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GEOTEKNISK INSTITUT

THE DANISH GEOTECHNICAL INSTITUTE

BULLETIN No. 28

J. BRINCH HANSEN

A REVISED AND EXTENDED FORMULA FOR BEARING CAPACITY

J. BRINCH HANSEN · SES INAN

TESTS AND FORMULAS CONCERNING SECONDARY CONSOLIDATION

COPENHAGEN 1970

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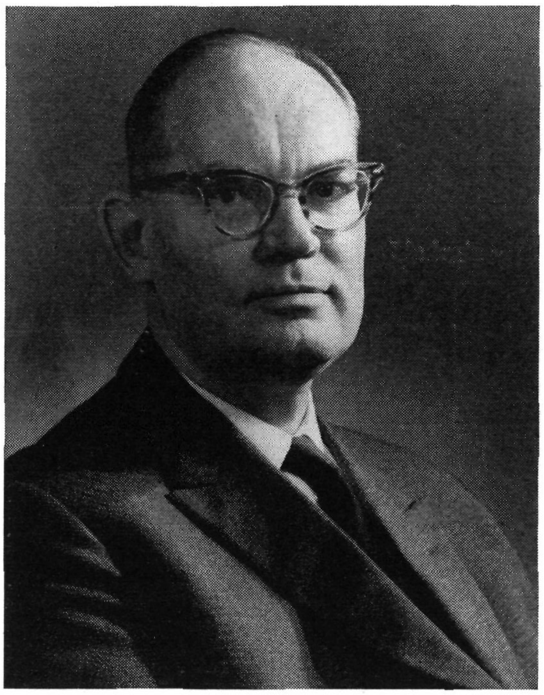

JØRGEN BRINCH HANSEN

* 29. July 1909 † 27. May 1969

J. Brinch Hansen was born in 1909. As an undergraduate after a few years' study at the University of Copenhagen he entered upon the study of civil engineering at the Technical University of Denmark, from which he graduated in 1935. Immediately after graduation he became a member of the staff of the well-known Danish firm of consulting engineers and contractors, Christiani and Nielsen Ltd. He was associated with this firm until 1955, from 1940 as sectional engineer and from 1953 as chief engineer and head of the central design department in Copenhagen. As a staff member of Christiani and Nielsen Ltd. during 20 years J. Brinch Hansen had the opportunity to participate in the solution of several unusual and difficult engineering assignments, the crucial problems of which were frequently in the field of Soil Mechanics.

The active and creative environment at Christiani and Nielsen Ltd. was important for J. Brinch Hansen's personal development. A decisive influence was certainly given by his superior during the first five years, chief engineer at the time (later Professor) A. E. Bretting, whose unvarying demand for an understanding supported by theoretical knowledge of the technical problems encountered in practice, combined with a practical feeling for what was feasible from an engineering point of view, J. Brinch Hansen adopted as a guide for his own work.

The engagement in construction projects of many different kinds caused J. Brinch Hansen to immerse himself into several technical subjects. Especially Soil Mechanics aroused his interest, and when he felt it necessary he sought to develop his own rational solutions to the foundation problems he encountered. The most prominent example of this is his doctoral thesis from 1953, "Earth Pressure Calculation", in which he, on the basis of the theory of plasticity, developed the first generally applicable method for the solution of most earth pressure problems in practice. As a rational basis for the use of the theory of plasticity he developed a limit design method for Soil Mechanics. Using a system of partial coefficients of safety, which are applied to the actual loads and strengths of materials, the problems are solved in a nominal state of failure.

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This principle makes possible the solution of even quite complicated failure problems in a rational and logical way.

In 1952 he became a member (1961 chairman) of a committee appointed by the Society of Danish Engineers to prepare a new Code of Practice for Foundation Engineering. In the final edition, which has been in use as Code of Practice in Denmark since April 1965, the principle of design on nominal states of failure was incorporated as an integral part. This is mainly due to the contributions made by J. Brinch Hansen.

In 1950 he became a member of the board of the Danish Society of Soil Mechanics, from 1956 in capacity of chairman. From 1965 he was Vice-President for Europe of the International Society of Soil Mechanics and Foundation Engineering.

When the Technical University of Denmark in 1955 divided the chair, then in Harbour Constructions and Foundation Engineering, into two separate chairs, J. Brinch Hansen was invited to accept the professorship in Soil Mechanics and Foundation Engineering. At the same time he became director of the Danish Geotechnical Institute, which is an independent consulting and research institution, affiliated to the Danish Academy of Technical Sciences. In both appointments he succeeded Professor, Dr. H. Lundgren, who during the previous five years had started modern research and development in several subjects of Soil Mechanics in Denmark.

J. Brinch Hansen took up his new duties with efficiency and enthusiasm. The greatly improved facilities for research work, which were now available to him, enabled him to become actively engaged in the study of a wider variety of important problems in Soil Mechanics, among other things: Bearing capacity of footings, lateral resistance of piles, general problems of stability, settlements in sand, secondary consolidation, and negative skin friction on piles. The bibliography of his work gives a good impression of the productiveness of his years as a professor and director of the Danish Geotechnical Institute. During the same time he was a consultant on several engineering projects, involving complicated foundation problems in Denmark as well as abroad. These included constructions of widely different types, such as tunnels, bridges, dry docks, and silos.

He was co-author of the first two modern Danish textbooks in Soil Mechanics, which are still being used at the Technical University of Denmark. He also in 1956 took the initiative to start the bulletin series of the Danish Geotechnical Institute.

In 1954 J. Brinch Hansen became a member of the Danish Academy of Technical Sciences. He was also a member of the German "Arbeitsausschuss Ufereinfassungen".

In 1965 he became Dr.ing.h.c. at the University of Ghent in Belgium. He was knight of the Dannebrog.

When J. Brinch Hansen in the early spring 1969 had to go to hospital he did not in any way seem seriously ill, and none of us who worked closely together with him could imagine that he would not get well. After a gallstone operation in March and a liver operation in April it became clear, however, that his illness was serious indeed. It caused his death on 27th May 1969.

The untimely death of J. Brinch Hansen is a great and painful loss to his friends all over the world as well as to the science of Soil Mechanics. Many kind letters of condolence and other expressions of sympathy have emphasized this.

It is clear, however, that the loss is felt most keenly —apart from the family—by the professors and students at the Technical University of Denmark and by the staff of the Danish Geotechnical Institute. It was primarily due to his versatile as well as profound scientific contributions that the work done at the Danish Geotechnical Institute was recognized by research workers in Soil Mechanics all over the world. He was in a class by himself, and as such he cannot be replaced.

We who have to carry on the work, feel deeply the loss of J. Brinch Hansen, his great knowledge and insight, and his ability to analyze and go directly to the kernel of the problems. Those who became closely connected with him, will also miss a loyal friend, ready to help. But in the loss we shall be glad that we have had the opportunity to work together with him during 14 years and to learn from his great experience. He had three main demands to himself and to others in connection with all activities: Quality, integrity, and accuracy. Those demands we shall adopt and try to honour in the work at the Danish Geotechnical Institute.

S. Thorning Christensen
Chairman of the Board
Danish Geotechnical Institute

J. Hessner
Director
Danish Geotechnical Institute

Bent Hansen
Head of Research Department
Danish Geotechnical Institute

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A Revised and Extended Formula for Bearing Capacity

by *J. Brinch Hansen*

(Reprint of Lecture in Japan, October 1968)

1. Original Formula.

A simple formula for the bearing capacity of a shallow foundation was developed around 1943 by Buisman, Caquot and Terzaghi. With the latter's notations it reads:

$$Q/B = \frac{1}{2}\bar{\gamma}BN_\gamma + \bar{q}N_q + cN_c \qquad (1)$$

This formula is developed for an infinitely long foundation of width B, placed upon the horizontal surface of soil with an effective unit weight $\bar{\gamma}$, a friction angle φ and a cohesion c. $\bar{q}$ is a unit surcharge acting upon the soil surface outside the foundation, and Q is the ultimate bearing capacity per unit length of this foundation, provided that it is loaded centrally and vertically.

2. Bearing Capacity Factors.

The exact formulas for N_q and N_c were indicated already by Prandtl:

$$N_q = e^{\pi \tan \varphi} \tan^2 (45° + \varphi/2) \qquad (2)$$

$$N_c = (N_q - 1) \cot \varphi \qquad (3)$$

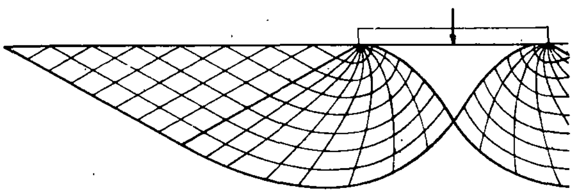

Fig. 1 Lundgren-Mortensen rupture figure for calculation of N_γ. Vertical load on heavy earth (no surface load).

The best available calculations of N_γ were made, first by Lundgren-Mortensen, and later by Odgaard and N. H. Christensen, using the rupture-figure shown in fig. 1. The results correspond closely to the empirical formula:

$$N_\gamma = 1.5 (N_q - 1) \tan \varphi \qquad (4)$$

Curves for all 3 factors are shown in fig. 2.

