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COMMON MISAPPLICATIONS OF THE FARADAY INDUCTION RULE

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COMMON MISAPPLICATIONS OF THE FARADAY INDUCTION RULE

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Abstract

Recent experiences with reviewers of a manuscript on railgun physics indicate widespread misapplications of the concepts pertaining to electromotive force, potential difference, and voltage. Contributing to this problem is the fact that instances of such misapplications can even be found in outstanding textbooks, such as the Feynmann lectures.

Most textbooks introduce the Faraday rule of induction using an armature propelled by an external force along a pair of rails in a magnetic field. This rail/armature configuration is reconsidered here. The discussion of the concepts is expanded to include railguns, where the armature is propelled along a pair of rails by internal forces generated by an applied rail current. Common misapplications of the Faraday rule are examined in terms of these configurations. Such distributed parameter problems are particularly susceptible to error. Correct analyses are offered.

Excessive reliance on the Faraday rule can mask the underlying physics of a problem. As an example, it is shown from the underlying physics that the potential distributions from flux creation within conducting loops are similar in origin to those described by the basic transmission line equations; it is also shown that the potentials due to armature motion arise from localized seats of emf.

Keywords: induced fields, emf, potentials, motional emf, railguns, Faraday rule, armatures.

TABLE OF CONTENTS

	<u>Page</u>
Introduction	1
The Faraday rule.....	1
Potentials and emfs	2
Feynmann et al [1] analysis of inductor /capacitor	4
Woodson and Melcher [4] analysis of a moving armature along a rail pair	6
Analysis of potential distributions in railguns.....	6
Summary	8
References	9
Appendix.....	10

LIST OF FIGURES

	<u>Page</u>
Figure 1. The standard, familiar, configuration used to introduce the Faraday law of induction.	2
Figure 2. A square conducting loop of unit side and area, in a uniform B field, with positive dB/dt	3
Figure 3. The basic experimental arrangement analyzed in Chapter 22 of Feynmann et al [1].	4
Figure 4. The basic railgun configuration.	6
Figure 5. One of the rails of a railgun near the armature	8

Introduction

The concepts of induced emfs are usually introduced in terms of a conducting armature sliding along a pair of conducting rails, where the entire assembly is immersed in a uniform magnetic field. Feynmann et al. [1] stress the following points regarding the physics of this rail/armature setup: a.) There are two independent physical processes that can induce an emf, namely, an emf associated with a flux change from a time rate of change of the magnetic flux density, B , and a motional emf, generated by a Lorentz force on the charge carriers within the moving metallic armature; b.) The “Faraday law” is remarkable in the sense that it combines these two entirely separate phenomena with one simple expression that computes the emf generated within any circuit from the total rate of change of flux within the loop. The correct magnitude of the emf is computed, regardless of whether the flux change is from true flux creation associated with a time rate of change in B , or an imagined flux creation, associated with a change in the area of the loop as a charged carrier traverses a B field; and c.) The “Faraday law” is not a true law of physics; for example, it does not apply with real bulk conductors that are susceptible to eddy currents. For this reason, Feynmann et al [1] recommend the use of the term “Faraday rule” rather than “Faraday law”. They further advise a focus on the underlying physics of the two separate phenomena to avoid pitfalls with reliance on the Faraday rule alone.

We show that another common problem with application of the Faraday rule is the distinction between emf and potential difference. This issue is addressed in detail by Page [2] and by Varney and Fisher [3]. The two concepts are frequently treated as synonymous. Part of the difficulty is that both emf and potential have the same units (volts). Our aim here is to expand on their discussions to show that any analyses derived from confusing these different physical quantities are invalid. We examine examples found in the physics textbook by Feynmann et al [1], despite their explicit warnings of possible pitfalls. We also examine examples from an engineering text by Woodson and Melcher [4].

Invalid analyses in textbooks are particularly effective in perpetuating erroneous analyses in the scientific community. Our experience with one of the reviewers for the 13th Electromagnetic

Launcher Workshop in Potsdam, who cites Reference [4], is included in the Appendix as a case study. Of course, correct treatments can be found in other texts, such as those by Reitz and Milford [5], and Panofsky and Phillips [6]. Since most readers are likely to be unfamiliar with this topic, we offer detailed introductory exercises as a primer.

