Failure Recovery for Wrench Capability of Wire-Actuated Parallel Manipulators

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Abstract—In this paper, the failure of wire-actuated parallel manipulators is examined considering their failure modes. A methodology for investigating the effect of actuator/wire failures on the force/moment capability of manipulators is presented, and the criteria for full and partial recovery from these failures are established. The methodology is also applicable for the cases that the minimum norm solution for the vector of wire tensions gives a negative value for tension by treating the corresponding wire as failed. The proposed methodology is valid for both planar and spatial wire-actuated parallel manipulators.

Keywords: Failure analysis; failure recovery; wire-actuated parallel manipulators

I. Introduction

In parallel manipulators, the mobile platform (end effector) is connected to the base by several legs/limbs/branches/wires. Therefore, considering the actuation, parallel manipulators could be categorized as solid-link manipulators (Figure 1(a)) and wire-actuated manipulators (Figure 1(b)). The solid-link parallel manipulators consist of kinematic chains of links with actuated (active) and passive joints; e.g., in Figure 1(a), the prismatic joints are actuated and revolute joints are passive. In the wire/cable-actuated manipulators (also referred to as the wire-suspended or cable-driven manipulators), the motion of mobile platform is controlled by wires/cables. Wires act in tension (could pull but not push) and cannot exert forces in both directions along their lines of action. Hence, in wire-actuated manipulators some form of redundancy, e.g., redundant wire or external wrench (force/moment), is required.

Parallel manipulators, including wire-actuated ones, could be designed to have high load capacity and dynamic characteristics; and low mass, cost and power consumption. Hence, their potential applications include both the terrestrial applications, such as manufacturing, entertainment, medical and service sectors; and the space applications. For some of these applications, fail-safe manipulators are crucial, e.g., when the device is used in surgery or in high speed operation. For tasks in hazardous environments and space/remote operations, human access to the manipulator could be very difficult, dangerous or impossible, while in some applications the downtime needs to be minimized.


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solutions/assembly modes [4, 5]; to allow the fixtureless calibration of manipulators [6]; and to facilitate the joint sensor fault detection, isolation and recovery [7]. Redundancy in actuation has been proposed to reduce the uncertainty/singularity configurations of parallel manipulators [8, 9]. In [10] the taskspace was partitioned into major and secondary tasks in order to complete the major task and optimize a secondary goal such as actuator fault tolerance. The reduced motion of parallel manipulators due to active joint jam and the design modification to compensate for the accuracy degradation were investigated in [11, 12]. In [13] the effect of losing a wire on the null space of the Jacobian matrix of a planar wire-actuated manipulator was considered. Methodologies for the fault tolerance of wire-actuated parallel manipulators are required to compensate for their performance degradation after failure.

In this article, the failure of wire-actuated parallel manipulators is studied. Failure modes of these manipulators are discussed in Section II. A methodology for recovering the lost force/moment capability due to the failure of wires and actuators is presented in Section III. The kinematics and static modelling of planar wire-actuated manipulators and simulation results for the loss of wire force are reported in Section IV. The article concludes with Section V.

II. Manipulator failure modes

Wire-actuated parallel manipulators could fail because of the failure of their components (e.g., wires, pulleys, spools and motors), subsystems (end effector) and systems (mechanical, electrical, software and controller). If any malfunctioning in the wire actuating mechanism affects the performance of manipulator such that the task cannot be completed as desired, then the manipulator is considered failed. Considering the mechanical system, wire-actuated parallel manipulators could fail because of the failure of a wire (wire breakage, wire jam, or undesired flexibility of wire), sensor failure, actuator failure and transmission failure. These failures could result in the loss of DOF, loss of actuation, and loss of motion constraint; in addition to loss of information, please refer to [3] for detailed discussion.

From the force point of view, the failure of manipulator occurs if the wire does not provide the required force/torque. For example, when the actuator force/torque is lost partially or fully or the actuator is saturated. This could also happen when the wire is broken or slack (zero tension), wire is jammed (constant length), or its actuating mechanism malfunctions such that a different (zero, constant or limited) force is provided by the wire. It should be noted that based on the task, the wire could be intentionally kept slack, i.e., its tension be set to zero, e.g., to avoid entangling of the wire with an obstacle or when the tension of a wire is not required.

III. Failure recovery methodology

In parallel manipulators, the mobile platform is connected to the base by a number of legs/branches/wires, e.g., refer to Figure 1. Because wires act in tension, to manipulate an m DOF rigid body suspended by wires, i.e., to fully constrain these manipulators, in the absence of gravity and external wrench, the number of wires/actuators of the manipulator should be larger than the DOF of manipulator (Figures 1(b) and 2). Hence, wire-actuated manipulators usually require some means of redundant actuation (e.g., redundant wires and/or gravity). In this article, when the number of wires n is equal to m+1, the manipulator will be referred to as “non-redundant”. For redundant actuation, one or more additional wires could be included, e.g., the 2 DOF translational manipulator of Figure 2(b) with four wires.

To provide fail-safe operation and fault tolerance, redundant wires could be incorporated in the design, or anchors could be mounted on slides, or combinations of these two could be used. After the failure of a wire, by adjusting the anchor positions of the remaining wires to new locations, the necessary platform wrench could be achieved for successful termination of the task. Alternatively, if the design supports the continuous motion of anchors along the slides, the required wrench of
mobile platform will be provided by the wires and the linear actuators used for moving the anchors.

For the $n$-wire-actuated parallel manipulators, the $n \times 1$ vector of wire forces, $\mathbf{r} = [r_1 \ldots r_n]^T$, is related to the $m \times 1$ vector of forces and moments (wrench) $\mathbf{F} = [F_1 F_2 \ldots F_m M_1 M_2 \ldots M_m]^T$ applied by the mobile platform, with the $m \times n$ transposed Jacobian matrix $\mathbf{J}^T$ as

$$\mathbf{F} = \mathbf{J}^T \mathbf{r} = \left[ \mathbf{J}_1^T \mathbf{J}_2^T \ldots \mathbf{J}_n^T \right] \mathbf{r} = \sum_{j=1}^{n} \mathbf{J}_j^T \tau_j$$

(1)

where $m \leq 6$ depending on the dimension of taskspace, e.g., $m = 3$ for planar motion. Column $j$ of $\mathbf{J}^T$, $\mathbf{J}_j^T$, corresponds to the wrench applied on the mobile platform by the $j$th wire.