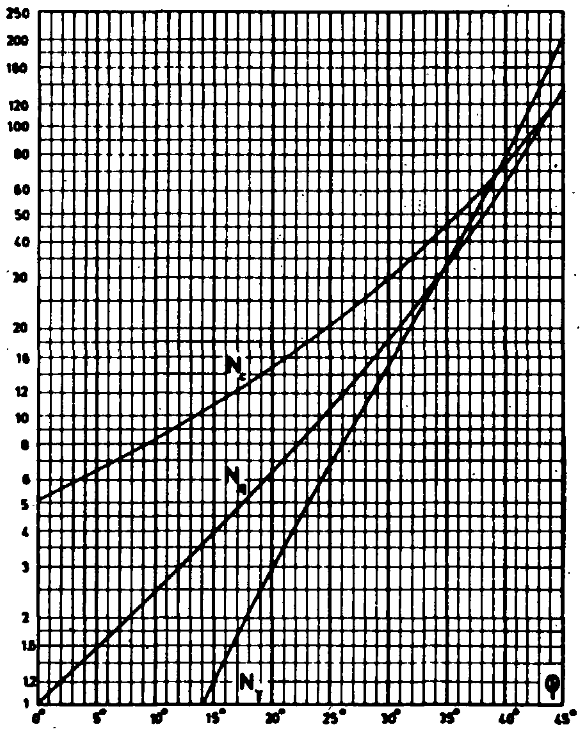

Fig. 2 Bearing capacity factors N_q, N_c, and N_γ as functions of φ.

Since N_q and N_c are calculated for one rupture-figure, and N_γ for another, the simple superposition implied by equation (1) must actually be an approximation. However, it is always on the safe side, and the error is usually less than 20 % (Lundgren-Mortensen).

3. Practical Cases.

Actual foundations deviate in several respects from the simple case considered above.

Thus, the load may be eccentric or inclined or both. The base of the foundation is usually placed at a depth D below the soil surface. The foundation has always a limited length L, and its shape may not even be rectangular. Finally, both the foundation base and the ground surface may be inclined.

Apart from the eccentricity, which is best taken

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into account by considering the so-called effective foundation area, all the other influences can be expressed by means of suitable factors to the 3 terms in the original formula.

The different factors can be found by considering rather simple cases, in which only one complication occurs at a time. When these factors are then used together for more complicated cases this will, of course, be an approximation.

For the different new factors we shall use the following symbols:

s shape factors.
d depth factors.
i inclination factors.
b base inclination factors.
g ground inclination factors.

4. Effective Foundation Area.

All loads acting from above upon the base of the foundation are combined into one resultant. It has a component V normal to the base, a component H in the base, and intersects the base in a point called the load centre.

Now, a so-called effective foundation area of rectangular shape is determined in such a way, that its geometric centre coincides with the load centre, and that it follows as closely as possible the nearest contour of the actual base area. A few examples are shown in fig. 3.

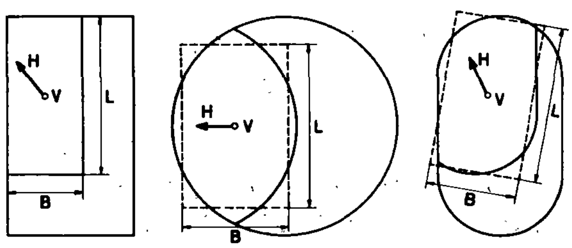

Fig. 3 Equivalent and effective foundation areas.

The short side of this equivalent rectangle is called B and the long one L. The effective foundation area is $A = BL$. For a strip foundation the effective width B will simply be twice the distance from the load centre to the nearest edge of the base.

Meyerhof, the writer and others have shown that the actual bearing capacity of an eccentrically loaded foundation will be very nearly equal to the bearing capacity of the centrally loaded effective foundation area. Consequently, in the following we shall only consider central loading, and the symbols B, L and A will always refer to the effective rectangle.

5. Extended Formulas.

Applying the 5 new kinds of factors to the original equation (1), we get the following formula (Brinch Hansen):

$$Q/A = \frac{1}{2}\bar{\gamma} B N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma + \bar{q} N_q s_q d_q i_q b_q g_q + c N_c s_c d_c i_c b_c g_c \tag{5}$$

$\bar{q}$ is now to be understood as the effective overburden pressure at base level.

In the special case of a horizontal ground surface, the ground inclination factors g disappear, of course, and the equation can then be written somewhat simpler:

$$Q/A = \frac{1}{2}\bar{\gamma} B N_\gamma s_\gamma d_\gamma i_\gamma b_\gamma + (\bar{q} + c \cot\varphi) N_q s_q d_q i_q b_q - c \cot\varphi \tag{6}$$

In the other speical case of $\varphi = 0$ ($\sim$ undrained failure in clay), it will theoretically be more correct to introduce additive constants instead of factors. Since the c-term is usually dominant, we may write:

$$Q/A = (\pi + 2)\, c_u (1 + s_c^a + d_c^a - i_c^a - b_c^a - g_c^a) \tag{7}$$

6. Load Inclination Factors.

An inclination of the load will always mean a reduced bearing capacity, and the reduction is often very considerable.

Exact formulas for i_q and i_c have been derived by several authors, using the rupture-figure shown in fig. 4. The rather complicated results can be approximated by means of simple empirical formulas, however (Brinch Hansen).

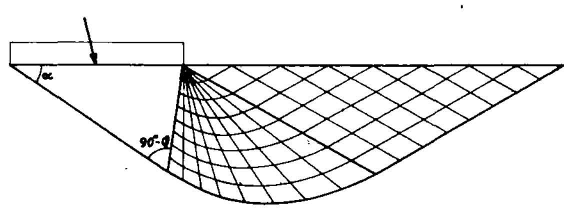

Fig. 4 Rupture figure for calculation of i_q and i_c. Inclined load on weightless earth (with vertical surface load).

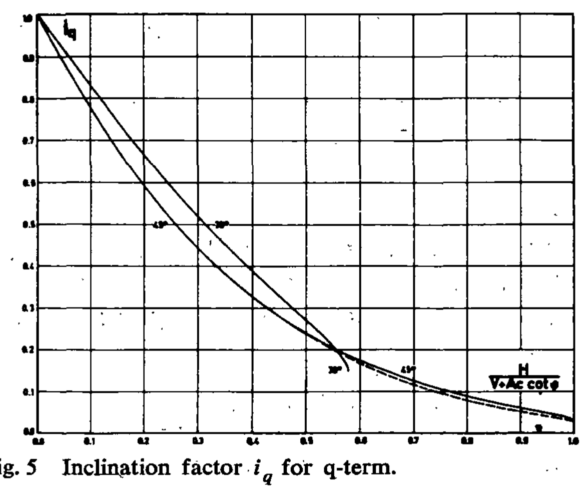

Fig. 5 Inclination factor i_q for q-term.

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In the case of $\varphi = 0$ we get:

$$i_c^a = 0.5 - 0.5\sqrt{1 - H/A\,c_u} \qquad (8)$$

For $\varphi = 30°$ and $45°$ the results are shown in fig. 5, The dotted line corresponds to the formula:

$$i_q = [1 - 0.5\,H : (V + Ac\cot\varphi)]^5 \qquad (9)$$

Calculations of i_γ have been made by Odgaard and N. H. Christensen, using the rupture-figure shown in fig. 6. Their results for $\varphi = 30°$ and $45°$ are shown in fig. 7. The dotted line corresponds to the formula:

$$i_\gamma = [1 - 0.7\,H : (V + Ac\cot\varphi)]^5 \qquad (10)$$

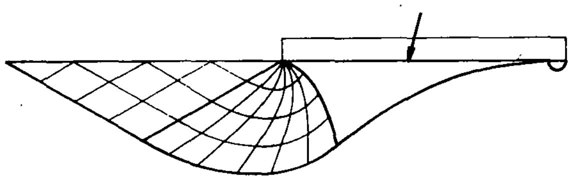

Fig. 6 Rupture figure for calculation of i_γ. Inclined load on heavy earth (no surface load).

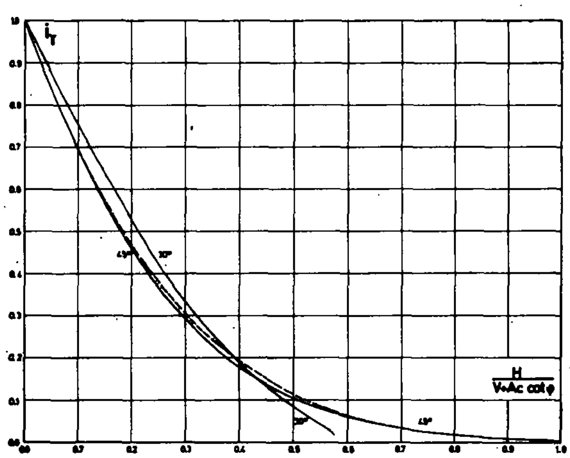

Fig. 7 Inclination factor i_γ for γ-term.

As we shall see later, i_γ must be modified if the foundation base is inclined.

To avoid misunderstandings it should be mentioned that (9) and (10) must not be used, if the quantity inside the bracket becomes negative. The bearing capacity will in such cases be negligible.

The above inclination factors are valid for a horizontal force $H = H_B$, acting parallel with the short sides B of the equivalent effective rectangle. If we substitute H by H_B in the equations (8–10), the corresponding factors may be termed i_{cB}^a, i_{qB} and $i_{\gamma B}$ respectively.

In the more general case, where there is also a horizontal force component H_L, acting parallel with the long sides L, we can find another set of factors i_{cL}^a, i_{qL} and $i_{\gamma L}$ by substituting H by H_L in (8–10).

The first set of factors (with second subscript B) should be used for investigating the usual failure along the long sides L, occuring when H_B is dominant. The second set (with second subscript L) are used for investigating a possible failure along the short sides B, which may occur when H_L is dominant.

7. Base and Ground Inclination.

The general case is shown in fig. 8. The slope angle is called β, and since the ground will usually be sloping away from the foundation, β shall be defined as positive in this case. The foundation depth D is measured vertically.

