Takagi-Sugeno Bilinear Based Model Non-fragile Guaranteed Cost Fuzzy Controller Design

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Abstract—In this paper, non-fragile characteristics existing in control systems are studied, and the existing sufficient condition for the non-fragile guaranteed cost control rate based on Lyapunov stable theory are derived, which can ensure the close-loop system asymptotically stability for any allowed uncertainties, and that performance index of the close-loop system doesn’t exceed some upper bound for a given performance function with quadratic form. And we also proposed the designing method for a novel non-fragile guaranteed cost fuzzy controller design for continuous time-delay fuzzy system based on Takagi-Sugeno bilinear model. In the last part, two simulation examples proved the effectiveness of this novel proposed method.

Index Terms—guaranteed cost; non-fragile; fuzzy controller; continuous time-delay; Takagi-Sugeno model

I. INTRODUCTION

Comparing with traditional control, fuzzy control has two much more advantages. One is that fuzzy control can realize the strategy of human control and experience effectively and simply in a lot of applications. The other one is that it is not necessary to know the precise exact mathematical model of the controlled objects. Takagi-Sugeno (T-S) fuzzy model is proposed in 1985 by Takagi T. and Sugeno M. [1], which brings far-researching impact on fuzzy control theory and its application, and makes the stability analysis of fuzzy systems to a new theoretical height.

When designing controller off-line, it is usually considered that the controller can calculate control signals accurately. However, the above condition cannot be always meet in actual system operating process and the controller parameters would be changed by many factors, such as the restraint of computer word length, the inherent deviation in Digital-Analog or Analog-Digital converting, the cutting-off deviation, and the parameters variations of electronic components. Usually the robust controller can work with controlled objects effectively, but doesn’t have fine perturbation control effect.

The conventional controllers are sensitive to its uncertainty, in other words, they are very fragile. Therefore, it needs to design non-fragile controllers to solve the uncertainty, which means that when a controller has parameter perturbation, it still could keep the close-loop system stable. Non-fragile controller design has attracted more and more researches in recent years [2].

Ref. [3] studied a series of non-fragile $H_{\infty}$ stable problems of the continues linear system, and proposed the design method by utilizing Riccati inequality. [4] designed non-fragile $H_{\infty}$ filter by Linear Matrix Inequality (LMI) [5] method when controllers had additional perturbation. And there are some other papers [6-8] studied the non-fragile controller design methods for fuzzy system, but most of them are based on T-S linear model, not T-S bilinear model. Combining with the Lyapunov stability theory [9], robust control theory and $H_{\infty}$ control theory [10], using LMI, this paper discussed the stability and stabilization problems of fuzzy systems based on T-S bilinear model in detail.

This paper is organized as follows: Section II discusses the system Description. Section III clarifies the novel design of non-fragile guaranteed cost fuzzy controller. Section IV presents simulation examples and Section V concludes.

II. SYSTEM DESCRIPTION

As the core of T-S system, the fuzzy rule sets is the most important factor to the precision and generalization of the system, but the number of fuzzy rules is very difficult to determine. The $i$-th fuzzy rule, $R_{i}$, for a nonlinear interconnected systems with a time-varying delay in both states and inputs, which are composed of a number of T-S fuzzy bilinear subsystems with interconnections, can be depicted as follows [11]: if $\delta_{i}(t)$ is $F_{u}$, $\delta_{j}(t)$ is $F_{v}$, ..., and $\delta_{i}(t)$ is $F_{u}$, then

\[
\dot{x}(t) = A_{x}x(t) + A_{d}x(t - d(t)) + B_{u}u(t) + B_{d}u(t - d(t)) + N_{x}(t)x(t)u(t) + N_{d}(t)x(t - d(t))u(t - d(t)),
\]

(1)
in which, \( x(t) = \phi(t), \ t \in [-\tau, 0] \) \( i = 1, 2, ..., s \); \( \phi(t) \) is the system initial state; \( s \) is the number of fuzzy rules; \( \delta_j(t) \) and \( F_i \) \( (i = 1, 2, ..., r) \) are premise variable and fuzzy set, respectively; \( x(t) \in \mathbb{R}^n \) and \( u(t) \in \mathbb{R} \) are state variable and control variable, respectively; \( d(t) \) is the time-delay term, which is a time-variant and differentiable function, and it satisfies that \( 0 \leq d(t) \leq \tau, \text{ } d(t) \leq \delta, \ \tau \) and \( \delta \) is known constants; \( A_i, A_{j0}, N_{ij}, N_j \in \mathbb{R}^{m \times m} \), and \( B_i, B_{j0} \in \mathbb{R}^{m \times 1} \) are known system matrices.