The Faraday rule

Figure 1 shows the standard, familiar configuration used to introduce the Faraday law of induction. A conducting armature lies across two parallel rails separated by distance h , and the assembly is immersed in a uniform magnetic flux density B . An external agent moves the armature with velocity v , as shown. The Faraday law states that the emf generated within the loop is given by the total rate of change of flux, $d\Phi/dt$, within the loop,

$$\text{emf} = \oint \mathbf{E}' \cdot d\mathbf{l} = -d\Phi/dt \quad (1)$$

Faraday’s experimental results with tests of this kind led to the generalized conclusion, that, with a moving conducting loop in a distributed magnetic field, B , the emf generated in the loop is given by Equation 1 where

$$d\Phi/dt = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_C (\mathbf{B} \times \mathbf{v}) \cdot d\mathbf{l} \quad (2)$$

The surface integral is over the loop surface area, S , and the line integral is over the perimeter, C , of S . The first term in Equation 2 is the true flux creation term, which arises from the instantaneous change in \mathbf{B} within a momentarily stationary area S . The second is the motional emf term, which arises from the Lorentz force on charges within a conductor on the perimeter C ; the conductor moves through a uniform field, \mathbf{B} . In the case of a static \mathbf{B} field, we have only motional emf, whose magnitude is Bvh for the setup in Figure 1.

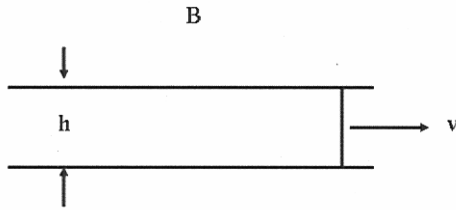


Figure 1. The standard, familiar, configuration used to introduce the Faraday law of induction.

A primary reason that Feynmann et al recommend use of the term “Faraday rule” rather than “Faraday law” can be illustrated by considering the consequences of replacing a thin wire armature with a bulk, flat sheet conductor across the rails in Figure 1; in that case, eddy currents will be set up within the bulk armature so that a meter between the conducting rails will register a lower value of potential and currents than those from the thin conductor. One would then have an erroneous measure of the induced emf.

Another caveat by Feynmann et al [1] is to focus on the underlying physics, because reliance on the Faraday rule alone can be misleading. A simple example relates to the location of the seat of emf in the textbook case shown in Figure 1 with a static B field. The Faraday rule only gives the magnitude of the emf generated within the complete circuit. Suppose one is interested in the rail-to-rail potential distribution generated by the motional emf from the moving armature. The distribution cannot be established without identifying the seat of emf. (“Seat of emf” refers to any circuit segment where a non-electrostatic field exists [5].). The seat of emf cannot be viewed as uniformly distributed along all conductors as is sometimes suggested [4] for this case. Here the seat of emf is entirely contained within the moving armature because the electromotive force on moving charges in the magnetic field is localized within the armature. Thus, the configuration is analogous to moving a battery along the rails.

Regarding the distribution of rail-to-rail potential differences in this open circuit case, a potential difference exists across the armature because of the charge separation caused by the motional emf within the armature. Assuming good contact, this potential difference is also present all along the rails and there are no potential gradients along x. Furthermore, only

electrostatic fields exist between the rails. If, instead, the same magnitude emf were generated by flux creation, a gradient in potential would be established along x (see the derivation of the transmission line equation below); the other main difference is that fields between the rails would now be a combination of electrostatic fields, \mathbf{E} , from the charge distribution, and induced fields, \mathbf{E}' , from $d\mathbf{B}/dt$. The two cases are therefore not equivalent. Again, the Faraday rule gives no measure of potential and field distributions because, contrary to what many believe (see Appendix), it can only give the net emf in the circuit from flux changes and/or motional effects.

Potentials and emfs

Page [2] and Varney and Fisher [3] address distinctions between potential difference and emf, and advocate precise definitions for each term. For our purposes Varney and Fisher’s more succinct definitions suffice:

(i) Emf is the integral of a non-electrostatic field, \mathbf{E}' , whereas potential difference is the integral of an electrostatic field, \mathbf{E} .

(ii) The emf integral is dependent on the path of integration (non-conservative field); the potential difference is independent of the path (conservative field).

The following simple exercises illustrate the importance of distinguishing between emf and potential difference. These exercises also provide a background for later discussions. The first exercise relates to the point made by Varney and Fisher [2], that there is an emf, but no potential differences in a conducting ring immersed in a uniform time varying magnetic field. See also discussions of emfs and potentials in conductors by Reitz and Milford [5] and Panofsky and Phillips [6].