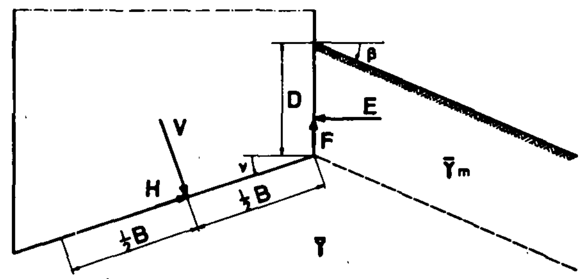

Fig. 8 General case of base and front of retaining wall with inclination of base and ground.

The effective width of the inclined base is called B, whereas V indicates the foundation load normal to the base and H the load in the base. ν is the angle between base and horizon.

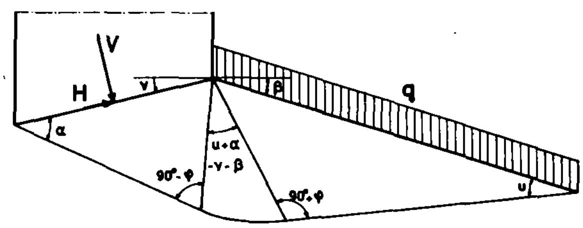

Fig. 9 Rupture figure for calculation of b_c, b_q, g_c and g_q. Frictionless earth or weightless earth with vertical surface load.

For the theoretical case of frictionless or weightless earth we get the rupture-figure shown in fig. 9. In the case of $\varphi = 0$ it gives the exact formulas:

$$b_c^a = \frac{2\nu}{\pi + 2} = \frac{\nu°}{147°} \qquad (11)$$

$$g_c^a = \frac{2\beta}{\pi + 2} = \frac{\beta°}{147°} \qquad (12)$$

For other friction angles we can find the following exact formula for b_q:

$$b_q = e^{-2\nu\tan\varphi} \qquad (13)$$

For b_γ we can develop an empirical formula by utilizing the fact that, for a vertical wall, we must have $K_\gamma = K_p$ according to Coulomb. This gives:

$$b_\gamma = e^{-2.7\,\nu \tan\varphi} \tag{14}$$

With regard to g_q, exact formulas can be developed. A closer inspection reveals that g_q is exactly the same function of tan β, as i_q is of H: $(V + Ac \cot\varphi)$. Hence we can write approximately:

$$g_q = [1 - 0.5 \tan\beta]^5 = g_\gamma \tag{15}$$

We have here assumed g_γ equal to g_q in accordance with the fact that, for a vertical wall and any ground inclination, we must have $K_\gamma = K_p$ according to Coulomb.

The above formulas should only be used for positive values of ν and β, the latter being smaller than φ. Also, $\nu + \beta$ must not exceed 90°.

In the case that the foundation base becomes a vertical wall ($\nu = 90°$), we shall, according to Coulomb's earth pressure theory, have $K_\gamma = K_p$, independent of the roughness of the wall. Therefore it will, in the general case, be necessary to let i_γ depend upon ν in such a way, that for $\nu = 90°$ we get $i_\gamma = i_q$. This is easily achieved by writing:

$$i_\gamma = [1 - (0.7 - \nu°/450°)\, H : (V + Ac \cot\varphi)]^5 \tag{16}$$

8. Shape Factors.

Theoretical values of the shape factors can hardly be indicated at present, since their calculation would require a 3-dimensional theory of plasticity. Thus, experimental evidence must be used.

Extensive and careful plate loading tests on sand have led de Beer to propose the empirical formulas:

$$s_\gamma = 1 - 0.4\, B/L \tag{17}$$

$$s_q = 1 + \sin\varphi \cdot B/L \tag{18}$$

As a result of loading tests on undrained clay Skempton found the value:

$$s_c^a = 0.2\, B/L \tag{19}$$

which can be shown to be a limiting case of (18).

The shape factors indicated above are actually only valid for vertical loading. In the other limiting case of the foundation sliding on its base, the shape can have little influence. For inclined loads we must, therefore, modify the formulas by introducing the inclination factors. And since failure can take place either along the long sides, or along the short sides, we shall need two sets of inclination factors. The following formulas are proposed (Brinch Hansen):

$$s_{cB}^a = 0.2\, i_{cB}^a\, B/L \tag{20}$$

$$s_{cL}^a = 0.2\, i_{cL}^a\, L/B \tag{21}$$

$$s_{qB} = 1 + \sin\varphi \cdot B i_{qB}/L \tag{22}$$

$$s_{qL} = 1 + \sin\varphi \cdot L i_{qL}/B \tag{23}$$

$$s_{\gamma B} = 1 - 0.4\, (B i_{\gamma B}) : (L i_{\gamma L}) \tag{24}$$

$$s_{\gamma L} = 1 - 0.4\, (L i_{\gamma L}) : (B i_{\gamma B}) \tag{25}$$

For the last two factors the special rule must be followed, that the value exceeding 0.6 should always be used.

9. Depth Effects.

Actual foundations are always placed at a certain depth D below the surface. One consequence of this is, that we must take into account the effective weight of the soil above base level. This is done by putting:

$$\bar{q} = \bar{\gamma}_m D \tag{26}$$

where $\bar{\gamma}_m$ is the average effective weight of the soil above base level.

Another effect is due to the fact that the soil above base level has a certain shearing strength. In the following this soil is assumed to be identical with the soil below base level. When it is inferior (or possesses no strength at all) the indicated depth effect will have to be reduced (or neglected completely).

At least one rupture-line will always pass through the soil layer above base level, and the effect will be an increase of the bearing capacity. This we express by means of a depth factor, which is used for foundations with mainly vertical loads.

If an upper Rankine zone is continued through the soil above the base without changing the rupture-figure below base level, the result will be an extra, horizontal force. Therefore, on foundations subjected to considerable horizontal loads (e.g. retaining walls), the simplest way to take the depth effect into consideration is to assume a passive earth pressure acting upon the side of the foundation.

10. Depth Factors.

The depth factor d_γ presents no problem, because we have always, according to definitions:

$$d_\gamma = 1 \tag{27}$$

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For small values of D/B it is easy to calculate d_q or d_c, because we can just extend the known rupture-figure for $D = 0$ up to the actual surface. In this way the following approximate formulas have been found (Brinch Hansen):

$$d_c^a = 0.4\, D/B \tag{28}$$

$$d_q = 1 + 2 \tan \varphi (1 - \sin \varphi)^2 \, D/B \tag{29}$$

These formulas may be used for $D \leq B$. For greater depth it is difficult to calculate the depth factors, but we know that they must ultimately approach an asymptotic value. I have, therefore, tentatively proposed the following formulas:

$$d_c^a = 0.4 \arctan D/B \tag{30}$$

$$d_q = 1 + 2 \tan \varphi (1 - \sin \varphi)^2 \arctan D/B \tag{31}$$

We can actually test these formulas by applying them, together with the formulas for the shape factors, to a square pile base at great depth ($D \to \infty$). This gives for $\varphi = 0$:

$$Q/A = (\pi + 2)\, c_u (1 + 0.2 + 0.4\, \pi/2) \doteq 9.4\, c_u \tag{32}$$

which is a well-known result for pile point resistance in clay.

For other friction angles we get:

$$Q/A = \bar{q} N_q (1 + \sin \varphi)(1 + \pi \tan \varphi (1 - \sin \varphi)^2) \tag{33}$$

For values of φ between 30° and 40° this formula gives the result $2.2\, \bar{q}\, N_q$ in very good agreement with Danish experience for pile point resistances in sand, provided that φ is taken as the friction angle in plane strain.

The above formulas are valid for the usual case of failure along the long sides L of the base, and formulas (30)–(31) give the corresponding depth factors d_{cB}^a and d_{qB}.

For the investigation of a possible failure along the short sides B we must use another set of depth factors:

$$d_{cL}^a = 0.4 \arctan D/L \tag{34}$$

$$d_{qL} = 1 + 2 \tan \varphi (1 - \sin \varphi)^2 \arctan D/L \tag{35}$$

11. General Formulas.

In the general case, where the horizontal force has both a component H_B parallel with the short sides B, and a component H_L parallel with the long sides L of the equivalent effective rectangle, we must use the following formulas:

$$Q/A \leq \begin{matrix} \frac{1}{2}\bar{\gamma} N_\gamma B s_{\gamma B} i_{\gamma B} b_\gamma \\ \frac{1}{2}\bar{\gamma} N_\gamma L s_{\gamma L} i_{\gamma L} b_\gamma \end{matrix} + \begin{matrix} (\bar{q} + c \cot \varphi) N_q d_{qB} s_{qB} i_{qB} b_q \\ (\bar{q} + c \cot \varphi) N_q d_{qL} s_{qL} i_{qL} b_q \end{matrix} - c \cot \varphi \tag{36}$$

This formula should be used in the following way. Of the two possibilities for the γ-term, the upper one should be used when $Bi_{\gamma B} \leq Li_{\gamma L}$, whereas the lower one should be used when $Bi_{\gamma B} \geq Li_{\gamma L}$. A check on the right choice is that $s_\gamma \geq 0.6$. Of the two possibilities for the q-term we must always choose the one giving the smallest numerical value.

In the special case of $\varphi = 0$ we must choose the smallest of the following two values:

$$Q/A \leq \begin{matrix} (\pi + 2)\, c_u (1 + s_{cB}^a + d_{cB}^a - i_{cB}^a - b_c^a - g_c^a) \\ (\pi + 2)\, c_u (1 + s_{cL}^a + d_{cL}^a - i_{cL}^a - b_c^a - g_c^a) \end{matrix} \tag{37}$$

12. Passive Earth Pressure.

For foundations subjected to considerable horizontal forces it has always been a question, whether it is permissible to assume a passive earth pressure acting on one vertical side of the foundation. The fear has been expressed that this would require too great horizontal movements of the foundation.

