that \( a \land b = \bot \) for all \( a, b \in B \),

\[
p(\bigvee B) = \sum_{b \in B} p(b).
\]

Then if there is an infinite subset \( B \subseteq A \) such that \( a \land b = \bot \) for all \( a, b \in B \) and every predicate \( b \in B \) is peculiar to the same non-negative degree, the sum \( p(\bigvee B) = \sum_{b \in B} p(b) \) must be zero or infinite. But if \( p(\bigvee B) = \sum_{b \in B} p(b) \) is infinite, this contradicts the fact that for all \( a \in A \), \( p(a) \leq 1 \) (since according to (non-negativity) \( p(\neg a) \leq 0 \) and \( p(\neg a) = 1 - p(a) \), so \( 1 - p(a) \leq 0 \) and \( p(a) \leq 1 \)). So if there is an infinite subset \( B \subseteq A \) such that \( a \land b = \bot \) for all \( a, b \in B \) and every predicate \( b \in B \) is peculiar to the same non-negative degree, then the sum \( p(\bigvee B) = \sum_{b \in B} p(b) \) is zero.

However, it seems as if there are infinite sets of predicates which all have the same non-negative degree of peculiarity. It seems, for example, that there is an infinite number of determinate shades of colour, which all have equal weight in determining degree of resemblance. Their equal degree of peculiarity cannot be positive, since then the degree of peculiarity of their disjunction ‘is coloured’ or the sum of their equal degrees of peculiarity would be an infinite number greater than one, contradicting the fact that all
degrees of peculiarity are real numbers less than one. So their equal degree of peculiarity must be zero, and the sum of their zero degrees of peculiarity or the degree of peculiarity of their disjunction ‘is coloured’ must be zero as well. But since there is a great deal of heterogeneity amongst the things that are coloured, the degree of peculiarity of ‘is coloured’ should be greater than zero.

These three problems arise partly from the intuition that certain predicates, such as those expressing the colours, should have equal weight in determining degrees of resemblance. It’s natural to generalize this intuition by suggesting that each of the $n$ atomic predicates has an equal peculiarity which if $n$ is finite is equal to $\frac{1}{n}$. Consequently, since according to (finite additivity) the peculiarity $p(a)$ of a predicate $a$ is the sum of the peculiarities $\frac{1}{n}$ of the $r$ inconsistent atoms it is a disjunction of, the peculiarity of each predicate of dimension $r$ will be $\frac{r}{n}$, so that every predicate of the same dimension will have a constant weight in determining degree of resemblance of $\frac{n-r}{n}$. Thus supposing that the predicates expressing the colours, for example, all have the same dimension, it would follow that the colours all have the same weight in determining degree of resemblance.

Generalized in this way, the proposal that the degree of resemblance between two different particulars is a weighted function of their common and uncommon properties does not escape the absurd conclusion that the degree of resemblance between two particulars is a fixed constant, which does not depend on the choice of the two particulars. Since in a finite lattice with $n$ atoms the number of predicates of dimension $r$ satisfied by two particulars which do not satisfy all the same predicates is $\binom{n-2}{r-2}$, the weight
of the properties in common between two particulars would be \( \sum_{r=2}^{n} \frac{n-r}{n} \binom{n-2}{r-2} \). And since the number of predicates of dimension \( r \) which are satisfied by only the first of two particulars which do not satisfy all the same predicates is \( \binom{n-2}{r-1} \), the weight of the properties of only one particular would be \( \sum_{r=1}^{n} \frac{n-r}{n} \binom{n-2}{r-1} \). Both are fixed constants depending only on the number of atomic predicates \( n \), and not on the choice of the two particulars. So the degree of resemblance between two different particulars would also be a fixed constant depending only on the number of atomic predicates \( n \), and not on the choice of two particulars.

In order to escape these problems within this approach, one not unnatural proposal is to weaken (finite additivity) to require that the disjunction of inconsistent predicates is not strictly equal to but merely greater than or equal to the sum of the disjunctions or, in other words, to: (finite increasivity) for all \( a, b \in A \) such that \( a \land b = \bot \), \( p(a \lor b) \geq p(a) + p(b) \) (or, equivalently, for every finite nonempty subset \( B \subseteq A \) such that \( a \land b = \bot \) for all \( a, b \in B \), \( p(\bigvee B) \geq \sum_{b \in B} p(b) \)). The peculiarity of a disjunction such as ‘is red or orange’, for example, is greater than the sum of the peculiarities of the disjuncts ‘is red’ and ‘is orange’. Likewise, the peculiarity of the ‘is red’ is greater than or equal to the sum of the peculiarities of the predicates which express all the determinate shades of red.