Our basic method of analysis makes use of the relation that the line integral of $\mathbf{E} + \mathbf{E}'$ minus the IR drop along a segment equals zero [5]. (R is the segment resistance, and the current is assumed to be uniform throughout the volume of the conductor.) Using the fact that the potential difference is the negative integral of E along the segment, while the emf is the positive integral of \mathbf{E}' along the segment [2], we have that the potential difference in a conducting segment

containing a seat of emf is given by the sum of the emf in the segment and the IR drop along the segment. With no current flow in the segment containing \mathbf{E} and \mathbf{E}' , we have $\mathbf{E} = -\mathbf{E}'$. In the case where there is current flow, but $\mathbf{E}' = 0$, we recover the familiar relation from lumped parameter circuits, $V = IR$.

For these exercises, assume the 2-D configuration shown in Figure 2, with a square conducting loop of unit side and area, in a uniform B field, with positive $d\mathbf{B}/dt$. In the first case, assume all the loop sides have the same resistance, R_n . An emf is generated by the non-electrostatic \mathbf{E}' within the loop; \mathbf{E}' is tangent to the loop surface and uniformly distributed along the loop. So, the magnitude of \mathbf{E}' can be computed from the line integral \mathbf{E}' around the entire perimeter (which gives the net emf in the circuit) and the surface integral of $d\mathbf{B}/dt$ over the unit area of the loop: $E' = (1/4) d\mathbf{B}/dt$. Since \mathbf{E}' is distributed uniformly along the loop, the seat of emf is distributed uniformly along the loop. The current is thus given by $I = \text{emf}/(4R_n) = d\mathbf{B}/dt/(4R_n)$. Suppose the emf from $d\mathbf{B}/dt$ is 8 volts. Then, by symmetry, each side acts as a 2 volt seat of emf. If R_n is one ohm, then a current of 2 amperes is generated in each side of the square, the continuity condition for current applies, and the problem is solved. The point is that there are no potential differences because there are no electrostatic charges and no electrostatic fields anywhere.

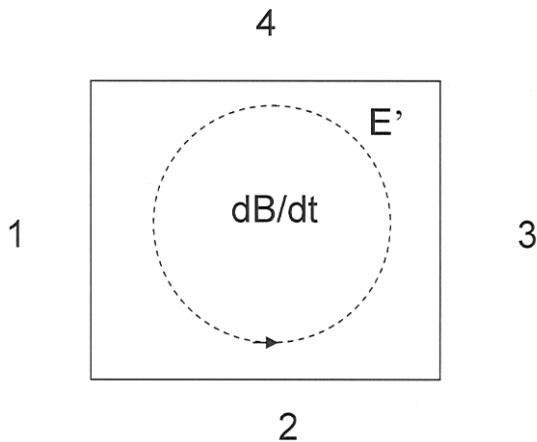


Figure 2. A square conducting loop of unit side and area, in a uniform B field, with positive $d\mathbf{B}/dt$.

For the second exercise, we show how electrostatic potentials are established in such cases. Assume the same emf exists, and non-uniformity is introduced by setting $R_1 = 8$ ohms,

with the resistance in the other sides essentially negligible: $R_2 = R_3 = R_4 = 0.001$ ohms. This physical setup is untenable with the emf alone (which is still uniformly distributed) because the 2 volt emf in each side will generate 0.25 amps in side 1, and 2000 amps in sides 2, 3, and 4. The physical process that establishes current continuity is an initial discontinuous current flow that establishes rapid charge separations and electrostatic potential differences throughout the circuit. Equilibrium is obtained when the currents generated by the electrostatic fields, \mathbf{E}_n , superimpose on the currents generated by the non-electrostatic \mathbf{E}' , to yield continuity in current flow.

Note that a non-uniformity in the circuit can also be introduced by a non-uniformity in the distribution of the seat of emf within the loop (e.g., local flux creation at one side of the square, or a battery in one side of the square). A similar process for generating electrostatic potentials occurs in transmission lines: non-uniform flux creation causes charge separation and an associated potential difference. This process is described by one of the basic pair of transmission line equations [1],

$$\partial I / \partial x = C_x \partial V / \partial t \quad (3)$$

where C_x is the capacitance per unit length. Non-uniform flux generation along the transmission line, and the resulting non-uniform seat of emf, induces the discontinuous current flow $\partial I / \partial x$. The resulting charge separation establishes an electrostatic potential ($\partial V / \partial t$). For transmission lines, the source of non-uniform flux creation can be a transient pulse or a continuous wave, for example.