A. Different wire forces

When wire $i$ is failed, its tension $\tau_{ci}$ will be different (instantaneously or permanently) than the desired value $\tau_i$, e.g., it may have zero force, $\tau_{ci} = 0$; or constant force, lower or higher force than is required, $\tau_{ci} \neq \tau_i$. If this level of wire tension does not affect the required force/moment of mobile platform, then the failed wire will not result in the failure of the manipulator. Otherwise, if the remaining wires cannot provide the lost wrench after the failure of wire $i$, the manipulator will be considered failed. As well, the calculated tension for a wire might be a negative value, which is not feasible. These cases will be examined in the following subsections.

A1. Single-wire failure

The force of wire $i$ will be zero, $\tau_{ci} = 0$, when wire $i$ is broken or slack (Figure 3), or the actuator of wire $i$ is failed. Wire $i$ could also have a non-zero input, $\tau_{ci} \neq 0$, different than the required value $\tau_i$, e.g., when the wire tension and/or the actuator torque reach the limit (maximum value). Then, the mobile platform wrench will be

$$\mathbf{F}_i = \left[ \mathbf{J}_1^T \mathbf{J}_2^T \ldots \mathbf{J}_i^T \ldots \mathbf{J}_{n-1}^T \mathbf{J}_n^T \right] \mathbf{r}_f$$

$$= \sum_{j=1}^{n} \mathbf{J}_j^T \tau_j - \mathbf{J}_i^T (\tau_i - \tau_{ci})$$

(2)

where $\mathbf{r}_f = [r_1 \ldots \tau_i \ldots \tau_{ci} \ldots \tau_{n-1} \tau_n]^T$ and the change in the wrench of mobile platform will be

$$\Delta \mathbf{F}_i = \mathbf{F} - \mathbf{F}_i = \mathbf{J}_i^T (\tau_i - \tau_{ci})$$

(3)

In other words, if at this pose the required input (force) from wire $i$ is $\tau_{ci}$, i.e., $\tau_i = \tau_{ci}$, there will be no change in the force/moment capability of the manipulator. However, if the required input from wire $i$ is different, i.e., $\tau_i \neq \tau_{ci}$, then the manipulator would be considered as failed unless the remaining wires could provide the lost wrench due to failed wire $i$.

When the tension of the failed wire $i$ is $\tau_{ci} \neq \tau_i$ if the lost wrench is fully recovered, i.e., the manipulator maintains its force/moment capability $\mathbf{F}$, the remaining wires must provide the required wrench for the mobile platform. If the correctional force $\Delta \mathbf{\tau}_\text{corr} = [\Delta \tau_1 \Delta \tau_2 \ldots 0 \ldots \Delta \tau_{n-1} \Delta \tau_n]^T$, to be provided by the remaining (healthy) wires, compensates for the lost force/moment partially or completely, then the recovered wrench of platform will be

$$\mathbf{F}_r = \mathbf{J}_i^T \tau_f + \mathbf{J}_i^T \Delta \mathbf{\tau}_\text{corr} = \mathbf{J}_i^T \tau_f + \mathbf{J}_i^T \Delta \mathbf{\tau}_\text{corr}$$

(4)

where column $i$ of the reduced Jacobian matrix is replaced by a zero vector, i.e.,

$$\mathbf{J}_i^T = \left[ \mathbf{J}_1^T \mathbf{J}_2^T \ldots 0 \ldots \mathbf{J}_{n-1}^T \mathbf{J}_n^T \right]$$

(5)

and the change in the wrench of mobile platform will be

$$\Delta \mathbf{F}_i = \mathbf{F} - \mathbf{F}_r = \mathbf{J}_i^T (\tau - \tau_f) - \mathbf{J}_i^T \Delta \mathbf{\tau}_\text{corr}$$

(6)

To fully compensate for the lost mobile platform wrench, i.e., for $\Delta \mathbf{F}_i = 0$, the correctional force provided by the remaining wires should be [14]

$$\Delta \mathbf{\tau}_\text{corr} = \mathbf{J}_i^T \mathbf{J}_i^{-1} (\mathbf{r} - \mathbf{\tau}_f)$$

(7)

where $\mathbf{r} - \mathbf{\tau}_f = [0 \ldots 0 \ldots (\tau_i - \tau_{ci}) \ldots 0 \ldots 0]^T$ is the lost wire force due to failure of wire $i$.

If $\mathbf{J}_i^T$ has full row-rank, i.e., $\mathbf{F}$ belongs to the range space of $\mathbf{J}_i^T$, $\mathbf{F} \in \mathfrak{R}(\mathbf{J}_i^T)$, the generalized (Moore-Penrose) inverse of $\mathbf{J}_i^T$ is

$$\mathbf{J}_i^{-T} = \mathbf{J}_i \left( \mathbf{J}_i^T \mathbf{J}_i \right)^{-1}$$

(8)

as the vector of wire forces is physically consistent (all entries have the same dimension of force, i.e., mass times acceleration).

When a combination of wires and joints are used, depending on the type of joint, the input vector might not be physically consistent. For example, if the manipulator has a combination of wires and active revolute joints in order to constrain the mobile platform, e.g., the
manipulator of [15], then when $\mathbf{J}^T_f$ has full row-rank a weighting metric would be required for calculating the generalized inverse of $\mathbf{J}^T_f$ as

$$\mathbf{J}^{ST}_f = \mathbf{W}_f \mathbf{J}_f (\mathbf{J}^T_f \mathbf{W}_f \mathbf{J}_f)^{-1}$$

(9)

Metric $\mathbf{W}_f$ is chosen such that $\mathbf{\tau}^T_f (\mathbf{W}_f^{-1} \mathbf{\tau})$ becomes physically consistent, e.g., refer to [16], for instance to minimize/maximize the strain energy of manipulator.

