Pedestrian route-choice and activity scheduling theory and models

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Received 29 January 2002; accepted 31 July 2002

Abstract

Among the most interesting and challenging theoretical and practical problems in describing pedestrians behavior are route choice and activity scheduling. Compared to other modes of transport, a characteristic feature of pedestrian route choice is that routes are continuous trajectories in time and space: since a pedestrian chooses a route from an infinite set of alternatives, dedicated theories and models describing pedestrian route choice are required.

This article puts forward a new theory of pedestrian behavior under uncertainty based on the concept of utility maximization. The main behavioral assumption is that pedestrians optimize some predicted pedestrian-specific utility function, representing a trade-off between the utility gained from performing activities at a specific location, and the predicted cost of walking subject to the physical limitations of the pedestrians and the kinematics of the pedestrian. The uncertainty reflects the randomness of the experienced traffic conditions.

Based on this normative theory, route choice, activity area choice, and activity scheduling are simultaneously optimized using dynamic programming for different traffic conditions and uncertainty levels. Throughout the article, the concepts are illustrated by examples.

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Keywords: Pedestrian travel behavior; Route choice; Activity scheduling; Destination choice; Dynamic programming

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0191-2615/ $ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/S0191-2615(03)00007-9
1. Background

Understanding pedestrian behavior is essential in the planning and the design of airports, public transport stations, shopping malls, etc., but also in public transit timetable design. Modeling tools can support infrastructure designers as well as public transport planners to optimize their plans. Furthermore, the management of pedestrian flows demands understanding of both the collective pedestrian flows as well as the individual pedestrian movements in the flow.

Pedestrian traffic and vehicular traffic are very different, justifying dedicated theory and model development. The differences pertain both to traffic operations, and to the travel behavior level. Motivated by the need for accurate dedicated pedestrian flow models, comprehensive theory and models for pedestrian activity scheduling, route determination in the two-dimensional space, and walking behavior have been developed. This paper focuses on the behavior at the tactical level. The walking behavior is discussed elsewhere (Hoogendoorn, 2001).

The article is outlined as follows. Section 2 discusses some empirical facts about pedestrian activity scheduling and route choice. Section 3 recalls previous theories and models describing pedestrian route choice behavior. Section 4 outlines the normative theory. Sections 5 and 6 respectively present the route choice and activity scheduling modeling approach. We recall the walker model from (Hoogendoorn, 2001) in Section 7. Model applications are described in Section 8. Section 9 summarizes the research results and discusses future research directions.

2. Empirical facts on pedestrian scheduling and route choice

In this article, pedestrian activity scheduling pertains to which, in which order, and where pedestrians perform activities. Pedestrian activity scheduling has not been studied comprehensively in the past. There are some indications that pedestrians somehow optimize the order in which they perform their activities, and that directness plays an important role (Helbing, 1997).

Hill (1982) has analyzed pedestrian strategies for choosing and describing routes. He concludes that, like most walking processes, route selection strategies are largely subconscious. Furthermore, directness is the most common reason for choosing a particular route. The route directness pertains not only to the length of the route, but also to its complexity (in terms of direction changes). Pedestrians appear to frequently choose the shortest route, albeit they are seldom aware that they are minimizing distance as a primary strategy in route choice (Senevarante and Morall, 1986; Guy, 1987). Other studies indicate that besides distance, pleasantness is an important route attribute together producing a high correlation with the route preferences (Bovy and Stern, 1990).

Other factors deemed important in route choice behavior are habit, number of crossings, pollution and noise levels, safety and shelter from poor weather conditions, and stimulation of the environment. The extent to which these route attributes play a substantial role his route choice behavior depends to a large extent on trip purpose (Bovy and Stern, 1990), e.g. scenery is very important for recreational trips, but it plays no role for work-related walking trips.

Cheung and Lam (1998) study pedestrians choosing between escalators and stairways in subway stations and its dependency on the differences in travel times. The authors showed that pedestrians are more susceptible to delays in the descending direction than in the ascending direction, and are more inclined to use the escalator in the latter case.
3. Modeling approaches to pedestrian travel behavior

Queuing models (Lovas, 1994) use Markov-chain models to describe how pedestrians move from one node of the network (mostly a room) to another. Random waiting times are incurred on the links, due to queues building up when pedestrian traffic demand is larger than the door capacity. Queuing models have been used mostly to describe pedestrian evacuation behavior from buildings.

Gipps (1986) and Hamacher and Tjandra (2001) describe pedestrian route choice through the walking facility by determining a finite number of routes through the walking infrastructure and applying basic discrete choice modeling. Similar to the model described in this article, the main theoretical assumption is that pedestrians make a subjective rational choice between alternatives. Contrary to the model described in this article, the number of choice options is finite. Verlander (1997) estimates discrete choice models using household-based diary data. Teklenburg et al. (1993) use a Space Syntax model, which is calibrated using pedestrian flow data.

Notwithstanding the practicality of assuming only a limited number of route alternatives, in real life pedestrians can choose between an infinite (and in fact, non-countable) number of routes. Using potential functions, Hughes (2002) accounts for this aspect, by describing the optimal walking direction to the destination (in terms of travel time) as a function of the current location $x$ of the pedestrian. However, the approach prohibits including general route attributes, such as walking distance or stimulation of the environment, as well as uncertainty in the traffic conditions expected by the pedestrians. The research described in this article has succeeded in remediating these issues, while at the same time establishing a solid theoretical basis for pedestrian route-choice and activity scheduling.

4. Normative pedestrian behavior theory

The main behavioral assumption here, is that all actions of the pedestrian, let it be performing an activity or walking along a certain route, will provide utility (or equivalently, induce cost) to him. The pedestrian will predict and optimize this expected utility, taking into account the uncertainty in the expected traffic conditions (similar to microeconomic consumer theory). It is well known that normative choice theory will not fully cover real-life human choice behavior. It does however provide a very convenient framework for modeling human decision making (Van Berkum and Van Der Mede, 1993). Moreover, several empirical studies have shown the applicability of utility-based approaches to pedestrian route choice (Hill, 1982; Bovy and Stern, 1990). Hence, its use in pedestrian behavior theory is justified.

The theory presented here differs from discrete choice theory in a number of ways. First and foremost, an infinite number of alternative routes is considered. Secondly, the randomness in the theory pertains to the uncertainty in the route that can be realised. In stochastic discrete choice theory, the randomness reflects both the fact that people do not always make the same decisions under the same circumstances and the analyst's lack of more precise knowledge about individuals' decision processes. Furthermore, discrete choice models are often used to describe the differences in observed choices made in a sample of individuals. The theory presented here is designed to describe choice behavior of individuals, and is there not directly representative for groups of
individuals. Nevertheless, the continuous theory is easily extended to include for instance taste variation.

