Analysis of an Economic Option Market based on Parimutuel Auctions.

Galo Nuno Barrau (galonuno@stanford.edu), Adam de la Zerda (adlz@stanford.edu),
Jianlin Wang (jianlin@stanford.edu), Victor Montiel Argaiz(vmontiel@stanford.edu)
Dept of Management Science & Engineering. Stanford University. MS&E 311 Optimization

***Abstract*— In this paper we study a method to implement economic option markets with *parimutuel digital auctions* pricing with linear programming techniques. This kind of auctions have represented a breakthrough in the financial domain over the last few years, since they have many positive features that provide agents with basic tools to hedge against risk on macroeconomics variables.**

Our study is mainly focused on two aspects. The first one the auction behaviour as the number of players decreases, and the second one is the performance of the auctions when we allow players to take also short positions on their bids. In both cases, we analyse the liquidity of the market, prices evolution and percentage of orders accepted. We have therefore proposed an extension to the basic worst-case model that allows to process both short and long positions.

Simulations were made based on a stochastic model that simulates different kinds of behaviors for bidders. The results show that the auction performs rather well, as the liquidity remains almost independent of the number of players and the overall performance increases with the introduction of short players.

***Index Terms*— Parimutuel Call Auction, Optimization, Linear Programming, Hedging.**

I. Introduction

A. The rational for economics derivatives

From a firm's perspective, unsystematic risks (i.e., risks that are intrinsic to the firm) can be easily hedged away through diversification. But systematic risks (i.e., risks attributable to the market or the economy) tend to remain even after extensive diversification. As conventional financial instruments have been of little help to hedge against those risks [1] [2], economic policy makers have often tried to control the negative effects of economic events with various tools using a top-down approach: central banks use interest rates; national governments use taxes and regulations and multilateral organizations (such as the World Bank and the IMF) use economic reform programs.

Economic derivatives are market-based instruments for the effective management of the risks associated with economic events. They may be used in combination with or even replace existing instruments or policy tools and they can be more adaptable to the changing needs of firms and nations. The basic idea is making a market for options on economic statistics, so that agents can hedge on well-defined economics events. Consequently, by developing a market on economic derivatives, governments could hedge fluctuations in tax revenues and corporations in their earnings whereas hedge funds and other speculators would trade and provide liquidity. Consequently, agents can directly relate the benefits of these instruments to their businesses, which enables an accurate hedging approach, and more importantly, a simultaneous management of basis risk.

Options on economic information pose two main challenges that have hindered their development for some years:

1) Economic data could be seen as a non-tradable underlying asset [3]. Pricing of this kind of instruments should not be based on the cash-flow of the underlying asset following any kind of no-arbitrage condition, but on the offer and demand of the instrument.
2) In a ”traditional market” approach, it is difficult to find discrete order matches and, consequently, there is no way to build volume and liquidity.

Fortunately, both problems can be solved with the Parimutuel Digital Call Auction (PDCA) technique developed by Goldman Sachs and Deutsche Bank [4]. The instruments are traded through a Universal Dutch Auction format. The auction format nonetheless uses standard trading conventions, whereby participants may buy and sell options by submitting limit order bids and offers. ICAP, [5] the international broker offers, the brokerage and distribution platform.

The first auction took place in October 2002. Since then, there have been 151 Economic Derivative Auctions, 62 Prepay Auctions and 27 Commodity Derivative Auctions. These auctions covered four data releases: ISM Manufacturing, Change in Non-Farm Payrolls, Initial Jobless Claims and Multiple Euro-zone HICP. More auctions are planned on other data sets.

B. Introduction to Parimutuel Digital Auctions

Traditionally, we define an auction as a public sale of some kind of property. In this paper, however, we are mainly interested in auctions where there is not such property, but instead, players are able to bid on future information or events. In this kind of auctions, the organizer offers a set of different states (or events) to the players where they can bid on. These states are mutually exclusive, i.e., only one of them will be realized when the event is revealed.

The procedure of the auction works as follows: initially the auction is open and the players bid on the different states according to their own probability estimations of the events. Each order has the next information: the state (or possibly states, if several states per order are accepted) they bid on, the quantity of bid orders and the maximum price they are willing to pay for those orders.