When the rank of $\mathbf{J}^T_f$ is less than $m$ then $\mathbf{J}^T_f$ will not have full row-rank, and in general, the lost wrench cannot be fully recovered. In this case, if $\mathbf{J}^T_f$ has full column-rank and the lost wrench is not in the range space of $\mathbf{J}^T_f$, the mobile platform wrench that best approximates the lost wrench in the least-square sense is calculated using the weighted left-generalized inverse of $\mathbf{J}^T_f$ as

$$\mathbf{J}^T_f = (\mathbf{J}_f \mathbf{W}_f \mathbf{J}^T_f)^{-1} \mathbf{J}_f \mathbf{W}_f$$

(10)

regardless of whether the vector of input forces/torques is physically consistent. The weighting metric $\mathbf{W}_f$ is chosen such that $\mathbf{F}^T (\mathbf{W}_f \mathbf{F})$ becomes physically consistent.

In equation (7), the generalized inverse $\mathbf{J}^{ST}_f$ maps the lost wrench, $\mathbf{J}^T_f (\mathbf{r}_i - \mathbf{r}_{ci})$, to the orthogonal complement of the null space of the reduced Jacobian matrix $\mathbf{J}^T_f$. Then $\mathbf{J}^T_f \mathbf{\Delta r}_{corr}$ (and $\mathbf{J}^T \mathbf{\Delta r}_{corr}$) maps this force/torque to the range space of $\mathbf{J}^T_f$ (generally a non-zero wrench). If $\mathbf{\Delta r}_{corr} = \mathbf{J}^{ST}_f (\mathbf{\tau} - \mathbf{\tau}_f) = \mathbf{0}$, i.e., when $\mathbf{J}^T_f (\mathbf{r}_i - \mathbf{r}_{ci}) = \mathbf{0}$ or $\mathbf{J}^T_f (\mathbf{r}_i - \mathbf{r}_{ci})$ belongs to the null space of $\mathbf{J}^{ST}_f$ (which is equivalent to the orthogonal complement of the range space of $\mathbf{J}^{ST}_f$), then there is no need for correcional force from the remaining wires.

It should be noted that if the correctional input was calculated using $\mathbf{\Delta r}_{corr} = \mathbf{J}^{ST}_f (\mathbf{\tau} - \mathbf{\tau}_f)$, in general, it would include a non-zero input for the failed wire $i$. This is because $\mathbf{J}^{ST}_f (\mathbf{\tau} - \mathbf{\tau}_f)$ projects the lost force, i.e., $\mathbf{\tau} - \mathbf{\tau}_f = [0 \ 0 \ ... \ (\mathbf{r}_i - \mathbf{r}_{ci}) \ ... \ 0 \ 0]^T$, to the orthogonal complement of the null space of $\mathbf{J}^T_f$. Therefore, even though it filters out the components that would not contribute to the wrench of platform it would assign a force to the failed wire $i$. For information on projections, refer to [17].

When $\mathbf{F} \in \mathbb{R}(\mathbf{J}^T_f)$ the deviation in the wrench of mobile platform will be zero using

$$\mathbf{\Delta F}_f = \mathbf{F} - \mathbf{F}_f = (1 - \mathbf{J}^T_f \mathbf{J}^{ST}_f) \mathbf{J}^T_f (\mathbf{\tau} - \mathbf{\tau}_f)$$

(11)

where $\mathbf{I} - \mathbf{J}^T_f \mathbf{J}^{ST}_f$ projects the lost wrench $\mathbf{J}^T_f (\mathbf{\tau} - \mathbf{\tau}_f)$ due to failure of wire $i$ to the orthogonal complement of the range space (i.e., to the null space) of $\mathbf{J}^T_f$, and hence, $\mathbf{\Delta F}_f = \mathbf{0}$, refer to Figure 4.

![Fig. 4. Projection of mobile platform wrench onto the orthogonal complement of range space of $\mathbf{J}^T_f$.](image)

When $\mathbf{J}^T_f$ does not have full row-rank and the lost wrench belongs to the range space of $\mathbf{J}^T_f$, then $\mathbf{\Delta F}_f = \mathbf{0}$. When the lost wrench belongs to the orthogonal complement of the range space of $\mathbf{J}^T_f$, the correctional input of the remaining wires cannot set the deviation in the mobile platform wrench to zero, $\mathbf{\Delta F}_f = \mathbf{0}$, refer to Figure 5. The lost wrench that cannot be recovered could be characterized considering the range space of $1 - \mathbf{J}^T_f \mathbf{J}^{ST}_f$, which is the same as the orthogonal complement of the range space of $\mathbf{J}^T_f$. Hence, the condition for partial recovery of the lost wrench after the failure of wire $i$, i.e., when $\mathbf{\Delta F}_f \in \mathbb{R}(\mathbf{J}^T_f)^\perp$, is

$$\mathbf{F}_{\perp_i} = (1 - \mathbf{J}^T_f \mathbf{J}^{ST}_f) \mathbf{F} \neq \mathbf{0}$$

(12)

![Fig. 5. Projection of mobile platform wrench onto the range space of $\mathbf{J}^T_f$.](image)

To identify if the lost wrench could be fully recovered, all the components of the wrench $\mathbf{F} = \mathbf{J}^T_f \mathbf{\tau}$ projected onto the orthogonal complement of the range space of $\mathbf{J}^T_f$ should be zero, i.e., the condition for full recovery is

$$\mathbf{F}_{\perp_i} = (1 - \mathbf{J}^T_f \mathbf{J}^{ST}_f) \mathbf{F} = \mathbf{0}$$

(13)
provided that the overall wire forces \( \mathbf{\tau}_f + \Delta \mathbf{\tau}_{\text{corr}} \) will not surpass the limit of the healthy wires and/or the corresponding overall actuator torques will not exceed their limit. Otherwise, the procedure could be applied for the wire corresponding to the entry of \( \mathbf{\tau}_f + \Delta \mathbf{\tau}_{\text{corr}} \) that reaches/exceeds the wire/actuator limit.