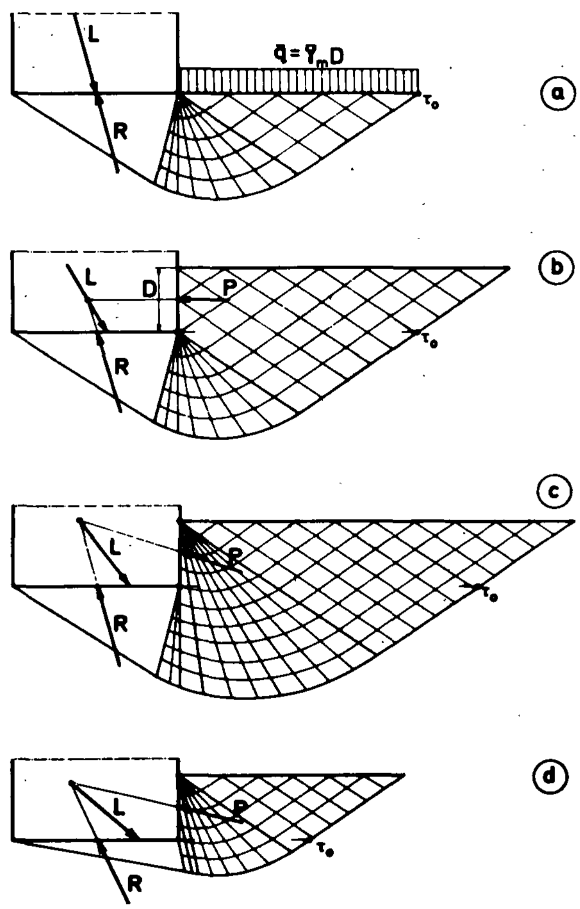

Fig. 10 Statically admissible rupture figures. Frictionless or weightless earth. Inclined load with fully developed passive earth pressure.

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Fig. 10 shows first, for the case of weightless earth, the rupture-figure for a surface foundation (a). For foundations below the ground surface two statically possible rupture-figures are shown, one for a smooth side (b) and another for a rough side (c).

In the considered case of weightless (or frictionless) earth the stresses in the outermost rupture-line at base level are exactly the same as for a surface foundation. The reaction R on the base is, consequently, also the same, but in addition we have now an earth pressure P acting on the vertical side.

Therefore, due to the depth D, the foundation can actually take up a total load L, determined so as to be in equilibrium with the two forces R and P. The simplest way to take this into account is to add the earth pressure P (vectorially) to the main foundation load L. The resultant R (components V and H) can then be treated in the usual way, Q being calculated by means of the appropriate bearing capacity formula. It should be noted that depth factors must not be used, when the passive earth pressure is taken into account as proposed.

For load inclinations up to a certain value the rupture-figure shown in fig. 10c will occur. It indicates that in this case the passive earth pressure can be calculated as for a perfectly rough wall. For greater load inclinations the rupture-figure shown in fig. 10d will occur, and to this will correspond a slightly lower earth pressure. A limiting case occurs, when the base starts to slide horizontally. To this corresponds the passive earth pressure on a rough wall, which is translated horizontally.

As will appear from fig. 10, it is one and the same rupture-figure, which is responsible for both the bearing capacity and the passive earth presure. Therefore, the passive earth pressure does not require other or greater movements of the foundation than does the bearing capacity.

On the other hand, we must limit the movements to allowable values. As regards the bearing capacity, this is usually ensured by dividing the ultimate bearing capacity with a safety factor of e.g. 2.0. For exactly the same reason we must also divide the ultimate passive earth pressure with a safety factor, e.g. 1.4.

I have already mentioned that for mainly vertical loads we use depth factors but assume no passive pressure on the side of the foundation, whereas for great horizontal loads we do the opposite.

In intermediate cases we may, as proposed by Bent Hansen, do the following. If the horizontal force H is smaller than the horizontal component of passive pressure on the whole height D, we calculate the height D_o necessary to develop a passive pressure just balancing the horizontal force. We can then calculate the foundation for vertical load only, and with depth factors corresponding to the remaining height $D - D_o$.

13. Soil Parameters.

In case of $\varphi_u = 0$ (saturated clay) we must, of course, for c_u use the relevant undrained shear strength. For long term calculations we must use the effective parameters $\bar{\varphi}$ and $\bar{c}$ as found in drained triaxial tests.

For sand we can usually, on the safe side, assume $c = 0$. As regards φ, it has been common practice to use the friction angle φ_{tr} found in ordinary triaxial tests. However, plate loading tests in several laboratories have shown that this leads to a severe underestimation of the bearing capacities. (Fig. 11).

Since the theoretical expressions for N_q and N_γ are developed for the case of plane strain, we must actually use the friction angle φ_{pl} found in a plane

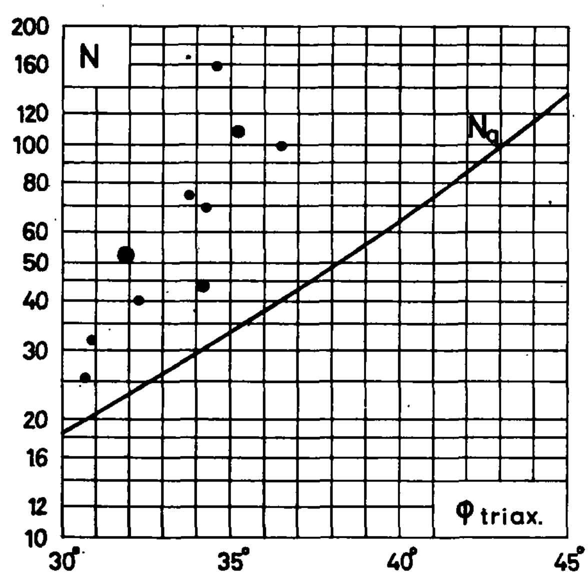

Fig. 11 Comparison of plate loading tests with calculated bearing capacities using friction angles from triaxial tests.

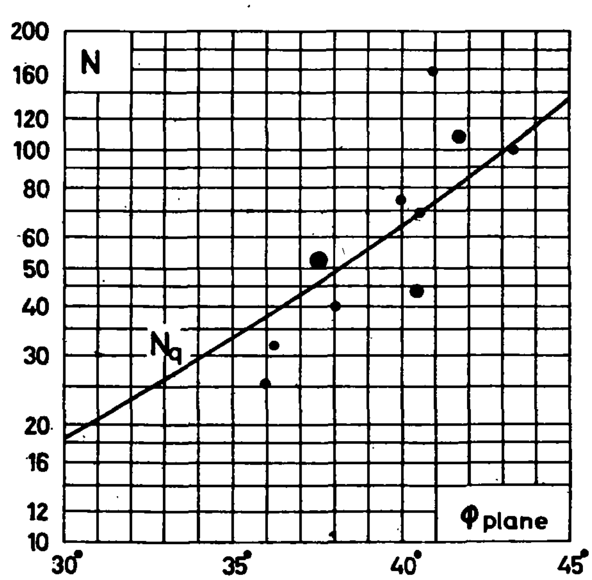

Fig. 12 Comparison of plate loading tests with calculated bearing capacities using friction angles corrected for effect of plane strain.

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strain test. Such tests have been made in several laboratories with somewhat different results. In Denmark we use always in our bearing capacity calculations the following value:

$$\varphi_{pl} = 1.1\,\varphi_{tr} \tag{38}$$

which should be on the safe side, since we have found factors up to 1.15.

This procedure has been found to give reasonable agreement between plate loading tests and the theoretical formulas. (Fig. 12).

14. Safety Factors.

The safety in bearing capacity problems is usually introduced as a total or overall safety factor F. This means that the bearing capacity Q should be calculated for the actual loads and actual soil parameters, and then the actual vertical load V on the foundation must not exceed Q/F.

In Denmark we prefer the system of partial safety factors. This means that we calculate a nominal bearing capacity Q_n for nominal loads and for nominal soil parameters defined by:

$$\begin{array}{ll} g_n = g_a & p_n = p_a \cdot f_p \\ c_n = c_a/f_c & \tan\varphi_n = \tan\varphi_a/f_\varphi \end{array} \tag{39}$$

The foundation should then be designed so that $V_n \leq Q_n$. The following values are used in Denmark:

$$f_p = 1.5 \qquad f_c = 1.75 \qquad f_\varphi = 1.25 \tag{40}$$

15. Example.

As a simple example we shall consider a couple of full-scale tests with foundation blocks, made by H. Muhs in Berlin.

The blocks had the dimensions $L = 2$ m, $B = 0.5$ m and $D = 0.5$ m. Base area $A = 1$ m². The soil was dense sand with $\bar{\gamma} = 0.95$ t/m³, and ground water level coincided with the ground surface. We can assume $c = 0$.

The first block was loaded centrally and vertically, and failed for $Q = 190$ t. If we estimate $\varphi_{pl} = 47°$ we find:

$$N_\gamma = 300 \qquad N_q = 190$$
$$d_{qB} = 1 + 2 \cdot 1.07 \cdot (1 - 0.732)^2 \cdot \arctan(0.5/0.5) = 1.12$$
$$s_{\gamma B} = 1 - 0.4 \cdot 0.5/2.0 = 0.90$$
$$s_{qB} = 1 + 0.732 \cdot 0.5/2.0 = 1.18$$

We can now calculate the ultimate bearing capacity:

$$Q = \frac{1}{2}\bar{\gamma} B N_\gamma d_{\gamma B} s_{\gamma B} + \bar{\gamma} D N_q d_{qB} s_{qB} =$$
$$\frac{1}{2} \cdot 0.95 \cdot 0.5 \cdot 300 \cdot 1 \cdot 0.90 + 0.95 \cdot 0.5 \cdot 190 \cdot 1.12 \cdot 1.18 =$$
$$64 + 120 = 184\,\text{t}\,(\sim 190)$$

If our formulas are correct, this shows that the plane friction angle was about 47°. To this corresponds a triaxial friction angle of 40°–42°, which is quite realistic for dense sand. In fact, Muhs measured $\varphi_{tr} = 40°$.