Returning to the second exercise, we use the fact that the potential drop across a segment is given by the sum of the emf and the IR_n product in the segment to solve this circuit problem. For the entire closed loop, the sum of the potential differences must be zero. So for the entire loop, the sum of the emfs (8V) plus the sum of the IR_n drops (8 ohms) equals zero. This gives $I = 1$ amp.

The charge separation across each segment produces a potential difference across each segment. Applying the general rule for the potential drop across a segment, we have V_1

=8V-2V. So the potential difference across the 8 ohm resistor from top to bottom is 6V. For sides 2, 3, and 4, the resistance is negligible, and we have $V_n \sim 0+2V$, so $V_2=V_3=V_4= \sim 2V$. The sum of the potential differences taken around the closed circuit is thus zero, as required.

Later we will address field and potential distributions in problems of this kind, so we address these in the present example. The rail-to-rail potential distribution from left to right is caused entirely by E' in segments 2 and 4 (segment resistances are negligible). Using the result that $E'=2V/m$ in each segment, the general expression for the potential at arbitrary x is given by

$$V(x) = V_B - (4V/m)x \quad (4)$$

In the present example, $V_B = 6V$.

The electrostatic field E corresponding to these potential distributions will vary from $-6V/m$ to $-2V/m$ (directed from top to bottom). Now consider the non-electrostatic field E' . Its vertical component varies from $-2V/m$ to $+2V/m$ from left to right across the square. The total electric field, $E+E'$, is the sum of the electrostatic and non-electrostatic fields, so its vertical component varies from $-8V/m$ to $0V/m$. The present exercises illustrate a general rule with non-symmetric circuits of this kind: the electric fields will generally be comprised of sums of electrostatic and non-electrostatic fields.

These exercises also show the differences between lumped parameter circuits and distributed parameter circuits. In lumped parameter circuits, electrical components are neatly packaged into separate units; potential differences across resistors are always given by the IR product, for example. This is not true for distributed parameter circuits as we have demonstrated. Some may find it disconcerting that the familiar $V=IR$ relation does not apply; currents can even flow through resistors with no potential differences across them.

The other main difference between lumped and distributed parameter problems is the need to constantly maintain the distinctions between emf and potential in distributed parameter circuit problems. Such distinctions are automatically set up in lumped circuit problems.

We assumed uniform currents in the cross section of the conductors (e.g., thin conductors) to simplify the exercises. This is not generally true; another inherent feature of distributed parameter problems is the need to consider the time dependence of current and field penetration into circuit components.

Feynmann et al [1] analysis of inductor/capacitor

Figure 3 shows the basic experimental arrangement analyzed in Chapter 22 of Feynmann et al [1]. An inductor coil is immersed in a time varying magnetic field. The inductor is connected to a set of capacitor plates which are located outside the field. Equilibrium exists and there is no current flow.

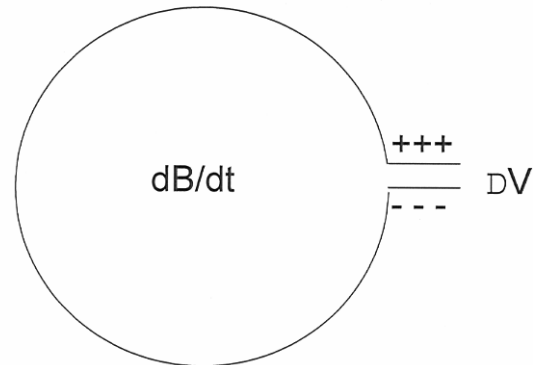


Figure 3. The basic experimental arrangement analyzed in Chapter 22 of Feynmann et al [1].

In their analysis, Feynmann et al compute the emf by performing a line integral along a path that passes through the inside of the conductor forming the inductor and traverses the air gap between the capacitor plates. Using the Faraday rule, dF/dt is equated to the line integral of E' . To obtain this line integral, the section of line integral of E' that passes through the core of the conductor is set to zero because, they claim, electric fields are zero inside conductors. Thus, the only section of the integral of E' that contributes to the line integral is said to be the small non-zero portion within the air gap. The integral of that non-zero portion is said to be the potential difference, but is not actually estimated. Instead the line integral of this portion is simply set equal to dF/dt . This is their proof that the emf generated in the inductor is equal to the potential difference across the capacitor plates.