A2. Multi-wire failure

The proposed method can be easily extended to the case that \( k \) wires have different force and the lost wrench is \( \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) = \mathbf{J}_f^T (\mathbf{\tau} - \mathbf{\tau}_f) \). In this case, \( k \) columns of \( \mathbf{J}_f^T \), corresponding to the wires with different inputs, are replaced by zeros resulting in \( \mathbf{J}_f^T \). The condition for zero error in the mobile platform wrench (if \( \mathbf{J}_f^T \) has full row-rank or the platform wrench is in the range space of \( \mathbf{J}_f^T \)), as a result of correctional input provided by the remaining wires, would be the same as equation (13). That is, when \( \mathbf{F} = \mathbf{R}(\mathbf{J}_f^T) \), then, \( \mathbf{F}_{\text{res}} = (\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\text{ST}}) \mathbf{J}_f^T \mathbf{\tau} = 0 \), and

\[
\Delta \mathbf{\tau}_{\text{corr}} = -\mathbf{J}_f^T \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) = \mathbf{J}_f^{\text{ST}} (\mathbf{\tau} - \mathbf{\tau}_f) \tag{14}
\]

It should be noted that even though the lost wire inputs might not be zero, the mobile platform wrench corresponding to the correctional force from the remaining wires will be zero when \( \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) = 0 \), i.e., the wrench produced by the failed wires cancel out, or when \( \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) \) is in the null space of \( \mathbf{J}_f^{\text{ST}} \).

If the lost mobile platform wrench cannot be fully recovered, i.e., \( \Delta \mathbf{F}_f \in \mathbf{R}(\mathbf{J}_f^T)^{\perp} \), \( \mathbf{F}_{\text{res}} = (\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\text{ST}}) \mathbf{F} \neq 0 \) and the recovered portion of wrench will be calculated using equation (11) as

\[
\mathbf{F}_r = \mathbf{F} - (\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f^{\text{ST}}) \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) \tag{15}
\]

In case the force of healthy wires cannot be changed after the failure of \( k \) wires, the error in the wrench that the platform could apply will be calculated as

\[
\Delta \mathbf{F}_f = \mathbf{J}_f^T \mathbf{J}_f^{\text{ST}} \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) \tag{16}
\]

where \( \mathbf{J}_f^T \mathbf{J}_f^{\text{ST}} \sum \mathbf{J}_f^T (\tau_i - \tau_{ci}) \) is the projection of the lost wrench to the range space of \( \mathbf{J}_f^T \).

B. Negative wire forces

Using equation (1), the solution of \( \mathbf{F} = \mathbf{J}_f^T \mathbf{\tau} \) for the vector of wire tensions is

\[
\mathbf{\tau} = \mathbf{J}_f^{\text{ST}} \mathbf{F} + (\mathbf{I} - \mathbf{J}_f^{\text{ST}} \mathbf{J}_f^T) \mathbf{k} = \mathbf{J}_f^{\text{ST}} \mathbf{F} + \mathbf{N} \lambda \tag{17}
\]

where \( \mathbf{J}_f^{\text{ST}} \mathbf{F} \) is the minimum norm or particular solution and \( (\mathbf{I} - \mathbf{J}_f^{\text{ST}} \mathbf{J}_f^T) \mathbf{k} \) and \( \mathbf{N} \lambda \) represent the homogenous solution. \( (\mathbf{I} - \mathbf{J}_f^{\text{ST}} \mathbf{J}_f^T) \mathbf{k} \) is the projection of the arbitrary vector \( \mathbf{k} \) onto the null space of \( \mathbf{J}_f^T \), while the columns of \( n \times (n - m) \) matrix \( \mathbf{N} \) correspond to the orthonormal basis of the null space of \( \mathbf{J}_f^T \).

The minimum norm solution could result in a negative value for a wire tension, then the \( n \times 1 \) free vector \( \mathbf{k} \), or \( (n - m) \times 1 \) free vector \( \lambda \) is chosen such that positive tension is maintained in the wires. If \( (\mathbf{I} - \mathbf{J}_f^{\text{ST}} \mathbf{J}_f^T) \mathbf{k} \) (or \( \mathbf{N} \lambda \)) does not result in positive tension in wires, e.g., positive tension in at least \( m \) wires \((m \leq n)\) and zero tension in the remaining \( n - m \) wires, when added to the minimum norm solution for any values of \( \mathbf{k} \) (or \( \lambda \)), then the mobile platform is outside the wrench closure workspace and cannot be controlled. If the task requires this pose then the manipulator will be considered failed.

B1. Zero external wrench

In the absence of external wrench (when there are no forces/moments applied by the mobile platform), \( \mathbf{F} = \mathbf{0} \), and the conditions for positive tension in wires are as follows. If the number of wires is \( n = m + 1 \), then the \( n \times 1 \) matrix \( \mathbf{N} \) that spans the null space basis of the transposed Jacobian matrix \( \mathbf{J}_f^T \) (here referred to as the null space vector) should have entries with consistent signs. When \( n > m + 1 \), the null space basis of \( \mathbf{J}_f^T \) is defined by \( n - m \) orthonormal vectors and at least one linear combination of these null space vectors should have entries with consistent signs (all non-negative or all non-positive).

When the minimum norm solution results in negative tension for wire \( i \), as long as the entries of the null space vector (or entries of the linear combination of the null space vectors) of \( \mathbf{J}_f^T \) have consistent signs, using the procedures presented in Section IIIA and treating wire \( i \) as the failed wire, the tension of wire \( i \) could be adjusted such that the negative value for \( \tau_i \), which results in the minimum norm solution for \( \mathbf{\tau} \), is set to zero or a positive value (finite value or a threshold in order to avoid slackness).

Similarly, when wire \( i \) is failed and the required mobile platform wrench is in the range space of \( \mathbf{J}_f^T \), provided that the entries of the null space vector of \( \mathbf{J}_f^T \) (or at least one linear combination of the null space vectors) have consistent signs, the correctional tension from the remaining wires could fully recover the required mobile platform wrench if the minimum norm solution for remaining wires results in negative tension for one or more wires. If the entries of the null space vector do not have consistent signs, then the platform wrench cannot be fully recovered. However, a zero (or a positive threshold) value for the wire with negative tension will result in the least-square error in the platform wrench.
B2. Non-zero external wrench

In the presence of external wrench, \( \mathbf{F} \neq \mathbf{0} \), with \( l \) components of wrench \( \mathbf{F} \) being nonzero, where \( l \leq m \leq 6 \), the conditions for positive tension in wires are as follows. Rearranging the force/moment balance equation (1) as \( \mathbf{J}^T \mathbf{\tau} - \mathbf{F} = \mathbf{0} \) and augmenting \( \mathbf{\tau} \) with \( l \) ones, to form the \( (n+l) \times 1 \) vector \( \mathbf{\tau}_{\text{aug}} \)

\[
\mathbf{J}^T \mathbf{\tau}_{\text{aug}} = \begin{bmatrix}
\mathbf{J}_1^T & \mathbf{J}_2^T & \cdots & \mathbf{J}_n^T & \mathbf{J}_{n+1}^T & \cdots & \mathbf{J}_{n+l}^T
\end{bmatrix} \mathbf{w}_{\text{aug}} = \mathbf{0}
\]  

