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Bose-Einstein vs. electrodynamic condensates: the question of order and coherence.

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Abstract

The remarkable recent experiments on ensembles of magnetically trapped ultracold alkali atoms have demonstrated the transition to a highly ordered phase, that has been attributed to the process of quantum-mechanical condensation, predicted long ago (1924) by Bose and Einstein. After having presented our argument against the above attribution, we show that the known phenomenology, including a discrepancy of about one order of magnitude with the predictions of Bose- Einstein condensation, is in good agreement with the electromagnetic coherence induced on the alkali atoms by the long range electrodynamic interactions. We also predict that for temperatures lower than a well defined T_{BEC}, the state predicted by Bose and Einstein coexists with the new coherent electrodynamic state.

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Bose-Einstein (BE) condensation, since its theoretical discovery [1],[2], has had a major role in elucidating subtle aspects of Quantum Field Theory (QFT). In particular it has provided a particularly simple and illuminating realization of the mechanism by which the simplest (and most idealized) of all many-body systems, the perfect gas, obeys Nernst's third principle of thermodynamics, which requires that entropy go to zero *with continuity* as the temperature tends towards absolute zero. A property that the classical, Maxwell-Boltzmann gas obviously doesn't possess.

But as far as its realization in Nature is concerned, up until 1995, no firm, generally accepted evidence was found in any low temperature physical system. Well known is the controversy as to whether He-II superfluidity is a (particularly striking) manifestation of BE condensation, a possibility keenly adversed by Landau, and dialectically dealt with by several other authors [3],[4]. Be that as it may, even ignoring the subtle theoretical problems which we shall take up below, the matter was definitely obscured by the fact that He-II and the low-temperature superconductors are dense interacting systems, and one can thus legitimately doubt that the non-interacting perfect quantum gas be an adequate dynamical model for them. In order to firmly establish the existence of BE-condensation one thus needs, everybody agrees, to study highly diluted atomic systems, and from the condition for BE condensation [1] [$n = \left(\frac{N}{V}\right)$ is the number density]

$$n_{BE} = \left(\frac{mT_{BE}}{2\pi}\right)^{\frac{3}{2}} 2.612 = \nu_{BE} \left(\frac{T_{BE}}{\mu K}\right)^{\frac{3}{2}} \tag{1}$$

for a density $n \cong 10^{14} cm^{-3}$ of Na-atoms of mass number $A = 23$, at an average distance $n^{-\frac{1}{3}} \cong 2000\mathring{A}$ (thus practically non-interacting), one computes $T_{BE} \cong 2.5\ \mu K$ an exceedingly low-temperature. One had thus to await for the development of sophisticated trapping and cooling techniques to be able to explore a region of phase-space where one could unambiguously probe the predictions of Bose and Einstein. And when the temperatures and densities that could be reached were in the above ball park punctually the sudden transition to a new state was observed, whose structure corresponded to the BE condensate [5].

The transition, in truth, was observed at densities about one order of magnitude lower than predicted, but this didn't ring any particular bell for, according to the authors, it could be accounted for by experimental inaccuracies: thus the announcement to the world that BE condensation, more than seventy years after its prediction, was an experimental reality. And the further observations at MIT [6] completely confirmed such findings and their theoretical interpretation. But this remarkable research program, progressing at a very swift pace, had another very big surprise in store: at the beginning of 1997 the same MIT group reported the observation [7] of strong interference fringes when two such condensates were let to diffuse one through the other, and their measured period $\Delta = 30\ \mu$ just corresponded to the quantum mechanical prediction

$$\Delta = \frac{2\pi}{|\vec{P}|}, \tag{2}$$

[1] We use natural units where $\hbar = c = k_B = 1$

where P is the relative momentum of the two condensates. Please note that, according to (2), the relative velocity of the condensates is in fact quite small: $v \cong 0.06\ cm/sec$.