The following is a list of the errors in their approach:

1. A major error is that Feynmann et al ignored their own advice to focus on the underlying physics.

2. It is not valid to choose a path through the conducting inductor loop where the electric field is really zero; and the complete line integral cannot be computed from the small line segment between the capacitor plates. If the authors are assuming that field and current diffusion into the metal is too slow for \mathbf{E}' to reach the selected integration path, then the Faraday rule is inapplicable. One cannot perform a line integral of \mathbf{E}' along a path where the field is shielded from the path. Its derivation from the Maxwell equation (equating the curl of \mathbf{E}' to $d\mathbf{B}/dt$) assumes the line integral can sense the unshielded field all along its perimeter.

Otherwise, if \mathbf{E}' is really zero inside the conductor, the conductor is not a seat of emf and there is no emf. Considering Figure 3, it is clear that if there were a real attempt to estimate the magnitude of the line integral of \mathbf{E}' within the gap, it would prove to be an insignificant fraction of the actual line integral, and the estimated magnitude could never equal dF/dt . (The gap length in real capacitors is so small that it would yield an infinitesimal contribution to this line integral.) The correct magnitude can only be obtained by performing the line integral along the entire seat of emf, which is along the entire inductor loop.

Perhaps the authors have confused the fact that the sum of the electrostatic and non-electrostatic fields are zero ($\mathbf{E} = -\mathbf{E}'$) in the loop because there is no current.

For a valid analysis, the emf must be computed from \mathbf{E}' , not from $\mathbf{E} + \mathbf{E}'$.

3. The line integral of \mathbf{E}' between the capacitor plates can never be the potential. That violates the definition of potential difference as discussed in the introduction. We reiterate: i) one cannot compute a potential from the line integral of a field that is path dependent and ii) the potential difference is the negative of the line integral of an electrostatic field \mathbf{E} associated with the charges accumulated on the capacitor plates. In the present example, that charge is a

result of the action of the distributed seat of emf generated by \mathbf{E}' all along the conducting loop.

4. Feynmann et al add to the confusion by locating the capacitor outside the \mathbf{B} field where the \mathbf{E}' field is conservative ($d\mathbf{B}/dt = 0$). They appear to suggest that one can locally define potentials from \mathbf{E}' fields at locations where $d\mathbf{B}/dt = 0$, because the curl of $\mathbf{E}' = 0$. That is false because the field must be conservative everywhere for a proper potential function. That is the reason electrostatic fields are specified in the definition of potential [2, 3].

The correct (but more tedious) proof is obtained using the procedures employed in the square loop exercises. The current is zero, so, perform the line integral of $\mathbf{E}' + \mathbf{E} = 0$ from one capacitor plate to another to obtain $\text{emf} = -DV$; emf is the line integral of \mathbf{E}' and $-DV$ is the negative of the line integral of \mathbf{E} . Distinctions between potential and emf and their related fields need to be maintained. In contrast to the erroneous assumption of Feynmann et al, we use the fact that the line integral of \mathbf{E}' along the short distance between the capacitor plates is negligible so that the line integral of \mathbf{E}' along the loop is essentially over a closed path through the conductors; we equate this net emf to $-d\mathbf{B}/dt$. Note that, in contrast to the uniformly distributed \mathbf{E}' , the localized charge concentration on the capacitor plates produces an intense, localized \mathbf{E} field in the capacitor. Thus, the integral of \mathbf{E} over the short distance between the plates is not negligible. The final step invokes the property of an electrostatic (conservative) field, i.e., the sum of the potential drops around a closed path is zero. This gives $+DV$ for the value of the integral across the capacitor plates, so $DV = \text{emf}$, as required.

As in the non-uniform square loop exercise, it is the non-uniformity in the inductor loop caused by the capacitor that produced the charge separation and its associated \mathbf{E} field. Consequently, there are potential differences between any two different points in this inductor loop. There would be no potential differences anywhere in the loop without the introduction of an element that affects the loop homogeneity (e.g., capacitor, resistor, or battery). For this configuration, the electric fields within the loop area are again given by $\mathbf{E} + \mathbf{E}'$. The \mathbf{E}' fields are tangential to the loop while the \mathbf{E} field (from an extension of the charge distribution at the

capacitor) is directed from the top half of the loop to the bottom half of the loop.