Some time before the event is happening or information is going to be released, the auction is closed, so that no new orders or modifications can be made.

Once the auction is closed, the organizer or auctioneer reviews all the orders and decides which orders to accept, as well as the final price of betting on each possible state (*state prices*). Once this information is made public, those bidders whose orders were accepted, i.e., whose price limit was greater or equal than the state price, will have to pay the cost of their bids according to the quantity and bidden states (*fair prices mechanism*).

The auction organizer should have some criteria (objective function to maximize) when he decides which orders to accept and at what price. Several approaches are possible here, depending on his own interest. For example, the auctioneer could desire maximize his income on the worst case, or, if he has a good probability estimation of the events, he could try to maximize the expected income or to minimize the risk or variance of the same quantity while assuring a minium expected value. On the other hand, the organizer might have other interests apart from making a big amount of money from the auction, for example, he might prefer accept the maximum number of orders, even if it could earn less money, or to charge a fee to the accepted bids, so that this encourages more people to play (potential future clients) or this keeps him in the auction markets with some other future interests.

Finally, the outcome of the future event is revealed and bidders on the winning state will be paid off a certain quantity of money depending on the number of accepted orders.

There are two fundamental features of parimutuel auction:

- The system should be self-funded [6], that is to say, all payments to the winners and the profit for the organizer should come from the collected money. This assures that the organizer never loss any money.
- The auctioneer should assure a fixed payment for the winning state. Therefore, the agents know beforehand their cash-flow if the bid event happens, which makes hedging easier.

Using this basic framework, we can add many other interesting features to the auction system that might encourage people to enter into the game [4]. For example, while the auction is open, auction organizer could broadcast some indicative data of received orders in real-time, such as prices, number of orders and probabilities estimations. He might even let the bidders modify their bids, according to this information, as long as the auction remains open. Another interesting possibility is to introduce some kind of priorities in accepting the orders, giving more priorities, for examples to earlier orders.

TABLE I

MATHEMATICAL NOTATION

m	Number of states to bid on
n	Number of bidders
w	Notional, money to pay to winning states
A	$\mathbb{R}^{mxn}$ Bids Matrix. $a_{ij} = 1$ represents player j bids on i
π_j	Limit price of player j
p_i	State price for event i
q_j	Maximum number of bids for participant j
x_j	Orders accepted for participant j
v_i	Estimated probability for state i

Finally, a large list of applications can be envisaged by organizing PDCA on different events. As we have mentioned, this kind of auctions can generate a market for financial instruments which allow traders to hedge against risk on macroeconomic variables. Also, as an example, energy companies could employ the system as a way to face the risk of variations in oil reserves, or auctions could be organizer on wars or terrorist attacks to let agents to cope with unexpected risk and to help government to forecast those kind of events. The next section explores further the possibilities of the system in the first example.

II. SOME MATHEMATICAL MODELS FOR PDCA

In this section, we are going to explain some simple mathematical models to calculate state prices under several criteria or objective function. Then, we will implement one of the models and we will comment the results. We will use the notation presented in Table I. For a complete discussion on the meaning of these parameters we recommend to refer to [6].

A. Robust Model

The linear program for the robust model is the following:

$$\begin{aligned} max\ \pi^T \cdot \mathbf{x} - w \cdot s \\ s.t.\ \mathbf{A} \cdot \mathbf{x} - \mathbf{e} \cdot s \leq 0 \\ \mathbf{x} \leq \mathbf{q} \\ \mathbf{x} \geq \mathbf{0} \end{aligned}$$

In consequence, what this model is trying to do is to maximize the income, that is to say, the difference between the money collected from orders $\pi^T \cdot \mathbf{x}$ and the money we have to payback to the winners in the worst case $w \cdot s$. At the same time, it establishes a limit in the number of orders accepted for each player $(\mathbf{q})$

B. Stochastic Model

$$\begin{aligned} max\ \pi^T \cdot \mathbf{x} - w \cdot \mathbf{v}^T \cdot \mathbf{s} \\ s.t\ \ \mathbf{A} \cdot \mathbf{x} - \mathbf{s} = \mathbf{0} \\ \mathbf{x} \leq \mathbf{q} \\ \mathbf{x} \geq \mathbf{0} \end{aligned}$$

In this model, we introduce some estimations of the probabilities to maximize better the profit. Now, the term $w \cdot \mathbf{v}^T \cdot \mathbf{s}$ is the expected amount of money to pay to bidders.

