

**ESTRAUS: A COMPUTER PACKAGE FOR SOLVING SUPPLY-DEMAND
EQUILIBRIUM PROBLEMS ON MULTIMODAL URBAN TRANSPORTATION
NETWORKS WITH MULTIPLE USER CLASSES**

Joaquín de Cea Ch., J. Enrique Fernández L., Valérie Dekock Ch.

Departamento de Ingeniería de Transporte, Pontificia Universidad Católica de Chile

Casilla 306, Santiago 22, CHILE

FAX: (56-2) 686 4820; e-mail jdc@ing.puc.cl

and

Alexandra Soto O.

Fernández & De Cea Ingenieros Limitada

Lota 2257, Providencia, Santiago, CHILE

FAX: (56-2) 234 1575; e-mail asoto@FDCconsult.com

ABSTRACT

In this paper we present a modeling approach for solving supply-demand equilibrium problems (with fixed trip generations and attractions) that combine different sets of demand choices and trip assignments, normally used for analyzing urban transportation networks with multiple user classes as well as pure and combined transportation modes, interacting over a physically unique network. The demand choices are supposed to have a hierarchical structure. When the trip distribution is variable, a doubly constrained entropy-maximizing model is considered at the first level of choice and a hierarchical logit model is used for the remaining demand choices (time of departure, mode choice, transfer point for combined modes, etc.). If the trip distribution is considered to be exogenous, the demand choices are modeled with a hierarchical logit. One of the model's main features is that it considers the effects of congestion on the road network as well as in each public transportation service network. Different problems are mathematically formulated as variational inequalities, with asymmetric cost functions, and all of them are solved following the diagonalization algorithm. Each iteration of the aforementioned procedure solves an optimization problem using Evan's algorithm. One of the mathematical formulations presented in this paper (distribution-modal split-assignment) has been implemented in the latest version of the computer model ESTRAUS, which is part of a battery of planning tools developed by the government of Chile to simulate the operation of alternative network configurations and evaluate strategic development plans for urban transportation systems. In the last part of the paper, some applications of ESTRAUS developed in Chile during the last years are briefly discussed.

1. INTRODUCTION

An important number of big size metropolitan areas have experienced increasing levels of vehicular congestion, which in many cases has contributed to the serious pollution problems that affect these cities. Because of this, government authorities must often face demands from the population and different interest groups in order to improve the operating conditions of the urban transportation systems and be pressured to build very expensive infrastructure projects (new metro lines, urban highways, etc.). Nevertheless, these options are very expensive for a developing country with very limited capital resources that have many alternative uses in different social areas such as education, health, housing and others.

In the Chilean case, because of the need to make rational decisions with respect to the future development of their main urban transportation systems (beginning with the transportation system of Santiago), the national government decided to implement a battery of planning tools, including computer models, to simulate the operation of alternative network configurations and evaluate development plans. Therefore, transportation planning models were implemented in Santiago in the context of a project called “Strategic Urban Transport Study (for Santiago)” known as ESTRAUS (see SECTU 1989). This name is presently given to the supply-demand equilibrium model, which is the most important part of the Santiago model. The description of ESTRAUS, including its mathematical formulation and solution algorithm, is the main objective of this paper.

The most important features of the model’s original version were the following:

- i) The model considers a simultaneous equilibrium formulation for trip distribution, modal split and assignment, in order to ensure consistency of the levels of service in the system for the different submodels. In this way, the levels of service used to estimate demands (i.e. total trips and trips by mode among zones) must be the same as the levels of service obtained when the O/D matrices by mode are loaded over their corresponding subnetworks (road network and transit networks).
- ii) The trip generation stage is exogenous; that is, trip productions and attractions are given as inputs of the model.
- iii) The model considers multiple pure and combined (combination of pure) modes, existing in the city of Santiago.
- iv) The congestion interactions between all the vehicles using the road network are explicitly considered (car, taxis and vehicles offering public transport services), given that all of them compete for the same common road capacity. The exceptions are the case of the exclusive bus lanes (separate links are coded in this case), the metro lines that operate over an independent network, and the interactions of the car part of the combined mode car-passenger/metro trips with other vehicles using the road network.

- v) Considering the importance of public transport in Santiago (trips on transit modes accounted at the mid-eighties for about 75% of the motorized trips in the city) the network model includes a detailed representation of the public transport system. Thus, transit lines and segments are explicitly considered and common lines are explicitly modeled. This first version of the model does not consider capacity constraints for the transit vehicles, given that at this time no reasonable transit equilibrium assignment models was available.
- vi) Demand is modeled using aggregate gravity functions (simply and doubly constrained) for trip distribution and disaggregate logit expressions (simple and hierarchical) for modal split. The specific formulation to be used would depend on the trip purpose modeled. This first version of the model considers that destination and mode choices are simultaneous (at the same hierarchical level).

The first application of ESTR AUS, developed for the city of Santiago, considered three trip purposes (work, study and others), thirteen user classes (combinations of household incomes and car ownership), seven pure transport modes (walking, car-driver, car-passenger, taxi, shared taxi, bus, and metro) and four combined modes (car-driver/metro, car-passenger/metro, bus/metro and shared taxi/metro). The following different modal networks and flow interactions were defined:

Car network. It constitutes the basic network (the road network) and it is used to assign the private car and taxi trip matrices. In order to consider the congestion interactions with buses and shared taxis it also includes the definitions of all the services of said modes. As a result, the travel cost functions for a given link depend on the total equivalent flow on that link (measured in pcu/h) and they are different for cars and for buses.

Bus network. It is used to assign the bus trip matrix. As mentioned above, the generalized costs of traveling by bus are, in this previous version of the model, independent of passenger flows.

Metro network. It is used to assign the metro trip matrix. In this case, given that the metro network is independent, the generalized travel costs are constant.

Shared taxi network. It is used to assign the shared taxi trip matrix. As in the case of the bus network, the generalized costs of traveling by shared taxi are independent of passenger flow.

Bus/metro network. It is used to assign the bus/metro and metro/bus trip matrices. It considers the combination of the bus and metro networks defined above, including the transfer links required to join them.

Shared taxi/metro network. It is used to assign the shared taxi/metro and metro/shared taxi trip matrices. It combines the shared taxi and metro networks defined above, introducing the necessary transfer links.

Car/metro network. It is used to assign the car/metro trip matrix. It is built combining the road network and the metro network.

The analytical formulation of the supply-demand transport equilibrium model was expressed in terms of a variational inequality, with non-separable cost and demand functions. The problem was heuristically solved using a diagonalization procedure, where the road network cost functions were diagonalized in the usual way, and a “quasi-diagonalization” method was used in order to make the demand functions invertible (for details see Fernández y De Cea, 1990).

During the last ten years, relevant theoretical improvements have been introduced to the model. The most important ones that have originated the new formulations described in this paper are:

- i) Capacity constraints are considered for vehicles of all public transport modes.
- ii) The interactions between the car part of a trip by mode car-driver/metro with the other vehicles using the road network are taken into account.
- iii) The demand side of the equilibrium model has a hierarchical structure. In this new version the destination and mode choices can be modeled simultaneously or sequentially (distribution first and mode choice second), depending on the values obtained for the calibration parameters of the demand models.
- iv) The hierarchical structure of the demand choices allows the introduction of other choices like departure time and transfer points for combined modes.

In addition, the algorithm's implementation has been permanently improved, in order to obtain good solutions in acceptable running times, using normal personal computers.

This paper is organized as follows: section 2, after this introduction, presents a very brief review of the literature, centered on the main contributions to the supply-demand equilibrium in transportation systems and to the analysis of departure time decisions. In section 3, the main modeling assumptions of the ESTRAUS model are discussed, the supply and demand submodels are described, and the equilibrium conditions are stated. Section 4 is devoted to the mathematical formulation of several problems and section 5 describes very briefly the solution algorithm implemented in ESTRAUS. Finally in section 6, some relevant applications of the model, in the context of the strategic planning process for the city of Santiago de Chile, are reported.

2. SUPPLY-DEMAND EQUILIBRIUM MODELS: A BRIEF REVIEW

Transportation equilibrium supply-demand models have been present in the literature for a long time, and an important amount of academic research has originated many different problem formulations and alternative solution algorithms during the last 25 years. Nevertheless, very few large-scale applications have been reported. Thus, many implementation problems are not well

understood and the importance of some modeling limitations present in the majority of these existing formulations have not been evaluated properly with experimental results.

As an alternative way of modeling supply-demand equilibrium in transportation systems using the traditional sequential four stage model (whose lack of consistency among the levels of service and flow values obtained at each stage is a very important shortcoming when congestion exists), Beckman et al. (1956) proposed the first equilibrium formulation with elastic demand for the simple case of separable demand and cost functions, and a single user class, in the form of an optimization problem.

After this work, particular types of distribution laws were studied in combination with user optimum trip assignment (combined trip distribution and assignment models). We have to mention here the papers by Brunooghe (1969), Florian et al. 1975 and Evans (1976). In particular Evans proposed an optimization problem of the Beckman type, combining a doubly constrained trip distribution model based on entropy maximization with user optimal trip assignment.

Because of the separable functions included in the objective functions of the above mentioned optimization models, they are not able to treat more general multimodal problems when there are modal (user classes) asymmetric interactions. Florian and Nguyen (1978) formulated a particular extension of the combined distribution and assignment model: a combined distribution, mode choice and assignment model, considering two (car and transit) independent modes, which results quite unrealistic for modeling systems where car and transit vehicles share the road infrastructure. This formulation is a convex optimization problem, which as the preceding ones can be solved using the Frank-Wolfe algorithm (see Frank and Wolfe, 1956) or the partial linear approximation algorithm proposed by Evans (1976).

Florian (1977) developed a two mode (private car and transit) network equilibrium model where the most important features are the distinction between the flow of vehicles and flow of transit passengers and the means of modeling the interaction between both types of vehicles that use the same road links of the network. In this case, non-separable demand functions are used to more realistically capture demand-side interdependence. The equilibrium is found by solving a sequence of problems like the one proposed by Beckman et al. for one mode (car), while parametrically varying the equilibrium travel costs of the other mode (transit) whose assignment is determined by an all-or-nothing technique.