Singleton fuzzification, product inference and center average fuzzification are adopted to deal with the input data, and the total model of fuzzy system can be derived as

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(\delta(t))[A_i x(t) + A_{j0} x(t - d(t)) + B_i u(t) + B_{j0} u(t - d(t)) + N_{ij} x(t)]
\]

(2)

where \( h_j(\delta(t)) = \frac{\omega_j(\delta(t))}{\sum_{i=1}^{r} \omega_i(\delta(t))} \) and \( \omega_j(\delta(t)) = \prod_{i,j} \mu_{ij}(\delta(t)) \).

\( \mu_{ij}(\delta(t)) \) is the membership function of \( \delta_j(t) \) in \( F_i \).

It is assumed that \( \omega_j(\delta(t)) \geq 0 \) and \( \sum_{i=1}^{r} \omega_i(\delta(t)) = 1 \). We use \( h_j, x_j(t) \) and \( u_j(t) \) to denote \( h_j(\delta(t)), x(t - d(t)) \) and \( u(t - d(t)) \), respectively.

When designing non-fragile controllers according to parallel distribution compensation (PDC) algorithm [12], the control rate of the \( i \)-th subsystem can be described as follows: if \( \delta_j(t) \) is \( F_{i1} \), \( \delta_j(t) \) is \( F_{i2} \), ..., and \( \delta_j(t) \) is \( F_{ir} \), then

\[
u_j(t) = \frac{\rho(K_i + \Delta K_i(t))(t-d(t)))x(t)}{1 + \rho^2(K_i + \Delta K_i(t)))} \]

(3)
in which, \( \rho > 0 \) is an unknown scalar; \( K_i \in \mathbb{R}^{m \times m} \) is an unknown controller gain; \( \Delta K_i(t) \) represents additional controller gain perturbation, which can be expressed as \( \Delta K_i(t) = H_i F_i(t) E_i \), where \( H_i \) and \( E_i \) are known constant matrices; \( F_i(t) \) is an unknown time-varying matrices, whose elements can be Lebesgue measurable [13], and for any \( i, f_i \), \( f_i(t) F_i(t) \leq 1, I \) is the identity matrix. Let \( \bar{F}_i = K_i + \Delta K_i(t) = K_i + H_i F_i(t) E_i \), then

\[
sin \theta_j = \frac{\bar{F}_i x(t)}{1 + \rho^2 \bar{F}_i \bar{K}_i}, \cos \theta_j = \frac{1}{1 + \rho^2 \bar{F}_i \bar{K}_i}, \text{ in which, } \theta_j \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right], \text{ and } i = 1, 2, ..., s. \ u_j(t) \text{ can be derived by }
\]

\[
u_j(t) = \rho(K_i + \Delta K_i(t))(t-d(t)))x(t)
\]

(4)
in which \( \sin \varphi \frac{\bar{F}_i x(t)}{1 + \rho^2 \bar{F}_i \bar{K}_i}, \cos \varphi \frac{1}{1 + \rho^2 \bar{F}_i \bar{K}_i} \), and \( i = 1, 2, ..., s \). 

\[
u_j(t) = \frac{\rho(K_i + \Delta K_i(t))(t-d(t)))x(t)}{1 + \rho^2(K_i + \Delta K_i(t)))}
\]

\[
u_j(t) = \rho \sin \varphi = \rho \cos \varphi(K_i + \Delta K_i(t))(t-d(t)))x(t)
\]

The state feedback control rate for system can be described by

\[
u_j(t) = \sum_{i=1}^{r} h_i \rho(\bar{K}_i x(t)) \]

(5)

under the effect of the control rate (5), the equation of the closed-loop system is

\[
u_j(t) = \sum_{i=1}^{r} h_i \rho(\bar{K}_i x(t)) + \Lambda_j \delta_j(t)
\]

(6)
in which, \( \Lambda_j = A_i + \rho \sin \theta \bar{N}_j + \rho \cos \theta \bar{B}_j \bar{K}_j \) and \( \Lambda_{j0} = A_{j0} + \rho \sin \varphi \bar{N}_j + \rho \cos \varphi \bar{B}_j \bar{K}_j \).

