

Private Money

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Abstract

A random matching environment is constructed where banks mitigate a mismatch between the timing of investment payoffs and when agents wish to consume. Claims on banks may serve as media of exchange, i.e. private money. Two problems can emerge with private money. First, there may exist welfare-dominated equilibria where banks hold low-return assets. Second, private media of exchange may be subject to lemons problems. In spite of these problems, the introduction of fiat money can decrease welfare, as this displaces private money and results in a crowding out of productive intermediation.

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1. INTRODUCTION

While most twentieth century monetary systems have evolved into arrangements dominated by government-supplied currency, privately-issued media of exchange were common in many countries during the 19th century and before. The success, or lack of success, of these historical experiments with privately-issued monies is subject to debate. For example, the Free Banking era in the United States (1837-1863) is characterized by some as chaotic, while others (e.g. Rolnick and Weber 1983, 1984, and Rolnick, Smith, and Weber 1997) argue that the system was more or less efficient.¹ There is less controversy about the Canadian (early 19th century to 1935) and Scottish (early 19th century) systems of privately-issued bank notes, which seemed to have worked well in providing a safe and widely-acceptable medium of exchange (see White 1984, Williamson 1989, Champ, Smith and Williamson 1996). Also, the Suffolk System, in place in New England during the Free Banking era, has been cited as an efficient monetary system with privately-issued bank notes (see Calomiris and Kahn 1995 and Rolnick, Smith, and Weber 1997).

Recently, interest in private money systems has revived, in part due to the technological innovations which permit the issue of new forms of private money. An example of such an innovation is the stored-value card, which is more sophisticated than the pieces of paper issued by banks in the regimes mentioned above, as physical objects do not need to change hands in executing a transaction. However, stored-value cards share many of the properties of circulating bank notes, in that they are private liabilities which permit transactions at dispersed locations.

In this paper, a random matching model of banking and monetary exchange is constructed to study the performance of economies with privately-issued money. The

¹Private money was certainly issued in the United States prior to the Free Banking era, but I mention the Free Banking era here because it has been studied relatively intensively.

random matching model we study is most closely-related to the monetary search models studied by Williamson and Wright (1994), Trejos and Wright (1995), and Velde, Weber, and Wright (1997). Other work on private money includes Williamson (1992), Champ, Smith and Williamson (1996), Cavalcanti and Wallace (1998), and Cavalcanti, Erosa and Temzelides (1997).

There are two sectors in the economy, a search sector and a banking sector, and agents are randomly allocated between these sectors in each period. Each agent receives a random preference shock each period, and either wishes to consume or does not, but an agent can produce consumption goods in any period (though not for her own consumption). In the search sector there are random pairwise matchings, and each matched pair consists of an agent who wishes to consume and one who does not. That is, there is an absence-of-double-coincidence problem. Agents who arrive in the banking sector can invest in two alternative technologies, one with a low funding cost and a low future payoff (the bad project), and one with a high funding cost and a high future payoff (the good project). It is efficient for these investment projects to be intermediated by banks, due to the random mismatch between when a project pays off and when an agent wishes to consume. Claims on banks can circulate as private money, in that an agent can arrive in the banking sector, fund an investment project in exchange for a claim to the future returns on a bank's portfolio, and then exchange this claim for goods in the search sector.

We confine our analysis to steady states. In the model, there are two potential problems with a private money system, i.e. a steady state where fiat money is not valued and claims on banks circulate. The first is a coordination failure, as in Cooper and John (1988). In their interactions with banks and in the search sector, agents are not able coordinate their investment activities, and due to the increasing returns associated with diversification, there is a strategic complementarity. Assuming full information, there may exist two steady states, one where banks have only good

investment projects in their portfolios, and one where all banks are bad. In each steady state, any individual prefers investing with the bank rather than investing independently in the alternative project.

The second problem that can arise in the model constructed here is related to the market failures of private money systems discussed by Friedman (1960). Friedman argued that the decentralized nature of monetary exchange makes private money peculiarly susceptible to private information frictions, which we interpret here as a lemons problem. That is, because monetary transactions tend to be small and take place at dispersed locations, it is very costly or impossible to verify the quality of all private circulating claims, and this may lead, as in Akerlof (1971), to the existence of a private money equilibrium where only low-quality claims circulate, or to the non-existence of a private money equilibrium. In the model, if two steady states with private money exist under full information, then with private information the bad steady state exists, but the good steady state may not.

In spite of the above problems with private money systems, the introduction of fiat money can be harmful. That is, in any steady state where private money circulates and fiat money is valued, an increase in the quantity of fiat money simply displaces an equal quantity of private money, and decreases welfare. The decline in welfare is due to the fact that private money supports welfare-enhancing intermediation. In an example, I show that there are cases where steady states exist with circulating private money when no equilibria with valued fiat money exist. Further, there may be a steady state equilibrium with valued fiat money where there is no private money, and an equilibrium with private money where fiat money is not valued, and the private money equilibrium dominates in welfare terms. Thus, private money may be a good thing even though there are some features of the banking system (coordination failures and lemons problems) which are typical candidate rationales for putting restrictions on private money.

The remainder of the paper is organized as follows. In Section 2 the model is constructed. In Sections 3 and 4 I consider the full information and private information versions of the model, respectively. Section 4 contains an analysis of an example, and Section 5 is a conclusion.

2. THE MODEL

There is a continuum of agents, each having preferences given by

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [\theta_t u(c_t) - \gamma x_t],$$

where c_t is consumption, x_t is production, $r > 0$, $\gamma > 0$, and θ_t is an i.i.d. preference shock with $\theta_t \in \{0, 1\}$, and $\Pr[\theta_t = 1] = \frac{1}{2}$. Assume that $u(\cdot)$ is increasing and strictly concave with $u(0) = 0$, $u'(0) = \infty$, and suppose that there exists some $\hat{q} > 0$ such that $u(\hat{q}) - \gamma\hat{q} = 0$.

Each period, an agent is either in the search sector, with probability π , or in the banking sector, with probability $1 - \pi$. Preference shocks and location shocks are i.i.d. over time and across consumers. If in the search sector, an agent is randomly matched with an agent having the other realization for the preference shock. That is, matches always occur such that one agent in a matched pair has $\theta_t = 1$ and the other has $\theta_t = 0$, so there is an absence-of-double-coincidence problem in the search sector. The role for a medium of exchange in this model is thus much like the role for money in Townsend's (1980) turnpike model.

If an agent is in the banking sector at the beginning of the period, then she has the opportunity to fund an investment project. These investment projects are indivisible and of two types. A good (bad) project requires an investment of γ_g (γ_b) goods and will ultimately yield a return of R_g (R_b) consumption goods in the future. Assume that $\gamma_g > \gamma_b$ and $R_g > R_b$. For either type of investment project, if the project was funded in period t then it will yield a return in period $t + k$, $k \geq 1$, with probability

α , conditional on not having paid off in any period $t + 1, t + 2, \dots, t + k - 1$.