A second block was also loaded centrally, but the load was inclined in the direction of the long sides L. At failure the forces $Q = V = 108$ t and $H_L = 39$ t. Passive pressure on the vertical side of the foundation was so small, that it could be neglected. Instead we reckon with depth factors.

Since $H_B = 0$ we find:

$$d_{qB} = 1.12 \qquad s_{qB} = 1.18 \qquad i_{\gamma B} = i_{qB} = 1 \qquad B i_{\gamma B} = 0.5$$
$$d_{qB} s_{qB} i_{qB} = 1.12 \cdot 1.18 \cdot 1 = 1.32.$$

For the other direction we find:

$$d_{qL} = 1 + 2 \cdot 1.07 (1 - 0.732)^2 \cdot \arctan(0.5/2.0) = 1.04$$
$$i_{\gamma L} = (1 - 0.7 \cdot 39/108)^5 = 0.235 \qquad L i_{\gamma L} = 0.47$$
$$i_{qL} = (1 - 0.5 \cdot 39/108)^5 = 0.370 \qquad L i_{qL} = 0.74$$

Since $B i_{\gamma B} > L i_{\gamma L}$, we must use (25) and the lower γ-term in (36):

$$s_{\gamma L} = 1 - 0.4 \cdot 0.47/0.5 = 0.625$$
$$s_{qL} = 1 + 0.732 \cdot 0.74/0.5 = 2.09$$
$$d_{qL} s_{qL} i_{qL} = 1.04 \cdot 2.09 \cdot 0.370 = 0.805$$

Since $d_{qL} s_{qL} i_{qL} < d_{qB} s_{qB} i_{qB}$, we must use the lower q-term in (36). Using the same values of N_γ and N_q as above, we get the ultimate, vertical bearing capacity:

$$Q = \frac{1}{2}\bar{\gamma} L N_\gamma d_{\gamma L} s_{\gamma L} i_{\gamma L} + \bar{\gamma} D N_q d_{qL} s_{qL} i_{qL} =$$
$$\frac{1}{2} \cdot 0.95 \cdot 2.0 \cdot 300 \cdot 1 \cdot 0.625 \cdot 0.235 +$$
$$0.95 \cdot 0.5 \cdot 190 \cdot 1.04 \cdot 2.09 \cdot 0.370 =$$
$$42 + 72 = 114\,\text{t}\,(\sim 108)$$

It will be seen that the new formulas explain the results quite well in this case.

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Tests and Formulas Concerning Secondary Consolidation

by *J. Brinch Hansen* and *Ses Inan*

(Reprint of Proc. 7th Int. Conf. Soil Mech., Mexico 1969, Vol. I, p. 45–53)

SYNOPSIS. An extensive series of Oedometer tests with a remoulded glacial lake clay was carried out in order to study the secondary consolidation. The effects of sample height, temperature, stress history, final load, load increment ratio and duration of previous steps were investigated. As a result, a comparatively simple empirical formula is proposed, which seems to describe all observed features of the secondary consolidation quite well. Some tests were also made with a normally consolidated intact clay, for which the same type of formula was found to apply. On the basis of this formula, a calculation method for secondary settlements of structures is proposed, and for a bridge in Denmark the secondary settlement is calculated and compared with actual measurements.

Introduction

Secondary consolidation is the name commonly used for compressions or settlements, which continue to develop after the practical disappearence of all excess pore pressures.

That secondary settlements may be as important as the primary ones, and that they can go on for many years, has f. inst. been proved in the case of the Aggersund Bridge in Denmark (Bjerrum, Jønson and Ostenfeld 1957). In a semi-logarithmic plot the time curve has been a perfectly straight line for at least 12 years.

Also in Oedometer tests, secondary consolidation has been proved to go on for at least 4 years (Cox 1936). Here, the time curve was slightly downwards concave.

Since the physical mechanisms and causes of secondary consolidation are not yet known, it is not apriori certain that secondary settlements of structures can be calculated on the basis of laboratory tests. Of course, the best way to investigate this problem is to compare observed secondary settlements with rational calculations.

Such calculations cannot be made in any reliable way, however, until we have a mathematical formula for the process of secondary consolidation, describing its dependence on the different loading steps and their duration, as well as on other pertinent parameters (void ratio, temperature etc.). Then we can, by means of suitable laboratory tests, determine the constants in this equation, and afterwards apply it to the conditions in situ.

The main purpose of the present paper is to propose such a formula. It has been developed so as to give reasonable agreement with at least one extensive series of consolidation tests, in which the effects of final stress, stress increment ratio, number of loading steps, duration of previous step etc. were investigated separately.

The main part of the described tests were carried out by Ses Inan at the DGI from 1966–68. The empirical formulas were developed by J. Brinch Hansen, whereas the detailed comparison between formulas and test results were made jointly.

Oedometer Tests.

This paper deals exclusively with one-dimensional compression of clay, as investigated in the Oedometer test. All tests were made in the new "inelastic" Oedometers of the DGI, in which both lateral yield and ring friction are minimized.

The tests were primarily made with a remoulded clay, both in order to get sufficiently reproducible results, and also because samples for comparative tests could be made practically identical.

The investigation was only concerned with primary loading (each loading higher than the previous one), not unloading and reloading.

It has been found practical to plot the time curves in a composite $\sqrt{t}$ – log t diagram (Brinch Hansen 1961). Fig. 1 shows such a diagram, in which the length of the $\sqrt{t}$ – part must be about 0.87 times (= 2 log e) the length of a decade in the log t – part in order to get a smooth time curve across the boundary line.

In such a diagram a normal time curve (for $\Delta p/p >$ 0.3) will consist of two straight lines, connected by a

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transition curve. The time scale should be adjusted so that the two straight lines will intersect approximately on the boundary line. The intersecting point defines the "consolidation time" t_c ($\sim T = \pi/4$) and the "primary compression" ε_c, whereas the "secondary compression" ε_s is the increase per time decade after dissipation of pore water pressures.

In the opinion of the authors, the secondary compression represents the real, rheological behaviour of the clay under the given conditions (stress, temperature, vibrations etc.). If the voids were empty, we would probably get a time curve as the one marked "rheological compression", but since it takes time to expel pore water from the voids, we get actually a "hydrodynamical retardation" of the compressions (see Fig. 1).

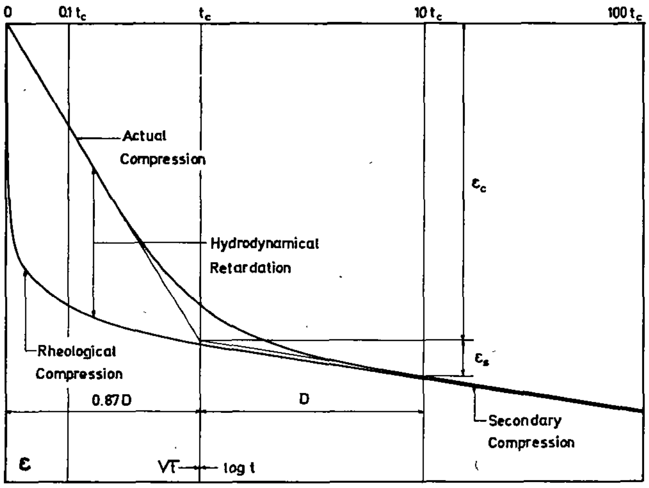

Fig. 1 Oedometer time curves.

Soil.

Soil samples have been taken from a brick factory pit near Nivaa, which is about 30 km north of Copenhagen. The samples were taken 4–5 m below ground surface, where the average field vane strength was about 8 t/m².

The soil is a late glacial lake clay, containing 46 % clay fraction, 54 % silt and 0.1 % sand. The natural water content was 23 %, the liquid limit 46 % and the plastic limit 19 % (all average values).

The remoulded samples were prepared by first crumbling the clay and then mixing it with water. The mixture was put into a rubber tube and stored there for about 10 days. Each day it was kneaded in the tube for some time.

Preliminary Tests.

Parallel tests were made with identical samples and loading procedures in order to investigate the reproducibility of the tests. This proved to be quite good, although a small amount of accidental scattering occurred.

Further parallel tests were made with different heights of the samples, from 13 to 46 mm, whereas the diameter was always 60 mm. The purpose was to investigate a possible effect of ring friction, and whether the secondary compression should be a function of sample height. The tests showed only small and non-systematic variations with the sample height, however. The consolidation times were, of course, longer for the higher samples.

Finally, parallel tests were made in two Oedometers at a constant temperature of 23° C and in two others at 36° C. The observed variations, both of total and of secondary compression, were small and non-systematic. In fact, the greatest deviation was found between two tests at the same temperature. However, the consolidation time was about 30 % longer at the lower temperature, as should be expected.

Number and Size of Loading Steps.

Two identical samples were subjected to two different loading procedures (Fig. 2). One sample was loaded in one step from 100 to 130 t/m², whereas the other sample was loaded in 3 steps (100–110–120–130 t/m²).

The result was that both total compression and secondary compression (indicated by the final inclination of the time curve) were nearly the same, independent of the number and size of the loading steps.

Conclusion: The void ratio e_n depends mainly on the final load p_n.

There is, however, a second order effect indicating a slight decrease of the total compression with the number of loading steps – or with the total time spent in the Oedometer. This effect may be due to ring friction.