For system (2), let performance index be

\[
u_j(t) = \int_{0}^{t} \left[ x^T(s) S x(s) + u^T(s) W u(s) ds \right]
\]

(7)

where \( S \) and \( W \) are known weighted symmetric positive definite matrix.

III. NON-FRAGILE GUARANTEED COST FUZZY CONTROLLER

**Theorem 1**: Considering time-delay of the fuzzy system (6) satisfies \( 0 \leq d(t) \leq \tau, d(t) \leq \sigma \), for given constants, \( \rho > 0 \) and \( \varepsilon > 0 \), if the conditions that \( P > 0, Q > 0, R > 0 \) and \( X_1, X_2, X_3, Y_1, Y_2, Y_3, K_i(i = 1, 2, ..., s) \) satisfy the inequality (8), then the closed-loop system (6) is asymptotically stable, the control rate (5) is a non-fragile guaranteed cost control rate, \( J_0 = \int_{0}^{t} x^T(s) S x(s) ds + \int_{t}^{T} u^T(s) R u(s) ds + x^T(0) P x(0) \), and

\[
\begin{cases}
T_{ii} + T_{j0}^* \leq 0, i = 1, 2, ..., s \\
T_{ij} + T_{j0}^* \leq 0, 1 \leq i \leq j \leq s
\end{cases}
\]

(8)
\[
T_0 = \begin{bmatrix}
T_{11,0} & * & *
\end{bmatrix}
\]
\[
T_{21,0} = \begin{bmatrix}
T_{21,1} & T_{22,1} & T_{33}
\end{bmatrix}
\]
\[
T_{11,0} = Q + X_1 + X_1^T + Y_1A_1 + A_1^T Y_1^T + 2\varepsilon_2\rho^2 Y_2^T
+ 3cN_1^T N_1 + 3c^{-1}(B_1\tilde{R}_1)(B_1\tilde{R}_1^T)
+ S + S^T \tilde{R}_1^T W \tilde{R}_1
\]
\[
T_{21,0} = -(\varepsilon_2 - \delta_2) - X_1 + Y_1A_1 + A_1^T Y_1^T
\]
\[
T_{33} = P + X_2 + Y_2A_2 - Y_2^T
\]
\[
T_0 = \begin{bmatrix}
T_{11,0} & * & *
\end{bmatrix}
\]
\[
T_{21,0} = \begin{bmatrix}
T_{21,1} & T_{22,1} & T_{33}
\end{bmatrix}
\]
(9)

**Proof:** Choose a Lyapunov function that
\[
V(x(t)) = x^T(t)P(x(t)) + \int_{s_0}^{t} \dot{x}^T(s)Qx(s)ds
+ \int_{s_0}^{t} \int_{s_0}^{s} \dot{x}^T(s)R\dot{x}(s)dsd\theta
\]
(10)

Taking the derivative of (10) with respect to \( x(t) \), we can obtain that
\[
\dot{V}(x(t)) = 2x^T(t)Px(t) + x^T(t)Q(x(t)) - (1 - \delta(t))x^T(t)Qx(t) + \tau \dot{x}^T(t)R\dot{x}(t)
- \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds
\]
(11)

Putting free weighted matrixes \( X = [X_1 \quad X_2 \quad X_3]^T \)
and \( Y = [Y_1 \quad Y_2 \quad Y_3]^T \) into system (6), and according to
Leibniz-Newton equation, it can be derived that
\[
\begin{align*}
2\eta^T(t)X & \left[ x(t) - x_s(t) - \int_{t-\delta(t)}^{t} \dot{x}(s)ds \right] = 0 \\
2\eta^T(t)Y & \sum_{i,j} h_i h_j \left[ \Lambda_{ij} \dot{x}(t) + \Lambda_{ij} \dot{x}_s(t) - \dot{x}(t) \right] = 0
\end{align*}
\]
(12)