Investment projects must be left behind in the banking sector, and an agent may either invest on her own, or deposit the investment project with a bank. There are two types of banks, with one specializing in good investment projects and the other in bad ones.² A bank exchanges investment projects for indivisible claims which can be redeemed on demand at any period in the future. The payoff to one of these claims, if redeemed, is a proportional share in the total return on the bank's portfolio in that period. That is, the bank divides the returns on its maturing investment projects equally among those who request redemption in the current period. In order to redeem a claim on a bank, an agent must return to the bank.

To rule out credit arrangements, I assume that agents who arrive in the banking sector are not able to communicate with each other, though each is able to communicate with the bank. Lack of communication can be imposed by assuming sequential service, as in Diamond-Dybvig banking models (e.g. Diamond and Dybvig 1983, Wallace 1988). The timing within the period then works as follows. Agents arrive in the banking sector and contact the banks sequentially, with agents having bank claims announcing whether or not they wish to redeem these claims. After all arrivals have visited the banks, banks determine the payoff to redeemed claims, and the agents in the banking sector again visit banks in sequence, this time receiving payoffs on any redeemed claims, and depositing new investment projects with banks.

I assume that an agent must always consume before producing again, and that in period 0 each agent is able to produce. Some fraction M of agents is endowed with one unit each of indivisible fiat money in period 0, and the inventory technology is such that an agent can not hold more than one unit of fiat money or more than one

²There is nothing to gain from allowing banks to diversify between good and bad investment projects. Claims on banks would then just be priced according to the type of project deposited at the bank.

bank claim, and cannot hold a bank claim and fiat money simultaneously.

3. FULL INFORMATION

Here, agents are assumed to be able to verify whether or not particular claims have investment projects backing them and they know whether these projects are good or bad. In this section, we want to determine the conditions under which intermediated claims will circulate, and when they will not. I then consider whether or not fiat money can be valued, and if valued fiat money can coexist with private money (circulating intermediated claims).

First, suppose that there are good and bad banks in the steady state. Let ϕ_i denote the steady state redemption value of a claim to a bank holding type i investment projects, where $i = g, b$. Then suppose that x_i is the quantity of projects held by the type i bank in the steady state, and y_i is the steady state quantity of investment in new type i projects. Then in the steady state, the quantity of new investment equals the quantity of projects that pay off in each period, or $\alpha x_i = y_i$. We also want the number of outstanding claims on the bank to be constant in the steady state, i.e. the number of redemptions that occur must equal the quantity of new investment, y_i . Therefore, since ϕ_i is the share in the bank's current payoff of an agent redeeming their claim, we have

$$\phi_i = \frac{\alpha x_i R_i}{y_i} = R_i.$$

That is, given perfect diversification by banks (on both sides of the balance sheet), in the steady state the flow of new investment projects into the bank's portfolio equals the number of maturing projects. Therefore, since the number of agents redeeming claims on the bank equals the number of new bank liability holders, the redemption value of bank claims is equal to the return on a maturing investment project.

Let q_i denote the price at which a claim on a type i bank trades in the steady state,

q_m the price of a unit of fiat money in terms of consumption goods, V_i the value of holding a claim on bank i at the end of the period, for $i = g, b$, V_m the value of holding fiat money, and V_p the value of being a would-be producer (an agent who holds no asset). Further, β is the fraction of agents who are would-be producers in the steady state, μ is the fraction of agents holding fiat money, and η is the fraction of agents holding claims on banks who hold good claims. Then, the Bellman equations for the consumer's problem imply

$$rV_i = \frac{\pi\beta}{2} \max [0, u(q_i) + V_p - V_i] + \frac{(1-\pi)}{2} \{u(R_i) + \max [V_p - V_i, V_g - \gamma_g - V_i, V_b - \gamma_b - V_i]\}, \quad (1)$$

for $i = g, b$,

$$rV_p = \frac{\pi(1-\beta-\mu)\eta}{2} \max [0, V_g - V_p - q_g] + \frac{\pi(1-\beta-\mu)(1-\eta)}{2} \max [0, V_b - V_p - q_b] + \frac{\pi\mu}{2} \max [0, V_m - V_p - q_m] + (1-\pi) \max [0, V_g - V_p - \gamma_g, V_b - V_p - \gamma_b], \quad (2)$$

$$V_m = \frac{\pi\beta}{2} \max [0, u(q_m) + V_p - V_m]. \quad (3)$$

In (1), an agent with a bank claim goes to the search sector with probability π , and then meets an agent who is a would-be producer with probability β . A necessary condition for trade to take place in this circumstance is that the agent with the bank claim wishes to consume and the would-be producer does not; this state occurs with probability $\frac{1}{2}$. A claim on a type i bank exchanges for q_i goods, and the holder of the bank claim will trade if the net gain from trade is non-negative at that price (we describe the game that will determine q_i below). With probability $1 - \pi$ the bank claim holder goes to the banking sector. If the agent does not want to consume ($\theta_t = 0$) then no action is taken and the agent is still holding the bank claim at the end of the period. Otherwise, with probability $\frac{1}{2}$ the agent wants to consume and

redeems the claim for R_i units of consumption, and then must make a decision as to investing in a good or bad bank, or doing nothing and so becoming a would-be producer. Equations (2) and (3) can be described similarly.

Steady States Where Fiat Money is not Valued

First consider the case where there is full information and fiat money is not valued, so that $q_m = \mu = 0$, and note that I am still assuming full information. Here, we want to derive conditions under which there is strictly positive investment intermediated by banks, and to determine when claims on banks do not circulate and when they do.

No Circulating Bank Claims.—

Suppose first that neither good nor bad bank claims circulate. In a steady state equilibrium where there are banks of type i , from (1) we get

$$V_i = \frac{(1 - \pi)}{2r} [u(R_i) - \gamma_i], \quad (4)$$

for $i = g, b$. To focus on interesting steady state equilibria, for now I will rule out steady states where claims on banks do not circulate by assuming that, if the bank's claims do not circulate, that no agent would invest in the bank, i.e. $V_i - \gamma_i < 0$, or

$$(1 - \pi)[u(R_i) - \gamma_i] - 2r\gamma_i < 0, \quad (5)$$

for $i = g, b$. Assuming that (5) holds is useful since this implies that intermediation only takes place if bank claims circulate. Thus, if steady state equilibria exist where there are banks, then the circulation of claims on banks is necessary to banks' viability. In Section 5 I consider an example where (5) is not imposed, so that all cases can be examined.

Bank Claims Circulate.—

Now, we will consider steady states where bank claims circulate and fiat money is not valued, so the only “money” in this economy is private money. In order to have trade between the holders of bank claims and would-be producers, we must have

$$V_p = V_i - \gamma_i, \quad (6)$$

otherwise no agent would want to invest in a bank, given the opportunity, or no agent would wish to be a would-be producer. Thus, whenever an agent redeems a bank claim, or arrives in the banking sector as a would-be producer, she is indifferent between investing in the bank and being a would-be producer at the end of the period.