4.1. Pedestrian behavioral levels

In our approach we distinguish choices at the following three levels:

1. Departure time choice, and activity pattern choice (strategic level);
2. Activity scheduling, activity area choice, and route-choice to reach activity areas (tactical level);
3. Walking behavior (operational level).

In this hierarchy, expected utilities at lower levels influence choices at higher levels. Choices at higher levels condition choice sets at lower levels. This article focuses on pedestrian behavior at the tactical level. The events and decisions causing the pedestrian to arrive at the walking facility are not considered; pedestrian arrival patterns, distinguishing groups of pedestrians having similar characteristics (activity sets, travel purpose, demographic characteristics, etc.) are assumed known a priori. With respect to the operational level, it is hypothesized that pedestrians adhere to their planned route (determined at the tactical level) as much as possible. In addition, walking is affected by interactions with other pedestrians or obstacles. For details, we refer to (Hoogendoorn, 2001).

The tactical level behavior is influenced by both external factors (e.g. presence of obstacles, stimulation of the environment), and internal (or personal) factors (e.g. time–pressure, attitudes of the pedestrian). Together with the expected traffic conditions (congestion, average speeds), the decisions at the tactical level serve as input for pedestrian’s walking behavior. In turn, the traffic conditions resulting from the pedestrian traffic demands and the collective walking behavior will affect the expected traffic conditions and thus the behavior at the tactical level.

4.2. Tactical level decision variables

Before we present the problem formulation, let us introduce the key decision variables for the pedestrian behavior at the tactical level

1. The activity schedule $S = \{i\}$, which is an ordered set describing which activities $i \in \Sigma$ are performed and in which order.
2. The velocity trajectory $v(\cdot)$ yielding the route $x(\cdot)$ through the facility.
3. The activity times $T_i$ at which activities $i \in S$ are performed, such that $x(T_i) \in A_{ij}$ for some $j$, where $A_{ij}$ denote the activity areas where activity $i$ can be performed.

A pedestrian arriving at the walking facility $\Omega \subset \mathbb{R}^2$ aims to perform (a type of) activities from the subjective activity choice set $\Sigma$, which consists of possible activities to perform in the facility, e.g. “buy a newspaper”, “buy a train ticket”, “wait at the train platform”, or “access the train”. The activities are generic, and reflect the purposes of the pedestrians in the facility.

The pedestrian chooses activities from the subjective activity choice set $\Sigma$, yielding the activity schedule $S = \{i\}$. $S$ describes which activities from $\Sigma$ are performed and in which order. The
pedestrian may have some freedom to choose where activity \( i \) from \( S \) is performed. This is modeled by considering different activity areas \( A_{ij} \subset \Omega \) with \( j = 1, \ldots, J_i \), where activity \( i \) can be performed. For example, a traveler can "buy a newspaper" at different ticket machines or ticket offices.

**Terminal time** \( T_i \) for activity \( i \) denotes the instant at which activity \( i \) is completed. That is, the difference \( T_i - T_{i-1} \) equals the expected walking time and the time needed to perform activity \( i \). Together with the route \( x(\cdot) \) through the facility, the terminal time determines the activity area \( A_{ij} \) chosen by the pedestrian, i.e. \( x(T_i) \in A_{ij} \) (assuming that the pedestrian will not move while performing the activity). The route \( x(\cdot) \) is a continuous function, uniquely determined by the velocity path \( v(\cdot) \)

\[
x(\cdot) := \{x(\tau) \in \Omega||t_0 \leq \tau \leq t_1 \text{ s.t. } \dot{x}(\tau) = v(\tau) \text{ and } x(t_0) = x_0\}
\]

### 4.3. Subjective utility optimization at the tactical level

We hypothesize that at the tactical level, a pedestrian makes a *simultaneous decision at time* \( t_0 \) minimizing the expected disutility or cost \( C \). This decision will cover all decision variables described in the previous section (activity patterns \( S \), terminal times \( T_i \), and the velocity path \( v(\cdot) \)). The expected disutility \( C = C(S, \{ T_i \}_{i \in S}, v(\cdot)|t_0, x_0) \) of the combined choice of a pedestrian entering the walking facility at instant \( t_0 \) at location \( x_0 \) stems from performing specific activities at certain activity areas, the order in which the activities are performed, and the expected cost of walking between the activity areas. We assume that the total disutility can be written as follows

\[
C(S, \{ T_i \}_{i \in S}, v(\cdot)|t_0, x_0) := \sum_{i \in S} C_i(T_i, v(\cdot)|T_{i-1}, x(T_{i-1})) + \psi(S)
\]

where \( C_i \) denotes the combined cost of walking from the location \( x(T_{i-1}) \in A_{i-1j} \) to \( x(T_i) \in A_{ij} \) where activity \( i \) is performed, and the cost (or utility \( U_{ij} \)) of performing activity \( i \) at activity area \( A_{ij} \); \( \psi \) denotes the cost of the activity scheduling (e.g. due to the specific order of activities). In the remainder, the components \( C_i \) are specified. Subjective utility optimization then yields that the pedestrian makes the following combined choice at the tactical level

\[
(S^*, \{ T_i \}_{i \in S}, v^*(\cdot)) = \arg \min C(S, \{ T_i \}_{i \in S}, v(\cdot)|t_0, x_0)
\]
1. Distance or travel time between origin and destination.
2. Proximity of obstacles or other physical obstructions; closeness to walls.
3. Number of sharp turns and rapid directional changes (route directness).
4. Expected number of interactions with other pedestrians (level-of-service).
5. Stimulation of environment, and attractiveness (e.g. ambience conditions, shopping windows, shelter in case of poor weather conditions).

Empirical studies (Bovy and Stern, 1990; Guy, 1987; Senevarante and Morall, 1986) have shown that these factors are not mutually consistent, while their importance in route choice will vary between different (homogeneous) groups of pedestrians.

4.4. Prevailing traffic conditions and dynamic pedestrian assignment

One important aspect in pedestrian activity scheduling and route choice is the inclusion of prevailing or future traffic conditions into the decision-making at the tactical level. We assume that pedestrians have information (visual information, experience) regarding prevailing and future conditions and choose their planned schedule and route accordingly. If we assume that only the current conditions are included, the route costs can simply be computed given prevailing conditions. It is likely that the pedestrian will reconsider his choices made at several times during his stay in the walking facility, thereby considering the observed in the current traffic conditions.