Aashtiani (1979) was the first author who proposed an optimization model for the supply-demand equilibrium problem with non-separable cost and demand functions to capture the interdependence (interactions) between different modes (user classes). When these interactions are supposed to be symmetrical, an optimization problem whose objective function can be expressed in terms of line integrals exists (this is an extension of the work developed by Dafermos, 1971 and 1972, for the traffic equilibrium problem with multiple user classes, with asymmetric interactions).

The lack of realism of the optimization problems formulated in order to model multimodal and multiclass equilibrium on transportation systems with symmetric interactions between users of

different modes or classes, gave place to a number of alternative and more realistic formulations. In fact, the work reported by Florian (1977) has been considered a particular form of a variational inequality formulation for supply-demand network equilibrium models with asymmetric link cost functions. This variational inequality formulation, which is an extension of the variational inequality developed by Smith (1979) for user optimization with fixed demand, has been independently developed by Florian (1979) and Dafermos (1980, 1982a) and later analyzed by Fisk and Nguyen (1982), Florian and Spiess (1982), and other authors.

As an alternative to the variational inequality formulation, Aashtiani (1979) and Aashtiani and Magnanti (1981) showed that the general asymmetric problem could be expressed as a non-linear complementary problem.

Concerning the solution procedures for these asymmetric supply-demand equilibrium problems, many algorithms have been suggested in the literature. The most popular and probably the most commonly used, because of its easy implementation, is the diagonalization method. For a description of this algorithm and its convergence conditions and properties see for instance Florian (1977), Abdulaal and LeBlanc (1979), Dafermos (1980 and 1982b), Pang and Chang (1982), Fisk and Nguyen (1982) and Florian and Spiess (1982).

Asmuth (1978), Aashtiani (1979), and Aashtiani and Magnanti (1981) present applications of fixed point, non linear complementarity and linear complementarity algorithms, while Dafermos (1980, 1982a) proposed projection methods to solve these kind of problems. Nguyen and Dupuis (1984) used the cutting plane algorithm to solve the traffic equilibrium problem with asymmetric cost functions, which is obviously applicable to solve supply-demand equilibrium problems with asymmetric cost functions.

A very good description of different methods for solving variational inequalities can be found in Harker and Pang (1987).

For the interested reader, Fernández and Friesz (1983) present a detailed review of the state of the art on this subject until the early eighties, including the main references about the existence and uniqueness of a variety of formulations related to the supply-demand transport equilibrium problem, not mentioned in this revision.

All the supply-demand equilibrium models combining trip distribution, mode choice and assignment, already mentioned, consider destination and mode choices to be simultaneous (at the same hierarchical level). Such is the case of the shipper model presented by Friesz et al. (1986) for predicting freight flows. A clear limitation of this kind of particular formulation is that the same value for the calibration parameters of destination and mode choice functions must be used.

The first mathematical formulation of a supply-demand network equilibrium model with sequential (hierarchical) rather than simultaneous destination and mode choices was proposed by Fernández et al. (1994). In this paper the authors present several approaches for formulating network equilibrium models with combined modes. One of these formulations considers a nested demand structure to model mode choice (car or car/metro) and the transfer point choice, which

allows the calibration of different parameters for the transfer point and mode choice functions. This approach is used by Fernández et al. (2000) to propose new equilibrium formulations for the intercity freight transportation problem. The general structure of the demand model considers hierarchical decisions for destination, mode carrier and transfer points when the mode is not pure.

Later on, Abrahamsson and Lundqvist (1999) develop nested combined models for trip destination, mode and route choices and implemented these models in the context of the Stockholm region. They consider a simple problem where the transit network and the road network are independent and no congestion effects exist over the transit network, whose travel impedance can be exogenously determined. They proposed three different models: the traditional nested (distribution, modal split and assignment) the reverse nested (modal split, distribution and assignment) and the simultaneous (trip distribution and modal split and assignment).

Finally, Florian et al. (1999) motivated by the original version of ESTRAUS (see Fernández and De Cea, 1990) developed the model STGO (Santiago), a combined distribution, modal split and assignment equilibrium model, with multiple mode and user classes, and hierarchical demand structure. The authors report that STGO reproduces the results of the original version of ESTRAUS (which does not consider capacity constraints in public transportation services). While the public transport assignment is based on the concept of strategy or hyperpath (Spiess and Florian 1989, and Nguyen and Pallottino, 1988) for the STGO model, the different versions of ESTRAUS have been based on the "transit route" concept (see Le Clerq, 1972 and Chriqui and Robillard, 1975). This difference is quite important when capacity constraints for the transit vehicles are introduced in the model. While ESTRAUS is formulated in terms of a variational inequality in the space of the arc flows (see De Cea y Fernández, 1993), any model based on the concept of hyperpath must be formulated as a variational problem in the spaces of arc and hyperpath flows. In the latter case, the solution algorithm of the model requires hyperpath enumeration, which is an important drawback of this modeling approach (see Wu et al., 1994).

Although several works concerning departure time choices are reported in the technical literature, this travel decision has not been integrated yet to supply-demand equilibrium models. Only quite recently, Dekock (2001) and Dekock et al. (2002) have described a simultaneous equilibrium model considering trip distribution, modal split and departure time choices. The basic idea of this model is that even if trip generations (and attractions) are fixed for a given period of time, users can choose the sub-period in which they travel, according to a logit model. The combined model considers a doubly constrained entropy-maximizing model, while modal split and departure time choices are modeled with a hierarchical logit model. Based on these works, two different models, depending on the relative values of the calibration parameters of the logit model, are described in detail in section 5.4 of this article.

In the following paragraphs a brief description of the main works regarding the departure time issue is given.

Hendrickson and Plank (1984) discuss why people should be interested in changing the time they make their trips and concludes that less congestion, low fares and parking availability are the main reasons that people have to change their departure time from the peak time to an earlier period.

Another interesting conclusion of this work is that departure time choice is more flexible than mode choice. So, given a high level of congestion, travelers would consider changing the time of their trips before thinking about a change in their mode. Concerning this subject, some works discuss the characteristics of users who can change their departure time. Johnston et al, (1989) argues that people with longer trips present more willingness to change the time when they make their trips than people traveling shorter distances. On the other hand, transit users present less flexibility than car users to change the departure time of their trips. Therefore, the level of service over the networks, the flexibility of the family activities, the household income level, type of work and age of travelers and the mode of transport used (car availability) are the main variables that explain user behavior concerning trip departure time decisions (see Chin, 1990 and Abkowitz, 1981, and the authors already mentioned).

Obviously, these variables may explain user choices in addition to their personal preferences. As a result, aggregated and disaggregated demand models, based on the random utility theory are normally used in this case. Abkowitz (1981), McCafferty and Hall (1982), Chin (1990) and Bianchi (1995) report several logit models, multinomial and hierarchical, calibrated with data of different cities. It is very interesting to note that the results obtained show, in general, that trip departure time decisions are more flexible than mode choices, which suggests that the most adequate order of travel decisions should be distribution, mode and then departure time. Nevertheless, this order for each particular case should be decided by the estimated values of the calibration parameters (McCafferty and Hall, 1982, obtained a model where the mode choice precedes the departure time decision).

More recently, Bhat (1998) presents a MNL-OGEV model (Multinomial Logit-Ordered Generalized Extreme Value) with a nested structure. The upper level of the decision tree corresponds to modal split (MNL) and the lower level to trip departure time (OGEV). The OGEV model belongs to the family of the models of Generalized Extreme Value (GEV), which are a relaxation of the MNL. Its main characteristic is that it recognizes the natural temporal order of the departure time alternative, allowing correlation between pairs of ordered alternatives depending on their temporal proximity.

3. BASIC HYPOTHESIS AND DEFINITIONS OF THE ESTRAUS MODEL

The main modeling assumptions of ESTRAUS are presented in this section of the paper. Although different combined problems will be analyzed, each of them requiring some specific definitions and notation, we will start defining the networks, the link cost functions, the equilibrium conditions and the associated notation, for the simultaneous distribution-modal split-assignment problem (D-MS-A). Later on, when other demand decisions are considered as part of the supply-demand equilibrium models, some new concepts and complementary notation will be redefined. Given that the D-MS-A problem is referred to a particular time period, this dimension (time) does not appear explicitly. Then, networks, cost functions, flows, origin-destination trips, etc., are all related to that period.

3.1 Networks and Cost Functions

The road network (the basic network) notation is represented by $G=(N,A)$, where N is the set of nodes and A the set of links. We define $c_a^{kp\tilde{m}}$ as link's a average operating cost (operating time or generalized cost), for users of class k , with trip purpose p , of private transportation mode \tilde{m} (e.g., car, taxi, etc.). This is a function of the summation of vehicle flows over all private transportation modes, user classes and trip purposes ($f_a^{kp\tilde{m}}$), as well as the fixed flow of public transportation vehicles (\bar{F}_a) on link a , all measured in equivalent vehicles (e.g., p.c.u.):

$$c_a^{kp\tilde{m}} = c_a^{kp\tilde{m}} \left(\sum_k \sum_p \sum_{\tilde{m}} f_a^{kp\tilde{m}}, \bar{F}_a \right) \quad (1)$$

It is important to note that, although the Jacobian of the cost functions vector is not diagonal, it does turn out to be symmetrical, given the functional form supposed for cost functions $c_a^{kp\tilde{m}}$ (every vehicle, from whichever user class, trip purpose and private transportation mode, produces the same impact on congestion). Nevertheless, the reader will note that this ‘‘symmetry’’ of the cost functions for private modes, which is a simplification, could be relaxed without changing the problem formulation and its solution algorithm. If more general cost functions are used, considering, for instance, that different classes of users of private modes produce different impacts on road congestion, the Jacobian of these cost functions will be asymmetric just like the one associated to the public transport cost functions. Then the combined problem is asymmetric, independent of the particular characteristics of the cost functions for private modes.

For every pure public transportation mode \bar{m} , we define the pure service networks as $G_{\bar{m}} = (N_{\bar{m}}, S_{\bar{m}})$, where $N_{\bar{m}}$ is the set of nodes ($N_{\bar{m}} = N$ for ground services that use the road network, such as buses, and $N_{\bar{m}} = N'$ where $N \cap N' = \mathcal{F}$ for independent public transportation services, (e.g., metro)) and $S_{\bar{m}}$ is the set of *transit links* that belong to mode \bar{m} . Next, the *public transportation link* concept is explained, which is basic for understanding how capacity constraint in vehicles is considered. For details on this subject see De Cea and Fernández (1993).