C. Risk Control

$$\begin{aligned} min\ \mathbf{s}^T \cdot \mathbf{V} \cdot \mathbf{s} \\ s.t\ \ \mathbf{A} \cdot \mathbf{x} - \mathbf{s} = \mathbf{0} \\ \pi^T \cdot \mathbf{x} - w \cdot \mathbf{v}^T \cdot \mathbf{s} \geq \mu \\ \mathbf{x} \leq \mathbf{q} \\ \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where $\mathbf{V} = diag(\mathbf{v}) - \mathbf{v} \cdot \mathbf{v}^T$ is the covariance matrix of the probabilities for each state.

Observe that this model has a non-linear objective function, and even if the constraints are linear, we cannot use exactly the same linear techniques to solve the problem. The model is trying to minimize the variance or risk of the profit and at the same time, assuring a minimum expected profit of μ.

III. Simulation of the robust linear programming model

For our simulations we have implemented the robust or worst-case model as the auctioneer faces no risk in it. The orders of the bidders have been generated randomly, as well as the price limits. Monte Carlo simulations have been used to observe the results of the auctions on the long-run.

The auction to be simulated has four states and each player bids on a state according with a discrete normal distribution in which states 2 and 3 have been assigned considerably more probability than the others.

Finally, for price limits π_j, we have used again a random distribution which tries to model different types of bidders. Price limits have two components. The first component is a bell curve which makes bidders put more money in the more probable winning states. This term could model the speculator behavior. On the other hand, the price is multiplied by a random factor (uniform distribution between 0 and 1) that tries to model other types of bidders. This is so because usually not all players follow the speculator behavior. For example, people that use the auction as a way to minimize risk (hedgers) might have other preferences in prices and bidden states different from the most probable, and others could just have these choices because of misinformation.

Although this ”perfect foresight” could be seen as a lost of generality, it is not as the real distribution only affects the due payments and the auctioneer profit, but not the liquidity, the income or the prices, which are the variables under analysis. Therefore, we can consider this assumption as a worst-case for the auctioneer as the agents commit no mistake in bidding to the winning state (in probabilistic terms).

Once that we have randomly generated the data for the auction, we run the linear programming model and calculate state prices and accepted orders. Then, we ”toss a coin” to decide which is the winning state, using the same probability distribution that bidders have used to make their bids, i.e., auction organizer and bidders have the same estimations of the probability distribution.

IV. Numerical results

The performance of the Parimutuel Auctions is a key aspect to the success of the economic option markets. The analysis of the former ones provides valuable insight on the behavior of the latter ones. Therefore we have decided to generate two different sets of simulations to analyze two crucial aspects of the markets:

1) On one hand, we are interested in the liquidity of the market. As the advantage of PDCA is that they can provide liquidity even with a reduced number of agents we analyze the performance of the auction with respect to the number of players.
2) On the other hand, we are interested in the performance of the market when agents take both long and short positions. PDCA is able to price the instruments even if all the agents take long positions, that is, they pay today the price (determined at the end of the auction) to obtain a future cash-flow w in the case of the realization of the outcome they bid for. However, it is also possible to allow players to take short positions where they obtain today an amount of money equal to the price of the events they bid for, providing they are willing to pay the fixed amount w if these events happen. This extension of the basic mechanism can be achieved by a little modification of the model assumptions.

A. Analysis of Market Liquidity

The main drivers of our ”liquidity metric” are the potential and actual amounts of money collected by the auctioneer (denoted as ”Potential Income” and ”Income” respectively), the average amount of money paid to the players (”Payments”) the auctioneers ”Profit” (amount collected minus amount paid) and the mean number of contracts awarded over the total number of bided ones (”Percentage of accepted bids”). All of them provide valuable information about how the auction behaves.