Assuming that the pedestrians have complete information concerning future flow conditions, we need to explicitly consider the consistency between activity scheduling/route choice and flow operations. While the behavior at the tactical level is dependent on the (future) flow operations, these will in turn depend on the activity scheduling and pedestrian route-choice. We then need to solve a user-optimal choice equilibrium problem.

4.5. Operationalization of pedestrian behavior theory

In the remainder, we will operationalize the outlined theory, proposing models describing pedestrian behavior at the tactical level. We derive models by applying dynamic programming theory. In Section 5, models are proposed for pedestrian route choice under uncertainty in continuous time and space for a single activity with multiple activity areas $A_{ij}$. Subsequently, we will propose models to describe how pedestrians determine the subjective optimal activity schedule (Section 6).

5. Optimal subjective route-choice under uncertainty

This section considers the combined route-choice and activity area choice of a pedestrian aiming to perform a given activity $i$ at either of the activity areas $A_{ij}$, with $j = 1, \ldots, J_i$, where activity $i$ can be performed. We assume that in simultaneously choosing the destination area and the route, the pedestrian will minimize the subjective expected disutility. In subjective utility theory, perceived disutility is generally assumed to be the weighted sum of different route attributes, such as (expected) travel time, travel time variance, distance traveled, safety, comfort, etc. Pedestrians will
often have to divert from their planned route, e.g. when interacting with another pedestrian, thus having to reconsider a new route. The theory should provide a framework in which the pedestrian’s flexible adaptation to other routes is included. This is solved mathematically by treating the route and activity area choice problem simultaneously for all locations \( x \) and instants \( t \), adopting the so-called expected minimum perceived disutility function \( W(t, x) \) in continuous time and space.

5.1. Pedestrian kinematics

To apply the approach, consider the location \( x \) (the state) and the velocity \( v \) (the control) of a pedestrian. To describe the route choice behavior, it is hypothesized that the pedestrian uses an internal model to estimate and predict the route costs. To this end, he will determine his current position \( x(t) \) at instant \( t \), reflected by \( \hat{x} \), and predict his future positions \( x(\tau) \) for \( \tau > t \) using the internal state prediction model:

\[
\begin{align*}
\text{dx} &= v \text{dt} + \sigma \text{dw} \quad \text{subject to } x(t) = \hat{x} \\
\end{align*}
\]

Here \( v = v(\tau) \) denotes velocity (i.e. speed and direction) of the pedestrian for \( \tau > t \). In this formulation, \( w \) denotes a standard Wiener process, meaning that for very small time periods \( [t, t+h) \), the increase \( w(t + h) - w(t) \) is a \( N(0, hI_m) \)-distributed random variable, where \( I_m \) denotes the \( m \times m \) identity matrix; \( \sigma = \sigma(x, v) \) is a \( 2 \times m \) matrix, reflecting the way in which the white noise vector \( w \) affects the location; \( \sigma'(w(t + h) - w(t)) \) is \( N(0, h\sigma') \)-distributed. The white noise term reflects the uncertainty in the expected traffic conditions and the resulting effects on the subject’s kinematics. The uncertainty reflects among other things lack of experience, observability and randomness of future conditions.

The speed \( ||v|| \) of the pedestrians is restricted by their physical limitations, and by the presence of other pedestrians in the flow. This is why the following restrictions are applied to the velocity \( v \)

\[
V_a(t, x) = \{ v \text{ such that } ||v|| \leq v_0(t, x) \} \subset \mathbb{R}^2
\]

where \( V_a \) denotes the set of admissible velocities. The dependence on \( t \) and \( x \) is used to describe that the maximum speed may change due to changing flow conditions, as well as differences in maximum speeds between different parts of the walking infrastructure. In illustration, the maximum speed on flat terrain will be higher than on a stairway or an escalator. The pedestrian \( p \) specific maximum speed also depends on the characteristics of \( p \) (age, gender, trip-purpose, luggage, etc.).

5.2. Generalized walking cost and activity utility

Let \( [t, t_1] \) denote the pedestrian’s planning period, where \( t \) and \( t_1 \) respectively denote the current time and the terminal time (planning horizon); let \( T_a \) denote the time of first arrival at either activity area \( A_{ij} \); and let \( T_i = \min(t_1, T_a) \). Consider the velocity (or control) path \( v_{[t, T_i]} \). In applying this control, the pedestrian predicts his location using the internal model (4). The predicted location \( x(\tau) \) is a stochastic variable. We hypothesize that given an arbitrary control path \( v_{[t, T_i]} \) and initial position \( x(t) = \hat{x} \), the pedestrian will determine the expected disutility or expected cost \( C_i \) of applying the intended velocity \( v_{[t, T_i]} \), according to the following cost definition.
\[
C_i(T_i, v_{[t, T_i]}|T_{i-1}, x(T_{i-1})) := E \left[ \int_t^{T_i} L(\tau, x(\tau), v(\tau)) d\tau + \phi(T_i, x(T_i)) \right] 
\]  
(6)

s.t. Eq. (4), where \( L \) and \( \phi \) denote the so-called running cost and the terminal cost respectively.

The running cost \( L(\tau, x(\tau), v(\tau)) \) reflects the costs that are incurred during a very small time period \([\tau, \tau + d\tau]\), given that the subject is located at \( x(\tau) \) and is ‘applying’ velocity \( v(\tau) \) to change his position. The terminal cost \( \phi(T_i, x(T_i)) \) reflect the cost due to ending up at position \( x(T_i) \) at the terminal time \( T_i \). These costs typically reflect the penalty that may be incurred when the subject does not arrive at the destination areas \( A_{ij} \) in time. Note that since the route costs are described by means of an integral (6), the theory assumes that the route costs are additive (i.e. the total route cost can be described by the sum of the costs of its constituent subroutes).