Consider two nodes A and B , such that there exists a set of L_1 transit services which allow a user to travel from A to B , without transferring from one line to another ($L_1 = \{l_1, l_2, l_3, \dots, l_n\}$). That is, between A and B operates a set of *line sections* (portion of a transit line between two not necessarily consecutive nodes of its itinerary), as shown in the following figure:

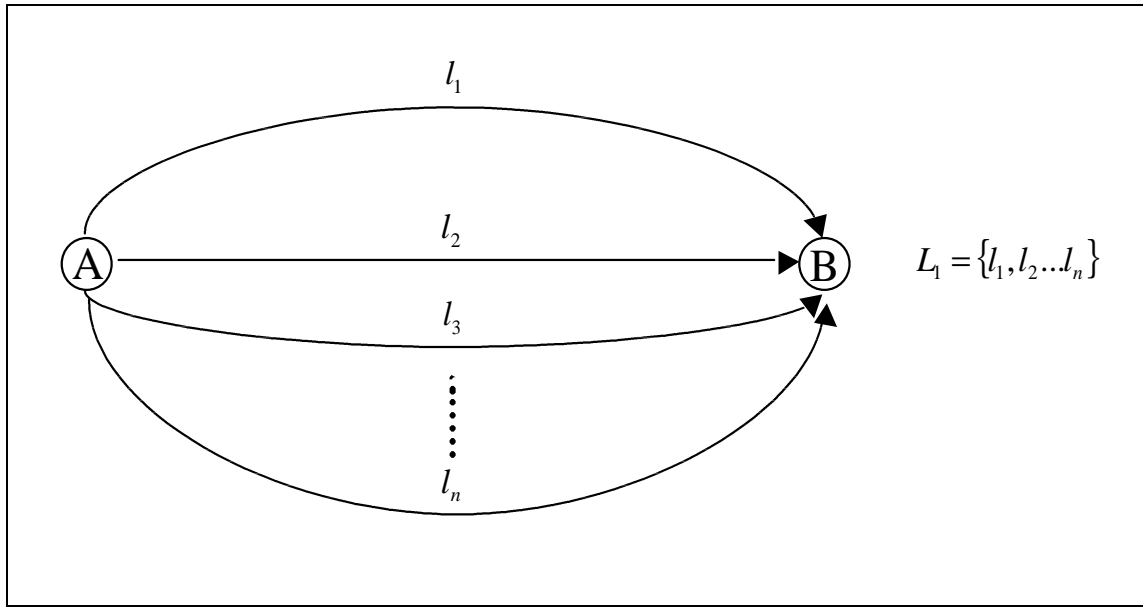


Figure 1: Line Sections between Two Nodes

In order to represent the fact that travelers select a sub-set of the L_1 called sub-set of *attractive* lines (see Le Clerq, 1972 and Chriqui and Robillard, 1975) to move from A to B , and at the same time consider the capacity constraints in these vehicles, the aforementioned situation is modeled in the following way (see De Cea & Fernández, 1993).

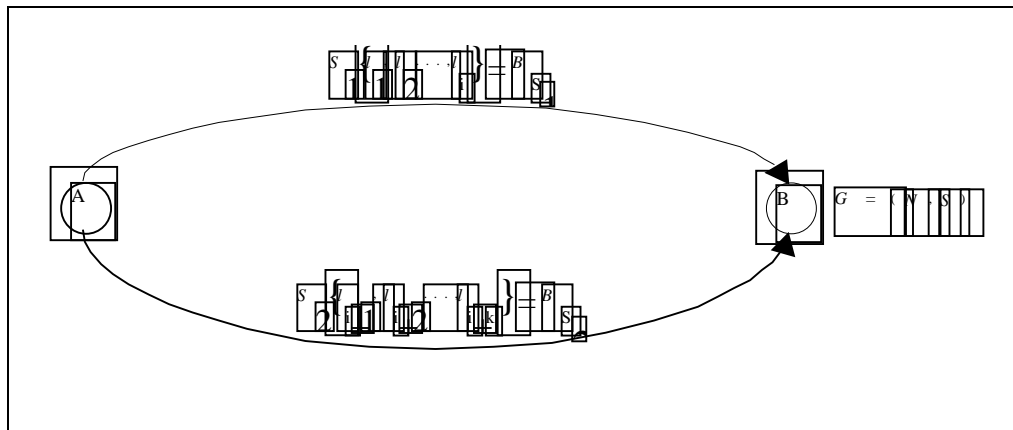


Figure 2: Public Transportation Links of Network $G(N,S)$

The first *public transportation link*, S_1 , represents the set of *fastest lines* B_{S_1} , which corresponds to the set of transit lines that minimize the generalized travel time (cost), without considering the vehicles capacity constraints. The second *public transportation link*, S_2 , represents the set of slower lines, B_{S_2} .

The generalized time (cost) functions of these public transportation links (sum of travel time, waiting time, transfer time, fare, etc.), depend on the vehicle flow over the road network as well as the passenger flow in the existing services, as is shown next:

$$c_s^{kp\bar{m}} = j^{kp\bar{m}} \left(\sum_k \sum_p \sum_{\bar{m}} f_a^{kp\bar{m}}, \bar{F}_a, \forall a \in l, l \in B_{s\bar{m}} \right) + (P_{TAR})^{kp\bar{m}} \cdot (TAR)_s^{\bar{m}} + (P_{WAIT})^{kp\bar{m}} \left[\frac{a^{\bar{m}}}{d_s^{\bar{m}}} + b^{\bar{m}} \left(\frac{V_s^{kp\bar{m}} + \tilde{V}_s^{kp\bar{m}}}{(CAP)_s^{\bar{m}}} \right)^{n^{\bar{m}}} \right] \quad (2)$$

where, besides the aforementioned notation:

$c_s^{kp\bar{m}}$: average or generalized cost on link s for users of class k , with trip purpose p , of public transportation mode \bar{m} (e.g. bus, subway, etc.).

$(P_{TAR})^{kp\bar{m}}$: fare multiplier for users of class k , with trip purpose p , of public transportation mode \bar{m} .

$(TAR)_s^{\bar{m}}$: fare related to public transportation link s of mode \bar{m} .

$(P_{WAIT})^{kp\bar{m}}$: waiting time multiplier for users of class k , with trip purpose p , of public transportation mode \bar{m} .

$a^{\bar{m}}, b^{\bar{m}}, n^{\bar{m}}$: calibration parameters of waiting time function, for public transportation mode \bar{m} .

$d_s^{\bar{m}}$: vehicle frequency of public transportation mode \bar{m} over the public transportation link s .

$(CAP)_s^{\bar{m}}$: capacity of public transportation link s of public transportation mode \bar{m} .

$V_s^{kp\bar{m}}$: passenger flow of class k , with trip purpose p , belonging to public transportation mode \bar{m} , and which use public transportation link s .

$\tilde{V}_s^{kp\bar{m}}$: passenger flow that competes with $V_s^{kp\bar{m}}$ for the capacity of transit lines belonging to B_s (flow with the same trip purpose, user class, and mode that belong to other public transportation links that compete or reduce the B_s line's capacity, plus flow from other purposes, classes, and modes that also compete for the B_s lines capacity).

It is easily seen, in this case, as we already mentioned, that the Jacobian of the cost function vector is non-diagonal and asymmetric.

In addition, the model considers the existence of combined modes, for example car/metro (private transportation/public transportation) or bus/metro (public transportation/public transportation). In each case, the union of the pure mode networks that compose them forms the combined mode network. In order to ease notation and understanding of the following formulation, combined modes (m^c) are considered to be formed by two public transportation modes. Nevertheless, it is important to stress that this does not limit the model's general use, since there is no problem in representing combined modes such as car-public transportation (as was mentioned in section one, the application of ESTRAUS for the city of Santiago considers combined modes like car driver-metro and car passenger-metro).

3.2 Equilibrium Conditions

Network equilibrium conditions.

The model's basic assumption, with respect to network flow equilibrium, is that for every mode, over its corresponding network, each user chooses his/her route according to Wardrop's first principle (i.e., every individual tries to minimize his/her average operating cost or generalized average trip cost). This gives place to the following equilibrium conditions:

$$C_r^{kpm^*} - u_w^{kpm^*} \begin{cases} = 0 & \text{si } h_r^{kpm^*} > 0 \\ \geq 0 & \text{si } h_r^{kpm^*} = 0 \end{cases} ; \quad \forall r \in P_w^m, w \in W, k, p \quad (3)$$

The above means that at equilibrium, routes with flow will have an equal (minimum) cost, while those without flow, will have an equal or greater cost than the minimum $u_w^{kpm^*}$. P_w^m represents the set of routes between origin-destination pair w for mode m , which can be a pure mode (private or transit mode) or any combined mode. When departure time decisions are considered, equilibrium conditions (3) must be true for every time period modeled. If departure time decisions are made before mode choice, index m should be replaced by hm (time period h and mode m). Alternatively, if departure time decisions are made after mode choice, m must be changed by mh . In these cases the notation used in equations (3) must be modified consequently.

According to the definitions of the link cost functions for both private and public transport networks and the conditions (3), the assignment part of the model described in this paper is consistent with a deterministic user equilibrium (DUE). This represents the state of the art in the field of the supply-demand network equilibrium models.

Although stochastic user equilibrium assignment models (SUE) have been reported to solve traffic assignment problems, no works have been published that present operational programs that model supply-demand network equilibrium using this approach. Implementation problems (the need for enumerating routes) and the fact that DUE tends to SUE when the level of congestion is important, seems to be the main reasons for this (see Sheffi, 1985).

Demand equilibrium conditions.

Before we detail the demand equilibrium conditions for the D-MS-A problem, the following figure illustrates one branch of the decision tree for a given origin-destination pair w , of a class k user, and with trip purpose p . From the tree's highest level *hang* as many branches, equivalent to the ones represented in the figure, as origin-destination pairs w , user classes k and trip purposes p , exist. The destination choices for each user class and trip purpose are represented in the uppermost level (not shown in the figure), while in the lowest level (actually in the two lowest levels), mode choices are represented in a bi-level logit structure.