Figure 1, 2 and 3 display our analyses. In the considered case, it can be seen how both the amount collected and the due payments grow linearly with the number of users. The average bid acceptance is around the 55% of the orders and the money collected is around a half of the maximum potential amount. Consequently we can affirm that this market provides fairly constant liquidity no matter the number of players, displaying a fairly constant behaviour for more than

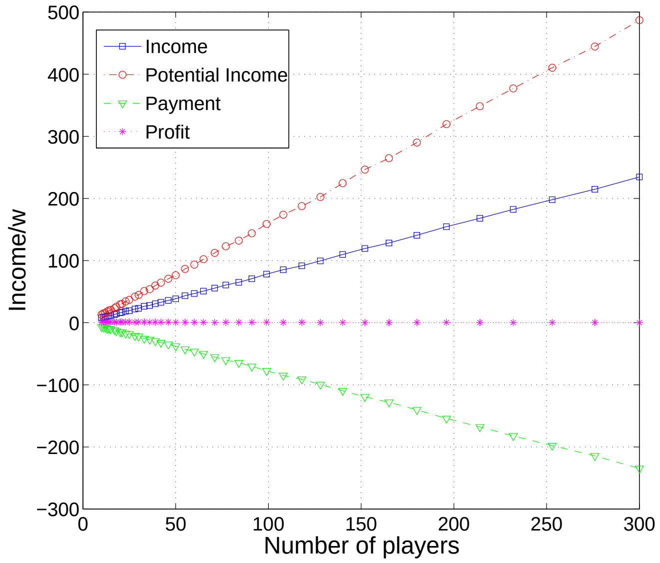

Fig. 1. **Evolution of income vs number of players**. Results for 200 MonteCarlo simulations. *Potential income* is the maximum money the auction organizer could collect if we charge the price limit quantity to each order. *Payment* is the total money that the auction organizer have to pay to winning bidders. *Profit* is the difference between the income and the payment, that is, the money the organizer earn in the auction. Money is relative to w.

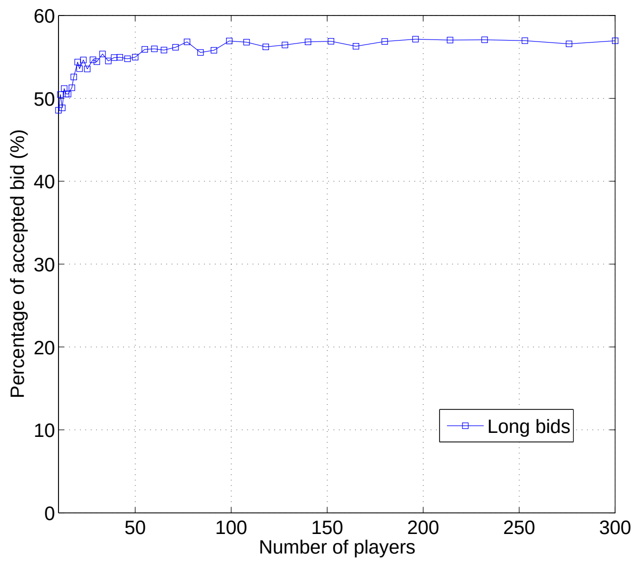

Fig. 2. **Percentage of bids accepted vs number of players**. Results for 200 MonteCarlo simulations. For this simulation, shorting is not allowed.

100 users. This is a desirable feature, as it means that players should expect to face a similar environment no matter their number.

B. Introducing short players

Once we have defined long and short players, we should also define what it is sometimes called ”buyers” and ”sellers”: A long-buyer is someone who pays the price (determined by the auction) to obtain a future cash-flow w in the case of the realization of the outcome he bids for (this definition is analogous to the one provided above for the long players, as all long players can be described as buyers, as we will explained shortly). A long-seller who bids for the same events would obtain w if those events do not happen. Therefore it can be easily shown that any long-seller can be modelled as a long-buyer who bids for all the other states, and consequently the long market can be fully characterized by taking into consideration only long-buyers.