5.3. Specification of the terminal cost

By definition, the terminal time \( T_i \) either equals the final time \( t_1 \) of the planning period or the first time \( T_a \) the pedestrian arrives at either of the activity areas \( A_{ij} \). As a result, the terminal costs \( \phi \) are described on a region in \([t, t_1] \times \Omega \); in mathematical terms:

\[
\phi(T_i, x(T_i)) = \begin{cases} 
- U_{ij}(T_i) & x(T_i) \in A_{ij} \quad T_i < t_1 \\
\phi_i & x(T_i) \notin \cup_{j} A_{ij} \quad T_i = t_1 
\end{cases} 
\]

(7)

The terminal cost \( \phi \) thus reflects the penalty \( \phi_i \) for not having arrived at any of the activity areas \( A_{ij} \) in time, i.e. before the end \( t_1 \) of the planning period, and the area specific utility \( U_{ij} \) of performing activity \( i \) at \( A_{ij} \). Note that to include preferred arrival times the utility \( U_{ij} \) depends on the terminal time \( T_i \). Also note that by including multiple activity areas in the terminal costs, the route choice and activity area choice problem in performing a certain \( i \) can be solved jointly.

5.4. Specification of the running costs

The running cost \( L \) reflects the different cost aspects relating to the route attributes considered by \( p \). For the sake of simplicity, we assume that the running cost is linear-in-parameters, i.e.

\[
L(t, x, v) = \sum_k c_k L_k(t, x, v) 
\]

(8)

where \( L_k \) denote the contribution of the different route attributes \( k \), and \( c_k \) denote the relative weights (importance of the attributes). However, linearity is not required for application of the approach described in the remainder of this article. Note that not all weights can be uniquely determined from real-life observed behavior, since only the relative importance of the weights is relevant. The parameters \( c_k \) will be different for different homogeneous groups, e.g. groups having different travel purposes. Alternatively, the parameters can reflect differences amongst pedestrians according to their age, gender, physical health, etc. The different cost factors \( L_k \) are described in the ensuing sections.

5.4.1. Expected travel time

Expected travel time is included in the expected route cost (6) by defining \( L_1 \) as follows

\[
L_1(t, x, v) = 1 
\]

(9)
Substitution of running cost \((9)\) yields the following contribution to the route cost \((6)\)

\[
E \left[ \int_t^{T_i} L_1(\tau, x(\tau), v(\tau)) d\tau \right] = E \left[ \int_t^{T_i} c_1 d\tau \right] = c_1 E[T_i - t] \tag{10}
\]

Eq. (10) shows that the contribution of the running cost factor \((9)\) is indeed equal to the expected travel time \(T_i - t\), multiplied by the weight \(c_1\). This weight factor \(c_1\) expresses \textit{time–pressure}, which in turn depends on for instance the trip purpose.

### 5.4.2. Discomfort due to walking too close to obstacles and walls

Obstacles are described by areas \(O_m \subset \Omega\), for \(m = 1, \ldots, M\). For any obstacle \(m\), we assume that the running cost component \(L_2\) is given by a monotonically decreasing function \(g\) of the distance \(d(x, O_m)\) between the location \(x\) of the pedestrian and the obstacle, i.e.

\[
L_2(t, x, v) = g_m(d(O_m, x)) = a_m \exp(-d(O_m, x)/b_m) \tag{11}
\]

The distance is defined by the minimum distance between the pedestrian location \(x\) and obstacle \(m\)

\[
d(x, O_m) = \min_{y \in O_m} \{||x - y||\} \tag{12}
\]

where \(||z||\) is the Euclidean length of a vector \(z\). Eq. (11) is based on the specification of the repellent force term of obstacles proposed by Helbing (1997). The parameters \(a_m > 0\) and \(b_m > 0\) are scaling parameters, describing the \textit{region of influence} of obstacle \(m\). Both \(a_m\) and \(b_m\) are dependent on the type of obstacle that is considered, e.g. they are different for building faces with and without a window, regular walls, trees, newsstands, etc. (HCM, 2000).

### 5.4.3. Walking at a certain speed (planned walking speed)

To describe that the planned walking speed \(||v||\) is a trade-off between the time remaining to get to the activity area in time and the energy use due to walking at a particular speed, we assume that this \textit{energy consumption} is a quadratic function of the pedestrian speed

\[
L_3(t, x, v) = \frac{1}{2} ||v||^2 = \frac{1}{2} v'v \tag{13}
\]

It is interesting to see that in practice, energy consumption, walking speed, and time–pressure appear to be closely connected: Weidmann (1993) shows that the ‘optimal energy level’ (e.g. minimal energy consumption per kilometer) is attained at walking speeds of 1.39 m/s, which is close to observed average speeds of 1.34 m/s.

### 5.4.4. Expected number of pedestrian interactions (discomfort due to crowding) and level-of-service

In including the \textit{cost of the expected pedestrian interactions}, we consider the function \(\zeta = \zeta(t, x)\), describing the expected number of interactions with other pedestrians at \((t, x)\). Note that for pedestrian flow operations, the frequency and the severity of interactions (or rather, physical contact) relates directly to the definition of the \textit{level-of-service} (HCM, 2000). In the remainder, it is assumed that \(\zeta(t, x)\) is some (non-linear) function of the density \(k(t, x)\)

\[
L_4(t, x, v) = \zeta(k(t, x)) \tag{14}
\]
We can argue that, depending on trip-purpose, age, gender, etc. pedestrians may be inclined to walk in areas where there are at least some pedestrians. In this case, small density values may have an attracting effect on pedestrians route choice behavior, yielding that $\zeta(k) < 0$ for some $k < k_{cr}$.

5.4.5. Stimulation of the environment

Finally, we consider the stimulating effects of the environment. This can be described relatively easy by considering the benefit $\gamma(t, x) \leq 0$ of walking at a certain location $x$ at instant $t$. Note that the benefit has a negative sign (negative cost). We have

$$L_5(t, x, v) = \gamma(t, x)$$

(15)

In substituting the contributions (9)–(15) into the running cost (8), we get

$$L(t, x, v) = c_1 + c_2 \sum_m a_m e^{-d(x, O_m)/b_m} + \frac{1}{2} c_3 v' v + c_4 \zeta(t, x) + c_5 \gamma(t, x)$$

(16)

Note that generally the weights $c_k$ as well as the weights $a_m$ cannot be determined uniquely from data, since only the ratio between the weights are determinant for pedestrian behavior. Without loss of generality, we can set $c_1 = 1$.

The approach is easily adapted when considering a heterogeneous population with taste-variation and differences in the physical abilities. In fact, these differences are the main reasons for including the different cost factors: empirical research has indicated large differences in how different types of pedestrians value route attributes (Bovy and Stern, 1990; Hill, 1982; Senevarante and Morall, 1986; Guy, 1987). The pedestrian population is stratified into homogeneous groups, and route-choice and activity scheduling is solved for each group. The characteristics of each homogeneous group is characterized by the utilities $U_{ij}$ gained when performing activity $i$ at $A_{ij}$, the weights $c_k$, and the maximum walking speed $v_0(t, x)$. For instance, when travel time is the dominant factor (e.g. for commuting pedestrians), $c_1$ will be relatively large compared to $c_2$, $c_4$ and $c_5$; when shopping, stimulation of the environment will be a relatively important attribute, which is expressed by $c_5$. Furthermore, the weights $c_k$ also depend on the situation: in case of an emergency, travel time (or distance) will be the dominantly important attribute. In the latter case however, the applicability of utility optimization approaches may be limited, depending on the evacuation situation.