The price q_i is determined by a take-it-or-leave-it offer made by the holder of the bank claim. This implies that

$$V_i - V_p - q_i = 0, \quad (7)$$

that is, the holder of the bank claim sets q_i so that the producer is indifferent between accepting and rejecting the offer. Then, (2), (7), and (8) imply that $V_p = 0$, so that (6) gives $V_i = \gamma_i$, and from (7) we have $q_i = \gamma_i$. Equation (1) then gives

$$r\gamma_i = \frac{\pi\beta_i}{2} [u(\gamma_i) - \gamma_i] + \frac{(1-\pi)}{2} [u(R_i) - \gamma_i], \quad (8)$$

where β_i denotes the fraction of agents in the population who are would-be producers in the type- i investment-project steady state equilibrium. Now, suppose that there is a steady state equilibrium where claims on good and bad banks circulate. Then, (8) must hold for $i = g, b$, with $\beta_g = \beta_b$, which happens only for a measure-zero region of the parameter space. We will therefore eliminate this case from consideration.

Next, consider an equilibrium where there are claims circulating only on banks investing in type i projects. For this equilibrium to exist, we require that it be in the interest of a holder of a bank claim to be willing to trade with a producer. That is, $u(q_i) + V_p - V_i \geq 0$, or

$$\gamma_i \leq \hat{q}. \quad (9)$$

Also, it must be in the interest of the holder of a bank claim to redeem the claim given the opportunity, i.e.

$$u(R_i) - \gamma_i \geq 0. \quad (10)$$

Given (8) we can solve for β_i as follows:

$$\beta_i = \frac{2r\gamma_i - (1 - \pi)[u(R_i) - \gamma_i]}{\pi[u(\gamma_i) - \gamma_i]}. \quad (11)$$

For this equilibrium to exist, we need $0 < \beta_i < 1$. Now, (5) and (9) guarantee that the numerator and denominator, respectively, on the right-hand side of (11) are positive, so $\beta_i > 0$. For $\beta_i < 1$ we need, from (11),

$$(1 + 2r)\gamma_i < (1 - \pi)u(R_i) + \pi u(\gamma_i). \quad (12)$$

Finally, for this equilibrium to exist, it cannot be in the interest of any agent to invest in the other technology, $j \neq i$. Since there are no banks of type j in existence in this steady state equilibrium, an agent investing in the type j technology must do this independently. Letting V_I denote the value associated with investing in the bad technology, and supposing first that a claim to this investment project would circulate, we have

$$rV_I = \frac{\pi\beta_i(1 - \alpha)}{2} [u(q_I) + V_p - V_I] + \pi\alpha(V_p - V_I) + (1 - \pi)\alpha \left[\frac{u(R_j)}{2} + V_p - V_I \right], \quad (13)$$

where q_I denotes the price at which the claim to the bad investment project would trade. Since q_I is determined by a take-it-or-leave-it offer by the holder of the claim on the project to a would-be producer, we have $V_I - V_p - q_I = 0$, or $V_I = q_I$. Then, substituting in (13) we obtain an equation determining q_I :

$$2(r + \alpha)q_I - (1 - \pi)\alpha u(R_j) = \pi\beta_i(1 - \alpha) [u(q_I) - q_I] \quad (14)$$

If there is a solution to (14) it is unique. We require that it be in the interest of the agent with a claim on the investment project to trade at the price q_I , which implies

that we must have $q_I \leq \hat{q}$, or using (14),

$$\hat{q} \geq \frac{(1 - \pi)\alpha u(R_j)}{2(r + \alpha)} \quad (15)$$

For it not to be in any agent's interest to invest in a type j investment project, in this case where the claim to the project could circulate, we must then have $V_I - \gamma_j \leq V_i - \gamma_i = 0$. But since $V_I = q_I$, this gives $q_I \leq \gamma_j$, or from (14),

$$2(r + \alpha)\gamma_j - (1 - \pi)\alpha u(R_j) \geq \pi\beta_i(1 - \alpha)[u(\gamma_j) - \gamma_j]. \quad (16)$$

There are parameter values satisfying (16) if and only if (15) is satisfied.

Next, consider the case where an agent considers investing in a type j project, but the claim to the project would not circulate. Then, we have

$$rV_I = \pi\alpha(V_p - V_I) + (1 - \pi)\alpha \left[\frac{u(R_j)}{2} + V_p - V_I \right],$$

and given that $V_p = 0$ we can solve for V_I to get

$$V_I = \frac{(1 - \pi)\alpha u(R_j)}{2(r + \alpha)}.$$

The incipient price at which a claim to the project would trade is $q_I = V_I$, given a take-it-or-leave-it offer by the holder of the claim to the project. For claims not to be traded, we require $q_I = V_I > \hat{q}$, i.e.

$$\hat{q} < \frac{(1 - \pi)\alpha u(R_j)}{2(r + \alpha)}, \quad (17)$$

and for agents to have no interest in investing in a type j project, we require $V_I \leq \gamma_j$, or

$$2(r + \alpha)\gamma_j - (1 - \pi)\alpha u(R_j) \geq 0. \quad (18)$$

But (9) implies that $\gamma_j < \hat{q}$, so that (17) implies that (18) does not hold. Thus, (15) is necessary for the steady state equilibrium with private money of type i to exist.

Proposition 1: There exist parameter values for which there are at least two steady state equilibria. In one of these fiat money is not valued and type g private money circulates, and in the other fiat money is not valued and type b private money circulates.

Proof. Suppose that $R_g = R_b$ and $\gamma_g = \gamma_b$. Then,

$$\hat{q} > \frac{(1 - \pi)u(R_g)}{2r + 1 - \pi} \quad (19)$$

is a necessary condition for (5) and (12) to be satisfied for $i = g, b$. We also require (10), or the sufficient condition

$$u(R_g) - \gamma_g > 0. \quad (20)$$

Then, if we choose

$$\gamma_g = \gamma_b \in \left(\frac{(1 - \pi)u(R_g)}{2r + 1 - \pi}, \hat{q} \right), \quad (21)$$

(5) and (12) are satisfied for $i = g, b$. Now, either (15) is not satisfied for $j = g, b$, or it is, but in the second case it is straightforward to show that (16) is satisfied for $(i, j) = (g, b), (b, g)$ (this is simply an implication of the fact that any agent would rather invest in a banking claim than invest independently in the same type of asset). Therefore, by continuity we can choose $(R_g, R_b, \gamma_g, \gamma_b)$ in any neighborhood of the set described by $R_g = R_b$, and satisfying (19)-(21), and the steady state equilibria with circulating type g and type b monies exist in that neighborhood. \square

Proposition 1 makes use of the strategic complementarity inherent in intermediation activity in the model. That is, given the existence banking in the steady state, any agent would strictly prefer to invest in a perfectly diversified bank rather than invest on her own in a project of the same type, given the stochastic mismatch between consumption needs and returns on investment projects. Therefore, if all projects are the same and a steady state equilibrium where private money circulates can be

supported, then we can make small changes in the startup costs of investments and in investment returns, and obtain multiple steady states. Thus, the strategic complementarity inherent in the increasing returns from diversification in intermediation can lead to multiple steady states. If agents coordinate on banks with bad (good) investment projects, it may be in no agent's interest to invest in a good (bad) project.