Woodson and Melcher [4] analysis of a moving armature along a rail pair

Woodson and Melcher [4] analyze the familiar textbook armature/rail problem shown in Figure 1. They present two solutions. In the first, they apply the Faraday rule and treat the motion of the armature as a uniform flux creation problem. Using the same approach as Feynmann et al, they obtain the potential across the left hand terminals by computing the emf from a line integral of \mathbf{E}' through the rails and armature where the electric field is said to be zero. The line integral is completed across the air gap at the left end in a same manner as in the inductor/capacitor case. Again, the line integral of \mathbf{E}' across the gap is set equal to the potential difference to obtain Bvh .

This analysis is invalid for all the same reasons enumerated above for the Feynmann et al. analysis of the inductor/capacitor. No distinction is made between conservative and non-conservative fields. What make this analysis especially meaningless is that there is no actual flux creation. The Faraday rule says that the sum of emfs in a loop containing motional emf can be computed as *if* there were flux creation equal to the rate at which the moving conductor encompasses the \mathbf{B} field. It needs to be stressed that the flux creation here is only imagined; it cannot be taken literally. The fallacy can be demonstrated further by considering the potential and field distributions from such imagined uniform flux creation. We have already mentioned in the example of Figure 1 that, for such cases, the potential distribution along a loop for a flux creation emf is entirely different that that for a pure motional emf. Thus, the two cases are not equivalent.

In their second approach, which is claimed to be equivalent to the first, Woodson and Melcher correctly treat this problem in terms of a motional emf that is induced in the moving armature. Again the answer is Bvh but that cannot be taken to mean the two processes are equivalent physically. Their second analysis is essentially the same as our treatment of the physical process in Figure 1. Woodson and Melcher's invalid analyses and their treatment of the two physical processes as equivalent can be

damaging to the unsuspecting scientific community. We provide our experiences in the following.

Analysis of potential distributions in railguns

The railgun problem shown in Figure 4 was analyzed in a manuscript submitted to the 13th Electromagnetic Launcher Symposium in Potsdam. The analysis uses the concepts outlined above. Appendix 1 is a compilation of the critical comments we received in e-mails from a reviewer regarding our analyses. Numerous private communications have shown that this reviewer's views are widely held. It can be seen that the reviewer's comments all reflect, directly or indirectly, the misconceptions contained in the two textbooks discussed earlier. In fact, reference [4] is cited by this reviewer as the authority on these matters. So, the following can be viewed as a case study of the damage that is caused by textbook errors.

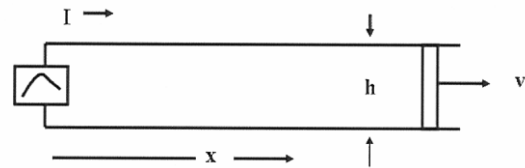


Figure 4. The basic railgun configuration. A power supply at the left supplies a current pulse that flows in the direction indicated.

First, we present our analyses of the distribution of potentials in a railgun. The interest in rail-to rail potentials in railguns stems from the fact that rail-to-rail arc formation is a problem in some systems, so a proper analysis of fields and potentials is needed. We then provide results derived from the methods found in the textbooks by Feynmann et al [1] and Woodson and Melcher [4].

Referring to Figure 4, a power supply provides a current pulse that flows in the direction indicated. There are two main sources of induced emfs in railguns, namely, current changes, dl/dt , and armature motion. First, consider emfs from current changes. The seat of emf from dl/dt is uniformly distributed along the railgun conductors, as in the square loop exercises, because \mathbf{E}' is uniformly distributed. For the positive dl/dt case, the potential change along length x , in the top rail is $-E'x$ and in the

bottom rails it is $+E'x$; the resistance term is again assumed to be negligible. Setting the sum of the potential differences to zero gives $V_B - E'x - V(x) - E'x = 0$. So the rail-to-rail potential for the initial positive dI/dt portion of the current pulse is

$$V(x) = V_B - 2E'x. \quad (5)$$

Note the similarity to the results of potential distribution the non-uniform square loop exercise.