The reason why the discovery of interference fringes with the period (2) is an extremely significant step forward in our knowledge of the physics of macroscopic systems of ultracold atoms is that it is the first experimental proof that these systems, at least in the phase space region probed by the MIT group, are *coherent*, i.e. are described by a *macroscopic wave-function* (or "order parameter")

$$\Psi(\vec{x},t) = \langle\Omega|\Psi(\vec{x},t)|\Omega\rangle \tag{3}$$

where $|\Omega\rangle$ is the state of the quantum field (V is the volume of the system and $b_{\vec{k}}$ are the quantum amplitudes associated to the modes of Fourier momentum $\vec{k}$)

$$\Psi(\vec{x},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k}} b_{\vec{k}}(t)e^{i\vec{k}\vec{x}}, \tag{4}$$

that collectively describes the atomic system. The important question now is whether BE condensation actually predicts that the system of two condensates, of momentum $-\frac{\vec{P}}{2}$ and $\frac{\vec{P}}{2}$ respectively, is described by the macroscopic wave-function

$$\Psi(\vec{x},t) = \sqrt{n}\left(e^{i\frac{\vec{P}}{2}\vec{x}} + e^{-i\frac{\vec{P}}{2}\vec{x}}\right) \tag{5}$$

so that its square, the density of the "matter waves", exhibits just the observed interference fringes.The answer is a very simple and definite no, at least for the state which the Bose-Einstein perfect gas falls into below the condensation temperature. Indeed, according to theory, such state contains a well defined number of atoms, so that we may set

$$|\Omega\rangle = |N_1\rangle_{\frac{\vec{P}}{2}}|N_2\rangle_{-\frac{\vec{P}}{2}} \tag{6}$$

and sandwiching the field (4) between the states (6) we obtain zero, instead of the observed (5), showing that the BE-condensate is indeed *ordered* but not *coherent*, and that order and coherence are two basically independent physical concepts. The reason for this (seemingly) surprising result is transparent: in absence of interactions among atoms, such as assumed in the perfect gas model and deemed perfectly reasonable for the actual condensates due to their very low density, [2] there is no physical mechanism to bring the phases of the atoms, that are ordered "by default", to "cohere" in a macroscopic coherent state, described by the "order parameter" (5). In other words, unless a long- range interaction among the atoms holds, that makes it energetically advantageous for the atoms to "align and lock" their phases, there is no reason whatsoever for the BE "order" to transmute itself into the observed "coherence". And for pretending that the discovery of "coherence" be just the experimental proof that the BE condensate is indeed coherent [7] and not that, instead, BE condensation is not what is being observed.

[2]As emphasized above, at the typical densities probed, $n \cong 10^{14}\ cm^{-3}$, the average interatomic distance is $a \cong 2000 \mathring{A}$, three orders of magnitude larger than the atomic size.

It must be stressed, however, that the above difficulties of BE-condensation are well present to the minds of the most thoughtful theoreticians, P.W. Anderson foremost among them [8], who believe that BE condensates may nevertheless become coherent as a consequence of their mutual interactions. However, this view has been challenged in Ref. [4], on the basis of a set of common sense arguments, that convincingly reject the notion that in a macroscopic system "ignorance may create coherence" It is only recently that a few papers have appeared claiming to have some proof or example in support of Anderson's conjecture that a BE condensate develops in due time a macroscopic phase [9], [10]. The fact that here one is dealing with some kind of paralogism or other is revealed by the claim in Ref. [9] that the phase appears spontaneously and kinematically, while it requires a definite interaction with the observer in Ref. [10], whose authors do not care to explain why in Ref. [9] one obtains for free what they get only after having generalized the Copenhagen contorsions to a truly macroscopic system, leaving unanswered the very reasonable objections of Leggett and Sols [4].