The corresponding utility (disutility) function is shown in each tree level. In the lowest level, each mode m has its associated disutility u_w^{kpm} (a linear function representing the generalized travel cost between pair w , for users of class k , with trip purpose p , using transportation mode m). For instance, in the case of car driver mode this cost considers travel time, operating costs, fares (when road pricing schemes are considered), etc. For the public transport modes, the specification of the disutility functions includes terms like access time, waiting time, in-vehicle travel time, fares and transfer penalties.

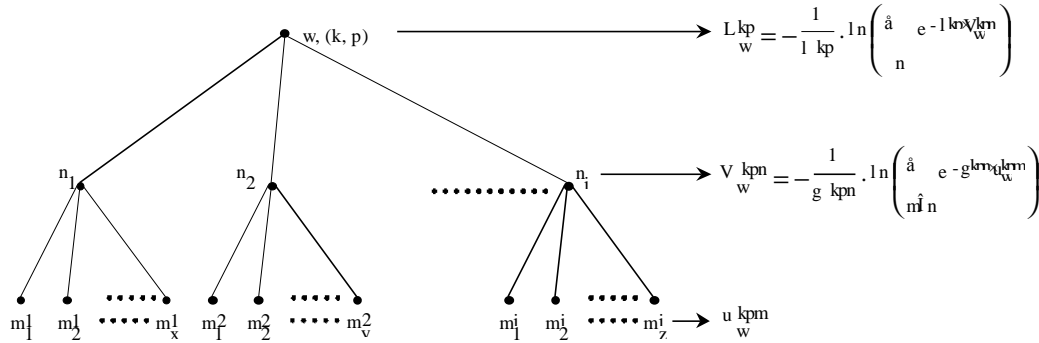


Figure 3: Hierarchical Structure of the Destination and Mode Choice Model

The set of modes are grouped in different nests (i.e. modal split with a bi-level hierarchical structure), with a V_w^{kpn} disutility (e.g., one nest for public transportation modes, and another for private transportation modes). The upper level is related to mode choice decisions, with composite costs L_w^{kp} . In equilibrium, the following must be true:

- Trip Distribution

$$\boxed{T_w^{kp*} = A_i^{kp} O_i^{kp} B_j^p D_j^p e^{-b^{kp} \cdot L_w^{kp*}}} \quad (4)$$

- Proportion of trips using mode m in nest n

$$P_w^{kpm*} = \frac{T_w^{kpm*}}{T_w^{kpn*}} = \frac{e^{-g^{kpn} u_w^{kpm*}}}{\sum_{m \in n} e^{-g^{kpn} u_w^{kpm*}}} \quad (5)$$

- Proportion of trips in nest n with respect to total trips

$$P_w^{kpn*} = \frac{T_w^{kpn*}}{T_w^{kp*}} = \frac{e^{-I^{kp} V_w^{kpn*}}}{\sum_{n'} e^{-I^{kp} V_w^{kpn*}}} \quad (6)$$

When the model includes departure time choices (for instance at the lower level of the hierarchical tree), in addition to conditions (4), (5) y (6) a logit expression like equation (5) should be added to the equilibrium conditions. This expression will represent the proportion of trips carried out within a given time period h , for users that already decided to use mode m . Figure 4 shows the decision's hierarchical tree for this particular case.

As in the previous case, the corresponding utility (disutility) function is shown at each tree level. In the lowest level, each time period h has its associated disutility u_w^{kpmh} (a linear function representing the generalized travel cost between pair w , for users of class k , with trip purpose p , using transportation mode m , during the time period h). In the same way, the disutilities of the upper level nests, are the corresponding combined disutilities of the alternatives included in the nest (V_w^{kpm} , Z_w^{kpn} and L_w^{kp} in figure 4).

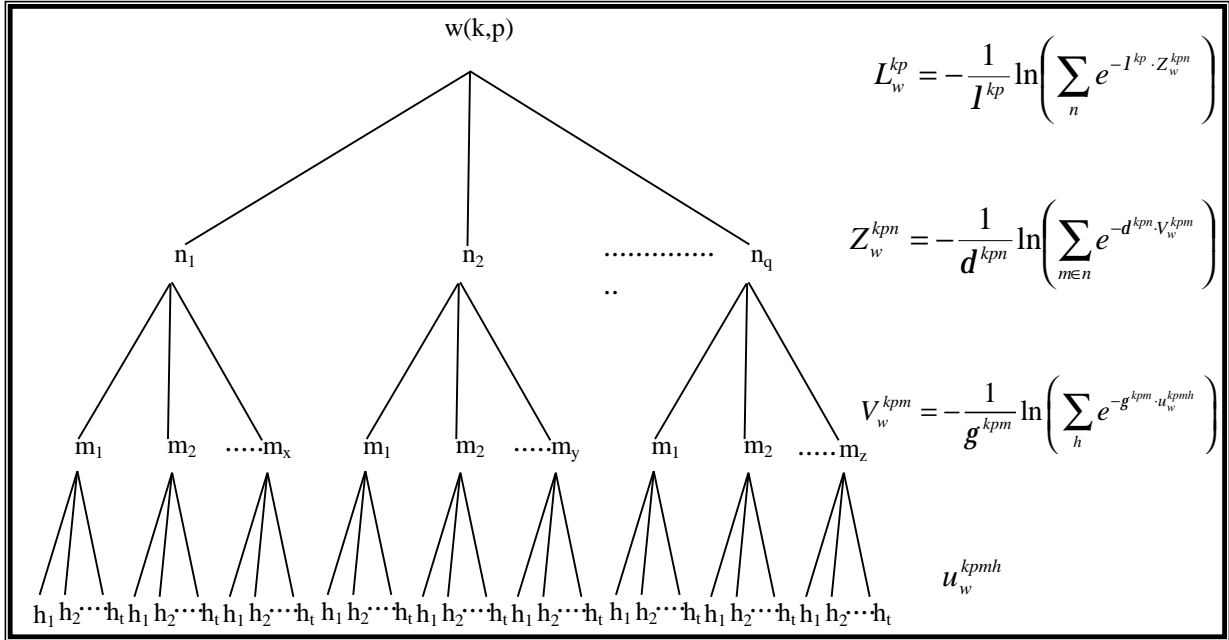


Figure 4: Hierarchical Structure of the Destination, Mode and Departure Time Choice Model

4. MATHEMATICAL FORMULATION

As was mentioned at the end of the introduction of this article, a general formulation for a variety of supply-demand equilibrium problems is presented in this section. The equilibrium conditions, like expressions (3) to (6) for the D-MS-A problem, can also be represented as a variational inequality of the following type:

$$c(X^*)^T (X - X^*) - g(T^*)^T (T - T^*) \geq 0, \quad \forall \text{ feasible } X, T \quad (\text{P-1})$$

where:

- X : vector of flow on links of the multimodal network
- X^* : vector of equilibrium flow on links of the multimodal network.
- T : vector of trips between origin-destination pairs of the multimodal network
- T^* : vector of equilibrium trips between origin-destination pairs of the multimodal network
- $c(X)$: column-vector of network link's cost functions (with non-diagonal and asymmetric Jacobian).

$g(T)$: vector of inverse demand functions or demand transformations (with a diagonal Jacobian).

Obviously, other combined problems (including departure time choices, for instance) can be formulated in the same way. Particularities of each of these problems, as we will see in the next section, are reflected in the flow vectors, trip vectors, cost functions and demand transformations. Another difference between different problems will be expressed in the set of constraints defining the feasible region for the decision variables.

The existence of an equilibrium solution for (P-1) requires that the cost functions be positive and continuous and that the O/D demand functions are non negative, continuous and upper-bounded (see Aashtiani, 1979). The uniqueness of the solution is based on the usual monotonicity conditions required for $c(X)$ and the demand functions $T(C)$.

5. SOLUTION ALGORITHM

5.1 Diagonalization of the Cost Functions

As explained in section 3.1, the link cost functions of private modes are not diagonal but symmetric, while the transit links cost functions are asymmetrical (i.e., Jacobian of the cost function vector is asymmetric). As a result of this, no equivalent optimization formulation of the Beckman type exists for the variational problem P-1.

As it has been discussed in section 2 of this paper, many different algorithms have been proposed in order to solve variational inequality problems. ESTRAUS uses the diagonalization approach, which as it was already mentioned is one of the most widely used methods for solving these types of problems. Dafermos (1982b), who referred to the diagonalization procedure as “relaxation algorithm” developed a global convergence criterion and established that a linear convergence rate occurs when the demand and cost functions are strongly monotonic and the cost functions yield a Jacobian matrix which is only mildly asymmetric.

At each iteration of the diagonalization algorithm, the cost functions $c_a^{kp\bar{m}}$ and $c_s^{kp\bar{m}}$ result in diagonalized cost functions $\hat{c}_a^{kp\bar{m}}$ and $\hat{c}_s^{kp\bar{m}}$, which depend only on their own flows, and the following variational inequality must be solved:

$$\hat{c}(X^*)^T (X - X^*) - g(T^*)^T (T - T^*) \geq 0, \quad \forall \text{ feasible } X, T \quad (\text{P-2})$$

Problem P-2 has an equivalent optimization problem. The functional form of its objective function and the constraints defining the feasible region of the decision variables will depend on the particular combined problem treated. In the following subsections several combined diagonalized problems are presented. Other problems can be formulated using the same modeling approach. The only restriction, at the moment, is that if distribution is a part of the combined model, destination choice must be supposed at the upper level of the hierarchical decisions tree. The other

choices (mode and departure time) can change their order in the decision tree.