5.5. Route choice and activity area choice in continuous space

Let us now formulate the combined route-choice and activity area (destination) choice problem. Eq. (6) shows that the expected (predicted) subjective cost $C$ is a function of initial instant $t$, location $x(t) = \hat{x}$, and velocity path $v(t, T)$.

5.5.1. Modeling principle and problem formulation

The subjective utility optimization paradigm implies that the pedestrian will choose the velocities $v(t, T)$ yielding the predicted route and used activity area that minimize the subjective expected cost (6) subject to the internal prediction model (4). The subjective utility optimization paradigm thus that the pedestrian determines the optimal velocity $v(t, T)$ satisfying
\[
v^*_t (t, \hat{x}; v_{[t, T_t]} \{ A_{ij} \}) = \arg \min_C \left( t, \hat{x}; v_{[t, T_t]} \{ A_{ij} \} \right) = \arg \min_E \left[ \int_{t}^{T_t} L(\tau, x(\tau), v(\tau)) d\tau + \phi(T_t, x(T_t)) \right]
\]

where \( L \) and \( \phi \) are given by Eqs. (8) and (7) respectively. To solve the path choice problem, let us define the so-called expected minimum perceived disutility function \( W(t, \hat{x}) \) (often referred to as the value function in optimal control theory) by the expected value of the costs upon applying the optimal velocity \( v^*_t \)

\[
W(t, \hat{x}) := E \left[ \int_{t}^{T_t} L(\tau, x^*(\tau), v^*(\tau)) + \phi(T_t, x^*(T_t)) \right]
\]

subject to

\[
dx^* = v^* dt + \sigma(x^*, v^*) dw \quad \text{subject to} \quad x^*(t) = \hat{x}
\]

To derive the dynamic programming equation, consider the period \([t, t + h] \). According to Bellman’s optimization principle (Bellman, 1957), we have

\[
W(t, \hat{x}) = E \left[ \int_{t}^{t + h} L(\tau, x^*(\tau), v^*(\tau)) + W(t + h, x^*(t + h)) \right]
\]

Eq. (20) describes that the expected minimal cost of walking from \((t, \hat{x})\) to \( A_{ij} \) equals the minimal expected cost of both walking from \((t, \hat{x})\) to \((t + h, x^*(t + h))\) and walking from \((t + h, x^*(t + h))\) to \( A_{ij} \). For small \( h \), the following approximation is valid

\[
E \left[ \int_{t}^{t + h} L(\tau, x(\tau), v(\tau)) \right] = L(t, x(t), v(t)) h + O(h^2)
\]

The random variate \( x(t + h) \) describing the predicted location at instant \( t + h \) subject to Eq. (4) can be expanded using a Taylor series

\[
x(t + h) = \hat{x} + hv(t) + \sigma \sqrt{h} w + O(h^{3/2})
\]

where \( \sigma \sqrt{h} w \) is a \( N(0, \sigma \sigma') \) distributed random variate. We can rewrite the expected value of the second term of the right-hand-side of Eq. (20)

\[
E[W(t + h, x(t + h))] = W(t + h, \hat{x} + hv) + h^2 \sum_{ij} \Theta_{ij}(x, v) \frac{\partial^2 W(t, \hat{x})}{\partial x_i \partial x_j} + O(h^{3/2})
\]

where \( \Theta(x, v) := \sigma(x, v) \sigma'(x, v) \). Substitution of Eqs. (21) and (23) into Eq. (20), using the appropriate Taylor series expansions, and taking the limit \( h \to 0 \) yields the so-called Hamilton–Jacobi–Bellman (HJB) or dynamic programming equation for decision making in continuous time and space under uncertainty

\[
-\frac{\partial}{\partial t} W(t, x) = H(t, x, \nabla W, \Delta W)
\]

with terminal conditions

\[
W(t_1, x) = \phi_i
\]
and boundary conditions
\[ W(T_i, x) = -U_{ij}(T_i) \quad \text{for} \quad x \in A_{ij} \quad \text{and} \quad T_i < t_i \] (26)

The Hamilton function \( H \) is an auxiliary function, defined by
\[
H(t, x, \nabla W, \Delta W) := \min_{v \in V_a(t, x)} \left\{ L(t, x, v) + \sum_i v_i \frac{\partial W}{\partial x_i} + \frac{1}{2} \sum_{ij} \Theta_{ij}(x, v) \frac{\partial^2 W}{\partial x_i \partial x_j} \right\}
\] (27)

Eq. (24) is the dynamic programming equation for the continuous stochastic dynamic user-optimal (CSDUO) route choice and activity area choice problem. Using terminal conditions (25) at \( t_1 \), the HJB equation can be solved backwards in time, subject to boundary conditions (26). The solution \( W(t, x) \) describes the minimum expected cost to the either of the activity areas for a pedestrian located at \( x \) at instant \( t \). From \( W(t, x) \), the optimal route can be determined easily, as is shown in the following section.

5.5.2. Optimal speed and direction

The optimal velocity \( v^* \) at time \( t \) satisfies
\[
v^*(t, x) = \arg \min \left\{ L(t, x, v) + \sum_i v_i \frac{\partial W}{\partial x_i} + \frac{1}{2} \sum_{ij} \Theta_{ij}(x, v) \frac{\partial^2 W}{\partial x_i \partial x_j} \right\}
\] (28)

subject to \( v^*(t, x) \in V_a(t, x) \). Assume that the level of uncertainty does not explicitly depend on the velocity, i.e. \( \Theta_{ij}(x, v) = \Theta_{ij}(x) \). It can then be shown that for the running cost definition (16), we find
\[
v^*(t, x) = V^*(t, x)e^*(t, x)
\] (29)

where the optimal speed \( V^* \) and optimal direction \( e^* \) are defined by
\[
V^*(t, x) := \min \{ c_3^{-1} \| \nabla W \|, v_0(t, x) \} \quad \text{and} \quad e^*(t, x) := -\nabla W / \| \nabla W \|
\] (30)