This characterisation of the weighting function still has a number of the desirable features of sparse conceptions of properties which deny there are properties corresponding to negative, disjunctive and conjunctive predicates, while avoiding some of the features which made such conceptions so ill-equipped to feature in the analysis of degrees of re-
semblance. Firstly, it still follows from (normalisation) that \( p(a \lor \neg a) = 1 \) since for all \( a \in A \), \( a \lor \neg a = \top \) and it follows from (finite increasivity) that \( p(a \lor \neg a) \geq p(a) + p(\neg a) \) since for all \( a \in A \), \( a \land \neg a = \bot \), so it follows from (normalisation) and (finite increasivity) that \( 1 \geq p(a) + p(\neg a) \) and so \( 1 - p(a) \geq p(\neg a) \). This captures the idea that if ‘is a raven’ has a high weight in determining the degree of resemblance of birds, ‘is a non-raven’ will have a low weight. But even if ‘is left-handed’, for example, has a low weight in determining degrees of resemblance, it no longer follows that ‘is right-handed’ must have a high weight, since the weight of each may be equally low.

Secondly, this characterisation still captures an asymmetry between conjunctive and disjunctive properties which sparse conceptions that maintained properties correspond to conjunctive but not to disjunctive predicates were unable to. For for every pair of predicates \( a, b \in A \) such that \( a \leq b \) or \( a \) entails \( b \), it follows that \( b = a \lor (b \land \neg a) \), and so according to (finite increasivity) \( p(b) \geq p(a) + p(b \lor \neg a) \) and so \( p(b) \geq p(a) \). In other words, it follows that if a predicate entails another, then the peculiarity of the former is less than that of the latter. So it still follows that for all \( a, b \in A \), \( p(a \land b) \leq p(a) \leq p(a \lor b) \) or, in other words, that the weight of a conjunction is greater than or equal to the weight of the conjuncts, whereas the weight of a disjunction is less than or equal to the weight of the disjuncts.

As well as having these advantages, this proposal lacks the counterintuitive consequences of it’s predecessor. Firstly, even if \( p(a) = 0 \) and \( p(b) = 0 \), it doesn’t follow that \( p(a \lor b) = 0 \), but only that \( p(a \lor b) \geq 0 \). Since the peculiarity of a conjunction is still less than or equal to the peculiarity of a conjunct, it’s still the case that \( 0 \leq p(a \land b) \leq p(a) = \)
0, 0 ≤ p(a ∧ ¬b) ≤ p(a) = 0 and 0 ≤ p(¬a ∧ b) ≤ p(b) = 0. But since it follows from (finite increasivity) only that \( p((a ∧ b) ∨ (a ∧ ¬b) ∨ (¬a ∧ b)) ≥ p(a ∧ b) + p(a ∧ ¬b) + p(¬a ∧ b) \), it no longer follows that \( p(a ∨ b) = 0 \), but merely that \( p(a ∨ b) ≥ 0 \). If ‘is red’ and ‘is green’, for example, are perfectly natural or not at all peculiar, ‘is red or green’ may still be less than perfectly natural or somewhat peculiar.

Secondly, even if \( p(a) = p(b) \) and \( p(c) = p(d) \) and if \( a ∧ c = ⊥ \) and \( b ∧ d = ⊥ \), it doesn’t follow that \( p(a ∨ c) = p(b ∨ d) \), since it only follows from (finite increasivity) that \( p(a ∨ c) ≥ p(a) + p(c) = p(b) + p(d) ≤ p(b ∨ d) \). So even if ‘is red’ and ‘is yellow’ are peculiar or natural to the same degree, and ‘is orange’ and ‘is purple’ are peculiar or natural to the same degree, it doesn’t follow that ‘is red or orange’ is peculiar or natural to the same degree as ‘is yellow or purple’. Rather, it follows that the degree of peculiarity of ‘is red or orange’ and the degree of peculiarity of ‘is yellow or purple’ are both greater than both the degree of peculiarity of ‘is red’ plus the degree of peculiarity of ‘is orange’ and also the degree of peculiarity of ‘is yellow’ plus the degree of peculiarity of ‘is purple’, but not that the degree of peculiarity of ‘is red or orange’ and the degree of peculiarity of ‘is yellow or purple’ are equally great.