5.2 Distribution-Modal Split-Assignment (D-MS-A)

The following optimization problem solves the variational problem (P-2), for the D-MS-A problem, at each iteration of the diagonalization algorithm used to solve problem (P-1). In ESTRAUS, this problem is solved using the EVANS algorithm.

$$\begin{aligned}
\min Z = & \sum_k \sum_p \sum_{\bar{m}} \sum_a \int_0^{f_a^{kp\bar{m}}} \hat{c}_a^{kp\bar{m}}(x) dx + \sum_k \sum_p \sum_{\bar{m}} \sum_s \int_0^{V_s^{kp\bar{m}}} \hat{c}_s^{kp\bar{m}}(x) dx \\
& + \sum_k \sum_p \sum_{m^c} \sum_s \int_0^{V_s^{kpm^c}} \hat{c}_s^{kpm^c}(x) dx \\
& + \sum_k \sum_p \frac{1}{b^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) - \sum_k \sum_p \frac{1}{I^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) \\
& + \sum_k \sum_p \sum_n \frac{1}{I^{kp}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) - \sum_k \sum_p \sum_n \frac{1}{g^{kpn}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) \\
& + \sum_k \sum_p \sum_n \frac{1}{g^{kpn}} \sum_{m \in n} \sum_w T_w^{kpm} (\ln T_w^{kpm} - 1)
\end{aligned} \tag{7}$$

subject to:

$$T_w^{kp} = \sum_n T_w^{kpn}, \quad \forall w, k, p \quad \left(u_w^{kp} \right) \tag{8}$$

$$T_w^{kpn} = \sum_{m \in n} T_w^{kpm}, \quad \forall w, k, p, n \quad \left(u_w^{kpn} \right) \tag{9}$$

$$T_w^{kpm} = \sum_{r \in P_w^m} h_r^{kpm}, \quad \forall w, k, p, m \quad \left(u_w^{kpm} \right) \tag{10}$$

$$O_i^{kp} = \sum_j T_w^{kjp}, \quad \forall i, k, p \quad \left(m_i^{kp} \right) \tag{11}$$

$$D_j^p = \sum_k \sum_i T_w^{kjp}, \quad \forall j, p \quad \left(h_j^p \right) \tag{12}$$

In addition to constraints (8)-(12), the constraints concerning non-negativity of flows, relationship between route flows and link flows, and distribution of public transportation link flow among transit line sections, must also be considered.

It is easy to see that, if trip distribution and modal split choices are considered to be made at the

same hierarchical level ($b^{kp} = I^{kp}$), the fourth and fifth terms of the objective function (7) disappear. On the other hand, if the modal split model were considered to be a multinomial logit, instead of hierarchical, then the sixth and seventh term in (7) would also disappear and constraints (8) and (9) collapse to $T_w^{kp} = \sum_m T_w^{kpm}$, $\forall w, k, p$, with dual variables (u_w^{kp}) .

As a result, the decision structure's general version illustrated in Figure 3, collapses to a simpler structure that depends on the calibrated values for parameters b^{kp} , I^{kp} y g^{kpn} . The latest operating version of ESTRAUS considers the case where $b^{kp} \leq I^{kp}$.

The diagonalized problem's optimality conditions are obtained constructing the Lagrangean L , with objective function (7) and constraints (8)-(12) multiplied by their corresponding dual variables. Then, deriving L with respect to h_r^{kpm} , results in Wardrop's equilibrium conditions for each pure or combined mode. In addition, if L is derived with respect to the trip demand variables T_w^{kp} , T_w^{kpn} and T_w^{kpm} , and equaled to zero, the problem's remaining equilibrium conditions are obtained (see Appendix).

5.3 Modal Split-Assignment (MS-A)

If trip distribution is considered to be exogenously determined, a combined equilibrium model can be formulated to predict trip matrices by user class, trip purpose and transport mode. This problem is similar to the D-MS-A, and is formulated like the variational inequality (P-1) with consistent flow vectors, trip vectors and constraints defining the feasible region of the decision variables. In this case, the optimization problem solved at each iteration of the diagonalization algorithm is the following:

$$\begin{aligned}
\min Z = & \sum_k \sum_p \sum_{\bar{m}} \sum_a \int_0^{f_a^{kp\bar{m}}} \hat{c}_a^{kp\bar{m}}(x) dx + \sum_k \sum_p \sum_{\bar{m}} \sum_s \int_0^{V_s^{kp\bar{m}}} \hat{c}_s^{kp\bar{m}}(x) dx \\
& + \sum_k \sum_p \sum_{m^c} \sum_s \int_0^{V_s^{kpm^c}} \hat{c}_s^{kpm^c}(x) dx \\
& + \sum_k \sum_p \sum_n \frac{1}{I^{kp}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) - \sum_k \sum_p \sum_n \frac{1}{g^{kpn}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) \\
& + \sum_k \sum_p \sum_n \frac{1}{g^{kpn}} \sum_{m \in n} \sum_w T_w^{kpm} (\ln T_w^{kpm} - 1)
\end{aligned} \tag{13}$$

s.a.:

$$T_w^{kp} = \sum_n T_w^{kpn}, \quad \forall w, k, p \quad (u_w^{kp}) \tag{14}$$

$$T_w^{kpn} = \sum_{m \in n} T_w^{kpm}, \quad \forall w, k, p, n \quad \left(u_w^{kpn} \right) \quad (15)$$

$$T_w^{kpm} = \sum_{r \in P_w^m} h_r^{kpm}, \quad \forall w, k, p, m \quad \left(u_w^{kpm} \right) \quad (16)$$

Additionally, the constraints concerning non-negativity of flows, relationship between route flows and link flows, and distribution of public transportation link flow among transit line sections, must be considered. Again, if the modal split model is multinomial instead of hierarchical, the fourth and fifth terms of the objective function (13) disappeared and constraints (14) and (15) collapses to $T_w^{kp} = \sum_m T_w^{kpm}, \forall w, k, p$, with dual variables (u_w^{kp}) . It should be noted that in this case the values T_w^{kp} are given (trips distributions are exogenous).

The optimality conditions of problem (13)-(16) are derived in the same way shown in the appendix for the D-MS-A.

5.4 Distribution-Modal Split-Departure Time-Assignment (D-MS-DT-A)

As we mentioned before, we consider a combined problem where trip generations and attractions are fixed for a given period of time (i.e. morning peak period). Within this period, based on the levels of service existing over the private and public networks during alternative sub-periods, users choose the time (sub-period) in which they travel, the mode used and the origin-destination pair of their trips. Distribution is represented by a doubly-constrained entropy maximizing model, mode and departure time choices are modeled with a hierarchical logit structure and assignment over each modal network in each alternative sub-period is consistent with Wardrop's first principle. Within a particular sub-period, travelers of different classes and trip purposes interact. So, congestion due to the physical capacity of the road network and the physical capacity of the public transport vehicles exists. Nevertheless, given the static nature of the model, traffic interactions between travelers belonging to different time subperiods in not considered.

The D-MS-DT-A is similar to the D-MS-A problem. In fact, it is formulated like the variational inequality (P-1), with consistent flow and trip vectors, cost functions and demand transformations. Additionally, The flow conservation constraints must consider in this case that the total trips, for any user class, trip purpose and mode, must be equal to the sum of these trips in each time sub-period, and that total O/D trips for any user class, trip purpose, mode and time sub-period must equal the sum of the trips on each used route, for this kind of trip.

As in the previous cases, to solve the D-MS-DT-A, the diagonalization algorithm could be used. The optimization problem solved at each iteration of the algorithm is the following (the notation used in this case is similar to that already defined, adding the index h to denote time period):

$$\begin{aligned}
\min Z = & \sum_k \sum_p \sum_m \sum_h \sum_a \int_0^{f_a^{kpmh}} \hat{c}_a^{kpmh}(x) dx + \sum_k \sum_p \sum_m \sum_h \sum_s \int_0^{V_s^{kpmh}} \hat{c}_s^{kpmh}(x) dx \\
& + \sum_k \sum_p \frac{1}{b^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) - \sum_k \sum_p \frac{1}{I^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) \\
& + \sum_k \sum_p \sum_n \frac{1}{I^{kp}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) - \sum_k \sum_p \sum_n \frac{1}{d^{kpn}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) \\
& + \sum_k \sum_p \sum_n \sum_m \frac{1}{d^{kpn}} \sum_w T_w^{kpm} (\ln T_w^{kpm} - 1) - \sum_k \sum_p \sum_n \sum_m \frac{1}{g^{kpm}} \sum_w T_w^{kpm} (\ln T_w^{kpm} - 1) \\
& + \sum_k \sum_p \sum_n \sum_m \frac{1}{g^{kpm}} \sum_h \sum_w T_w^{kpmh} (\ln T_w^{kpmh} - 1)
\end{aligned} \tag{17}$$

s.a.:

$$T_w^{kp} = \sum_n T_w^{kpn} \quad \forall w, k, p \quad (u_w^{kp}) \tag{18}$$

$$T_w^{kpn} = \sum_{m \in n} T_w^{kpm} \quad \forall w, k, p, n \quad (u_w^{kpn}) \tag{19}$$

$$T_w^{kpm} = \sum_h T_w^{kpmh} \quad \forall w, k, p, m \in n \quad (u_w^{kpm}) \tag{20}$$

$$T_w^{kpmh} = \sum_{r \in P_w^{pmh}} h_r^{kpmh} \quad \forall w, k, p, m \in n, h \quad (u_w^{kpmh}) \tag{21}$$

$$O_i^{kp} = \sum_j T_w^{kp} \quad \forall i, k, p \quad (m_i^{kp}) \tag{22}$$

$$D_j^p = \sum_k \sum_i T_w^{kp} \quad \forall j, p \quad (h_j^p) \tag{23}$$

In addition to constraints (18)-(23), the non-negativity of flows, the relationships between route flows and link flows, and the distribution of transit link flows among transit line sections, for each period of time h , must be considered.