substitute (12) into (11), then
\[
\dot{V}(x(t)) = 2x^T(t)Px(t) + x^T(t)Q(x(t)) - (1 - \delta(t))x^T(t)Qx(t) + \tau \dot{x}^T(t)R\dot{x}(t)
- \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds
\]
\[
+ 2\eta^T(t)X \left[ x(t) - x_s(t) - \int_{t-\delta(t)}^{t} \dot{x}(s)ds \right]
\]
\[
+ 2\eta^T(t)Y \sum_{i,j} h_i h_j \left[ \Lambda_{ij} \dot{x}(t) + \Lambda_{ij} \dot{x}_s(t) - \dot{x}(t) \right]
\]
\[
\leq 2x^T(t)Px(t) + x^T(t)Q(x(t)) - (1 - \delta(t))x^T(t)Qx(t) + \tau \dot{x}^T(t)R\dot{x}(t)
- \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds
\]
\[
+ 2\eta^T(t)X \left[ x(t) - x_s(t) - \int_{t-\delta(t)}^{t} \dot{x}(s)ds \right]
\]
\[
+ 2\eta^T(t)Y \sum_{i,j} h_i h_j \left[ \Lambda_{ij} \dot{x}(t) + \Lambda_{ij} \dot{x}_s(t) - \dot{x}(t) \right]
\]
\[
+ \sum_{i,j} h_i h_j \left[ \dot{x}(t)Sx(t) + \rho^2 x^2(t)\tilde{R}_1^T \cos \theta W \tilde{R}_1 \cos \theta x(t) \right]
\]
\[
- \dot{x}^T(t)Sx(t) + \dot{x}^T(t)u(T)Wu(t)
\]
(13)

in which, \( \eta^T(t) = \begin{bmatrix} \dot{x}(t) & \dot{x}_s(t) & \ddot{x}(t) \end{bmatrix} \). Thus
\[
\dot{V}(x(t)) \leq \sum_{i,j} h_i h_j \eta^T(t)\tilde{T}_{ij} \eta(t) - \int_{t-\delta(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds
- 2\eta^T(t)X \int_{t-\delta(t)}^{t} \dot{x}(s)ds - \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t)
\]
\[
\leq \sum_{i,j} h_i h_j \eta^T(t)\tilde{T}_{ij} + \tau XR^1 X^T \eta(t)
\]
\[
- \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t)
\]
\[
- \int_{t-\delta(t)}^{t} \eta^T(t)X + \dot{x}^T(t)R \eta(t)
\]
\[
\leq \sum_{i,j} h_i h_j \eta^T(t)\tilde{T}_{ij} + \tau XR^1 X^T \eta(t)
\]
\[
+ \sum_{i,j} h_i h_j \eta^T(t)\tilde{T}_{ij} + 2\tau XR^1 X^T \eta(t)
\]
\[
- \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t)
\]
(14)

where
\[
\tilde{T}_{ij} = \begin{bmatrix}
\tilde{T}_{11,ij} & * & *
\tilde{T}_{21,ij} & \tilde{T}_{22,ij} & \tilde{T}_{33}
\end{bmatrix}
\]
(15)

For \( \tilde{R}_1^T \cos \theta \), \( \cos \theta W \tilde{R}_1 \leq \tilde{R}_1 W \tilde{R}_1 \) and \( \tilde{R}_1^T \cos \theta \), \( \cos \theta W \tilde{R}_1 \)
\( + \tilde{R}_1^T \cos \theta \), \( \cos \theta W \tilde{R}_1 \leq \tilde{R}_1 W \tilde{R}_1 + \tilde{R}_1 W \tilde{R}_1 \), it can be derived that
\[
\dot{V}(x(t)) + \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t)
\]
\[
\leq \sum_{i,j} h_i h_j \eta^T(t)\tilde{T}_{ij} + \tau XR^1 X^T \eta(t)
\]
\[
+ \sum_{i,j} h_i h_j \eta^T(t)\tilde{T}_{ij} + 2\tau XR^1 X^T \eta(t)
\]
\[
\dot{V}(x(t)) + \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t)
\]
(16)

According to Schur complement theorem, it can be derived from (8) that
\[
\begin{bmatrix}
T_{ij} + \tau XR^1 X^T & < 0
\end{bmatrix}
\]
\[
\begin{bmatrix}
T_{ij} + \tau XR^1 X^T & < 0
\end{bmatrix}
\]
(17)

thus
\[
\dot{V}(x(t)) \leq - \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t) \leq 0
\]
(18)

It can be concluded that the close-loop system (6) is asymptotically stable.