Suppose that we use the mean level of expected utility in the steady state as a welfare measure. That is, welfare in a banking steady state where all banks are of type i is given by

$$W = \beta_i V_p + (1 - \beta_i) V_i = (1 - \beta_i) \gamma_i,$$

where β_i is defined by (11). Since $\gamma_g > \gamma_b$, the holders of bank claims are better off in the steady state with good banks than in the steady state with bad banks. Would-be producers are indifferent between the two steady states, as $V_p = 0$ in either case. However, in general $\beta_g \neq \beta_b$, and given the assumptions we have made thus far, we could have $\beta_g \leq \beta_b$ or $\beta_g > \beta_b$. Our interest is in the case where $(1 - \beta_g) \gamma_g > (1 - \beta_b) \gamma_b$, since this defines a steady state equilibrium with type g private money as “good.” Using (11), we get

$$\begin{aligned} & \gamma_g \left\{ \frac{\pi [u(\gamma_g) - \gamma_g] - 2r\gamma_g + (1 - \pi) [u(R_g) - \gamma_g]}{\pi [u(\gamma_g) - \gamma_g]} \right\} \\ & \geq \gamma_b \left\{ \frac{\pi [u(\gamma_b) - \gamma_b] - 2r\gamma_b + (1 - \pi) [u(R_b) - \gamma_b]}{\pi [u(\gamma_b) - \gamma_b]} \right\}, \end{aligned} \tag{22}$$

and we will assume in what follows that (22) holds. Given (22) and Proposition 1, there exist parameter values such that steady state equilibria with good and bad private money exist, and the good private money steady state dominates in welfare terms.

Steady States with Valued Fiat Money

Now, consider steady state equilibria where $q_m > 0$ and $\mu > 0$. There are potentially four types of these steady state equilibria with valued fiat money: there is strictly positive investment but private money does not circulate; private money circulates; and investment is zero.

First, considering steady states with valued fiat money and positive investment where private money does not circulate, (4) determines the value of a claim on a bank of type i . We must have $V_i - \gamma_i \geq 0$, and for fiat money to be valued, agents with fiat money must be able to trade with would-be producers. But from (2) we have $V_p = 0$, so to guarantee that agents will choose to be would-be producers, we must have $V_i - \gamma_i = 0$. But then this steady state equilibrium exists only as a hairline case, and we can ignore it.

Next, consider a money and banking steady state where banks hold investment projects of type i . Here, $V_i = \gamma_i$ and $V_p = 0$ as in the banking steady state analyzed in the previous section, and $\beta = \beta_i$ as determined in (11). The price of fiat money in terms of consumption goods is determined by a take-it-or-leave-it offer made by a fiat money-holder when she meets a would-be producer who does not want to consume, that is $V_m - V_p - q_m = 0$, or $V_m = q_m$. Then, from (3), q_m is determined by

$$rq_m = \frac{\beta_i \pi}{2} [u(q_m) - q_m], \quad (23)$$

and (23) has a unique solution for q_m , with $q_m < \hat{q}$, so that it will be in the interest of the fiat money-holder to trade given the price determined by the take-it-or-leave-it offer. The conditions for existence of this steady state equilibrium are the same as for the steady state equilibrium with private money and $q_m = 0$, except that we now require $\beta_i < 1 - \mu$, so that (12) is altered to

$$2r\gamma_i - (1 - \pi) [u(R_i) - \gamma_i] < (1 - \mu)\pi [u(\gamma_i) - \gamma_i]. \quad (24)$$

Therefore, if this steady state equilibrium exists, the steady state equilibrium with type i private money and $q_m = 0$ exists, but we can always make μ sufficiently large that (24) will not hold, since the left-hand side of (24) must be positive for the steady state equilibrium to exist, and the right-hand side is decreasing in μ and equal to zero when $\mu = 1$. Thus, if the quantity of fiat money in existence is sufficiently large, it can drive out private money.

Welfare in the money and banking equilibrium is given by

$$W = \beta V_p + (1 - \beta - \mu)V_i + \mu V_m = (1 - \beta - \mu)\gamma_i + \mu q_m,$$

so that money improves welfare (i.e. W is strictly increasing in μ) if and only if $q_m > \gamma_i$, since q_m does not depend on μ . From (11) and (23), $q_m > \gamma_i$ if and only if $u(R_i) - \gamma_i < 0$. But if the steady state equilibrium with private money and $q_m > 0$ exists, then $u(R_i) - \gamma_i > 0$, from (10). Therefore, the introduction of fiat money cannot increase welfare if fiat money coexists with circulating bank claims. This is because fiat money simply crowds out bank claims one-for-one, and bank claims are superior to fiat money if they circulate. That is, fiat money does not have better properties than private bank claims as a medium of exchange, but private bank claims have the advantage that they permit welfare-enhancing production. Of course, investment projects may be too costly to finance, in which case, if $\gamma_i \geq \hat{q}$, then $u(\gamma_i) - \gamma_i \leq 0$, and steady state equilibria with private money do not exist.

This result contrasts in an interesting way with results from commodity money economies (e.g. Burdett, Trejos, and Wright 1998). Here, the introduction of fiat money can be a bad thing because it crowds out private money and eliminates productive intermediation. In commodity money models, which can be considered private money models, it is typically the case that replacing commodity money with fiat money is a good thing. This is because fiat money eliminates the cost of producing the private money, and frees up commodities for consumption purposes.

Note that the fact that there may be a coordination problem in banking need not imply a role for fiat money. That is, even if two steady states exist, introducing fiat money will decrease welfare in either steady state. While one type of private money may be preferred to another, bad private money is preferred to fiat money.