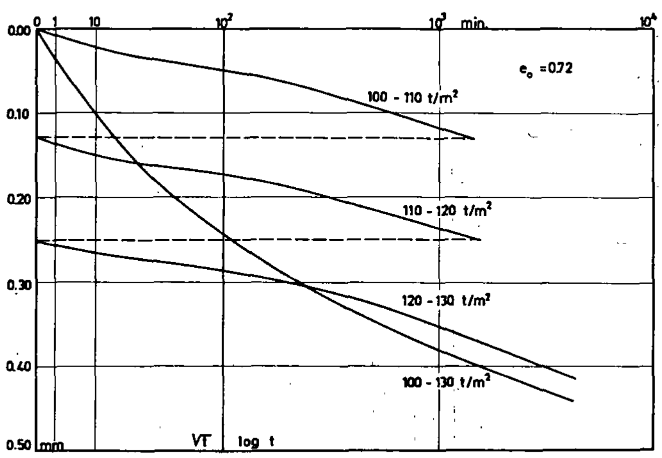

Fig. 2 Different number and size of loading steps.

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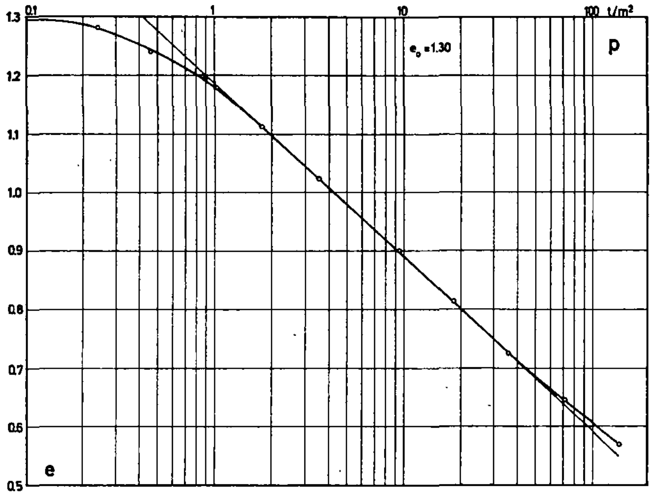

Fig. 3 *e-p*-curve in semi-logarithmic plot.

Void Ratio – Load Function.

In order to study the relation between load and void ratio we must consider loading steps of equal duration t_o (here chosen equal to 24 hours = 1440 min.). At the end of an arbitrary step n we measure a void ratio e_n corresponding to the load p_n.

For a typical test, fig. 3 shows the usual semi-logarithmic plot of e against p. The curve is seen to be quite straight between 1.5 and 40 t/m², but it is curved at both ends.

However, as already pointed out by Terzaghi, the curvature for low loads can be eliminated by plotting e against $p + p_o$, where p_o is a constant load. And the curvature for high loads can apparently be eliminated by using a double-logarithmic diagram.

Such a plot is shown in fig. 4, the upper curve representing the same test as in fig. 3. With $p_o = 1.1$ t/m² a straight line is seen to approximate the test results for the whole stress interval quite well. Accordingly, the void ratio – load relationship can be expressed as:

$$e_n = e_o \left[1 + \frac{p_n}{p_o}\right]^{-a} \qquad (1)$$

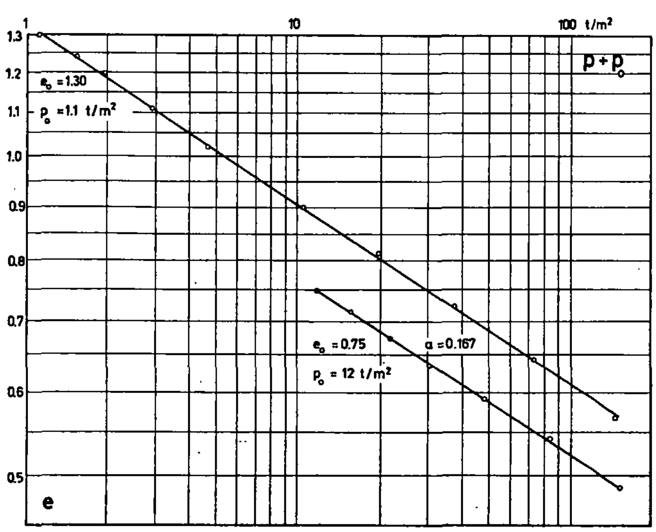

Fig. 4 $e - (p + p_o)$ – curves in double logarithmic plot.

The power a is found as the inclination of the straight line. Since, in fig. 4, a vertical decade is equal to 4 times a horizontal decade, the inclination should be divided by 4, giving here $a = 0.167$.

In fig. 4, the lower curve represents the results of another test, this time with a much lower initial void ratio e_o. With $p_o = 12$ t/m² a straight line is obtained, and with the same inclination, i.e. the same value of a.

If the best possible straight line is determined for each test separately, it is usually found that the power a increases slightly with e_o. However, it is a quite good approximation to assume a constant a and determine p_o accordingly, as in fig. 4.

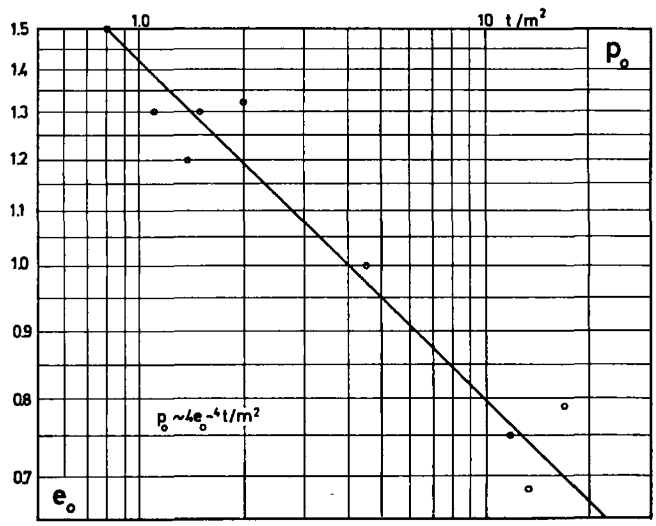

Fig. 5 Relation between p_o and e_o.

p_o is evidently a function of e_o, and the values found in 9 different tests are plotted in fig. 5. The results can be approximated by a straight line, corresponding to the formula:

$$p_o = 4\, e_o^{-4} \text{ t/m}^2 \qquad (2)$$

Duration of Previous Step.

Two identical samples were subjected to the same loads, but with different durations in a certain step (Fig. 6). Both samples were loaded from 50 to 100 t/m², but in one case for 1500 min. and in the other for 6000 min. After this, they were both loaded to 110 t/m².

The result was that both the total compression and the secondary compression (indicated by the final inclination of the time curve) were nearly the same,

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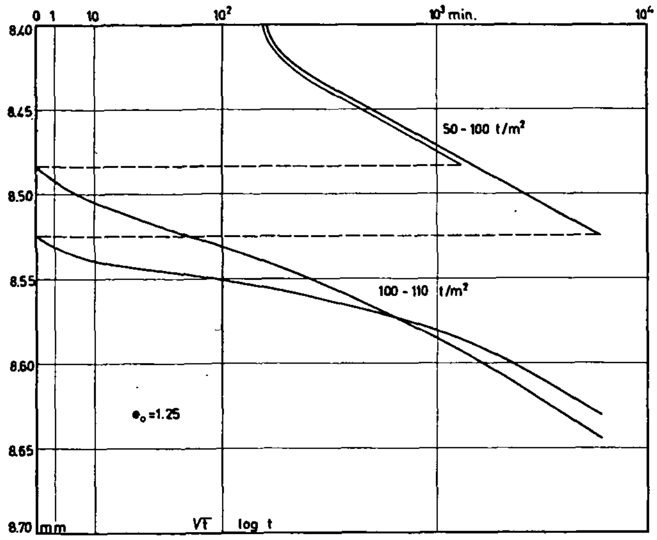

Fig. 6 Different durations of previous step.

independent of the duration of the previous step. In other words: what was gained in the previous step was lost in the following, and vice versa.

Conclusion: The void ratio e_n depends mainly on the duration Δt_n of the load p_n.

There is, however, a second order effect indicating a slight decrease of the total compression with the total time spent in the Oedometer. This effect may be due to ring friction.

Large Load Increment Ratios.

For load increment ratios $\Delta p/p > 0.3$ it is usually found that the secondary part of the time curve (for $\Delta t > 10\ t_c$) is very nearly a straight line in a semi-logarithmic plot. Since e varies very little during secondary consolidation, a straight line will also be obtained in a double-logarithmic plot.

Fig. 7 shows an example with $\Delta p/p = 1.0$. In order

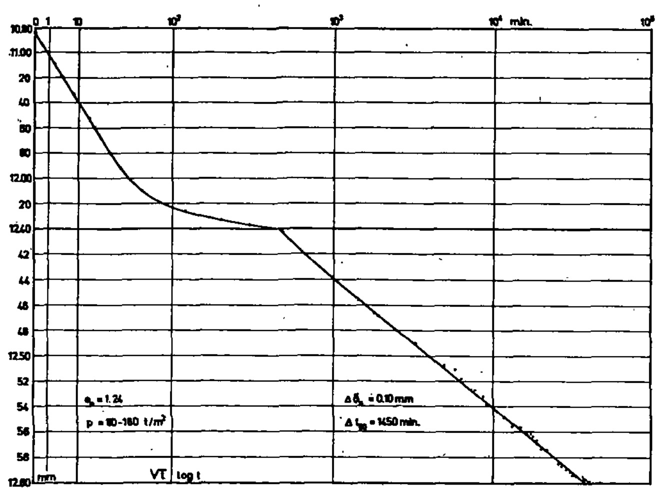

Fig. 7 Time curve for large load increment ratio.

to see the deviations from the straight line, the lower part of the diagram has been magnified 10 times in relation to the upper part. The duration of the considered loading step was 4 weeks.