To corroborate the view of these latter authors we would like to note that if one assumes, as everybody does, that BE condensation initially produces a state like (6), then any perturbation of the general type:

$$H_{SR} = \frac{1}{2}\int d^3x \int d^3y \Psi(\vec{x},t)^\dagger \Psi(\vec{x},t) V_{SR}(\vec{x}-\vec{y}) \Psi(\vec{y},t)^\dagger \Psi(\vec{y},t) \tag{7}$$

where $V_{SR}(\vec{x}-\vec{y})$ is a short-range interaction potential, will lead to an evolution equation for the order parameter:

$$i\dot{\Psi}(\vec{x},t) = \langle\Omega|[H_{SR}, \Psi(\vec{x},t)]|\Omega\rangle \tag{8}$$

whose RHS can easily be shown to vanish for a state of the type (6), a result that can be directly related to the gauge-invariance of the Hamiltonian (7), which, as emphasized time and again by Anderson himself, prevents the state (6) from acquiring a non-trivial (relative) phase. Thus, the only way for the atoms' condensate to develop an "order parameter" (5) is that there exists extra non-perturbative long- range interactions leading to a new ground state where the atomic systems are in phase, and are described by a *coherent superposition* of states of the type (6). Or, put differently, what has been experimentally revealed is not a BE-condensate but a peculiar coherent state [see Eq. (5)] whose "raison d'être" can only be found in a long-range interaction totally extraneous, not only to the simple perfect gas of Bose and Einstein, but to modern condensed matter theory as well. We shall call this state Coherent Electrodynamic Condensate (CEC). In this paper we shall demonstrate that the recent experimental findings can be perfectly understood and explained in terms of the QED coherent interactions among the ultracold alkaline atoms, whose general theory appears in a recently published book [11].

We write the Coherence Equations (CE) for the matter and the e.m. field amplitudes in a box of volume V, as [11]

$$i\dot{a} = \frac{e}{\omega}\left(\frac{N}{V}\right)^{\frac{1}{2}} \frac{1}{(2\omega)^{\frac{1}{2}}} \langle 0|\vec{\epsilon}^{\,r}_{\vec{k}} \cdot \vec{J}|\alpha\rangle \alpha^r_{\vec{k}} b_{\alpha\vec{k}},$$

$$i\dot{b}_{\alpha\vec{k}} = \frac{e}{\omega}\left(\frac{N}{V}\right)^{\frac{1}{2}} \frac{1}{(2\omega)^{\frac{1}{2}}} \langle\alpha|\vec{\epsilon}^{\,\star r}_{\vec{k}} \cdot \vec{J}|0\rangle \alpha^r_{\vec{k}} a,$$

$$-\frac{1}{2}\ddot{\alpha}^r_{\vec{k}} + i\dot{\alpha}^r_{\vec{k}} = \frac{e}{\omega}\left(\frac{N}{V}\right)^{\frac{1}{2}} \frac{1}{(2\omega)^{\frac{1}{2}}} \langle 0|\vec{\epsilon}^{\star r}_{\vec{k}} \cdot \vec{J}|\alpha\rangle b_{\alpha\vec{k}} a, \tag{9}$$

where a is the (coherent) amplitude of the matter (atom) field in the ground state, $b_{\alpha\vec{k}}$ the amplitude for the excited state $|a\rangle$ and Fourier momentum $\vec{k}$, ω the frequency of the transition with matrix element $\langle 0|\vec{J} \cdot \vec{\epsilon}^r_{\vec{k}}|\alpha\rangle$, and $\alpha^r_{\vec{k}}$ the e.m. amplitude of momentum $|\vec{k}| = \omega$, and polarisation r with $\vec{k} \cdot \vec{\epsilon}^r_{\vec{k}} = 0$. The time-derivative is with respect to the adimensional time $\tau = \omega t$. By restricting the system (9) to a "Coherence Domain" (CD), a cubic region of side equal to the wavelength $\lambda = \frac{2\pi}{\omega}$, for each of the 6 independent Fourier components $\vec{k} = \omega[(1,0,0),(-1,0,0),...]$, the last two equations of the system (9) become:

$$\begin{aligned} i\dot{b}_{\alpha\vec{k}} &= g\alpha^{\star r}_{\vec{k}} a, \\ -\frac{1}{2}\ddot{\alpha}^r_{\vec{k}} + i\dot{\alpha}^r_{\vec{k}} &= g b_{\alpha\vec{k}} a, \end{aligned} \tag{10}$$

where the coupling constant is

$$g^2 = \frac{e^2}{m_e^2}\left(\frac{N}{V}\right)\frac{1}{4\omega^2} f, \tag{11}$$

and f is the oscillator strength for the transition $0 \leftrightarrow \alpha$. The very low value of $\left(\frac{N}{V}\right)$, and the correspondingly low value of the coupling constant g clearly imply that the coherent process involving the e.m. field modes and the atomic transitions with frequency ω is *weak*, i.e. the coherent amplitudes $\alpha^r_{\vec{k}}$ and $b_{\alpha\vec{k}}$ are of $O\left(\frac{1}{N_{CD}^{\frac{1}{2}}}\right)$, where N_{CD} is the number of atomic systems to be found (in the average) in the Coherence Domain. This latter fact puts an obvious constraint on the density necessary for the development of QED coherence in the macroscopic system, namely:

$$n \geq n_{CE} = \frac{1}{x_c\lambda^3} = \frac{n_0}{x_c} \tag{12}$$

where $x_c (0 \leq x_c \leq 1)$ is the fraction of atomic systems involved in the coherent process. The constraint (12) then simply means that one must have (in the average) at least *one* coherent system per CD. By solving the system (9) -(10) with the appropriate boundary conditions [11] for $n > n_0$ one obtains the following expression for the "gap" associated to the Coherent Electrodynamic Condensate (CEC):

$$\delta = \frac{3\alpha}{8\pi^2}\frac{\omega^2}{m_e} f x_c = \delta_0 x_c \tag{13}$$

Please note that δ_0 refers to the maximum value of the gap, which, if one takes into account the necessary spatial variation [3], implies an average value $\bar{\delta_0} \cong \frac{1}{2}\delta_0$.

[3] See ref. [11], Ch. 3.

One should also notice the linearity of δ in x_c, stemming from the collective, many-body nature of the coherent e.m. interaction.

Eqs. (12) and (13) embody the main physical features of the CEC, which can now be brought to bear upon the recent experiments. In Table I we report the relevant parameters for the three alkali atoms that have been studied so far [4] and we add also a line with our prediction for atomic hydrogen.

TABLE I.The main parameters of the CEC for different atomic systems

Atom	$\lambda(A)$	$\omega(eV)$	f	$n_0\ cm^{-3}$	$\delta_0(\mu K)$	$\nu_{BE}\ \frac{cm^{-3}}{(\mu K)^{\frac{3}{2}}}$	$T_{BEC}(\mu K)$
Na	5889.9	2.13	.64	$4.89\ 10^{12}$	18.7	$5.62\ 10^{13}$	0.20
	5895.5	2.13	.32	$4.87\ 10^{12}$	9.35		
Li	6707	1.87	.50	$3.31\ 10^{12}$	11.25	$9.40\ 10^{12}$	0.49
	6707	1.87	.25	$3.31\ 10^{12}$	5.623		
Rb	7800	1.61	.67	$2.11\ 10^{12}$	11.17	$4.02\ 10^{14}$	0.03
	7947	1.58	.33	$2.12\ 10^{12}$	5.58		
H	1215.67	10.34	.44	$5.57\ 10^{14}$	200	$5.08\ 10^{11}$	105

According to coherent QED [11] for $T \neq 0$ the CEC consists of two fluids, the coherent one, whose fraction is x_c , and the incoherent one which lives in the interstices of the Coherence Domains (CD) whose fraction is given by (m is the mass of the atom):

$$(1 - x_c) = \frac{1}{n}\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} f\left(\frac{\bar{\delta}}{T}x_c\right) \tag{14}$$

where

$$f(x) = \frac{2}{\sqrt{\pi}}\int_0^\infty dt \frac{t^{\frac{1}{2}}}{(e^{t+x} - 1)} \tag{15}$$

is a well-known function directly related to the sum over states of a Bose gas. The condition for BE condensation can be read off also from (14) once we put $x_c = 0$ and recall that $f(0) = 2.612$, thus obtaining Eq. (1).