In the case where the fields are generated by loop currents, Equation (5) can be rewritten using the total loop inductance, L , and the inductance per unit length, L_X . For large railgun length to height ratio, x_A/h , we obtain the net emf from the Faraday rule, $dF/dt = 2E'x_A = Ldl/dt = L_X x_A dl/dt$. Using E' obtained from the Faraday rule, we have from Equation 5,

$$V(x) = V_B - L_X x dl/dt. \quad (6)$$

The rail-to-rail potential is reduced below the power supply potential everywhere along the rails.

For negative dI/dt portion of the current pulse, E' is directed along the current flow direction and the same expression holds. The rail-to-rail potential now is higher than the power supply potential everywhere along the rails.

At the peak value of the pulse, $dI/dt=0$ and $V(x) = V_B$ all along the rails.

Taking the derivative of Equation (6) yields

$$dV/dx = - L_X dl/dt. \quad (7)$$

Generalizing this expression to account for the fact that the V and I can be a functions of x and t , as discussed in the discussion of the square loop problem, we have

$$\partial V/\partial x = L_X \partial I/\partial t. \quad (8)$$

This is the second of the basic pair of transmission line equations [1]. These results indicate that the concepts underlying the transmission line equations have broad application to any distributed parameter circuit problem. The flux creation that generates E' within the circuit does not need to be restricted to loop currents for Equation (5) to apply. Thus,

the potential distribution in the non-uniform square loop problem has the same origin as this second transmission line equation, for example.

Equation (8) is also the fundamental reason why motional emf cannot be physically equivalent to the emf generated by uniform flux creation as claimed in the textbook by Woodson and Melcher [4]. There must always be a potential gradient when the emf is generated by flux creation; the exception, of course, is the uniform conducting loop where there are no potentials.

Now we present the erroneous "textbook" approach advocated by our reviewer and others. In an attempt to circumvent the limited scope of the Faraday rule and derive field and potential distributions, the line integral is performed along a closed loop that passes through the body of the armature and continues to an arbitrary position x inside the rails. The electric field is set to zero within these conductors (see Appendix). The integration path then exits the rails at x and continues in the space from one rail to the other where E' is said to be non zero. The distinction between conservative and non-conservative fields is ignored and the rail-to-rail potential at x is computed from the $E'h$ product. That potential is then equated to the flux enclosed by the loop, i.e., $L_X (x_A - x) dl/dt$. The same procedure is used for the negative dI/dt case. This potential from flux creation is added to the potential from the power supply to obtain (see Appendix),

$$V(x) = V_B + L_X (x_A - x) dl/dt. \quad (9)$$

This result is clearly different from our result (Equation 6). One demonstration that this equation is invalid is the fact that at the armature ($x = x_A$), there is no effect from the inductance of the rail/armature loop. Thus, the presence of an inductance beyond the power supply terminals (terminal potential= V_B) has no effect on the current through the armature and through the system. This violates Lenz's law which says that in circuits where currents generate a magnetic field, an emf will be established to cause counter currents. Our Equation 6, by contrast, is compatible with Lenz's law.

The other clue that something is amiss with this approach is the claim that E' is perpendicular to the top and bottom rails all along the length of the railgun and that its value varies from zero at x_A to a maximum at $x=0$. This is false, as can be

seen by simply considering the symmetry of the problem. The actual \mathbf{E}' field from $d\mathbf{B}/dt$ is more complicated. It resembles the \mathbf{E}' field in Figure 2; it is tangential to the conductors everywhere. The total field between the rails is $\mathbf{E} + \mathbf{E}'$. As in the square loop exercises, the electrostatic field, \mathbf{E} , is directed between the top and bottom rails while the vertical component of \mathbf{E}' changes sign at $x_A/2$.

For a simple, direct demonstration of the lack of validity of this commonly used, but inappropriate, Faraday rule approach, apply it to the square loop exercises above. Since the results depend only on dF/dt , identical, erroneous distributions of fields and potentials will be obtained for both the uniform and non-uniform cases.

The final topic in this case study relates to the proper treatment of the emf associated with armature motion in railguns. Figure 5 shows one of the rails near the armature. The element of rail dx represents the new length of current-carrying rail generated by the motion of the armature along x in time dt . The new field $d\mathbf{B}$, generated by the appearance of the new current-carrying rail segment dx is given by the Biot and Savart law of field creation. For a rod-shaped segment, $d\mathbf{B}$ is highly localized, varying as the inverse square of the distance from the segment. The same local flux creation occurs behind the armature at the other rail.