Thirdly, suppose (finite increasivity) is strengthened to (general increasivity), according to which for every subset, finite or infinite, \( B ⊆ A \) such that \( a ∧ b = ⊥ \) for all \( a, b ∈ B \), \( p(\bigvee B) ≥ \sum_{b ∈ B} p(b) \). Then even if there is an infinite subset \( B ⊆ A \) such that \( a ∧ b = ⊥ \) for all \( a, b ∈ B \) and every predicate \( b ∈ B \) is natural to the same non-negative degree, the sum \( \sum_{b ∈ B} p(b) \) must still be zero or infinite. But since according to (general increasivity) \( p(\bigvee B) ≥ \sum_{b ∈ B} p(b) \), if \( \sum_{b ∈ B} p(b) \) is infinite, this contradicts the fact that for all \( a ∈ A \),
\( p(a) \leq 1 \) and so \( \sum_{b \in B} p(b) \) is still zero. But it does not follow from (general increasivity) and the fact that \( \sum_{b \in B} p(b) = 0 \) that \( p(\bigvee B) = 0 \), but merely that \( p(\bigvee B) \geq 0 \), which is unexceptionable.

Suppose, for example, that there is an infinite number of determinate shades of colour, which all have equal non-negative weight in determining degree of resemblance. Their equal degree of peculiarity still cannot be positive, since then the degree of peculiarity of their disjunction ‘is coloured’ or the sum of their equal degrees of peculiarity would be an infinite number greater than one, contradicting the fact that all degrees of peculiarity are real numbers less than one. So their equal degree of peculiarity must be zero, and the sum of their equal degrees of peculiarity must be zero as well. But it doesn’t follow from this and (general increasivity) that the degree of peculiarity of their disjunction ‘is coloured’ must be zero as well, but only that the degree of peculiarity of their disjunction ‘is coloured’ must be greater than zero, which is unexceptionable.

These three problems arose partly from the intuition that certain predicates, such as those expressing the colours, should have equal weight in determining degrees of resemblance. It’s natural to generalize this intuition by suggesting that each of the \( n \) atomic predicates has an equal peculiarity which if \( n \) is finite is equal to \( s \leq \frac{1}{n} \). For example, it’s natural to suggest that each of the \( n \) atomic predicates has an equal peculiarity of zero. Then according to (finite increasivity) the peculiarity \( p(a) \) of a predicate \( a \) is greater than the sum of the peculiarities of the \( r \) inconsistent atoms it is a disjunction of, the peculiarity of each predicate of dimension \( r \) will be greater than \( sr \). But this peculiarity could be a different peculiarity greater than \( sr \) for each predicate, so

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it wouldn’t follow that each predicate of dimension \( r \) has the same weight in determining degree of resemblance. As a result, it no longer follows that the degree of resemblance between two different particulars is a fixed constant, which does not depend on the choice of the two particulars. Thus, there is an intuitive revision to the third premise which escapes the barely believable conclusion, and the problem appears to be resolved.

4 Conclusion

In order to escape the barely believable conclusion that the degree of resemblance between two different particulars is a fixed constant, independent of the choice of the two particulars, one must deny either the first premise, that the number of (possible) predicates satisfied by two particulars which do not satisfy all the same predicates is a fixed constant equal to the number of (possible) predicates satisfied by only the first, the second premise, that the number of (possible) predicates satisfied by two particulars is the number of properties they have in common and the number of (possible) predicates satisfied by only one is the number of properties they have not in common, or the third premise, that the degree of resemblance between particulars is a function of the number of properties they have in common and the number of properties they have not in common. This section concludes by reconsidering each option in light of the preceding discussion, and arguing tentatively for the third.

The rationale for the first premise, that the number of (possible) predicates satisfied by two particulars which do not satisfy all the same predicates is a fixed constant equal to the number of (possible) predicates satisfied by only the first, was divided into a
provable part, that the number of elements in a complete boolean lattice with \( n \) atoms is \( 2^n \), and a very plausible part, that the (possible) predicates ordered by entailment are a complete boolean lattice. So one may deny the first premise either by rejecting the provable part, and denying that the number of elements in a complete boolean lattice with \( n \) atoms is \( 2^n \), or denying the very plausible part, that the (possible) predicates ordered by entailment are a complete boolean lattice.

The only controversial assumption of the proof that the number of elements in a complete boolean lattice with \( n \) atoms is \( 2^n \) was the axiom of choice, which was required to prove the existence of the atoms in the infinite case. Since it’s easy in the infinite case to form the impression that infinity is the source of the problem, and since the axiom of choice is associated with many counterintuitive results concerning infinity (such as the Banach-Tarski paradox), it’s tempting to attempt to escape the barely believable conclusion in the infinite case by denying the axiom of choice. But since the conclusion of the argument is equally barely believable even in the finite case, and since the axiom of choice is not needed for the proof in the finite case, denying the axiom of choice does not lead to a general solution of the problem. Given this, it seems most likely that the axiom of choice is not the source of the problem even in the infinite case.