In the case of a CEC the condition is rather different, we must first of all take for the density n the value n_{CE} given by (12), and determine x_c at the transition through:

$$\frac{1 - x_c}{x_c} = \left(\frac{n_{BE}}{n_0}\right)\frac{f\left(\frac{\bar{\delta}}{T}x_c\right)}{2.612} \tag{16}$$

which clearly holds only when $n > n_0$. Thus when $n_{BE} < n_0$, and this happens when the temperature T is below the value

$$T_{BEC} = \frac{2\pi}{m}\left(\frac{n_0}{2.612}\right)^{\frac{2}{3}} \tag{17}$$

[4] Observations of condensates of Li-atoms have been reported in Ref. [12]

for $n_{BE} < n < n_0$ our theory predicts an *incoherent* BE-condensate, in agreement with the theory of Bose and Einstein.

The existence for $T < T_{BEC}$ of an interval of densities where the CEC cannot occur, leaving as the only possibility the formation of an incoherent BEC, is a most significant prediction of our theory, which allows the test of the predicted coherence properties of the two condensates as well as of the occurrence of a phase transition between them when $n = n_0$.

In Fig. 1 we show our predictions for the phase diagram for an ensemble of both Na and Rb atoms. One sees that below a line, which only in a limited range of densities is different from the line given by Eq. (1), we predict condensation, which exhibits the coherence properties, typical of the electrodynamic condensate, only when $n \geq n_0 = \frac{1}{\lambda^3}$, where λ is the wavelength of the line with largest oscillator strength.

Apart from the prediction that the MIT experiments are in the CEC-region, which has been dramatically verified by the observation of strong interference between two condensates [7], our phase- diagram predicts a large deviation from the "kinematics" of BEC for the Boulder experiments [5], while for MIT [6] no such deviation occurs. Indeed, a glance at Fig. 1 shows that when the condensate first appears in the Boulder experiments, i.e. for $T = 170\ nK$ and $n = 2.6\ 10^{12}\ cm^{-3}$, we are far away (almost an order of magnitude) from $n_{BE} = 2.94\ 10^{13}\ cm^{-3}$ predicted by the Bose-Einstein theory, while our theory predicts that at $T = 170\ nK$ the transition density is about $n_0 = 2.11\ 10^{12}\ cm^{-3}$, in full agreement with observations. On the other hand, in the case of the MIT experiments, where the transition is observed for $T \cong 2\mu K$, we predict it to occur for $n \cong n_{BE}$, in agreement with experiment.Finally we note that, according to our theory, the maximum temperature T_{BEC} for which a phase- transition between BEC and CEC may be observed is 190 nK and 30 nK for Na and Rb atoms respectively.

In conclusion, we have shown that QED is capable to finally solve the puzzle raised by the discovery of coherence in condensates of highly diluted, ultracold alkali atoms. Having provided further arguments against the likelihood (and reasonableness) of coherence in BEC's, we have outlined the theoretical bases for the emergence of Coherent Electrodynamic Condensates (CEC's) and described their main features. We have then shown that the phase diagram is essentially modified, there appearing for $T < T_{BEC}$ a line of transition between the BEC and the CEC, and in addition deviations from the "kinematics" of BEC that affect the Boulder [6] but not the MIT [7] experiments

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FIGURE CAPTION

Figure 1: The phase diagram for the condensation of Rb and Na Atoms. The lines are the boundaries of the regions where the Coherent Electrodynamics Condensation (CEC) occurs.