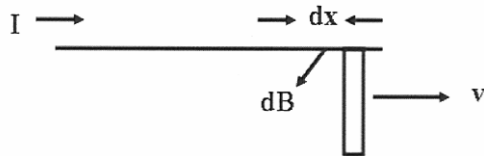


Figure 5. One of the rails of a railgun near the armature. The element of rail dx represents the new length of current carrying rail generated by the motion of the armature along x in time dt .

As we have discussed throughout, a potential gradient is established wherever flux creation occurs in these systems. In contrast to the case where flux creation is uniformly distributed along the rails, so that the potential varies uniformly along x , localized flux creation and its associated localized seat of emf will cause a localized rail-to-rail potential reduction (in the vicinity of the rail segments, dx). This localized reduction is a compressed version of the

potential reduction along x for uniform flux creation.

The Faraday rule can be used to obtain the net emf in this local flux creation case. (Assume there is no dI/dt component.) It is clear that we are dealing with flux creation given that new flux continuously appears in the space behind the moving armature. Since the net flux creation rate in the loop must equal the average B field in the loop times the area swept out by the moving armature, we have, $emf=Bvh$. The emf is also given by Bvh in the motional emf case and in the Woodson and Melcher uniform flux creation scenario. The physical processes are very different in the three cases illustrating that the crucial underlying physics is completely masked if one relies only on the Faraday rule.

Our description of the emf associated with armature motion is unacceptable to our reviewer and to others in the railgun community. They accept Woodson and Melcher's erroneous portrayal of motional emf as equivalent to uniform flux creation between the rails. Some view the effect of armature motion as producing uniform flux creation within the rail/armature loop. The reviewer (see Appendix), on the other hand, rejects the model of local flux creation as the source of emf from armature motion. Instead, he treats it as purely motional emf so that his predicted rail-to-rail potential distribution is constant from breech to armature (as we described in the discussion of motional emf). We have attempted to show that the basic physics of armature motion is incompatible with the common beliefs that the associated emf arises from either uniform flux creation or motional emf.

Summary

The Faraday induction rule is highly susceptible to misinterpretation and misapplication. A related difficulty is the confusion that exists with the concepts of emf and potential differences. We show that this confusion is reflected in serious errors in several well known textbooks. We also describe experiences with a reviewer who used one of these textbooks in his critique of the some of the analyses we present here. The consequences of misapplications of these basic concepts are illustrated in analyses of fields and potentials in distributed parameter circuits. We use rail/armature configurations

encountered in elementary physics textbooks and in railguns as examples.

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Appendix

The following are quotes from a reviewer of our manuscript on the topic of railgun potential and field distributions for the 13th EML Symposium. These quotes were obtained from an extended and widely distributed e-mail correspondence.

1. "...I had said that the $L \, dI/dt$ portion of the voltage drop (and E field) was zero at the armature and maximum at the breech terminals. I should have said the 'magnitude of the $L \, dI/dt$ portion of the voltage drop'; "
2. "I still don't believe in your "flux creation" scenario for the development of motional EMF..."
3. "...the speed voltage or $L \, dI/dt$ portion (*motional emf*) of the voltage (or E field) ... is constant from breech to armature." (Italics added by the authors).
4. "Why worry about the "seat of EMF"? The time changing flux through the area proscribed by the railgun conductors produces a voltage across the terminals of the launcher."
5. "The key to using Faraday's integral law is picking the contour where you know E (*is zero*) for most of the distance so that you can find it across the terminals. " (Italics added by the authors).
6. "The transmission line equations really don't apply in this case, since the electromagnetic transit time from breech to armature is short compared to other times of interest;"
7. "So if the energy comes out of the power supply, why does the flux get created at the armature? (It doesn't, nor does it have to). The EMF doesn't need a "seat" (except if you are an electrochemist..."
8. "Yes I do buy into Faraday's law, since it describes what's going on. If evaluated correctly it allows you to evaluate the electric fields and the voltages they produce in magnetoquasistatic systems. It's not a cop-out. Labeling things as "motional emf" and "flux creation is a cop-out. "
9. "Remember that it is the changing magnetic flux which is producing the electric field E (and thus the voltage you can measure at the breech terminals) ..."