The remaining steady state equilibrium to consider is the one where there is valued fiat money and no investment. Here, $V_p = \mu \frac{\pi}{2}(V_m - V_p - q_m) = 0$, since q_m is determined by a take-it-or-leave-it offer by the holder of fiat money to the would-be producer. We then have $V_m = q_m$, and q_m is determined by

$$rq_m = \frac{(1 - \mu)\pi}{2} [u(q_m) - q_m], \quad (25)$$

so that there is a unique solution for q_m , with $q_m < \hat{q}$, and the holder of fiat money is then willing to trade at this price. For this to be an equilibrium, it must be the case that agents do not wish to use either investment technology. As for the private money equilibria discussed in the previous section, we need only consider the case where a claim to an investment project would circulate if such an investment project were undertaken. The value of investing in technology i , supposing that the claim to the technology is tradeable at price q_i , is denoted V_I^i , which is determined by

$$rV_I^i = \frac{\pi(1 - \mu)(1 - \alpha)}{2} [u(q_i) + V_p - V_I^i] + \pi\alpha(V_p - V_I^i) + (1 - \pi)\alpha \left[\frac{u(R_i)}{2} + V_p - V_I^i \right].$$

Then, given that q_i is determined by a take-it-or-leave-it offer by the owner of the claim to the project, we have $V_I^i = q_i$, and substituting for V_p and V_I^i in the above equation, we obtain an equation solving for q_i , that is

$$2(r + \alpha)q_i^i - (1 - \pi)\alpha u(R_b) = \pi(1 - \mu)(1 - \alpha) [u(q_i^i) - q_i^i],$$

and we require that $q_i^i \leq \hat{q}$ in order for this to be consistent with the claim circulating, i.e.

$$\hat{q} \geq \frac{(1 - \pi)\alpha u(R_i)}{2(r + \alpha)}. \quad (26)$$

For it not to be in the interest of an agent to invest, we require

$$2(r + \alpha)\gamma_i - (1 - \pi)\alpha u(R_i) \geq \pi(1 - \mu)(1 - \alpha) [u(\gamma_i) - \gamma_i]. \quad (27)$$

What we have shown is that, with full information, the only case that can be made for the introduction of fiat money is that steady state equilibrium would otherwise not exist, for example if the costs of investment are too high, or if there exists a steady state equilibrium with private money and an equilibrium with valued fiat money and no investment, and the second equilibrium dominates the first in terms of welfare. In Section 5 I explore this further by way of example.

4. PRIVATE INFORMATION

Private information frictions are often given as a rationale for the introduction of government-supplied fiat money, and for legal restrictions on the issue of private money, so it is useful to consider such frictions here. I will suppose that there is private information about the quality of investment projects. In particular, an agent accepting a banking claim does not know the quality of the investment projects in the bank's portfolio, or the number of projects the bank is holding. However, everyone is able to detect the difference between a claim on a bank with no projects in its portfolio (a claim which is costless to create and therefore worthless) and a claim on a bank having a positive number of projects. Further, when an agent makes a deposit of an investment project in a bank, we assume that she knows the type of all the other projects the bank is holding in its portfolio.³

As in the full information case, I consider steady states where claims on banks circulate. Given the information structure, all such claims must trade at the same

³There is a potential problem here, in that an agent might be a would-be producer and be offered a claim on a bank for which she knows the quality of the portfolio, as she previously deposited in the same bank. However, we can easily rig the environment so that this possibility is ruled out, with the analysis going through in exactly the same way.

price, q . Notation will be the same as in the previous section, except that we need to allow for the fact that an agent may produce in exchange for a note of unknown quality, in which case we let V_u denote its value. Also, η ($1 - \eta$) will denote the probability that such a note is a claim on a good (bad) bank, where $0 \leq \eta \leq 1$. The Bellman equations from the agent's dynamic optimization problem give

$$rV_i = \frac{\pi\beta}{2} \max[0, u(q) + V_p - V_i] + \frac{(1-\pi)}{2} \{u(R_i) + \max[V_p - V_i, V_g - \gamma_g - V_i, V_b - \gamma_b - V_i]\}, \quad (28)$$

for $i = g, b$,

$$rV_p = \frac{\pi(1-\beta-\mu)}{2} \max[0, V_u - V_p - q] + \frac{\pi\mu}{2} \max[0, V_m - V_p - q_m] + (1-\pi) \max[0, V_g - V_p - \gamma_g, V_b - V_p - \gamma_b], \quad (29)$$

$$V_m = \frac{\pi\beta}{2} \max[0, u(q_m) + V_p - V_m]. \quad (30)$$

$$V_u = \frac{\pi\beta}{2} \max[0, u(q) + V_p - V_u] + \frac{(1-\pi)}{2} \left\{ \begin{array}{l} \eta u(R_g) + (1-\eta)u(R_b) \\ + \max[V_p - V_i, V_g - \gamma_g - V_i, V_b - \gamma_b - V_i] \end{array} \right\} \quad (31)$$

Steady States with Strictly Positive Investment

First consider the steady state equilibria where fiat money is not valued. As before, we will consider only those steady state equilibria where private money circulates. Again, this implies that $V_i - \gamma_i = V_p$ if there are type i banks in the steady state, and given take-it-or-leave-it offers by holders of bank claims to would-be producers, (29) implies that $V_p = 0$, so that $V_i = \gamma_i$ if there are type i banks.

Suppose first that there are both good and bad banks in the steady state. Then, equation (28) gives

$$r(\gamma_g - \gamma_b) = \frac{\pi\beta}{2}(\gamma_b - \gamma_g) + \frac{(1-\pi)}{2} [u(R_g) - u(R_b) - \gamma_g + \gamma_b], \quad (32)$$

which is satisfied only for a measure zero subset of the parameter space, so I will ignore this case.

The other two cases, where there are only good banks, or only bad banks, are identical to the steady states studied in the full information case, except that there are different incentives for agents to invest in the technology not used by banks in equilibrium. That is, if there are only type i banks in the steady state, an agent contemplating investing in the alternative technology knows that the claim to this project cannot be distinguished from a claim on a type i bank. Letting V_I^j denote the value of investing in an investment project of type $j \neq i$, we have

$$rV_I^j = \frac{\pi\beta_i(1-\alpha)}{2} [u(q) + V_p - V_I^j] + \pi\alpha(V_p - V_I^j) + (1-\pi)\alpha \left[\frac{u(R_j)}{2} + V_p - V_I^j \right], \quad (33)$$

where β_i is determined by (11). Now, we require that $V_I^j - \gamma_j \leq 0$ in the steady state or, given that $q = \gamma_i$ and $V_p = 0$ from (33) we have

$$2(r + \alpha)\gamma_j - (1 - \pi)\alpha u(R_j) \geq \pi\beta_i(1 - \alpha)[u(\gamma_i) - \gamma_j], \quad (34)$$

for $j \neq i$. Now, compare (34) with (16). For $i = g$ and $j = b$ the left-hand sides of (16) and (34) are identical, but the right-hand side of (34) is greater than the right-hand side of (16). For $i = b$ and $j = g$, the left-hand sides of (16) and (34) are identical, and the right-hand side of (34) is less than the right-hand side of (16). Thus, private information is favorable to bad banks, but unfavorable to good banks, as one might anticipate. In other words, if the full information steady state equilibrium with bad private money exists, then so does the private information steady state with bad private money. If the private information steady state with good private money exists then so does the full information steady state with good private money.

Further, it is straightforward to show, given (16) and (34), that if full information steady state equilibria with good private money and with bad private money exist,

then with private information the good private money equilibrium may not exist. Therefore, there are conditions under which private information leads to a standard lemons problem, in that the steady state equilibrium with private money and highest welfare ceases to exist due to the private information friction and the only private money steady state is one with bad banks.