For the considered large load increment ratios we propose the relation:

$$e_n = e_o \left[1 + \frac{p_n}{p_o} \left[\frac{\Delta t_n}{t_o} \right]^c \right]^{-a} \tag{3}$$

where Δt_n is the duration of the step with load p_n. For t_o is chosen the constant value of 24 hours = 1440 min.

Since we usually find $c < 0.05$ and $a < 0.3$, equation (3) will give a very nearly straight secondary time curve in both semi- and double-logarithmic plots.

As compared with the usually employed relationships involving log p and log Δt, equation (3) has the

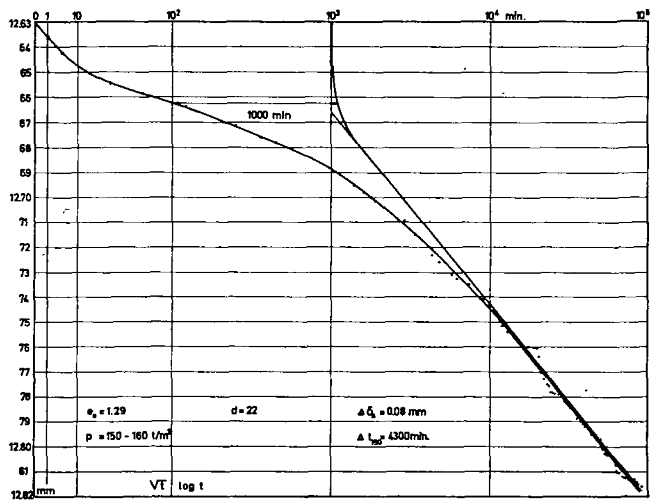

Fig. 8 Time curve for small load increment ratio.

theoretical advantage of giving a finite compression (to $e = 0$) for both $p = \infty$ and $\Delta t = \infty$. Moreover, it gives $e = e_o$ for both $p = 0$ and $\Delta t = 0$ but is, of course, not intended to be used for such small values of Δt.

Small Load Increment Ratios.

For load increment ratios $\Delta p/p < 0.2$ a different shape of the time curve is usually found, involving an inflexion point if plotted in the $\sqrt{t} - \log t$ diagram.

Fig. 8 shows a typical example with $\Delta p/p = 0.07$. The duration of the considered loading step was 9 weeks. It will be seen that also here the time curve eventually becomes straight, but only after a much longer time ($\Delta t > 100\ t_c$).

However, by adding a certain constant time t_n^o to the actual times Δt_n, it is possible to transform the time curve to a straight line for the usual duration of the secondary compression ($\Delta t > 10\ t_c$). In fig. 8 the best value of t_n^o is found by trial to be 1000 min.

This means that, for a small load increment ratio, we can write:

$$e_n = e_o \left[1 + \frac{p_n}{p_o} \left[\frac{\Delta t_n + t_n^o}{t_o} \right]^c \right]^{-a} \tag{4}$$

Of course, we can also use this formula for large load increment ratios, if t_n^o is given sufficiently small values.

For one case, namely zero load increment (after previous loading steps with large load increments), it is easy to indicate the value of t_n^o, because then the last two steps (with identical loads $p_{n-1} = p_n$) can also be considered as one step with a total duration of $\Delta t_{n-1} + \Delta t_n$. Consequently we have in this case $t_n^o = \Delta t_{n-1}$ (the duration of the previous loading step).

A simple formula, which gives the correct result both for zero and for large load increment, is:

$$t_n^o = \left[\frac{p_{n-1}}{p_n} \right]^d \Delta t_{n-1} \tag{5}$$

Since we usually find $d = 20 - 30$, t_n^o will become very small for large load increment ratios, as required.

For further investigation of the validity of formula (5), tests were made on identical samples with different load increments in the last step. Fig. 9 shows the time curves for 3 such tests, all with $p_{n-1} = 150$ t/m², but with $\Delta p = 5$, 15 and 30 t/m² respectively. By adding $t_n^o = 30$, 300 and 1300 min. respectively in the 3 cases, the indicated 3 straight lines were obtained. By insertion in formula (5) we find in all 3 cases the same $d = 24$.

Tests were also made with the same small load increment in the last step, but with different durations of the previous step. Fig. 10 shows the time curves for 2 such tests (in fact the same as in fig. 6). Both tests have $p_{n-1} = 100$ t/m² and $p_n = 110$ t/m², but Δt_{n-1} was 1500 and 6000 min. respectively. Straight lines are obtained with $t_n^o = 150$ and 600 min. respectively, t_n^o being proportional with Δt_{n-1} as required by formula (5). In both cases we find $d = 25$.

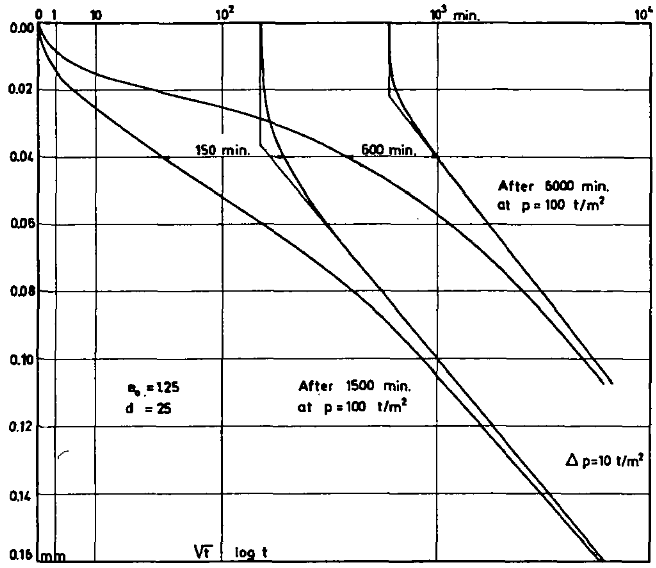

Fig. 10 Different durations of previous step.

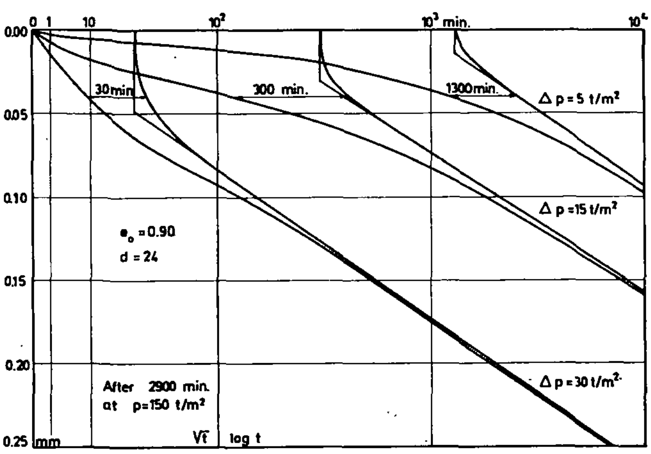

Fig. 9 Different load increments.

The Complete Equation

Since we can have small load increment ratios in more than one step, and since all previous loading steps give, in principle, a contribution to any later time curve, we must actually extend formula (5) to the following:

$$t_n^o = \sum_{i=1}^{i=n-1} \left[\frac{p_i}{p_n} \right]^d \Delta t_i \tag{6}$$

which should be used in combination with (4). The final result can be written in one equation:

$$e_n = e_o \left[1 + \frac{p_n}{p_o} \left[\sum_{i=1}^{i=n} \left[\frac{p_i}{p_n} \right]^d \cdot \frac{\Delta t_i}{t_o} \right]^c \right]^{-a} \tag{7}$$

This equation may be said to imply a "superposition" of the different loading steps, but it is a superposition of loading times, not of load increments. In the senior author's previous paper on secondary consolidation (Brinch Hansen 1961) it was assumed, that a simple superposition could be made of the time effects of all individual load increments. This assumption is now seen to be unwarranted and, consequently, the conclusions in the above-mentioned paper are not correct.

Equation (7) describes with good approximation the observed shapes of secondary time curves, both for small and large load increment ratios. Consequently,

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the distinction made by some other authors between "Type I", "Type II" and "Type III" curves is really quite arbitrary and unnecessary.

By differentiation of (4) with regard to log Δt_n we can find the secondary compression per time decade:

$$\varepsilon_s = \frac{2.3\,a\,c\,e_n}{1+e_o} \cdot \frac{1-(e_n/e_o)^{1/a}}{1+t_n^o/\Delta t_n} \tag{8}$$

or, using for e_n the approximate value (1):

$$\varepsilon_s = \frac{2.3\,a\,c\,e_o\,(1+p_n/p_o)^{-a}}{(1+e_o)\,(1+p_o/p_n)\,(1+t_n^o/\Delta t_n)} \tag{9}$$

For a large load increment ratio, e.g. $\Delta p/p = 1.0$, the time t_n^o becomes insignificantly small compared with measured values of Δt_n. If the load p_n is also large compared to p_o, we get the simple approximate expression:

$$\varepsilon_s \approx \frac{2.3\,a\,c\,e_n}{1+e_o} \tag{10}$$

This equation shows that ε_s is not only independent of the load increment ratio, but depends also rather little on the actual load and does, in fact, decrease slightly with this load, except for small loads, where equation (8) indicates an increase with further load. Several investigators have reported experimental results in agreement with this.