The partial derivatives \( \nabla W \) are the marginal cost of \( x \): if \( x \) changes by a small amount \( \delta x \), the change in the total minimal cost \( W(t, x) \) equals \( \nabla W \delta x \). Eq. (30) shows how the optimal direction \( e^*(t, x) \) points in the direction in which the optimal cost decreases most rapidly. Upon walking into this direction, Eq. (30) shows that the optimum speed \( V^*(t, x) \) depends on the rate \( \| \nabla W(t, x) \| \) at which the minimum cost \( W(t, x) \) function decreases in the optimal direction \( e^* \), the relative cost \( c_3 \) of walking at high speeds, and the maximum admissible speed \( v_0(t, x) \) at position \( x \) and time \( t \): when the expected minimum perceived disutility function \( W(t, x) \) decreases very rapidly, the pedestrian will walk at the maximum speed \( v_0(t, x) \). When either \( W(t, x) \) decreases very slowly in the optimal walking direction, pedestrians tend to walk at a speed lower than \( v_0(t, x) \). This may be the case when the time–pressure is low: in line with empirical observations, where walking speed for pedestrians having higher time–pressure (e.g. commuters) are farther from the ‘energy consumption–optimal’ walking speeds than in case the time–pressure is lower (e.g. shopping pedestrians).

5.5.3. Numerical solution approach

The dynamic programming Eq. (24) can be solved by discretizing the area \( \Omega \) into small \( \delta \times \delta \)-cells, and considering approximate solutions on this lattice at fixed time instants \( t_k = hk \) (i.e. \( \delta \) is the spatial step size, and \( h \) is the temporal step size). We can show that the resulting problem is a
Markov diffusion process in two dimensions with nearest-neighbor transitions that are determined by the stochastic differential Eq. (4) (Fleming and Soner, 1993). Solving this (discrete) stochastic dynamic programming problem is related to solving Eq. (24) by replacing the partial derivatives with the appropriate finite differences. Let $u_i$ denote the unit vector in the $i$th dimension ($i = 1, 2$). The forward finite difference $\Delta^+_x W$ and backward finite difference $\Delta^-_x W$ are defined by

$$\Delta^\pm_x W := \delta^{-1}[W(t,x \pm \delta u_i) - W(t,x)] \quad \text{for} \quad i = 1, 2 \tag{31}$$

For the second-order terms, we then use the following approximations

$$\Delta^2_{x_i} W := \delta^{-2}[W(t,x + \delta u_i) - 2W(t,x) + W(t,x - \delta u_i)] \quad \text{for} \quad i = 1, 2 \tag{32}$$

$$\Delta^-_{x_ix_j} W := \pm \frac{1}{2} \delta^{-2}[W(t,x + \delta u_i + \delta u_j) + 2W(t,x) + W(t,x - \delta u_i - \delta u_j)]$$

$$\pm \frac{1}{2} \delta^{-2}[W(t,x + \delta u_i) + W(t,x + \delta u_j) + W(t,x - \delta u_i) + W(t,x - \delta u_j)]$$

In numerically approximating Eq. (24), the following solution approach is proposed (Fleming and Soner, 1993):

$$W(t-h,x) = W(t,x) - hH(x, \Delta^\pm_{x_i} W, \Delta^2_{x_i} W, \Delta^-_{x_ix_j} W) \tag{33}$$

where the numerical Hamiltonian is defined by

$$H(x, \Delta^\pm_{x_i} W, \Delta^2_{x_i} W, \Delta^-_{x_ix_j} W) = \min_{v \in V_x(t,x)} \left\{ L(t,x,v) + \sum_i (v_i^+ \Delta^+_x W - v_i^- \Delta^-_x W) \right.$$  
$$\left. + \frac{1}{2} \sum_i \Theta_i(x,v) \Delta^2_{x_i} W + \frac{1}{2} \sum_i (\Theta^+_i(x,v) \Delta^+_x W - \Theta^-_i(x,v) \Delta^-_x W) \right\} \tag{34}$$

where $\alpha^+ := \max\{x, 0\}$ and $\alpha^- := \min\{x, 0\}$. This numerical solution approach has been adopted in the dedicated microscopic pedestrian flow simulation model NOMAD (Hoogendoorn, 2001).

**Example 1** (Route choice and activity area choice for free-flow conditions in Schiphol Plaza). This example considers Schiphol Plaza, which is a multi-purpose multi-modal transfer station. Fig. 1 shows a snapshot of the microscopic simulation model NOMAD described in (Hoogendoorn, 2001). In this figure, exits E1–E5 indicate exits from Schiphol Plaza; escalators E6 and E7 indicate exits to the train platforms. V1 and V2 depict the locations of the newspaper vendors. The colors reflect pedestrians having distinct activity schedules; the color intensity reflects gender (light: female, dark: male).

Fig. 2a and b respectively show the expected minimum perceived disutility functions $W$ and example routes for pedestrians using the escalators E6 or E7 to get to the train platform and pedestrians using either of the exits E1–E5 to get outside. The expected minimum perceived disutility functions have been determined using the numerical solution approach described in Section 5.5.3, with route weights $c_1 = 1$, $c_2 = 10$, $c_3 = 1.5$, $c_4 = c_5 = 0$, and $d_m = 1$ and $b_m = 0.1$ (for all obstacles $m$). Eq. (29) shows that the optimal routes are perpendicular to the iso-expected minimum perceived disutility function curves depicted in the figures. Fig. 2a shows three exemplar
Fig. 1. Snapshot from Schiphol Plaza simulation using the NOMAD model (Hoogendoorn, 2001).

Fig. 2. Expected minimum perceived disutility functions $W$ and example routes describing combined route-choice and activity area choice for (a) leaving Schiphol Plaza via either of the escalators E6 and E7 to the train platforms and (b) leaving Schiphol Plaza via either of the exits E1–E5. The numbers indicate the generalized walking time (in seconds).
routes that all lead to the escalator E6. In this case, combined route-choice/activity area choice yield different routes but the same activity area. Fig. 2b shows three exemplar routes leading to the exits. Not all routes lead to the same exit in this case.

5.6. Pedestrian route choice and activity area choice for congested operations

Section 5.5 showed how we can determine the optimal walking paths for pedestrians under the assumption that pedestrian flow conditions are time-independent (and no time restrictions apply). However, in practice pedestrian traffic conditions will deteriorate due to other pedestrians walking in the same area. This deterioration will restrict the maximum speed \( v_0(t,x) \) at which pedestrians can walk, and increase the discomfort experienced in crowded areas. Pedestrians will reconsider the former route choice by including the additional delays due to the prevailing congestion into their path choice. To include prevailing and future conditions in the pedestrian pathfinding behavior, two options are considered in the ensuing:

1. Pedestrians consider instantaneous information regarding prevailing conditions.
2. Pedestrians anticipate on future conditions, by incorporating the anticipated behavior of other pedestrians and the resulting expected future traffic conditions.