The optimality conditions of this diagonalized problem are obtained like is shown in the Appendix for the D-MS-A. Concerning the network equilibrium conditions, for every mode m , in each sub-period h the following must be true (note that we have maintained the original notation, introducing the index h to identify a given sub-period of time):

$$C_r^{kpmh^*} - u_w^{kpmh^*} \begin{cases} = 0 & \text{si } h_r^{kpmh^*} > 0 \\ \geq 0 & \text{si } h_r^{kpmh^*} = 0 \end{cases} ; \quad \forall r \in P_w^{mh}, w \in W, k, p \quad (24)$$

Finally, the demand equilibrium conditions are:

- Trip Distribution

$$T_w^{kp^*} = A_i^{kp} O_i^{kp} B_j^p D_j^p e^{-b^{kp} \cdot T_w^{kp^*}} \quad (25)$$

- Proportion of trips in nest n with respect to total trips

$$P_w^{kpn^*} = \frac{T_w^{kpn^*}}{T_w^{kp^*}} = \frac{e^{-l^{kp} \cdot Z_w^{kpn^*}}}{\sum_{n'} e^{-l^{kp} \cdot Z_w^{kpn'^*}}} \quad (26)$$

- Proportion of trips using mode m in nest n

$$P_w^{kpmn^*} = \frac{T_w^{kpmn^*}}{T_w^{kpn^*}} = \frac{e^{-d^{kpn} \cdot V_w^{kpmn^*}}}{\sum_{m' \in n} e^{-d^{kpn} \cdot V_w^{kpm'^*}}} \quad (26)$$

- Proportion of trips in time period h using mode m

$$P_w^{kpmh^*} = \frac{T_w^{kpmh^*}}{T_w^{kpmn^*}} = \frac{e^{-g^{kpm} \cdot u_w^{kpmh^*}}}{\sum_{h' \in h} e^{-g^{kpm} \cdot u_w^{kpmh'^*}}} \quad (28)$$

These results prove that if the diagonalization algorithm converges, an equilibrium solution is obtained which satisfies the conditions established for the variational problem (P-1) for the D-MS-DT-A.

5.5 Distribution-Departure Time-Modal Split-Assignment (D-DT-MS-A)

It is clear that the order of the travel decisions (particularly departure time and modal split) depend in practice of the values of the calibration parameters g^{kph} and d^{kphn} . In fact, the model presented

above, where modal split precedes departure time, supposes that for every nest n , $g^{kph} < d^{kphn}$. On the other hand, if we suppose that departure time choice precedes modal split decision (D-DT-MS-A model), the diagonalized optimization problem becomes:

$$\begin{aligned}
\min Z = & \sum_k \sum_p \sum_h \sum_m \sum_a \int_0^{f_a^{kpmh}} \hat{c}_a^{kphnm}(x) dx + \sum_k \sum_p \sum_h \sum_m \sum_s \int_0^{v_s^{kpmh}} \hat{c}_s^{kphnm}(x) dx \\
& + \sum_k \sum_p \frac{1}{b^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) - \sum_k \sum_p \frac{1}{I^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) \\
& + \sum_k \sum_p \sum_h \frac{1}{I^{kph}} \sum_w T_w^{kph} (\ln T_w^{kph} - 1) - \sum_k \sum_p \sum_h \frac{1}{g^{kph}} \sum_w T_w^{kph} (\ln T_w^{kph} - 1) \\
& + \sum_k \sum_p \sum_h \sum_n \frac{1}{g^{kph}} \sum_w T_w^{kphn} (\ln T_w^{kphn} - 1) - \sum_k \sum_p \sum_h \sum_n \frac{1}{d^{kphn}} \sum_w T_w^{kphn} (\ln T_w^{kphn} - 1) \\
& + \sum_k \sum_p \sum_h \sum_n \frac{1}{d^{kphn}} \sum_m \sum_w T_w^{kphnm} (\ln T_w^{kphnm} - 1)
\end{aligned} \tag{29}$$

s.a.:

$$T_w^{kp} = \sum_h T_w^{kph} \quad \forall w, k, p \quad (v_w^{kp}) \tag{30}$$

$$T_w^{kph} = \sum_n T_w^{kphnm} \quad \forall w, k, p, h \quad (v_w^{kph}) \tag{31}$$

$$T_w^{kphn} = \sum_{m \in n} T_w^{kphnm} \quad \forall w, k, p, h, n \quad (v_w^{kphn}) \tag{32}$$

$$T_w^{kphnm} = \sum_{r \in P_w^{hnm}} h_r^{kphnm} \quad \forall w, k, p, h, n, m \quad (v_w^{kphnm}) \tag{33}$$

$$O_i^{kp} = \sum_j T_w^{kp} \quad \forall i, k, p \quad (m_i^{kp}) \tag{34}$$

$$D_j^p = \sum_k \sum_i T_w^{kp} \quad \forall j, p \quad (h_j^p) \tag{35}$$

As in the previous case, in addition to constraints (29)-(35), the non-negativity of flows, the relationships between route flows and link flows, and the distribution of transit link flows among transit line sections, for each period of time h , must be considered. The optimality conditions of this problem are obtained in the same way as before.

It must be noted that for these two problems considering departure time choice, it has been

supposed that for every mode the order of the mode and departure time is the same (MS-DT or DT-MS). This is not a limitation of the modeling approach. The reader should note that similar formulations will be obtained in “mixed” cases where for some modes (nests) the departure time decision precedes mode choice and for others the order is the opposite.

6. SOME APPLICATIONS OF ESTRAUS

During the last years ESTRAUS has been used in Chile as a very important tool for analyzing and evaluating strategic projects and development plans for the city of Santiago, whose population is near 5,500,000 inhabitants.

After its first application in the late eighties, the model was used in 1990-1991 to evaluate the metro line 5, which began its operation in 1997. It is important to mention that the demand of this line (the load profile) was over estimated by the model in about 15%-20% in the morning peak period, which could be considered quite acceptable for these type of predictions. Part of this over estimation can be explained by two facts. Firstly, when the line was modeled, traveling by metro was less expensive than traveling by bus (metro’s main competitive transit mode). However, when the metro line began its operation this situation had changed and metro fares were higher than bus fares. Secondly, the project was simulated with a set of feeder bus services in its southern terminal. The fare of this bus/metro services was around 70% of the sum of bus and metro fares. Nevertheless, these services were never implemented.

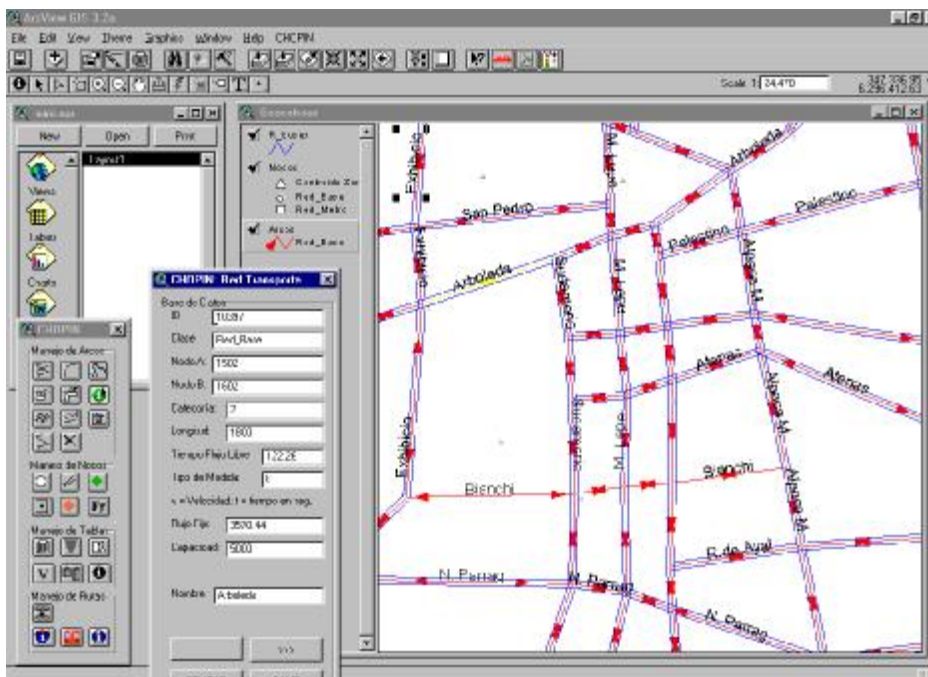
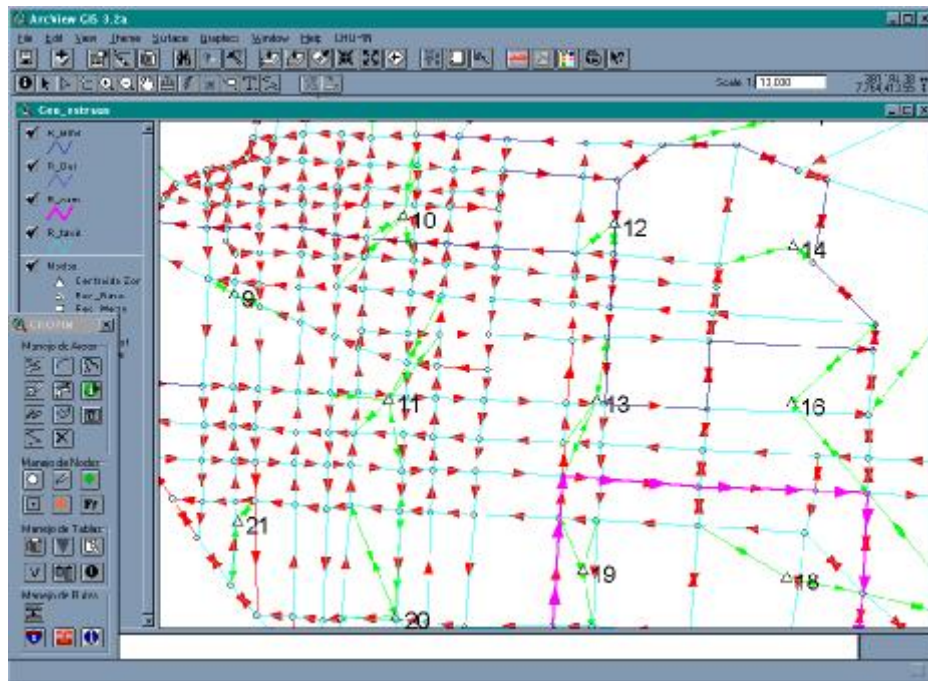
In addition, the model has been used to:

- Perform a social and private evaluation of a series of urban highways private concessions.
- Perform a social evaluation of the strategic plan for the city of Santiago for the period 1995-2010.
- Analyze a development plan of the public transport system for the city of Santiago, considering new metro lines, a set of exclusive transit corridors (tramways), suburban railway services, etc.
- Test the impact of different road pricing approaches and parking fares.
- Test the impact of alternative transit service structures and different fare systems.

At the present time, the model is being used to study development plans for the metropolitan areas of Valparaíso and Concepción. In both cases the population is near 1,000,000 inhabitants.