Likewise, the most controversial assumption of the plausible part is that the boolean lattice of (possible) predicates under the relation of entailment is a complete lattice or, in other words, that every infinite set of (possible) predicates has a disjunction and a conjunction. Since it’s easy in the infinite case to form the impression that infinity is the source of the problem, and since the existence of arbitrary infinite disjunctions and
conjunctions is associated with other counterintuitive results (such as the in principle possibility of reducing all supervenient (possible) predicates or properties to their subvening bases), it’s tempting to escape the barely believable conclusion in the infinite case by denying that the boolean lattice of (possible) predicates under the relation of entailment is complete and that every infinite set of (possible) predicates has a disjunction and conjunction. But since the conclusion of the argument is equally barely believable even in the finite case, and since the assumption that the lattice of (possible) predicates is complete is not needed for the proof in the finite case, denying the assumption of completeness does not lead to a general solution of the problem. Given this, it seems most likely that completeness is not the source of the problem even in the infinite case.

The second premise, that the number of (possible) predicates satisfied by two particulars is the number of properties they have in common and the number of (possible) predicates satisfied by only one is the number of properties they have not in common, was motivated by an abundant conception of properties, according to which there is a property corresponding to every (possible) predicate. So one may motivate denying the second premise by adopting a sparse conception of properties. I considered two kinds of sparse conceptions of properties above. The first denied the existence of properties corresponding to uninstantiated or coextensive predicates. The second denied the existence of properties corresponding to negative, disjunctive and conjunctive predicates. I also considered David Armstrong’s extremely influential conception of sparse properties, according to which there are instantiated conjunctive, but no negative, disjunctive or uninstantiated properties. Surprisingly, none of these sparse conceptions were able
to escape the barely believable conclusion that the degree of resemblance between two different particulars is a fixed constant, which does not depend on the choice of the two particulars. The exception was a modification of Armstrong’s conception proposed by John Bacon. But that modification seemed poorly motivated and unintuitive.

It does not follow from these results that there is no sparse conception of properties which escapes the barely believable conclusion (I am more optimistic, for example, about a proposal due to Peter Gardenfors (2000) and elaborated by Graham Oddie (2005)). And for any particular lattice of (possible) predicates, it seems possible to identify which (possible) predicates in the lattice contribute towards resemblance between the particulars which satisfy them, and so which (possible) predicates correspond to predicates. But it does not seem possible to clarify which (possible) predicates contribute towards resemblance using the resources available here, so I am pessimistic about the prospects of resolving the problem by adopting a sparse conception of properties.

The third premise, that the degree of resemblance between particulars is a function of the number of properties they have in common and the number of properties they have not in common, was motivated by the analysis of resemblance as having properties in common, which suggests that the more properties particulars have in common, the more they resemble each other, and the more properties particulars have not in common with each other, the less they resemble each other. One way to avoid the barely believable conclusion, while retaining what is intuitive about this premise, is to argue that the degree of resemblance is a function of the weights of the properties they have in common and the weights of the properties they have not in common. One may think
of these weights as subjective degrees of importance, as Nelson Goodman (1972) does, or objective degrees of naturalness, as David Lewis (1983) does.

Revising the premise that the degree of resemblance between two different particulars is a function of their number of common and uncommon properties to the thesis that the degree of resemblance between two different particulars is a weighted function of their common and uncommon properties must be at least as good a way to avoid the conclusion that the degree of resemblance between two different particulars is constant if defined as adopting a given sparse conception of properties, because if the predicates which correspond to a property according to a given sparse conception have weight one and the predicates which do not correspond to a property according to that sparse conception have weight zero, then the sums of the weights of the predicates in common and not in common between two particulars according to this revision will be the same as the number of predicates in common and not in common according to the given sparse conception.

But perhaps unsurprisingly, adding rather natural assumptions about the weights of various (possible) predicates, while recapturing some of what was intuitive about sparse conceptions of properties which deny the existence of properties corresponding to negative and disjunctive predicates, led to a conception of degrees of importance or naturalness which still failed to escape the barely believable conclusion that the degree of resemblance between two different particulars is a fixed constant, independent of the choice of the two particulars. In this case, however, a natural weakening of the assumptions seemed to lead to a better conception, according to which the weight of a predicate
is always non-negative, the weight of predicates necessarily satisfied by everything is zero, and the weight of disjunctions of inconsistent predicates is always less than or equal to the weight of their disjuncts. This is the solution about which I'm most optimistic.

References