(18)

where for a 6 DOF manipulator, when \( l = 6 \)
\( \mathbf{W}_1 = [-F_x, 0, 0, 0, 0, 0] \), \( \mathbf{W}_2 = [0, -F_y, 0, 0, 0, 0] \),
\( \cdots \), and \( \mathbf{w}_{\text{aug}} = [r_1, \cdots, r_n, 1, 1, 1, 1]^T \). For a 3 DOF planar manipulator, when \( l = 3 \) and the plane of motion is defined by the X-Y plane, \( \mathbf{W}_1 = [-F_x, 0, 0] \),
\( \mathbf{W}_2 = [0, -F_y, 0] \), \( \mathbf{W}_3 = [0, 0, -M_z] \) and \( \mathbf{\tau}_{\text{aug}} = [r_1, \cdots, r_n, 1, 1, 1]^T \).

The null space basis of \( \mathbf{J}_{\text{aug}}^T \) is defined by \( n + l - m \) vectors and at least one linear combination of these \( (n+l) \times 1 \) null space vectors should have non-negative values for the first \( n \) entries and positive values for the last \( l \) entries, e.g., refer to [13] for the case that \( \mathbf{F} \) corresponds to the gravitational force.

When the tension of wire \( i \) is negative for the minimum norm solution, and the first \( n \) entries of the null space vector (or of the linear combination of the null space vectors) of \( \mathbf{J}_{\text{aug}}^T \) are non-negative and the last \( l \) entries are positive, then the tension of wire \( i \) could be adjusted such that the negative value for \( \tau_i \), which results in the minimum norm solution for \( \mathbf{\tau} \), is set to zero (or a positive value).

Similarly, when wire \( i \) is failed and \( \mathbf{F} \in \mathcal{Z}(\mathbf{J}_i^T) \), provided that the entries of the null space vector of \( \mathbf{J}_{\text{aug}}^T \) fulfill this condition, the correctional tension from the remaining wires could fully recover the required mobile platform wrench if the minimum norm solution for \( \mathbf{\tau} \), the null space solution for \( \mathbf{J}^T \mathbf{\tau} - \mathbf{F} = \mathbf{0} \), is called as the condition for the minimum tension solution for \( \mathbf{\tau} _\text{aug} \), which results in the minimum norm solution for \( \mathbf{\tau} \) and the remaining wires results in negative tension for one or more wires.

B3. Formulation of null space vector

The null space vector of \( \mathbf{J}^T \) (or \( \mathbf{J}_{\text{aug}}^T \), \( \mathbf{J}_{\text{aug}}^T \)) could be identified by different methods. Using Cramer’s rule, e.g., [18], when \( n = m+1 \) vector \( \mathbf{N} \), which spans the null space basis of \( \mathbf{J}^T \), is calculated as

\[
\mathbf{N} = \begin{bmatrix}
(-1)^{n-1} \det(M_1) & (-1)^{n-2} \det(M_2) & \cdots & \det(M_n)
\end{bmatrix}^T
\]  

(19)

where \( \mathbf{M}_i \) is the \( i \)-th \( m \times m \) submatrix (minor) of the transposed Jacobian matrix, formed by removing the \( i \)-th column of \( \mathbf{J}^T \). For wire-actuated parallel manipulators, all entries of vector \( \mathbf{N} \) should be either non-negative or non-positive, e.g., \((-1)^{i-1} \det(M_i) \geq 0 \) for \( i = 1, \cdots, n \).

IV. Case Study

To model the wire-actuated parallel manipulators, a fixed reference frame \( \Psi(X,Y,Z) \) is assigned to the base, with origin at point 0, and a moving reference frame \( \Gamma(X',Y',Z') \) is attached to the center of mass, point \( P \), of the mobile platform. For the translational wire-actuated manipulators, the mobile platform shrinks to point \( P \) (a point mass). The position vector of point \( P \) in the base frame is \( \mathbf{p} = [p_x, p_y, p_z]^T \). The orientation of the mobile platform with respect to the base frame \( \Psi(X,Y,Z) \) is given by Euler angles.

![Fig. 6. Parameters of planar wire-acted parallel manipulators.](image-url)
\[
A_{\Psi,\Gamma} = \begin{bmatrix} 
\cos \phi & -\sin \phi & p_x \\
\sin \phi & \cos \phi & p_y \\
0 & 0 & 1
\end{bmatrix}
\] (20)

which is in terms of the mobile platform position \( p = [p_x, p_y]^T \) and orientation \( \phi \).

When only wire actuators are used, for non-redundant translational 2 DOF manipulators \((m = 2); \) two translations on the plane, in the absence of gravity and external force, three wires are required \((n = 3)\). For non-redundant 3 DOF planar manipulators \((m = 3); \) two translations on the plane and a rotation about an axis normal to the plane, in the absence of gravity and external wrench, four wires are required \((n = 4)\). For redundant actuators, if only single wire actuators are utilized one or more wires should be added; while if hybrid actuation is considered the anchors may be mounted on linear actuators.