Applying ESTRAUS to the city of Santiago, involves 13 user classes, 3 trip purposes and 11 transport modes (that means that 439 trip matrices are estimated by the model). The city is divided into nearly 450 zones and the road network is represented by around 2,500 nodes and 8,000 links. The bus system has 740 unidirectional lines and there are about 400 lines of shared taxis. Considering this scenario, the execution time of one simulation with ESTRAUS on a 1,5 GHz and 512 Mb Pentium IV PC computer under Linux, takes about 12 minutes per iteration. For a typical peak hour simulation, convergence errors of 0,5% for the O/D trip differences and 1% for the link flows differences are reached in about 20-25 iterations. To reduce both errors to a maximum of 0.1%, between 100 and 120 iterations are required.

Concerning the computer package, different GUI's (graphic user interfaces) have been developed. For the current version of the model, interfaces with TRANSCAD and ARCVIEW have been developed, which gives ESTRAUS interactive-graphic capabilities, both for network edition and for result analysis. The figures that follow show some example snapshots of these interfaces. Finally, a GUI is currently being developed using MAP OBJECT's graphic libraries, to provide ESTRAUS with its own graphic capabilities.



REFERENCES

- AASHTIANI, H.Z. (1979). *The multimodal traffic assignment problem*. Ph. D. Dissertation. Alfred P. Sloan School of Management. M.I.T. , Cambridge, Massachusetts.
- AASHTIANI, H.Z. and MAGNANTI, T.L. (1981). Equilibria on a congested transportation network. *SIAM Journal of Algebraic and Discrete Methods* 2, pp. 213-226.
- ABDULAAL, M. and LEBLANC, L.J. (1979). Methods for combining modal split and equilibrium assignment models. *Transportation Science* 13, pp.292-214.
- ABKOWITZ, M.D. (1981). An analysis of the commuter departure time decision. *Transportation*, 10, pp. 283-297.
- ABRAHAMSSON, T. and LUNDQVIST, L. (1999). Formulation and estimation of combined network equilibrium models with applications to Stockholm. *Transportation Science* 33(1), pp.80-100.
- ASMUTH, R.L. (1978). Traffic network equilibria. Technical Report, SOL 78-2, Systems Optimization Laboratory, Stanford University, Stanford, California.
- BECKMAN, M.J., McGUIRE, C.B. and WINSTEN, C.B. (1956). *Studies in the Economics of Transportation*. Yale University Press, New Haven, Connecticut.
- BHAT, CH. R. (1998). Analysis of travel mode and departure time choice for urban shopping trips. *Transportation Research* 6B, pp. 361-371.
- BIANCHI, R.A. (1995). Modelación de la elección de horario de viaje usando preferencias declaradas. *M.Sc. Thesis*, Pontificia Universidad Católica de Chile.
- BRUYNOOGHE, M. (1969). *Un modèle intégré de distribution, de trafic sur un réseau*. Technical Report, Departement de Recherche Operationnelle et Infoematique, Arcueil.
- CHRIQUI, C. and ROBILLARD, P. (1975). Common bus lanes. *Transportation Science* 9, pp.115-121.
- CHIN, A.T.H. (1990). Influences on commuter trip departure time decisions in Singapore. *Transportation Research* 24A, pp. 321-333.
- DAFERMOS, S. (1971). An extended traffic assignment model with applications to two-way traffic. *Transportation Science* 5, pp. 366-389.
- DAFERMOS, S. (1972). The assignment problem for multiclass-user transportation networks. *Transportation Science* 6, pp.73-87.

DAFERMOS, S. (1980). Traffic equilibrium and variational inequalities. *Transportation Science* 14(1), pp.42-54.

DAFERMOS, S. (1982a). The general multimodal network equilibrium problem with elastic demand. *Networks* 12(1).

DAFERMOS, S. (1982b). Relaxation algorithms for the general asymmetric traffic equilibrium problem. *Transportation Science* 16(2), pp.231-240.

DE CEA, J. and FERNANDEZ, J.E. (1993). Transit assignment for congested public transport systems: an equilibrium model. *Transportation Science* 27(2), pp. 133-147.

DE CEA, J. and FERNANDEZ, J.E. (1996). Equilibrio simultáneo oferta-demanda en redes multimodales congestionadas: formulación matemática y algoritmo de solución. *Apuntes de Ingeniería* 19(4), pp. 17-30.

DEKOCK, V., DE CEA, J. and FERNÁNDEZ, J.E. (2001). Equilibrio simultáneo de distribución, partición modal y asignación con elección horaria de viajes (paper to be presented to CIT 2002, July 2002, Santander, Spain).

DEKOCK, V. (2001). Modelo de equilibrio simultáneo con elección de destino, modo y horario de viaje: formulación matemática y algoritmo de solución. MSc. Thesis, Engineering School, Pontificia Universidad Católica de Chile.

EVANS, S. (1976). Derivation and analysis of some models for combining distribution and assignment. *Transportation Research* 10, pp. 37-57.

FERNANDEZ, J.E. and FRIESZ, T.L. (1983). Equilibrium predictions in transportation markets: the state of the art. *Transportation Research B*, 17B(2), pp. 155-172.

FERNANDEZ, J.E. and DE CEA, J. (1990). An application of equilibrium modelling to urban transport planning in developing countries: the case of Santiago de Chile. En: H.E. Bradley (ed.) *Operational Research 90*, Pergamon Press PLC.

FERNANDEZ, J.E., DE CEA, J., FLORIAN, M. and CABRERA, E. (1994). Network equilibrium models with combined modes. *Transportation Science* 28(3), pp. 182-192.

FERNANDEZ, J.E., DE CEA, J. and SOTO, A. (2000). Multimodal equilibrium models for predicting intercity freight flows. Submitted to *Transportation Research B*.

FISK, C. and NGUYEN, S. (1982). Solution algorithms for network equilibrium with asymmetric user costs. *Transportation Science* 16(3), pp. 361-381.

FLORIAN, M., NGUYEN, S. and FERLAND, J. (1975). On the combined distribution

assignment of traffic. *Transportation Science* 9(1), pp. 43-53.

FLORIAN, M. (1977). A traffic equilibrium model of travel by car and public transit modes. *Transportation Science* 11(2), pp. 166-179.

FLORIAN, M. and NGUYEN, S. (1978). A combined trip distribution modal split and assignment model. *Transportation Research* 12, pp. 241-246.

FLORIAN, M. (1979). *Asymmetrical variable demand multi-mode traffic equilibrium problems: existence and uniqueness of solutions and a solution algorithm*. Divicao de Intercambio de Edicoes, INDO/0/0, Pontificia Universidad Catolica de Rio de Janeiro, Brasil.

FLORIAN, M. and SPIESS, H. (1982). The convergence of diagonalization algorithms for asymmetric network equilibrium problems. *Transportation Research* 16B, pp. 447-483.

FLORIAN, M., WU, J.H. and HE, S. (1999). A multi-class multi-mode variable demand network equilibrium model with hierarchical logit structures. *IX Congreso Chileno de Ingeniería de Transporte, 18-22 Octubre 1999*, Santiago, Chile.

FRANK, M. and WOLFE, P. (1956). An algorithm for quadratic programming. *Naval Research Logistics Quarterly* 3, pp. 95-110.

FRIESZ, T.L., GOTTFRIED, J.A. and MORLOK, E.K. (1986). A sequential shipper-carrier network model for predicting freight flows. *Transportation Science* 12(2), pp. 80-91.

HARKER, P.T. and PANG, J.S. (1987). *Finite-Dimensional Variational Inequality Nonlinear Complementarity Problems: A Survey of Theory, Algorithms and Applications*. Report 87-12-06. The Wharton School, University of Pennsylvania, 1987.

HENDRICKSON, C. and PLANK, E. (1984). The flexibility of departure times for work trips. *Transportation Research* 18A(1), pp. 25-36.

JHONSTON, R.H., BATES, J.J. and ROBERTS M. (1989). A survey of peak spreading in London: methodology and initial results. *Proceedings 17th PTRC Annual Meeting, University of Sussex, UK*.

LE CLERQ, F. (1972). A public transport assignment method. *Traffic Engineering and Control*, 13(6), pp. 91-96.

McCAFFERTY, D. and HALL, F.L. (1982). The use of multinomial logit analysis to model the choice of time to travel. *Econ. Geography* 58(3).

NGUYEN, S. and DUPUIS, C. (1984). An efficient method for computing traffic equilibria in networks with asymmetric transportation costs. *Transportation Science* 18, pp. 185-202.

NGUYEN, S. and PALLOTTINO, S. (1988). Equilibrium traffic assignment for large scale transit networks, *European Journal of Operation Research*, 5, pp. 396-404.

PANG, J.S. and CHANG, D. (1982). Iterative methods for variational and complementarity problems. *Mathematical Programming* 24, pp. 284-313.

SECTU (1989). *Strategic urban transport study for Santiago: Final Report*.

SHEFFI, Y. (1985). **Urban Transportation Networks**. Prentice Hall, Inc. New Jersey.

SMITH, M.J. (1979). The existence, uniqueness and stability of traffic equilibria. *Transportation Research* 13B, pp. 295-304.

SPIESS, H. and FLORIAN, M. (1989). Optimal strategies : a new assignment model for transit networks. *Transportation Research B*, 23(2), 83-102.

WU, J.H., FLORIAN, M. and MARCOTTE, P. (1994). Transit equilibrium assignment: a model and solution algorithms. *Transportation Science*, 28(3), 193-203.