The quadrature of (18) from 0 to \( T \) can be obtained by
\[
\int_{0}^{T} \left[ - \dot{x}^T(t)Sx(t) + \dot{u}^T(t)Wu(t) \right] dt \leq -V(x(T)) + V(x(0))
\]
(19)

For \( V(x(t)) \geq 0 \), and \( \dot{V}(x(t)) < 0 \), thus
\[
\lim_{T \to \infty} V(x(T)) = c, \ c \text{ is a nonnegative constant. When } T \to \infty.
\]

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\[ J \leq x'(0)Px(0) + \int_{0}^{t} x'(s)Qx(s)ds + \int_{0}^{t} \int_{0}^{1} x'(r)R(x(s))dsd\theta \]  \hspace{1cm} (20)

Therefore, system (6) is asymptotically stable, and the performance index satisfies (20). Theorem 1 is proved completely.

It is assumed that the free weighted matrices, \( Y_i, Y_j \), and \( Y_k \), are nonsingular, and \( Y_i^T = \lambda Z \) (\( i = 1, 2, 3, \ Z = P^T \), \( \lambda > 0 \)) is positive constant. Using \( \Theta = \text{diag}\{Y_1, Y_2, Y_3, Y_4\} \) left and \( \Theta^T = \text{diag}\{Y_1^T, Y_2^T, Y_3^T, Y_4^T\} \) right multiplying (8), and denoting that \( \Omega = Y_i^T Q Y_i^T \), \( \bar{R} = Y_i^T R Y_i^T \) and \( \bar{X}_i = Y_i^T X Y_i^T \) (\( i = 1, 2, 3 \)), we can obtain that

\[
\begin{bmatrix}
\bar{T}_{11,i} & * & * & * \\
\bar{T}_{21,i} & \bar{T}_{22,i} & * & * \\
\bar{T}_{31,i} & \bar{T}_{32,i} & \bar{T}_{33,i} & * \\
\tau \bar{X}_1 & \tau \bar{X}_2 & \tau \bar{X}_3 & -\tau \bar{R}
\end{bmatrix} < 0, i = 1, 2, \ldots, s, \hspace{1cm} (21)
\]

\[
\begin{bmatrix}
\bar{T}_{11,j} & * & * & * \\
\bar{T}_{21,j} & \bar{T}_{22,j} & * & * \\
\bar{T}_{31,j} & \bar{T}_{32,j} + \bar{T}_{33,j} & 2\bar{T}_{33,j} & * \\
2\tau \bar{X}_1 & 2\tau \bar{X}_2 & 2\tau \bar{X}_3 & -2\tau \bar{R}
\end{bmatrix} < 0, 1 \leq i < j \leq s, \hspace{1cm} (22)
\]

where

\[
\bar{T}_{11,i} = \Omega + \bar{X}_i + \bar{X}_i^T + \lambda A Z + \lambda Z A_i^T + \lambda^2 Z S Z
\]

\[
+ 2\alpha\rho^2 I + 3\alpha^{-1}\lambda^2 (N_i Z)^T (N_i Z), \hspace{1cm} (23)
\]

\[
\bar{T}_{21,i} = -\lambda^2 Z + \bar{X}_i + \lambda A Z + \lambda Z A_i^T,
\]

\[
\bar{T}_{22,i} = -(1 - \delta)\Omega - \bar{X}_j - \bar{X}_j^T + \lambda A_{i,j} Z + \lambda Z A_{i,j}^T
\]

\[
+ 2\alpha\rho^2 I + 3\alpha^{-1}\lambda^2 (N_{i,j} Z)^T (N_{i,j} Z), \hspace{1cm} (25)
\]

\[
\bar{T}_{31,i} = \lambda^2 Z + \bar{X}_i + \lambda A Z - \lambda Z,
\]

\[
\bar{T}_{32,j} = -\bar{X}_j + \lambda A_{i,j} Z - \lambda Z,
\]

\[
\bar{T}_{33,j} = \tau \bar{R} - 2\lambda Z + 2\alpha\rho^2 I. \hspace{1cm} (28)
\]