For velocity analysis, the rate of change of wire lengths (and velocity of linear actuators) \( \dot{i} \) is related to the translational and rotational velocity of mobile platform \( \dot{V} = [v_x, v_y, \phi]^T \), with the Jacobian matrix \( J \)

\[
\dot{i} = -J\dot{V}
\] (21)

To solve for the velocity of mobile platform \( V \), when matrix \( J \) is not square

\[
\dot{V} = -J^+ \dot{i}
\] (22)

Considering the velocity relation \( \dot{i} = -J\dot{V} \) and the \( n \times m \) Jacobian matrix \( J \), where \( n > m \), vector \( \dot{i} \) is physically consistent (all entries have the same dimension of length/time). Hence, even though for the general motion the entries of the mobile platform velocity vector are not unit consistent (include rotational and translational velocities), as long as \( J \) has full column-rank there is no need for a weighting metric on the task-space velocity. Therefore, matrix \( J^T \) is the un-weighted generalized inverse of matrix \( J \) and is calculated as

\[
J^T = (J^TJ)^{-1} J^T
\] (23)

When the actuation is provided by \( n \) wires the \( n \times 3 \) Jacobian matrix will be

\[
J = \begin{bmatrix}
\cos \alpha_1 & \sin \alpha_1 & v_1 \\
\vdots & \vdots & \vdots \\
\cos \alpha_n & \sin \alpha_n & v_n
\end{bmatrix}
\] (24)

and \( \dot{i} = [\dot{i}_1, \cdots, \dot{i}_n]^T \). The direction cosines corresponding to the axis of wire \( i \) are calculated as \( \cos \alpha_i = l_{ix}/l_i \) and \( \sin \alpha_i = l_{iy}/l_i \), and \( v_i \) is the moment of wire axis with respect to the origin of \( \Gamma (X',Y',Z') \), point \( P \), formulated in \( \Psi(X,Y,Z) \) as

\[
v_i = -\cos \alpha_i (b_{iy} - p_y) + \sin \alpha_i (b_{ix} - p_x)
\] (25)

For the 2 DOF translational manipulators, the mobile platform velocity vector is \( \dot{V} = [v_x, v_y]^T \) and the Jacobian matrix \( J \) reduces to

\[
J = \begin{bmatrix}
\cos \alpha_1 & \sin \alpha_1 \\
\vdots & \vdots \\
\cos \alpha_n & \sin \alpha_n
\end{bmatrix}
\] (26)

B. Force analysis

For the planar parallel manipulators, the relationship between the vector of wire forces (and forces of linear actuators) \( \tau = [\tau_1, \cdots, \tau_n]^T \) and the wrench applied by the mobile platform, \( F \), is \( F = J^T \tau \), where column \( i \) of matrix \( J^T \), i.e., \( J_i^T \), corresponds to the wrench that wire \( i \) (or linear actuator) applies on the mobile platform, i.e.,

\[
F = J^T \tau = J_1^T \tau_1 J_2^T \tau_2 \cdots J_{n-1}^T \tau_{n-1} J_n^T \tau_n = \sum_{j=1}^{n} J_j^T \tau_j.
\]

Considering the force relation, \( F = J^T \tau \) and the \( 3 \times n \) matrix \( J^T \), where \( n > 3 \), the vector of wire forces (and forces of linear actuators) is physically consistent (all entries have the same dimension of force, i.e., mass times acceleration). Therefore, to formulate \( \tau = J^{gT} F + (I - J^{gT} J) k = J^{gT} F + N \lambda \), even though the entries of the mobile platform wrench vector are not unit consistent, as long as \( J^T \) has full row-rank (i.e., \( F \) belongs to the range space of \( J^T \)) there is no need for a weighting metric and \( J^{gT} \) is calculated as

\[
J^{gT} = J (J^T J)^{-1}
\] (27)

When only wires are used, referring to Figure 7, in the absence of gravity, the static force and moment equations are

\[
\sum_{i=1}^{n} \tau_i \cos \alpha_i = F_{\text{ext}}
\] (28.1)

\[
\sum_{i=1}^{n} \tau_i \sin \alpha_i = F_{\text{ext}}
\] (28.2)

\[
\sum_{i=1}^{n} \tau_i v_i = M_{\text{ext}}
\] (28.3)

or,

\[
\begin{bmatrix}
F_{\text{ext}} \\
F_{\text{ext}}^T \\
M_{\text{ext}}
\end{bmatrix} = J \begin{bmatrix}
\tau_1 \\
\vdots \\
\tau_n
\end{bmatrix}
\] (29)

where \( F = [F_{\text{ext}}, F_{\text{ext}}^T, M_{\text{ext}}]^T \) and

\[
J^T = \begin{bmatrix}
\cos \alpha_1 & \cdots & \cos \alpha_n \\
\sin \alpha_1 & \cdots & \sin \alpha_n \\
v_1 & \cdots & v_n
\end{bmatrix}
\] (30)
where column $i$ of the $3 \times n$ matrix $J^T$, i.e., $J^T_i$, represents the axis of wire $i$. When the plane of motion is a vertical plane and the combined weight of platform and its payload, $mg$, is not negligible, the applied wrench by the platform is $\mathbf{F} = [\mathbf{F}_{ext}, (\mathbf{F}_{ext} - mg) \mathbf{M}_{ext}]^T$. Hence, the correctional force to be provided by wires 1, 3 and 4 is

$$\Delta \mathbf{r}_{corr} = J^T \Delta \mathbf{r}_2 = [0.470 \quad 7.572 \quad 13.149]^T$$

(34)

Then, the overall wire forces will be

$$\mathbf{r}_f + \Delta \mathbf{r}_{corr} = [1.881 \quad 13.453 \quad 25.454]^T$$

(35)

which produces the original platform wrench.

C2. 3 DOF four-wire manipulator

For the parallel manipulator of Figures 2(a) and 7, the taskspace dimension is $m = 3$ and the number of wires is $n = 4$. The coordinates of the base attachment points $A_i$, $i = 1, ..., 4$, in the base frame are respectively $(-2, -1.5), (2, -1.5), (2, 1.5)$ and $(-2, 1.5)$. The position of connection points $B_i$ on the mobile platform is set at a constant radius of $r_{BFP} = 0.25$ meters. The angular coordinates, $\theta_i$, $i = 1, ..., 4$, of the wire connections to the mobile platform are respectively $180^\circ, 0^\circ, 0^\circ$ and $180^\circ$.