APPENDIX

Diagonalized problem's optimality conditions (D-MS-A)

The optimality conditions of the optimization problem defined by objective function (7) and constraints (8)-(12), are obtained deriving the lagrangean L , with respect to h_r^{kpm} and to the trip demand variables T_w^{kp} , T_w^{kpn} and T_w^{kpm} . When L is derived with respect to h_r^{kpm} and the derivatives are equaled to zero, Wardrop's equilibrium conditions for each pure or combined mode are achieved. In addition, if L is derived with respect to T_w^{kp} , T_w^{kpn} and T_w^{kpm} , and the derivatives are equaled to zero, the problem's remaining equilibrium conditions are obtained. We show in detail how the demand equilibrium conditions are obtained.

$$\begin{aligned}
\min L = & \sum_k \sum_p \sum_{\bar{m}} \sum_a \int_0^{f_a^{kp\bar{m}}} \hat{c}_a^{kp\bar{m}}(x) dx + \sum_k \sum_p \sum_{\bar{m}} \sum_s \int_0^{V_s^{kp\bar{m}}} \hat{c}_s^{kp\bar{m}}(x) dx \\
& + \sum_k \sum_p \sum_{m^c} \sum_s \int_0^{V_s^{kpm^c}} \hat{c}_s^{kpm^c}(x) dx \\
& + \sum_k \sum_p \frac{1}{b^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) - \sum_k \sum_p \frac{1}{l^{kp}} \sum_w T_w^{kp} (\ln T_w^{kp} - 1) \\
& + \sum_k \sum_p \sum_n \frac{1}{l^{kp}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) - \sum_k \sum_p \sum_n \frac{1}{g^{kpn}} \sum_w T_w^{kpn} (\ln T_w^{kpn} - 1) \\
& + \sum_k \sum_p \sum_n \frac{1}{g^{kpn}} \sum_{m \in n} \sum_w T_w^{kpm} (\ln T_w^{kpm} - 1) + \sum_k \sum_p \sum_w u_w^{kp} (T_w^{kp} - \sum_n T_w^{kpn}) \\
& + \sum_k \sum_p \sum_n \sum_w u_w^{kpn} (T_w^{kpn} - \sum_{m \in n} T_w^{kpm}) + \sum_k \sum_p \sum_n \sum_m \sum_w u_w^{kpm} (T_w^{kpm} - \sum_{m \in n} T_w^{kpn}) \\
& + \sum_i \sum_k \sum_p m_i^{kp} (O_i^{kp} - \sum_j T_w^{kjp}) + \sum_j \sum_p h_j^p (D_j^p - \sum_k \sum_i T_w^{kip})
\end{aligned} \tag{A.1}$$

Deriving L respect to T_w^{kp} , T_w^{kpn} and T_w^{kpm} we obtain:

$$\frac{\partial L}{\partial T_w^{kp}} = \frac{1}{b^{kp}} \ln T_w^{kp} - \frac{1}{l^{kp}} \ln T_w^{kp} + u_w^{kp} - m_i^{kp} - h_j^p \quad \forall w, k, p \tag{A.2}$$

$$\frac{\partial L}{\partial T_w^{kpn}} = \frac{1}{l^{kp}} \ln T_w^{kpn} - \frac{1}{g^{kpn}} \ln T_w^{kpn} - u_w^{kp} + u_w^{kpn} \quad \forall w, k, p, n \tag{A.3}$$

$$\frac{\partial L}{\partial T_w^{kpm}} = \frac{1}{g^{kpn}} \ln T_w^{kpm} - u_w^{kpn} + u_w^{kpm} \quad \forall w, k, p, m \quad (\text{A.4})$$

In equilibrium, and if T_w^{kp} , T_w^{kpn} , $T_w^{kpm} > 0$, then the following must be true:

$$\frac{1}{b^{kp}} \ln T_w^{kp} - \frac{1}{l^{kp}} \ln T_w^{kp} + u_w^{kp} - m_i^{kp} - h_j^p = 0 \quad \forall w, k, p \quad (\text{A.5})$$

$$\frac{1}{l^{kp}} \ln T_w^{kpn} - \frac{1}{g^{kpn}} \ln T_w^{kpn} - u_w^{kp} + u_w^{kpn} = 0 \quad \forall w, k, p, n \quad (\text{A.6})$$

$$\frac{1}{g^{kpn}} \ln T_w^{kpm} - u_w^{kpn} + u_w^{kpm} = 0 \quad \forall w, k, p, m \quad (\text{A.7})$$

From expression (A.7):

$$T_w^{kpm} = e^{g^{kpn}(u_w^{kpn} - u_w^{kpm})} = e^{g^{kpn}u_w^{kpn}} e^{-g^{kpn}u_w^{kpm}} \quad \forall w, k, p, n \quad (\text{A.8})$$

Using (A.8) in (9) we obtain

$$T_w^{kpn} = \sum_{m' \in n} T_w^{kpm'} = e^{g^{kpn}u_w^{kpn}} \sum_{m' \in n} e^{-g^{kpn}u_w^{kpm'}} \quad \forall w, k, p, m \quad (\text{A.9})$$

Finally, by dividing expressions (A.8) and (A.9) the **mode trip proportions in its nest** are obtained.

$$P_w^{kpm} = \frac{T_w^{kpm}}{T_w^{kpn}} = \frac{e^{-g^{kpn}u_w^{kpm}}}{\sum_{m' \in n} e^{-g^{kpn}u_w^{kpm'}}} \quad \forall w, k, p, m \quad (\text{A.10})$$

On the other hand, adding (A.6) and (A.7):

$$\frac{1}{l^{kp}} \ln T_w^{kpn} - \frac{1}{g^{kpn}} \ln T_w^{kpn} - u_w^{kp} + \frac{1}{g^{kpn}} \ln T_w^{kpm} + u_w^{kpm} = 0 \quad \forall w, k, p, n, m \in n \quad (\text{A.11})$$

$$T_w^{kpm} = e^{-g^{kpn} \left(\frac{1}{l^{kp}} \ln T_w^{kpn} - \frac{1}{g^{kpn}} \ln T_w^{kpn} - u_w^{kp} \right)} e^{-g^{kpn}u_w^{kpm}} \quad \forall w, k, p, n, m \in n \quad (\text{A.12})$$

Adding (A.12) $\forall m' \in n$ and applying natural logarithm we obtain:

$$\sum_{m' \in n} T_w^{kpm'} = T_w^{kpn} = e^{-g^{kpn} \left(\frac{1}{I^{kp}} \ln T_w^{kpn} - \frac{1}{g^{kpn}} \ln T_w^{kpn} - u_w^{kp} \right)} \sum_{m' \in n} e^{-g^{kpn} u_w^{kpm'}} \quad \forall w, k, p, n \quad (\text{A.13})$$

$$\frac{1}{g^{kpn}} \ln T_w^{kpn} = -\frac{1}{I^{kp}} \ln T_w^{kpn} + \frac{1}{g^{kpn}} \ln T_w^{kpn} + u_w^{kp} - V_w^{kpn} \quad \forall w, k, p, n \quad (\text{A.14})$$

From (A.14):

$$V_w^{kpn} = -\frac{1}{I^{kp}} \ln T_w^{kpn} + u_w^{kp} \quad \forall w, k, p, n \quad (\text{A.15})$$

and from (A.15):

$$T_w^{kpn} = e^{-I^{kp} V_w^{kpn}} e^{I^{kp} u_w^{kp}} \quad \forall w, k, p, n \quad (\text{A.16})$$

Using (A.16) in (8):

$$T_w^{kp} = \sum_{n'} T_w^{kpn'} = e^{I^{kp} u_w^{kp}} \sum_{n'} e^{-I^{kp} V_w^{kpn'}} \quad \forall w, k, p \quad (\text{A.17})$$

and dividing (A.16) by (A.17):

$$\boxed{P_w^{kpn} = \frac{T_w^{kpn}}{T_w^{kp}} = \frac{e^{-I^{kp} V_w^{kpn}}}{\sum_{n'} e^{-I^{kp} V_w^{kpn'}}}} \quad \forall w, k, p, n \quad (\text{A.18})$$

which represents the **proportion of trips in nest n with respect to total trips**.

Reorganizing equation (A.5):

$$\ln T_w^{kp} = b^{kp} \left(\frac{1}{I^{kp}} \ln T_w^{kp} - u_w^{kp} + m_i^{kp} + h_j^p \right) \quad \forall w, k, p \quad (\text{A.19})$$

$$T_w^{kp} = e^{b^{kp} \left(\frac{1}{I^{kp}} \ln T_w^{kp} - u_w^{kp} \right)} e^{b^{kp} m_i^{kp}} e^{b^{kp} h_j^p} \quad \forall w, k, p \quad (\text{A.20})$$

On the other hand, from (A.17):

$$T_w^{kp} = e^{I^{kp} u_w^{kp}} \sum_{n'} e^{-I^{kp} V_w^{kpn'}} \quad \forall w, k, p \quad (\text{A.21})$$

Applying natural logarithm to (A.21) and reorganizing:

$$\ln T_w^{kp} = I^{kp} u_w^{kp} + \ln \sum_{n'} e^{-I^{kp} V_w^{kpn'}} \quad \forall w, k, p \quad (\text{A.22})$$

$$I_w^{kp} = u_w^{kp} - \frac{1}{I^{kp}} \ln T_w^{kp} \quad \forall w, k, p \quad (\text{A.23})$$

Replacing (A.23) in (A.20):

$$T_w^{kp} = e^{-b^{kp} L_w^{kp}} e^{b^{kp} m_i^{kp}} e^{b^{kp} h_j^p} \quad \forall w, k, p \quad (\text{A.24})$$

which can be written as:

$$\boxed{T_w^{kp} = A_i^{kp} O_i^{kp} B_j^p D_j^p e^{-b^{kp} L_w^{kp}}} \quad \forall w, k, p \quad (\text{A.25})$$

where:

$$A_i^{kp} O_i^{kp} = e^{b^{kp} m_i^{kp}} \quad \forall i, k, p \quad (\text{A.26})$$

$$B_j^p D_j^p = e^{b^{kp} h_j^p} \quad \forall j, k, p \quad (\text{A.27})$$

and

$$A_i^{kp} = \left[\sum_j B_j^p D_j^p e^{-b^{kp} L_w^{kp}} \right]^{-1} \quad \forall i, k, p \quad (\text{A.28})$$

$$B_j^p = \left[\sum_i \sum_k A_i^{kp} O_i^{kp} e^{-b^{kp} L_w^{kp}} \right]^{-1} \quad \forall j, k, p \quad (\text{A.29})$$

All the above proves that if the diagonalization algorithm converges, an equilibrium solution is obtained which satisfies the conditions established by our simultaneous equilibrium problem (equations 3,4,5 and 6 in section 3.2). It is important to add, that in all the applications developed by the authors, considering large networks with asymmetric cost functions (equilibrium assignment in capacity constrained public transportation networks; trip distribution, modal split and combined assignment in interurban freight networks and urban passenger networks), the algorithm has converged adequately. In many empirical works for different kind of asymmetric problems, starting from different initial solutions for a given case, the same equilibrium solution has been obtained.