Valued Fiat Money

The analysis of steady states with valued fiat money is identical to that for full information. As was the case there, fiat money cannot increase welfare as long as it coexists with circulating bank claims. The introduction of fiat money simply displaces an equal quantity of private money, and the net productive benefit of financial intermediation is lost. Thus, in spite of the fact that there can be bad outcomes in banking and the creation of private money due to strategic complementarities and private information frictions, fiat money may not be able to achieve better outcomes.

The introduction of fiat money can permit the existence of preferred steady states only if steady states with private money do not exist, or if in steady states with private money the private sector produces a sufficiently suboptimal quantity of circulating media of exchange. However, any steady where fiat money improves welfare will have the property that there is no circulating private money.

5. AN EXAMPLE: ONE PROJECT TYPE AND PIECEWISE-LINEAR UTILITY

In this section, we consider a tractable example where we can consider in more detail the characterization of steady state equilibria and the evaluation of welfare across equilibria. Since there is never more than one investment technology used in the steady state, it is convenient to assume here that only one such technology

exists, and thus eliminate the complicated issue of existence of equilibrium when there are two investment technologies. We suppose that the single investment technology requires γ units of the consumption good to fund, and that it pays off $R = \kappa\gamma$ units, where $\kappa > 1$.

The major simplification is to assume that the utility function takes the form

$$\begin{aligned} u(q) &= \delta_1 q, \text{ for } 0 \leq q \leq q^* \\ u(q) &= \delta_1 q^* + \delta_2 (q - q^*), \text{ for } q \geq q^*, \end{aligned}$$

where $0 < \delta_2 < 1 < \delta_1$ and $q^* > 0$. Thus, $u(c)$ is piecewise linear, and the function $u(q) - q$ is tent-shaped. We then have

$$\hat{q} = \frac{(\delta_1 - \delta_2)q^*}{1 - \delta_2}. \quad (35)$$

There will be three relevant cases to consider, which differ according to whether γ and $\kappa\gamma$ are less than or greater than q^* .

Case 1: $0 < \gamma < q^*$, $0 < \kappa\gamma \leq q^*$

Suppose that fiat money is not valued, and consider a steady state equilibrium where claims on banks do not circulate. Then, from (5), such a steady state equilibrium exists if and only if

$$2r - (1 - \pi)(\delta_1\kappa - 1) \leq 0. \quad (36)$$

Alternatively, suppose a steady state where fiat money is not valued and private money circulates. Then, given the restrictions we have assumed on parameters, the counterparts of (9) and (10) hold. Solving for β as in (11), we have

$$\beta = \frac{2r - (1 - \pi)(\delta_1\kappa - 1)}{\pi(\delta_1 - 1)}, \quad (37)$$

and for $0 < \beta < 1$ we require

$$2r - (1 - \pi)(\delta_1\kappa - 1) > 0, \quad (38)$$

and

$$2r - \delta_1[(1 - \pi)\kappa + \pi] - 1 < 0. \quad (39)$$

Here, since we have only one type of investment project, we do not need to include conditions guaranteeing that the other type of project is not used in equilibrium.

Now, suppose fiat money is valued, and private money circulates in the steady state. Then β is determined by (37) as before, and the price of fiat money is determined by (23). For a solution, we must have $q_m \geq q^*$, and (23) gives

$$q_m = \frac{\beta\pi(\delta_1 - \delta_2)q^*}{2r + \beta\pi(1 - \delta_2)}. \quad (40)$$

But then from (37) and (40), $q_m \geq q^*$ implies that

$$-(1 - \pi)(\delta_1\kappa - 1) \geq 0,$$

which does not hold, since $\delta_1 > 1$ and $\kappa > 1$. Therefore, in this case there is a steady state equilibrium where private money does not circulate if (36) holds, a steady state where private money circulates if (38) and (39) hold, and there is no steady state equilibrium where fiat money is valued and private money circulates.

Now, suppose that there is no private banking in the steady state, but fiat money is valued. Then from (25), we get

$$q_m = \frac{(1 - \mu)\pi(\delta_1 - \delta_2)q^*}{2r + (1 - \mu)\pi(1 - \delta_2)}, \quad (41)$$

and we require that (41) solve for $q_m \geq q^*$, which implies that

$$\mu \leq 1 - \frac{2r}{\pi(\delta_1 - 1)}. \quad (42)$$

Therefore, since μ must be strictly positive in this steady state equilibrium, a necessary condition for existence of this steady state is

$$2r - \pi(\delta_1 - 1) < 0. \quad (43)$$

Further, in this equilibrium, agents must not have an incentive to invest independently or, from (27),

$$2(r + \alpha) - (1 - \pi)\alpha\delta_1\kappa - \pi(1 - \mu)(1 - \alpha)(\delta_1 - 1) \geq 0. \quad (44)$$

Now, suppose that fiat money is valued and there is private banking, but private money does not circulate. Then condition (36) must hold, and the fact that private money circulates implies that $V_p = V - \gamma$, where V is the value associated with holding a claim on a bank. But (2) implies that $V_p = 0$ (would-be producers get no surplus from trading with money-holders given take-it-or-leave-it offers) and $V - \gamma = 0$ only when (36) holds with equality, which is a hairline case. Thus, in general, fiat money is not valued in case 1.

Case 2: $0 < \gamma \leq q^*$, $\kappa\gamma > q^*$

As for the previous case, we first suppose that there is positive investment in the steady state, but claims on banks do not circulate. Then, from (5), the steady state equilibrium exists if and only if

$$2r - (1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1] \leq 0. \quad (45)$$

Next, suppose that there is positive investment in the steady state and private money circulates. Then, following the same steps as in case 1, we have

$$\beta = \frac{2r - (1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1]}{\pi(\delta_1 - 1)}, \quad (46)$$

and a steady state equilibrium exists if and only if

$$2r - (1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1] > 0, \quad (47)$$

and

$$2r - (1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1] - \pi(\delta_1 - 1) < 0. \quad (48)$$

In a steady state equilibrium where private money circulates and fiat money is valued, β is determined by (46) and q_m is determined by (40). Therefore, similar to case 1, this steady state equilibrium exists if and only if

$$-(1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1] \geq 0, \quad (49)$$

which does not hold, since $q^* \geq \gamma$ and $\kappa\gamma > q^*$. Therefore, this steady state equilibrium does not exist. Following similar arguments to case 1, a steady state with banking where private money does not circulate and fiat money is valued also does not exist. The analysis of the steady state where fiat money is valued and there is no investment is identical to case 1, except that we replace (44) with

$$2(r + \alpha) - (1 - \pi)\alpha \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa \right] - \pi(1 - \mu)(1 - \alpha)(\delta_1 - 1) \geq 0. \quad (50)$$

Case 3: $\gamma \geq q^*$, $\kappa\gamma > q^*$

Here, in a steady state equilibrium with positive investment and non-circulating claims on banks, the results are identical to case 2, i.e. this steady state exists if and only if (45) holds. In a steady state equilibrium where fiat money is not valued and private money circulates, we obtain

$$\beta = \frac{2r - (1 - \pi) \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1 \right]}{\pi \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2 - 1 \right]} \quad (51)$$

and a steady state equilibrium exists if and only if

$$2r - (1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1] > 0, \quad (52)$$

and

$$2r - (1 - \pi)[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1] - \pi \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2 - 1 \right] < 0. \quad (53)$$