Negative wire tension

For the four-wire manipulator of Figure 2(a), when the mobile platform pose is $\mathbf{p} = [0 \quad 0]^T$ meters and $\varphi = 30^\circ$ the transpose of the Jacobian matrix is

$$\mathbf{J}^T = \begin{bmatrix} -0.792 & 0.739 & 0.792 & -0.739 \\ -0.611 & -0.674 & 0.611 & 0.673 \\ 0.033 & -0.238 & 0.033 & -0.238 \end{bmatrix}$$

(36)

For $\mathbf{F} = [1.056 \quad 25.681 \quad -12.965]^T$ applied by the mobile platform, i.e., for a force of $[1.056 \quad 25.681]^T$ Newtons and a moment of $-12.965$ N-m about the Z direction, using $\mathbf{F} = \mathbf{J}^T \mathbf{r}$, the minimum norm wire forces are

$$\mathbf{r} = \mathbf{J}^T \mathbf{F} = [-13.720 \quad 16.694 \quad 6.280 \quad 36.694]^T$$

(37)

with negative tension for wire 1, which is not allowed. The null space vector of $\mathbf{J}^T$ is $\mathbf{N} = [1.000 \quad 0.139 \quad 1.000 \quad 0.139]^T$. Because all entries of $\mathbf{N}$ are positive, this pose is in the wrench closure workspace of manipulator.

The augmented Jacobian matrix is

$$\mathbf{J}^T_{aug} = \begin{bmatrix} -0.792 & 0.739 & 0.792 & -0.739 & 0 & 0 \\ -0.611 & -0.674 & 0.611 & 0.673 & 0 & -25.681 \\ 0.033 & -0.238 & 0.033 & -0.238 & 0 & 12.965 \end{bmatrix}$$

(38)

A null space vector of $\mathbf{J}^T_{aug}$ is $\mathbf{N}_{aug} = [1.000 \quad 0.452 \quad 0.486 \quad 0.511 \quad 0.324 \quad 0.013 \quad 0.016]^T$ and that of $\mathbf{J}^T_{aug}$, after removing the first column of $\mathbf{J}^T_{aug}$, is $\mathbf{N}_{aug} = [0.504 \quad 1.000 \quad 0.699 \quad 0.614 \quad 0.029 \quad 0.019]^T$, where the first four (and three) positive entries of $\mathbf{N}_{aug}$ (and $\mathbf{N}_{aug}$) correspond to the wires and the last three positive entries correspond to the platform wrench. Therefore, the negative tension for wire 1, which is calculated using the 2-norm solution, could be set to a positive value by using the homogenous solution or employing the methodology presented in Section III. If the minimum allowable wire tension is 2 Newtons, treating wire 1 as a failed wire
\[ \tau_f = [2 \ 16.694 \ 6.280 \ 36.694]^T \]  \hspace{1cm} (39) \\

and the wrench of mobile platform is calculated as 
\[ \mathbf{F}_f = \mathbf{J}^T \tau_f = [-11.394 \ 16.083 \ -12.443]^T. \]

The additional forces to be provided by wires 2, 3 and 4, should be 
\[ \Delta \tau_{corr} = \mathbf{J}_f^T \mathbf{J}_f^{-1} \tau_i = [0 \ 2.191 \ 15.720 \ 2.191]^T \]  \hspace{1cm} (40) \\

Then, the overall wire forces will be 
\[ \tau_f + \Delta \tau_{corr} = [2 \ 18.885 \ 22.000 \ 38.885]^T \]  \hspace{1cm} (41) \\

which results in 
\[ \Delta \mathbf{F}_f = (\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f) \mathbf{J}^T (\tau - \tau_f) = 0, \]

and hence, produces the original platform wrench of 
\[ \mathbf{F} = \mathbf{J}^T (\tau_f + \Delta \tau_{corr}) = [1.056 \ 25.681 \ -12.965]^T. \]

Single wire failure

When the mobile platform pose is \( p = [1 \ 0]^T \) meter and \( \varphi = 0^\circ \) the transpose of Jacobian matrix is 
\[ \mathbf{J}^T = \begin{bmatrix} -0.878 & 0.447 & 0.447 & -0.878 \\ -0.479 & -0.894 & 0.894 & 0.479 \\ 0.120 & -0.224 & 0.224 & -0.120 \end{bmatrix} \]  \hspace{1cm} (42) \\

and the null space vector of \( \mathbf{J}^T \) is \( \mathbf{N} = [0.509 \ 1.000 \ 1.000 \ 0.509]^T. \)

When the wrench \( \mathbf{F} = [-21.534 \ 23.310 \ -1.355]^T \) is applied by the mobile platform, i.e., for a force of \( [-21.534 \ 23.310]^T \) Newtons and a moment of \(-1.355 \) Newton-meters about the Z direction, a null space vector of \( \mathbf{J}_{\text{avg}}^T \) is 
\[ \mathbf{N}_{\text{avg}} = [0.039 \ 0.012 \ 0.090 \ 1.000 \ 0.039 \ 0.023 \ 0.075]^T. \]

Using \( \mathbf{F} = \mathbf{J}^T \tau \), the minimum norm wire forces is 
\[ \tau = \mathbf{J}^T \mathbf{F} = [-5.262 \ -9.961 \ 0.040 \ 24.738]^T \]  \hspace{1cm} (43) \\

For \( \lambda = 16.399 \)
\[ \tau = \mathbf{J}^T \mathbf{F} + \mathbf{N} \lambda = [0.001 \ 0.372 \ 10.372 \ 30.001]^T \]  \hspace{1cm} (44) \\

When wire 1 is slack (\( \tau_i = 0 \)) the wrench of mobile platform is calculated as 
\[ \mathbf{F}_f = \mathbf{J}^T \tau_f = [2 \ 18.885 \ 22.000 \ 38.885]^T. \]

A null space vector of \( \mathbf{J}_{\text{avg}}^T \), after removing the first column of \( \mathbf{J}_{\text{avg}}^T \), is 
\[ \mathbf{N}_{\text{avg}} = [0.157 \ 0.311 \ 1.000 \ 0.031 \ 0.026 \ 0.063]^T. \]

Therefore for the given platform wrench, this pose is in the wrench closure workspace. As well, the projection of platform wrench \( \mathbf{F} \) on the range space of \( \mathbf{J}_f^T \) is a zero vector, which indicates that the failure of wire 1 could be fully recovered by the remaining wires assuming that the correctional force will not result in a wire/actuator force/torque exceeding the limit.