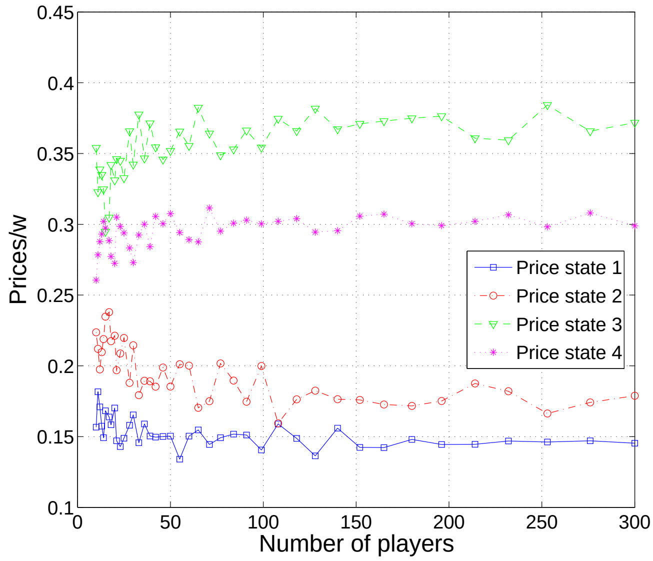

Fig. 3. **State Prices vs number of players**. Results for 200 MonteCarlo simulations. Prices are relative to w.

The problem with short players is that they obtain a certain cash-flow w now (after the auction is closed) but they will pay only if the states they short happen. Therefore, in a market with only short players (short-buyers according to what we have mentioned before) a worst-case model would not offer any bid as it is not possible to pay w in all the states if we collect only the price $p_i \leq w$ in only one of them. As we can see in figures 4 and 5, results are consistent with that possibility.

To simulate short players we should allow matrix $\mathbf{A}$ to take negative values. Therefore an entry $a_{i,j} = -1$ represents player j shorting state i. The limit price j will also be negative as it represents now the minimum amount that the player is willing to receive in exchange of paying w if the bided state happens. As a useful comment, entry values between -1 and $+1$ would be useful to generate all kind of composite instruments, such as capped calls or puts or similar, both for short or long positions.

Results presented in Figure 5 show how the inclusion of short players improve the liquidity of the market as the number of accepted long-bids increase with the percentage of short players. The total amount collected in figure 4 is

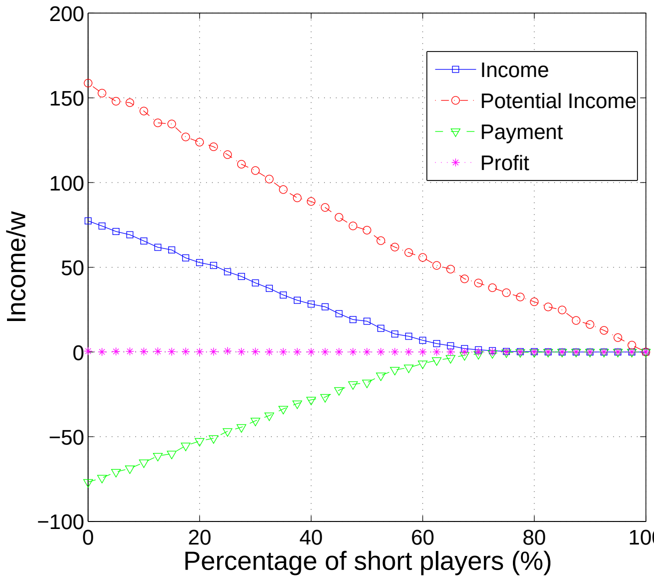

Fig. 4. **Income vs percentage of short players**. Results for 200 MonteCarlo simulations. *Income* is the money collected by auction organizer . *Potential income* is the maximum money the auction organizer could collect if we charge the price limit quantity to each order. *Payment* is the total money that the auction organizer have to pay to winning bidders. *Profit* is the difference between the income and the payment, that is, the money the organizer earn in the auction. Money is relative to w