Next, suppose that there is a steady state equilibrium where fiat money is valued and private money circulates. Then β is determined by (51) and we can solve for q_m using

(40) to get

$$q_m = \frac{\beta\pi(\delta_1 - \delta_2)q^*}{2r + \beta\pi(1 - \delta_2)}. \quad (54)$$

Since we require $q_m \geq q^*$, we must then have, from (51) and (54),

$$2r(\delta_1 - \delta_2)\left(1 - \frac{q^*}{\gamma}\right) - (1 - \pi)(\delta_1 - 1) \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1 \right] \geq 0. \quad (55)$$

Further, for the equilibrium to exist, we must have $\beta < 1 - \mu$, i.e. the fraction of private-money-holders in the population must be strictly positive, so from (51),

$$2r - (1 - \pi)\left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa - 1\right] - \pi(1 - \mu) \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2 - 1 \right] < 0. \quad (56)$$

Now, suppose that there exists a steady state equilibrium where private money circulates. Therefore, (52) and (53) hold. Then, we can always choose μ to be sufficiently small so that (56) holds. However, if γ is sufficiently close to q^* , then (55) will not hold, and the steady state with valued fiat money and circulating private money does not exist. If γ is sufficiently close to \hat{q} , then from (35) and (52), (55) holds. A necessary condition for existence of the steady state equilibrium where private money circulates and fiat money is valued is that the steady state equilibrium with circulating private money exist. However, if the steady state equilibrium with circulating private money exists, there may or may not exist a steady state equilibrium where there is circulating private money and valued fiat money.

As in the previous two cases, there is no steady state equilibrium with valued fiat money, positive investment, and non-circulating claims on banks. The analysis of the steady state where fiat money is valued but investment is zero is identical to cases 1 and 2, except that the counterpart of conditions (44) and (49) is

$$2(r + \alpha) - (1 - \pi)\alpha \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2\kappa \right] - \pi(1 - \mu)(1 - \alpha) \left[(\delta_1 - \delta_2)\frac{q^*}{\gamma} + \delta_2 - 1 \right] \geq 0.$$

Numerical Example

For illustration, consider an example where we fix parameter values, excluding r and γ , and then determine the regions of the parameter space for which particular equilibria exist. We choose parameters arbitrarily as follows: $\delta_1 = 2$, $\delta_2 = 0$, $\pi = .5$, $q^* = 1$, and $\alpha = .5$. The choice of κ will then be irrelevant (given that $\delta_2 = 0$) and from (35) we have $\hat{q} = 2$. We will consider case 3, which has the richest possibilities in terms of equilibria. Figure 1 shows, in (r, γ) space, how the parameter space is subdivided, according to which equilibria exist.

In Figure 1, aa is determined by (53) with equality, bb is determined by (52) with equality, cc is determined by (56) with equality and $\mu = .25$, and dd is determined by (43) with equality. Let equilibrium 1 denote the steady state equilibrium where there is strictly positive investment, fiat money is not valued, and private money does not circulate, equilibrium 2 the steady state where private money circulates and fiat money is not valued, equilibrium 3 the steady state where private money circulates and fiat money is valued, and equilibrium 4 the steady state with zero investment where fiat money is valued. The parameter space is then subdivided as follows:

- Region A : Zero investment and fiat money is not valued.
- Region B : Equilibria 2,3.
- Region C : Equilibrium 2.
- Region D : Equilibrium 1.
- Region E : Equilibria 1 and 4.
- Region F : Equilibria 2 and 4.
- Region G : Equilibria 2, 3, and 4.

Figure 1: Existence of Equilibria

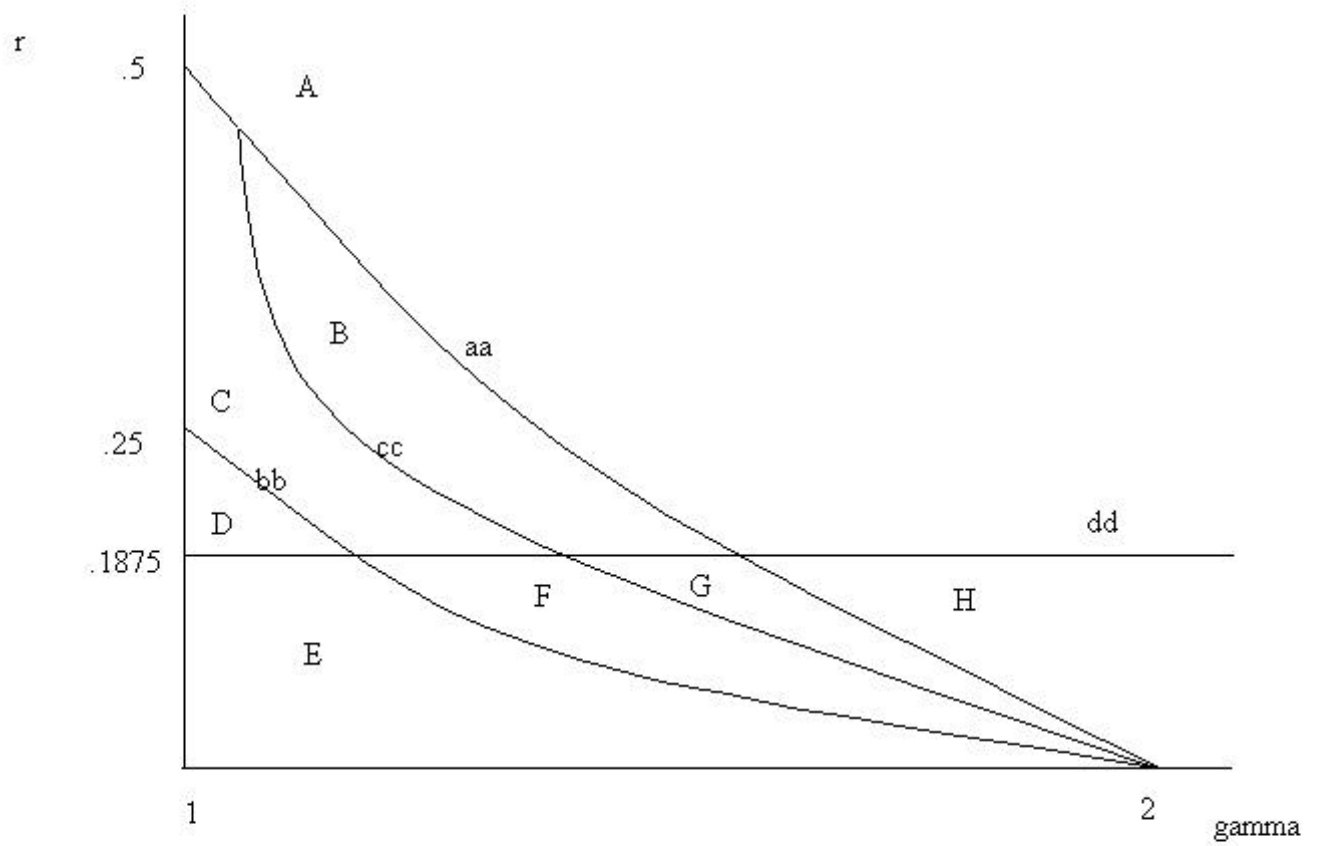


FIG. 1.