If p is a continuous function of t, the void ratio at the time t_n will, of course, be:

$$e_n = e_o\left[1+\frac{p(t_n)}{p_o}\left[\int_o^{t_n}\left[\frac{p(t)}{p(t_n)}\right]^d \cdot \frac{dt}{t_o}\right]^c\right]^{-a} \tag{11}$$

Determination of Constants.

As already described, p_o and a can be determined by plotting $p + p_o$ against e in a double-logarithmic diagram. e should for each loading step be the value obtained after a time $\Delta t = t_o = 1440$ min, and it should be corrected for initial compressions (bedding effects), if any.

If p_o is already known, either from an empirical formula such as (2), or by other means (see later), we can find a from (1):

$$a = \frac{\log(e_o/e_n)}{\log(1+p_n/p_o)} \tag{12}$$

Inversely, if a should be known, we can find p_o from:

$$p_o = p_n : \left[(e_o/e_n)^{1/a} - 1\right] \tag{13}$$

In order to get a reliable determination of the constant c, we should preferably use a loading step with a load increment ratio around 1.0, a rather great load p_n and a rather long duration ($\Delta t_n > 100\ t_c$). An example is shown in fig. 7. If the secondary curve is straight, we can measure the compression ε_s per time decade and then find c from (10):

$$c = \frac{(1+e_o)\,\varepsilon_s}{2.3\,a\,e_n} \tag{14}$$

If more accuracy is required, we must use (9) or (8) instead.

The last constant d can only be determined by means of a loading step with a load increment ratio around 0.1, and a rather long duration ($\Delta t_n > 200\ t_c$). The previous loading step should have a load increment ratio around 1.0. An example is shown in fig. 8. By trial we find the constant t_n^o which, when added to the observed times, gives the longest possible straight line. If the duration of the previous loading step was Δt_{n-1}, we can find d from (5):

$$d = \frac{\log(\Delta t_{n-1}/t_n^o)}{\log(p_n/p_{n-1})} \tag{15}$$

Evaluating our tests with the remoulded glacial lake clay in the way described, we have found the following average values of the 3 constants:

$a = 0.16 \qquad c = 0.022 \qquad d = 24$

Apart from the already mentioned slight dependence of a on e_o, we have not been able to establish any systematic variations with e_o. The values from different tests show, of course, some scattering, but it is quite a good approximation to assume a, c and d to be real constants for the clay investigated (standard deviation ± 15 %).

Influence of Primary Consolidation.

Until now we have tacitly assumed that each load increment Δp becomes effective instantly, i.e. already for $\Delta t = 0$. Actually, the effective load increment increases from zero to full value during primary consolidation. However, since we have so far only considered values of $\Delta t > 10\ t_c$, the error has been insignificant. But for values of Δt between t_c and $10\ t_c$ a correction must be made.

During primary consolidation the effective stress varies, not only with time, but also with distance from draining boundaries. However, for simplicity we consider here an average effective stress, which varies only with time. Moreover, we assume the following variation during the consolidation time t_{ci}:

$$p_i(t) = p_{i-1} + (p_i - p_{i-1})\sqrt{t/t_{ci}} \tag{16}$$

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We define now an "equivalent" time t_{ei} by the condition that a constant effective load p_i, acting for a time t_{ei}, should have the same effect as the varying effective load during the consolidation time t_{ci}. This gives, by means of (11):

$$\int_0^{t_{ci}} \left[p_{i-1} + (p_i - p_{i-1}) \sqrt{t/t_{ci}}\right]^d dt = p_i^d t_{ei} \qquad (17)$$

The integration is easy, but the resulting expression a little complicated. Since (16) represents an approximation anyway, the following simplified result may be sufficiently accurate:

$$t_{ei} = t_{ci} : \left[1 + \frac{d}{2}(1 - p_{i-1}/p_i)\right] \qquad (18)$$

This formula is actually correct in both the limits $p_{i-1} = 0$ and $p_{i-1} = p_i$.

We can now use the general formula (7), when we insert, instead of Δt_i, the "effective" time $\Delta t_i - t^{ci} + t_{ei}$.

Especially, the void ratio e_{cn} at the end of primary consolidation in step n can be found from (7) by inserting for Δt_n the effective time t_{en}. The primary compression in this step is then:

$$\varepsilon_c = \frac{e_{n-1} - e_{cn}}{1 + e_o} \qquad (19)$$

if the compression is related to the original height of the sample or layer (corresponding to void ratio e_o).

Normally Consolidated Intact Clay.

A similar, but less extensive series of Oedometer tests has been carried out with intact samples of the normally consolidated clay found under the earlier mentioned Aggersund Bridge.

It was found that, also in this case, the general formula (7) gave a quite accurate description of the test results. The average values of the constants were for this clay:

$$a = 0.22 \qquad c = 0.040 \qquad d = 20$$

The constant load p_o was generally found to be approximately 1.5 times the effective vertical overburden pressure in situ (see later).

Since the expansion of a normally consolidated clay is usually rather small, we can assume as an approximation, that the void ratio e_o measured on the intact sample is the same found in situ under the effective pressure p_b before construction.

Equation (7) describes the compression of an intact sample in the Oedometer and gives, correctly, $e_n = e_o$ for $p_n = 0$. The equation describing the compression in situ should, for $p_n = p_b$ and the given geological history, also give $e_n = e_o$. Moreover, for great pressures the two equations should give practically the same final void ratio.

These conditions can be satisfied if we, for the compression in situ, omit the unity sign in (7) and simply write:

$$e_n = e_o \left[\frac{p_n}{p_o}\left[\sum_{i=1}^{i=n}\left[\frac{p_i}{p_n}\right]^d \cdot \frac{\Delta t_i}{t_o}\right]^c\right]^{-a} \qquad (20)$$

where t_o is chosen arbitrarily equal to 24 hours, whereas e_o is the void ratio in situ before construction.

For the determination of p_o we must consider the geological history of the deposit. If the clay at the considered depth has been loaded by increasing effective loads p_1, p_2 etc. up to p_b, for corresponding lengths of time Δt_1, Δt_2 etc. to Δt_b, we must, in order to make (20) give $e_n = e_o$ for $p_n = p_b$, assume:

$$p_o = p_b \left[\sum_{i=1}^{i=b}\left[\frac{p_i}{p_b}\right]^d \cdot \frac{\Delta t_i}{t_o}\right]^c \qquad (21)$$

This p_o might appropriately be called the "effective preconsolidation pressure". Due to the secondary consolidation it exceeds the overburden pressure p_b, in the investigated case with about 50 %.

Using formula (6) we can write equation (20) simpler:

$$e_n = e_o \left[\frac{p_n}{p_o}\left[\frac{\Delta t_n + t_n^o}{t_o}\right]^c\right]^{-a} \qquad (22)$$

By differentiation with regard to log Δt_n we find the secondary compression per time decade:

$$\varepsilon_s = \frac{2.3\, a\, c\, e_n}{(1 + e_o)\,(1 + t_n^o/\Delta t_n)} \qquad (23)$$

By using for e_n the value given by (20), but neglecting the time effect herein, we find approximately:

$$\varepsilon_s \sim \frac{2.3\, a\, c\, e_o (p_o/p_n)^a}{(1 + e_o)\,(1 + t_n^o/\Delta t_n)} \qquad (24)$$

Calculation of Settlements.

First we must determine e_o, and then the constants a, c and d as previously described. If the geological history is known, we can calculate p_o by means of (21); otherwise we must find it from the Oedometer tests as described.

The soil under the foundation is now divided into a number of horizontal layers as usual in a settlement calculation. The compression of each layer should be calculated separately, and added later. The vertical loads p_b before and p_a after construction are calculated for the middle of each layer.

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For each layer the time t_{ca} necessary for about 90 % primary consolidation is calculated (or estimated), and the corresponding equivalent time t_{ea} calculated from (18).

Next, t_a^o is calculated from (6). It should, in principle, include all previous loading steps, also the geological ones. However, for not too small load increments in the last step the previous steps will have a negligible effect. On the other hand, they may give a significant contribution for small load increments, and in the limiting case of no load increment they give the only contribution to the secondary compression.

We calculate now e_{ca} from (22), inserting for Δt_n the time t_{ea}. The total primary compression is then:

$$\varepsilon_c = \frac{e_o - e_{ca}}{1 + e_o} \tag{25}$$

Up to the consolidation time t_c the primary compression can be assumed equal to $\varepsilon_c \sqrt{t/t_c}$.

Finally we can, for any later time Δt_a, calculate the secondary compression ε_s per time decade from (23) or (24), inserting for Δt_n the effective time $\Delta t_a - t_{ca} + t_{ea}$.

Comparison with Measured Settlements.

For the Aggersund Bridge, ε_s was calculated from (23), using the values of *a*, *c* and *d* found by Oedometer tests. The calculated secondary settlement of the abutment was 20 cm per time decade.

The corresponding measured value was 35 cm, but it should be noticed that the calculation, being based upon Oedometer tests, cannot take any regard to the actual lateral yield under the abutment. In view of this, the agreement may be considered satisfactory. At least, the calculated value is of the right order of magnitude, and the deviation between measured and calculated values goes in the right direction.

Conclusions

1) By means of an extensive series of Oedometer tests with a remoulded clay it has been shown, that a comparatively simple empirical formula as (7) can describe satisfacorily all main features of the secondary consolidation observed in the tests.

2) The same type of formula seems to be valid for Oedometer tests with a normally consolidated intact clay.

3) The formula (7) for Oedometer tests can be modified to a form (20), describing the secondary consolidation in nature under structures, provided that lateral yield is negligible.

4) A method for calculating secondary settlements of structures is indicated, and is shown to give fair agreement with measured secondary settlements of a bridge abutment in Denmark.

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