In the remainder of this article, the case where pedestrians react on current flow conditions is considered. Anticipation on future conditions involves using predictions of the pedestrian traffic conditions in the walking infrastructure, which can be achieved by simultaneously solving the HJB equation (24) and a dynamic model describing pedestrian traffic conditions (Hoogendoorn, 2001).

To include the reaction to current traffic conditions, average walking speeds \( \bar{v}(t,x) \) are determined based on the average speeds of the pedestrians in the microscopic simulation model (Hoogendoorn, 2001), or any other model describing pedestrian flow operations. These speeds are subsequently used to restrict the speeds according to

\[
v_0(t,x) = \bar{v}(t,x)
\]  

Secondly, the \( L_4(t,x,v) = \gamma(t,x) \) is updated to include the current traffic conditions in the running cost function \( L \). Generally, it will suffice to determine the average traffic concentration \( k(t,x) \), and use \( r(t,x) = \gamma(k(t,x)) \). Having updated both the maximum speed and the interaction cost, we can again solve the HJB equation numerically. We hypothesize that pedestrians will not continuously reconsider their route-choice, but only at discrete time intervals, say at each 10s.

Example 2 (Route choice for congested conditions). Fig. 3 shows the case where pedestrians enter at \( x_1 = 0 \) and walk towards the destination at \( x_1 = 40 \) m and \( 17.5 \) m \( \leq x_2 \leq 22.5 \) m. We assume that the speed is constant (1.5 m/s). According to the optimal control law, pedestrians walk in the direction perpendicular to the iso-cost curves. Fig. 3a shows the minimal route costs \( W(t,x) \) and the resulting route choice for free-flow conditions. Fig. 3b shows the optimal cost function \( W(t,x) \), when the reduction in speed caused by congestion is taken into account. Clearly, pedestrians are inclined to change their route to avoid the congestion.
5.7. Role of uncertainty

The level of uncertainty is described by the matrix \( \Theta = \sigma \alpha' \) reflecting the variance of the paths predicted by the internal state prediction model (4) of the pedestrian choosing his route. The HJB Eq. (24) shows that uncertainty yields a second-order term \( \frac{1}{2} \sum_{ij} \Theta_{ij}(x, v^i) D_{x^i} W \) causing smoothing of solutions \( W(t, x) \) of the HJB equation (see Fig. 4). This smoothing implies that

Fig. 3. Expected minimum perceived disutility function \( W(t, x) \) determined by considering reduction in average speeds.

Fig. 4. Expected minimum perceived disutility functions for leaving Schiphol Plaza via escalators for uncertainty levels (a) \( \sigma_0 = 0.01 \) and (b) \( \sigma_0 = 0.25 \). Optimal paths are perpendicular to iso-expected minimum perceived disutility function curves.
unfavorable conditions tend to spread out spatially. As a result, the expected minimum perceived disutility function $W$ will have a higher value nearby obstacles and walls when uncertainty is higher, since a pedestrian is not sure whether he will walk near obstacles or walls in the future (which would yield a high walking cost). We will therefore observe that the expected minimum perceived disutility function in very narrow passageways tends to increase very quickly, and thus that the pedestrian will be less inclined to use narrow passageways.

**Example 3 (Route choice under uncertainty).** Let us reconsider the Schiphol Plaza case for pedestrians aiming to walk to either of the escalators E6 or E7. For this particular example, the level-of-uncertainty was constant, i.e. $\sigma(x, v) = \sigma_0$. Fig. 4a and b show the combined route choice and activity area choice behavior for different uncertainty levels. Fig. 4b clearly shows that when future conditions are less certain, pedestrians are inclined to avoid narrow passageways or walking close to obstacles, yielding different route/activity area choices.

6. Pedestrian activity scheduling

This section outlines an approach to solve the joint activity scheduling and route choice problem described by Eq. (3). This is achieved by solving the combined route choice activity area choice problem described in Section 5 for fixed activity schedules. In turn, all feasible activity schedules are listed and the optimal schedule is chosen.

6.1. Fixed activity-order problem

The approach discussed in this paper is easily extended to a *fixed-order activity schedule* considering activity schedules $S = \{1, \ldots, I\}$, where for each activity $i \in S$, several activity areas $A_{ij}$ may be considered. In assuming that pedestrians make a simultaneous path-choice and activity area choice decision, we need to consider the next activity $i + 1$ (and the respective activity areas $A_{i+1j}$) into the path and activity area choice associated with activity $i$. This is achieved by application of the dynamic programming principle due to Bellman (1957), claiming that “at each moment of the control interval, the remaining control of an optimal control law, is optimal with respect to the current state determined by the preceding control actions”. This implies that to determine the optimal path and activity area choice, we need to solve the problem backwards in time, starting by computing the expected minimum perceived disutility function $W_i(t, x)$ for the final activity $I$ in $S$. Having determined $W_{i+1}(t, x)$, we compute $W_i(t, x)$ for activity $i$ by solving the HJB equation

$$-rac{\partial}{\partial t} W_i(t, x) = H(t, x, \nabla W_i, \Delta W_i)$$

using boundary conditions

$$W_i(T_i, x) = W_{i+1}(T_i, x) - U_{ij}(T_i) \quad \text{for } x \in A_{ij}$$
Note that since we have assumed that activity $i + 1$ is completed, we need not impose an additional penalty for not being able to reach any of the activity areas $A_{ij}$.

**Example 4 (Route choice in Schiphol Plaza for fixed activity schedules).** Let us consider the following pedestrian groups:

1. Pedestrian buying an item at either vendor V1 or V2 (see Fig. 1) before using the escalators E6 or E7 to get to the train platform.
2. Pedestrians buying an item at either vendor V1 or V2 before exiting Schiphol Plaza using one of the exits E1–E5.

Fig. 5a and b show that vendor 1 is more attractive to pedestrians continuing their trip by train than to pedestrians leaving Schiphol Plaza by foot. When pedestrians leave using either of the exit, they will be more inclined to buy a newspaper at vendor 2.