- Region H : Equilibrium 4.

Welfare

As in the general case, steady state equilibria where fiat money is valued and private money circulates are always dominated in welfare terms by the steady state where fiat money is not valued and private money circulates. Thus, the key welfare issue is how steady state equilibria with circulating private money and a price of zero for fiat money compare to steady state equilibria where private money does not circulate and fiat money is valued. First, note in the numerical example above that, with reference to Figure 1, in regions B and C private money circulates but no equilibrium exists with valued fiat money where private money does not circulate. Second, in regions E and H , private money does not circulate but there exists a steady state equilibrium with valued fiat money. That is, given the cost of starting an investment project, γ , the discount rate may be sufficiently large that the net gain from trading fiat money for goods is insufficient to support an equilibrium with valued fiat money (regions B and C). As well, given the discount rate, the startup cost of an investment project may be low enough, implying a high rate of return on investment, that agents will never trade away claims on banks in spite of the fact that fiat money may trade at a positive price (region E). Further, if the startup cost of an investment project is sufficiently high, then there is no investment, but fiat money may be valued in equilibrium (region H).

The regions where we can compare welfare in equilibria where exchange is supported either by fiat money or by private money, but not both, are regions F and G . Here, it is straightforward to show, given the numerical example above, that there exist cases where the private money equilibrium dominates the fiat money equilibrium, and where the reverse holds.

6. CONCLUSION

A matching model was constructed which permits banking arrangements and circulating private money. Through diversification, banking mitigates the mismatch between the times at which investment projects pay off and the times at which agents wish to consume, and claims on banks can be used as media of exchange.

Relative to a fiat money system, a private money system has two problems in this model. First, there can be a coordination problem, in that there may exist multiple steady states which can be ranked in terms of welfare. In the “good” steady state, the investment projects held by banks are more productive than in the “bad” steady state. Second, there can be private information concerning the quality of assets in banks’ portfolios, which can give rise to a lemons problem. For example, with full information there may exist two steady states with banks. In one steady state there are only good banks and in the other only bad banks. However, given the same parameter values and private information, the steady state with bad banks exists but the steady state with good banks may not.

In spite of these problems with private money systems, it need not be the case that the introduction of fiat money can improve matters. In fact, there exists the possibility that fiat money can only decrease welfare. In any steady state with circulating private money and valued fiat money, private media of exchange are displaced one-for-one by fiat money. Welfare declines as the quantity of fiat money increases, due to the fact that private money is backed by productive assets. There is the potential that the introduction of fiat money can increase welfare, but only if the quantity of private money produced is sufficiently suboptimal. Further, the steady state with highest welfare is either one with private money only, or with fiat money only; steady states where private money and fiat money serve as media of exchange are always dominated in welfare terms.

Simply stated, the conclusion of this paper is that private monetary arrangements allow for the intermediation of investment, while fiat money systems do not, so that private money is superior. This result can hold up even if there are frictions in the functioning of the private banking system, so the case for restrictions on private money issue is not compelling. There are other good reasons for permitting private money issue that we have not explored here, for example the fact that private money systems can be self-regulating in supplying an “elastic currency” in the face of cyclical shocks (see Sargent and Wallace 1982 and Champ, Smith and Williamson 1996). These issues certainly seem worthy of more research.

REFERENCES

- Burdett, K., Trejos, A., and Wright, R. 1998. “Cigarette Money,” working paper.
- Calomiris, C. and Kahn, C. 1995. “The Efficiency of Self-Regulated Payments Systems: Learning From the Suffolk System,” *Journal of Money, Credit and Banking* 28, 766-797.
- Cavalcanti, R., Erosa, A., and Temzelides, T. “Private Money and Reserve Management in a Random Matching Model,” working paper, University of Miami, University of Western Ontario, and University of Iowa.
- Cavalcanti, R. and Wallace, N. “A Model of Private Bank-Note Issue,” working paper, University of Miami and Pennsylvania State University.
- Champ, B., Smith, B., and Williamson, S. 1996. “Currency Elasticity and Banking Panics,” *Canadian Journal of Economics* 29, 828-864.
- Diamond, D., and Dybvig, P. 1983. “Bank Runs, Liquidity, and Deposit Insurance,” *Journal of Political Economy* 91, 401-419.

- Friedman, M. 1960. *A Program For Monetary Stability*, Fordham University Press, New York.
- Rolnick, R., Smith, B. and Weber, W. 1997. "The United States' Experience with State Bank Notes: Lessons for Regulating E-Cash," working paper, Federal Reserve Bank of Minneapolis.
- Rolnick, R. and Weber, W. 1983. "New Evidence on the Free Banking Era," *American Economic Review* 73, 1080-1091.
- Rolnick, R. and Weber, W. 1984. "The Causes of Free Bank Failures: A Detailed Examination," *Journal of Monetary Economics* 14, 267-292.
- Sargent, T. and Wallace, N. 1982. "The Real Bills Doctrine vs. the Quantity Theory: A Reconsideration," *Journal of Political Economy*.
- Townsend, R. 1980. "Models of Money with Spatially Separated Agents," in *Models of Monetary Economies*, edited by J. Kareken and N. Wallace, Federal Reserve Bank of Minneapolis, Minneapolis, MN, pp. 265-304.
- Velde, F., Weber, W., and Wright, R. 1997. "A Model of Commodity Money, with Applications to Gresham's Law and the Debasement Puzzle," working paper, Federal Reserve Bank of Chicago, Federal Reserve Bank of Minneapolis, and University of Pennsylvania.
- Wallace, N. 1988. "Another Attempt to Explain an Illiquid Banking System: The Diamond and Dybvig Model with Sequential Service Taken Seriously," Federal Reserve Bank of Minneapolis *Quarterly Review* 12, 3-16.
- White, L. 1984. *Free Banking in Britain: Theory, Experience, and Debate, 1800-1845*, Cambridge University Press, New York.

- Williamson, S. 1989. "Restrictions on Financial Intermediaries and Implications for Aggregate Fluctuations: Canada and the United States, 1870-1913," in *NBER Macroeconomics Annual 1989*, edited by Olivier Blanchard and Stanley Fischer, MIT Press, Cambridge, MA.
- Williamson, S. 1992. "Laissez Faire Banking and Circulating Media of Exchange," *Journal of Financial Intermediation* 2, 134-167.
- Williamson, S. and Wright, R. 1994. "Barter and Monetary Exchange Under Private Information," *American Economic Review* 84, 104-123.