According to Schur complement theorem, (21) and (22) are equal to the following two equations respectively:

\[
\begin{bmatrix}
\Phi_{1,ij} & * & * & * \\
\Phi_{2,ij} & \Phi_{2,ij} & * & * \\
\Phi_{3,ij} & \Phi_{3,ij} & \Phi_{3,ij} & * \\
\Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij}
\end{bmatrix} < 0, i = 1, 2, \ldots, s, \hspace{1cm} (32)
\]

\[
\begin{bmatrix}
\Phi_{1,ij} & * & * & * \\
\Phi_{2,ij} & \Phi_{2,ij} & * & * \\
\Phi_{3,ij} & \Phi_{3,ij} & \Phi_{3,ij} & * \\
\Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij}
\end{bmatrix} < 0, 1 \leq i < j \leq s, \hspace{1cm} (33)
\]

in which,

\[
\bar{F}_{1,ij} = Q + X_i + X_i^T + \lambda Z A^T + 2\alpha\rho^2 I
\]

\[
\bar{F}_{2,ij} = -(1 - \sigma)\Omega - \bar{X}_j - \bar{X}_j^T + \lambda Z A_{i,j} + \lambda Z A_{i,j}^T + 2\alpha\rho^2 I. \hspace{1cm} (31)
\]

It can be concluded that if the positive definite symmetric matrices, \( Z, \ \bar{Q}, \ \bar{R} \), and the matrices, \( \bar{X}_i, \ \bar{X}_j, \ K_{ij} \) (\( i = 1, 2, \ldots, s \)), satisfy (29) and (30), then according to Theorem 1, the system (6) is asymptotically stable, and the control rate is a non-fragile guaranteed cost control rate.

According to the above analysis, we made the following controller designing method.

\textbf{Theorem 2: Considering time-delay of the fuzzy system (6) satisfies 0 \leq d(t) \leq T, d(t) \leq \tau, \ for given constants, \( \rho > 0, \lambda > 0 \) and \( \delta > 0 \), if the conditions that \( Z > 0, \bar{Q} > 0, \bar{R} > 0 \) and \( \bar{X}_i, \ \bar{X}_j, \ \bar{X}_k, \ M \) (\( i = 1, 2, \ldots, s \)) and constant \( c > 0 \) satisfy LMI (32) and (33), then the system (6) is asymptotically stable, the control rate (5) is a non-fragile guaranteed cost control rate, \( J_0 = \int_{0}^{T} x'(s)Qx(s)ds + \int_{0}^{T} \int_{0}^{1} x'(s)R\hat{x}(s)dsd\theta + x'(0)P(0)x(0) \), the controller gain is depicted as \( K_i = M_i Z_i^{-1} \), and \( J_0 = x'(0)Z^{-1}x(0) + \int_{0}^{T} \int_{0}^{1} x'(s)Z^{-1}\bar{Q}Z^{-1}x(s)dsd\theta \).

And

\[
\begin{bmatrix}
\Phi_{1,ij} & * & * & * \\
\Phi_{2,ij} & \Phi_{2,ij} & * & * \\
\Phi_{3,ij} & \Phi_{3,ij} & \Phi_{3,ij} & * \\
\Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij}
\end{bmatrix} < 0, i = 1, 2, \ldots, s, \hspace{1cm} (32)
\]

\[
\begin{bmatrix}
\Phi_{1,ij} & * & * & * \\
\Phi_{2,ij} & \Phi_{2,ij} & * & * \\
\Phi_{3,ij} & \Phi_{3,ij} & \Phi_{3,ij} & * \\
\Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij} & \Phi_{4,ij}
\end{bmatrix} < 0, 1 \leq i < j \leq s, \hspace{1cm} (33)
\]

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where

$$
\Phi_\omega = \text{diag}[-\delta_1, -\delta_1, -\delta_1, -\delta_1, -\delta_1, -\delta_1, -\delta_1, -\delta_1],
$$

$$
\Phi_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
$$

$$
\Phi_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
$$

$$
\Phi_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
$$

For $K_i = M_i Z^{-1}$, thus

$$M_i = K_i Z.$$

Put (35) into (34), and according to Lemma A.6 [14, 17], it can be derived that

$$\Phi_\omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} = \Gamma < 0 \text{ } (36)
$$

It can be known from (36) that (29) and (30) can be derived by (32) and (33). According to the equivalence between (29), (30) and (21), (22), Theorem 2 is proved completely.