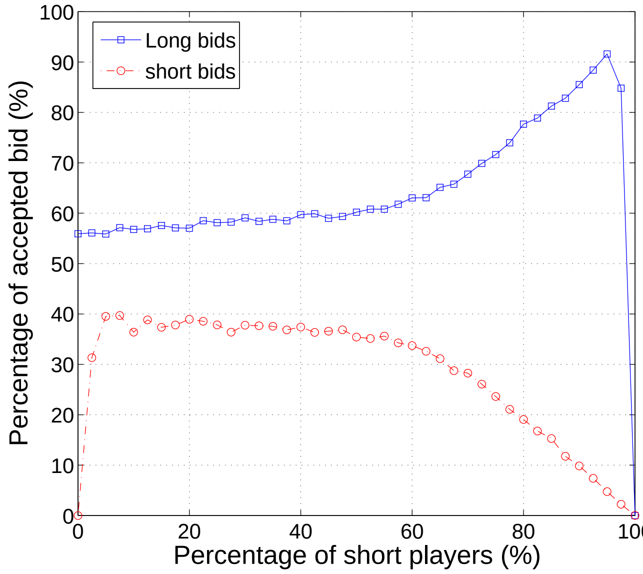

Fig. 5. **Percentage of accepted bids vs percentage of short players**. Results for 200 MonteCarlo simulations. *Long bids* represents the percentage of long bids accepted in the auction. *Short bids* represents the percentage of short $(a_{i,j} = -1)$ bids accepted in the auction.

reduced as money flows directly from long to short players at the auction specified prices (in some sense the auction is providing the reference prices at which transaction occurs). As proportion of short players surpass the long players one the market starts to reject short players and increase the ratio of accepted long ones, in an attempt to obtain money to pay the short players.

To sum up, the possibility of allowing shorting would beneficial for the agents as it provides more hedging possibilities. The PDCA presented in [6] is able to absorb any percentage of short players (except 100% which no self-funded model should accept) and displays a fairly constant behavior if they constitute less than the 50% of the market.

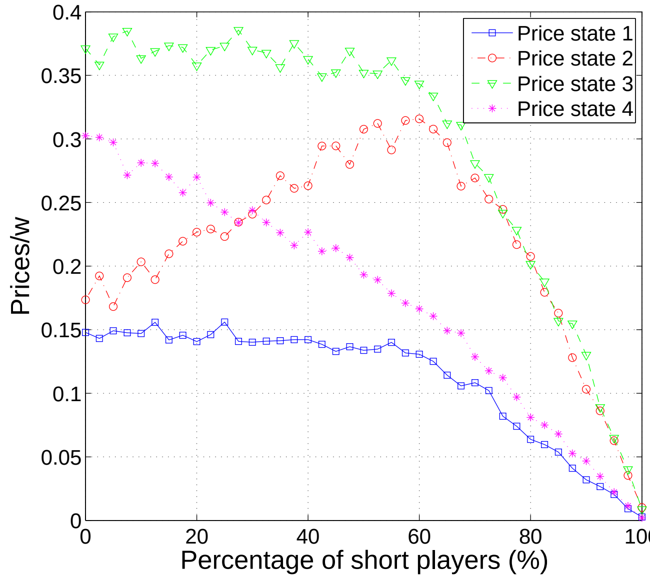

Fig. 6. **State prices versus percentage of short players**. Results for 200 MonteCarlo simulations. Prices are relative to w.

V. Conclusions and Further Research

In this paper we have analyzed the possibility of implementing economic option markets by applying Parimutuel Auctions (in particular a robust, worst-case model like the one presented in [6]). By constructing a software simulator we have shown that these approach yields an excellent performance, as its liquidity is independent of the number of agents, who can be either buyers or sellers. We have shown how a straightforward extension of the model by allowing negative entries to **A** can cope with the issue of short-players and how this modified model solves satisfactorily this new problem and provides liquidity in both markets (long and short) as long as the proportion of long players is considerable (where ”considerable” can be defined according to additional specifications, but it does not need to be above 50%).

Finally, an interesting line of research would be to analyze the behavior of the agents and the auction results when the auctioneer provides reference prices before the auction is finished, as the players could try to use them to infer both the implied probabilities and the behavior of the other participants.

REFERENCES

[1] V. K. Bansal, J. F. Marshall, and R. P. Yuyuenyongwatana, "Hedging business cycle risk with macro economic swaps some preliminary evidence," *Journal of Derivatives 1(3) 50 58*, 1994.

[2] ——, "Macroeconomic derivatives more viable than first thought," *Global Finance Journal 8(1) 101 110*, 1995.

[3] *Swaps and Financial Derivatives: Products, Pricing Application and Risk Management,3rd Ed*, 2004.

[4] D. Cass, "Goldman and deutsche to launch economic data options," *Risk*, 2002.

[5] http://www.economicderivatives.com.

[6] M. Peters, A. M.-C. So, and Y. Ye, "A convex parimutuel formulation for contingent claim markets," *NSF grant DMS-030661*.