We hypothesize that pedestrians schedule their activities such as to maximize the combined utility of activities performed, the activity order, and locations, and their routes. In choosing which activities are performed, we have distinguished between *discretionary* and *mandatory* activities, operationalized by putting large penalties on not being able to complete a mandatory activity in time. Some activities can only be performed once others are completed. To represent these dependencies, in Hoogendoorn (2001) activity trees are described in which different activity layers are distinguished that describe the required order of the activities (activities can only be started when activities at higher layers are completed).

![Fig. 5. Combined route-choice and activity area choice for (a) pedestrians buying an item before heading towards escalators E6, E7 and (b) pedestrians buying an item before leaving via either of the exits.](image)
6.2. Activity order choice

Consider a sequence of two fixed-order activities, where activity 2 follows activity 1. Let $A_{1j}$ and $A_{2j}$ denote the activity areas for activities 1 and 2 respectively. We hypothesize that a pedestrian will take into account both activities upon planning the route, i.e. in planning activity 1, he will consider that activity 2 will need to be performed afterwards. To this end, we first solve the route choice problem for activity 2, yielding the expected minimum perceived disutility function $W_2(t,x)$. Secondly, the pedestrian plans the primer activity by determining the path that is stipulated by $W_{12}(t,x)$, which is a solution of the HJB equation

$$-\frac{\partial}{\partial t} W_{12}(t,x) = H(t,x,\nabla W_{12}, \Delta W_{12})$$

with boundary/terminal conditions

$$W_{12}(t_1,x) = \phi_{12} \quad \text{and} \quad W_{12}(T_{12},x) = W_2(T_{12},x) - U_{1j}(T_{12}) \quad \text{for } x \in A_{1j} \quad \text{and} \quad t < t_1$$

Clearly, the terminal conditions describe how the (optimal) cost $W_2$ of walking to the second activity area are considered by the pedestrian when walking from any location $x$ to the first activity area $A_{1j}$. The approach may be easily extended when a sequence of more than two activities have to be considered. Note that in most practical situations, the number of activities that pedestrians take into account during planning is generally limited.

The case, where the order of the activities is not fixed is equivalent to solving two fixed activity order problems and determining the minimum of $W_{12}(t,x)$ and $W_{21}(t,x)$. At any location $x(t)$, $\min\{W_{12}(t,x), W_{21}(t,x)\}$ determines both the optimal direction and speed, as well as the optimal order of the activities 1 and 2. In case of three or more activities, we need to consider the minimum of all combinations of activity sequences. This implies that, although the approach is conceptually very straightforward, in practice the number of combinations can become very large.

Conceptually, the inclusion of discretionary activities is equally simple. In illustration, consider the situation where activity 1 is mandatory, while activity 2 is discretionary. To determine the order of the activities as well as whether activity 2 will be performed or not, the pedestrian at $x(t)$ will determine the minimum of $W_{12}$, $W_{21}$, and $W_1$. If it turns out that if $W_1$ is optimal, activity 2 is skipped.

7. Modeling pedestrian flow operations

This article focuses on the behavior of pedestrians at the tactical level. Hoogendoorn (2001) describes how, given the planned activity schedule and route, pedestrian walking behavior can be described under the hypothesis that pedestrians aim to minimize the predicted cost of their behavior. The subjective disutility stems from cost incurred when deviating from the planned (i.e. optimal) path, discomfort caused by walking too close to (or even touching) other pedestrians and obstacles, and frequent and severe acceleration and deceleration. The optimal behavior will represent a trade-off between these factors. Models are determined from this normative walking
theory by applying an optimal control approach or the theory of differential games. This yields closed form expressions describing how pedestrians react to other pedestrians in their direct environment.

8. Model applications

The approach describes pedestrian activity scheduling and route choice for different types of pedestrians with distinct perceptions of route attributes. Contrary to network-based approaches, routes are continuous in time and space. Applications of the model are manifold. For one, the approach is used to model choice behavior in the pedestrian microsimulation model NOMAD (Hoogendoorn, 2001). Stand-alone applications of the model are however also possible to predict route choice in infrastructure facilities, such as transfer stations and shopping malls. For planning purposes, dynamic user-equilibrium solutions of the pedestrian assignment problem can be used to forecast pedestrian flows. Such predictions are valuable to reveal bottlenecks in infrastructure design, to predict average transfer times (walking time from egress to access locations, given expected traffic conditions), or the optimal location of a ticket machine or newspaper stand. For instance, the Schiphol Plaza example shows which vendor location is preferable from the viewpoint of passing pedestrian flows, given pedestrian origin-activity demands. This way, infrastructure design, platform allocation, time tables, etc. can be optimized. This pertains to regular circumstances, as well as to emergency conditions (albeit different models are required to describe walking operations). Practical application of the models will require calibration (and validation) of the model. This can be done by considering previous empirical studies showing the relative importance of different route attributes for varying trip purposes, and subsequently estimating the relevant weights. Comparing (qualitatively) the resulting path flows and speeds with observations reveals whether changes in the weights are needed. Given the fact that distance (or travel time) is the most important route attribute, we expect that model calibration is relatively straightforward, at least for distinct groups of pedestrians (e.g. commuters).

9. Summary and future research

This article puts forward a new theory based on the assumption that pedestrians are subjective utility maximizers: they schedule their activities, the activity areas, and the paths between the activities (which, on the contrary to other transportation networks, are continuous functions in time and space) simultaneously to maximize the predicted utility of their efforts and walking. The utility reflects a trade-off between the utility of completing an activity, and the cost of walking towards the activity areas. In turn, the latter results from different factors, such as the travel time, discomfort of walking too close to obstacles and walls, stimulation of the environment, etc. Uncertainty pertaining to the predictability of the future conditions is included by assuming that the predicted routes are realizations of random processes. The effect of prevailing traffic conditions on pedestrian choice behavior have been considered as well. To operationalize the theory, different techniques from stochastic mathematical optimal control theory have been applied successfully. The different concepts have been illustrated by examples.
The main contribution of the article is the joint description of activity scheduling and route-choice behavior under uncertainty, hypothesizing that the pedestrian can choose between an infinite number of candidate routes, which are continuous paths in time and space.

Future research is directed towards developing methods to efficiently solve the dynamic pedestrian assignment problem. Moreover, different applications of the approach will be considered, such as the routing control of automated guided vehicles for container handling in terminals, and partially autonomous drones (e.g. for garbage collection).

Acknowledgements

This research is funded by the Social Science Research Council (MaGW) of the Netherlands Organization for Scientific Research (NWO). The authors are very grateful for the constructive suggestions and critical comments of the anonymous reviewers.

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