IV. SIMULATION EXAMPLES

Let's consider the following examples to verify above methods and conclusions.

Example 1: $R^1$ and $R^2$ are fuzzy bilinear systems, which are depicted as follows:

(1) $R^1$: if $x_i$ is $F_1$, then

$$\dot{x}(t) = A_1 x(t) + A_{12} x_2(t) + B_1 u(t) + B_{12} u_2(t) + N_1 x(t) u(t) + N_{12} x_2(t) u_2(t).$$

(2) $R^2$: if $x_i$ is $F_2$, then

$$\dot{x}(t) = A_2 x(t) + A_{21} x_1(t) + B_2 u(t) + B_{21} u_1(t) + N_2 x(t) u(t) + N_{21} x_1(t) u_1(t).$$

in which, the system matrices are $A_1 = \begin{bmatrix} 32 & 9 \\ 38 & 5 \end{bmatrix}$, $N_2 = \begin{bmatrix} 3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 10 & 0 \\ 5 & 2 \end{bmatrix}$, $A_{21} = \begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix}$, $N_{12} = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}$, $N_{21} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}$, $B_{12} = B_{21} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$.

The positive definite matrices of the system performance index are given as $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $W = 1$.

The controller additional perturbations are given as $H_1 = H_2 = 0.1$, $E_{11} = \begin{bmatrix} 0.05 & -0.01 \end{bmatrix}$, and $E_{12} = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}$.

The membership functions are chosen as

$$\mu_{\xi_1}(x_i) = \begin{bmatrix} 1 + \exp(-2 \xi_1(t)) \end{bmatrix} \text{ and } \mu_{\xi_1}(x_i) = 1 - \mu_{\xi_1}(x_i).$$

The parameter values are $\rho = 0.5$, $\lambda = 2$, $\delta = 0.1$, $\tau = 1$, and $\sigma = 0$. According to Theorem 2, a group of feasible solutions can be efficiently solved by means of

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Matlab LMI control box as $\mathbf{P} = \begin{bmatrix} 31.0161 & 3.4644 \\ 3.4644 & 3.0513 \end{bmatrix}$, 
$\mathbf{Q} = \begin{bmatrix} 32.9547 & -4.8267 \\ -4.8267 & 23.0104 \end{bmatrix}$, $\mathbf{R} = \begin{bmatrix} 28.3777 & -0.3193 \\ -0.3193 & 27.5910 \end{bmatrix}$, $\varepsilon = 11.0364$, 
$K_1 = [-1.0341, -1.1562]$, and $K_2 = [-0.8412, -0.2774]$. The initial value is chosen as $\phi(t) = [2.8, -2.6]^T$ ($t \in [-1, 0]$).

We can get the response curve and control curve for the state variables of systems, which are shown in Fig. 1 and Fig. 2. And they depict that the non-fragile controller can guarantee the close-loop system to be asymptotically stable, and $J_0 = 798.9433$.

Example 2: A Van de Vusse continuous stirred tank reactor (CSTR) model is given by

$$
\begin{align*}
\dot{x}_1 &= -50x_1 - 10x_1' + u(10 - x_1) + u(t - 2) + u(t - 2)(0.5x_1(t - 2) + 0.2x_1(t - 2) + 5x_1(t - 2)) + u(t - 2)(0.3x_1(t - 2)) + 10x_1(t - 2) - 5x_1(t - 2), \\
\dot{x}_2 &= 50x_2 - 100x_2 - u(t - 2) + u(t - 2)(0.5x_2(t - 2) + 5x_2(t - 2)) + 10x_2(t - 2) - 5x_2(t - 2) \quad . \quad (39)
\end{align*}
$$