To fully recover from the failure of wire 1, the wire forces are adjusted such that 
\[ \Delta \mathbf{F}_f = \mathbf{J}_f^T (\tau - \tau_f) - \mathbf{J}_f^T \Delta \tau_{corr} = \mathbf{J}_f^T \tau_i - \mathbf{J}_f^T \Delta \tau_{corr} = 0. \]

Hence, the correctional force to be provided by wires 2, 3 and 4 (to be added to the minimum norm solution) should be 
\[ \Delta \tau_{corr} = \mathbf{J}_f^T \mathbf{J}_f^{-1} \tau_i = [0 \ 10.330 \ 10.330 \ 5.262]^T \]  \hspace{1cm} (45) \\

Then, the overall wire forces will be 
\[ \tau_f + \Delta \tau_{corr} = [0 \ 0.370 \ 10.370 \ 30.000]^T \]  \hspace{1cm} (46) \\

which results in 
\[ \Delta \mathbf{F}_f = (\mathbf{I} - \mathbf{J}_f^T \mathbf{J}_f) \mathbf{J}^T (\tau - \tau_f) = 0, \]

and hence, produces the original platform wrench of 
\[ \mathbf{F} = \mathbf{J}^T (\tau_f + \Delta \tau_{corr}) = [-21.534 \ 33.310 \ -1.355]^T. \]

C3. Six-Wire Manipulator

For the six-wire parallel manipulator of Figure 8, the coordinates of the base attachment points \( A_i, i = 1, \ldots, 6 \), in the base frame are respectively \((-2, -1.5), (2, -1.5), (2, 1.5), (2, 1.5), (-2, 1.5) \) and \(-2, 1.5) \). The position of connection points \( B_i \) on the mobile platform is set at a constant radius of \( r_{\text{base}} = 0.25 \) meters. The angular coordinates, \( \theta_i, i = 1, \ldots, 6 \), of the wire connections to the platform are respectively \( 180^\circ, 180^\circ, 0^\circ, 0^\circ, 0^\circ, 180^\circ \).

When the mobile platform pose is \( \mathbf{p} = [0.5 \ -1]^T \) meters and \( \varphi = -30^\circ \) the transpose of Jacobian matrix is 
\[ \mathbf{J} = \begin{bmatrix} -0.899 & 0.974 & 0.974 & -0.899 & -0.547 & -0.475 \\ -0.438 & -0.224 & -0.162 & 0.754 & 0.837 & 0.880 \\ 0.207 & -0.073 & 0.088 & 0.245 & 0.113 & -0.131 \end{bmatrix} \]  \hspace{1cm} (47) \\

For \( \mathbf{F} = [45.951 \ 74.774 \ 23.202]^T \) applied by the mobile platform, using \( \mathbf{F} = \mathbf{J}^T \tau \), the minimum norm wire forces is reported in Table 1. Different failure cases have been investigated for this manipulator. Due to space limitation, two cases are reported here. In case 1, the required platform wrench is in the range space of \( \mathbf{J}_f^T \), as indicated by \( \mathbf{F}_f^{\perp} = [0 \ 0 \ 0]^T \), and hence, the manipulator fully recovers the lost wrench by adjusting the tension of the remaining healthy wires. In case 2, the three healthy wires (wires 3, 4 and 5) are connected to the same attachment point on the mobile platform and cannot provide/resist rotation of platform about that point. In this case, \( \mathbf{F}_f^{\perp} = [45.967 \ -47.232 -3.210]^T \) indicates that the
required platform wrench is not in the range space of $J_f^T$, and hence, cannot be fully recovered. To reduce the error in the platform wrench, the correctional forces from wires 3, 4 and 5, which result in the minimum norm solution and approximate the lost wrench in the least-square sense, is $\Delta \tau_{corr} = [0 \ 0 \ -2.065 \ -1.227 \ -2.454 \ 0]^T$, using an identify matrix as the weighting metric. The deviation (least square error) in the platform wrench after applying the correctional forces, i.e., the error of recovered wrench, is $\Delta F_r = \left( I - J_f^T J_f^{ST} \right) J_f^T (\tau - \tau_f) = \left[ -2.630 \ 2.097 \ 1.395 \right]^T$, and the recovered wrench of platform is $F_r = [48.581 \ 72.676 \ 21.808]^T$.

V. Conclusion

Failure analysis of wire-actuated parallel manipulators was discussed considering their failure modes, i.e., zero and different/limited wire forces. A methodology, based on the projection of the lost wire force onto the orthogonal complement of the null space of the Jacobian matrix of the failed manipulator, was presented for the full and partial recovery of the lost wrench of the mobile platform. In addition, conditions on the lost platform wrench that could not be recovered were established. The minimum norm solution for wire tension vector could result in negative value for wire tension. By treating the wire with negative tension as failed, the methodology could also be utilized to set the tension to a positive value if the considered pose is in the wrench closure workspace of manipulator.

References


Table 1 Example failures for the six-wire parallel manipulator.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\tau = J_f^{ST} F$</th>
<th>$F_f = J_f^T \tau_f$</th>
<th>$\Delta \tau_{corr} = J_f^{ST} \sum J_f^T \tau_i$</th>
<th>$\tau_f + \Delta \tau_{corr}$</th>
<th>$\Delta F_r = [I - J_f^T J_f^{ST}] J_f^T (\tau - \tau_f)$</th>
<th>$F_{\text{rec}} = [I - J_f^T J_f^{ST}] J_f^T \tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$[2.457 \ -1.917 \ 23.424 \ 68.455 \ 32.820 \ 0.112]^T$</td>
<td>$[24.758 \ 78.041 \ 21.009]^T$</td>
<td>$[0.607 \ 0 \ 0 \ 17.201 \ -19.093 \ 0]^T$</td>
<td>$[3.064 \ 0 \ 85.656 \ 13.727 \ 0]^T$</td>
<td>$[0 \ 0 \ -2.630 \ 2.097 \ 1.395]^T$</td>
<td>$[0 \ 0 \ 0 \ 45.969 \ -47.232 \ -3.210]^T$</td>
</tr>
<tr>
<td>2</td>
<td>$[50.081 \ 75.321 \ 22.568]^T$</td>
<td>$[0 \ 0 \ -2.065 \ -1.227 \ -2.454 \ 0]^T$</td>
<td>$[0 \ 0 \ 21.359 \ 67.528 \ 30.366 \ 0]^T$</td>
<td>$[2.630 \ 2.097 \ 1.395]^T$</td>
<td>$[45.969 \ -47.232 \ -3.210]^T$</td>
<td></td>
</tr>
</tbody>
</table>