The values of balance point for (39), $[x_1, u_1]$ are shown in Tab. 1. We choose these points as the expectantly operating points, $[x', u']$. According to the modeling method in [15, 16], the system (39) turns into

(1) $R^1$: if $x_1$ is about 2.0422, then

$$
\dot{x}_1 = A_1x_1(t) + A_{21}x_1(t) + B_1u_1(t) + B_{21}u_{d1}(t) + N_1x_1(t)u_1(t) + N_{21}x_1(t)u_{d1}(t), \quad (40)
$$

(2) $R^2$: if $x_1$ is about 3.6626, then

$$
\dot{x}_1 = A_1x_1(t) + A_{21}x_1(t) + B_1u_1(t) + B_{21}u_{d1}(t) + N_1x_1(t)u_1(t) + N_{21}x_1(t)u_{d1}(t), \quad (41)
$$

(3) $R^3$: if $x_1$ is about 5.9543, then

$$
\dot{x}_1 = A_1x_1(t) + A_{21}x_1(t) + B_1u_1(t) + B_{21}u_{d1}(t) + N_1x_1(t)u_1(t) + N_{21}x_1(t)u_{d1}(t), \quad (42)
$$

The positive definite matrices of the system performance index are given as $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $W = 1$.

The controller additional perturbations are given as $H_1 = H_2 = H_3 = 0.1$, $E_{u1} = [0.05 \ -0.01]$, $E_{u2} = [0.02 \ 0.01]$, and $E_{u3} = [-0.01 \ 0]$. The parameter values are $\rho = 0.45$, $\lambda = 1.02$, $\delta = 0.11$, $\tau = 2$, and $\sigma = 0$. According to Theorem 2 and Matlab LMI control box, a group of feasible solutions can be obtained as $\mathbf{P} = \begin{bmatrix} 7.5659 & -1.3007 \\ -1.3007 & 6.4906 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 14.1872 & -1.9381 \\ -1.9381 & 13.0104 \end{bmatrix}$, $\mathbf{R} = \begin{bmatrix} 8.3691 & -1.3053 \\ -1.3053 & 7.0523 \end{bmatrix}$, $\varepsilon = 1.8043$, $K_1 = [-0.4233, -0.5031]$, $K_2 = [-0.5961, -0.7049]$, and $K_3 = [-0.4593, -0.3874]$. We choose $x' = [3.6626 \ 2.5443]$ and $u' = 77.7272$ as the expectantly operating points. The initial value is chosen as $\phi(t) = [1.2, -1.8]^T$ ($t \in [-2, 0]$).

We can get the state response curve in Fig. 3 and Fig. 4. And they depict that the close-loop system under the proposed non-fragile controller is asymptotically stable, and $J_0 = 197.4552$. 

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SYSTEM BALANCE POINT</strong></td>
</tr>
<tr>
<td>$x_1'$</td>
</tr>
<tr>
<td>2.0422</td>
</tr>
<tr>
<td>3.6626</td>
</tr>
<tr>
<td>5.9543</td>
</tr>
</tbody>
</table>

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And we also proposed the designing method for non-fragile guaranteed cost control rate with quadratic form, performance index of the non-fragile characteristic should be considered deeply in the process of system design. A novel non-fragile guaranteed cost fuzzy controller design is asymptotically stable, and for a given performance index and for any allowed uncertainties, the closed-loop system doesn’t exceed some upper bound. We also proposed the designing method for non-fragile guaranteed cost fuzzy controller.

V. CONCLUSION

The controller gain perturbations always exist in actual control systems, which may result in instabilities. Therefore, non-fragile characteristics should be considered deeply in the process of system design. A novel non-fragile guaranteed cost fuzzy controller design for continuous time-delay fuzzy system based on Takagi-Sugeno bilinear model is studied. When the controller has additional perturbation, we derived the existing sufficient condition for the non-fragile guaranteed cost control rate based on Lyapunov stable theory. This condition ensures that for any allowed uncertainties, the close-loop system is asymptotically stable, and for a given performance function with quadratic form, performance index of the close-loop system doesn’t exceed some upper bound. We also proposed the designing method for non-fragile guaranteed cost fuzzy controller.

REFERENCES


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