

Chiral symmetry breaking and topology for all N

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We investigate spontaneous chiral symmetry breaking in $SU(N)$ gauge theories at large N using overlap fermions. The exact zero modes and the low-lying modes of the Dirac operator provide the tools to gain insight into the interplay between chiral symmetry breaking and topology. We find that topology indeed drives chiral symmetry breaking at $N = 3$ as well as at large N . By comparing the results on various volumes and at different lattice spacings we are able to show that our conclusions are not affected by finite volume effects and also hold in the continuum limit. We then address the question whether the topology can be usefully described in terms of instantons.

1. INTRODUCTION

The $SU(3)$ gauge fields of QCD possess non-trivial topological properties and these properties are related to the zero modes of the Dirac operator. The importance of the non-trivial topology partly stems from the fact that it leads to the axial $U(1)$ anomaly and therefore to a massive η' in the chiral limit. On the other hand, topology is not necessarily involved in the spontaneous breaking of chiral symmetry. Through the Banks-Casher relation we know that the chiral condensate is proportional to the density of small eigenmodes of the Dirac operator, $\langle\psi\psi\rangle \sim \lim_{\lambda \rightarrow 0} \rho(\lambda)$. A popular scenario is therefore to assume that the non-vanishing density is due to exact zero modes of the Dirac operator which interact with each other and are lifted away from zero eigenvalue producing in this way $\lim_{\lambda \rightarrow 0} \rho(\lambda) \neq 0$. These near-zero modes would therefore have a topological origin since they emerged from the topological zero modes.

It is an interesting and important question to ask whether this scenario is indeed true for $SU(3)$ and, moreover, whether it remains valid in the large N limit [1]. While we believe that the chiral symmetry of QCD is spontaneously broken also at $N = \infty$, it is not clear at all whether the topological charge density is localised, as suggested by

semi-classical considerations, or whether it corresponds to a 'continuous' topological charge distribution due to large vacuum fluctuations from confinement, as argued by Witten [2].

In order to address these questions we resort to a non-perturbative lattice calculation where we examine the local chiral and topological structures of the low-lying fermion modes. The modes serve as probes of the non-perturbative vacuum that are insensitive to ultraviolet fluctuations and therefore they allow us to determine their topological content.

As an important side remark we note that any statement we are able to make in quenched QCD at large N readily applies to full QCD at large N as well.

2. LATTICE CALCULATION

We generated three ensembles of gauge field configurations using the pure gauge Wilson action with $N = 2, 3, 4, 5$ at two different lattice spacings $a \simeq 0.12$ and 0.09 fm and two different volumes $V \simeq 4.0$ and 13.6 fm⁴. The lattice spacing is set by the string tension $a\sqrt{\sigma} = 0.261$ and 0.196 , respectively, for all gauge groups. We then calculated all eigenmodes of the chirally symmetric overlap Dirac operator $D(0)\psi_{\pm} = \lambda_{\pm}\psi_{\pm}$ with $\text{Im}\lambda_{\pm} < 520$ MeV.

We observe very large autocorrelation of the topological charge at small a and large N . It is

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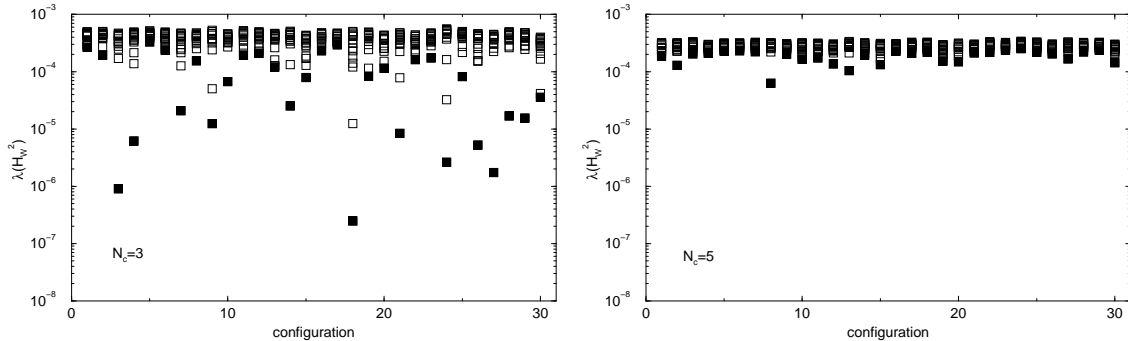


Figure 1. Lowest 15 eigenvalues of H_W^2 at fixed lattice spacing $a \simeq 0.09$ fm and fixed volume $V = 4.0$ fm⁴ for $N = 3$ (left) and 5 (right).

due to the suppression of topological charge fluctuations at the cut-off (dislocations) and was already observed in [1]. It is also reflected in the eigenvalue distribution of the hermitian Wilson Dirac operator H_W entering the overlap operator construction. The striking effect of the disappearance of dislocations, and consequently the small eigenvalues, is exemplified in fig. 1 where we compare the lowest 15 eigenvalues $\lambda(H_W^2)$ for the ensembles with $a = 0.09$ fm and $V = 4$ fm⁴ at $N = 3$ and 5. The behaviour indicates that for large enough N the cost for the overlap operator will follow the naively expected scaling with V and N , i.e., $\propto N^2$ and N^3 for the fermionic and gauge part, respectively. This is in contrast to the SU(3) case where we encounter additional costs due to the increasing number of small eigenvalues $\lambda(H_W^2)$. Together with the intriguing finite size scaling in [3], this motivates a revisit of the large N reduction using overlap fermions [4].

3. RESULTS

The chiral condensate and thus the eigenmode density is expected to scale with V and N . While we observe the correct volume scaling, the correct scaling with N begins to set in only at the finer lattice spacing. Furthermore we seem to observe large scaling violations with a for this quantity.

In order to investigate now the interplay between topology and chiral symmetry breaking we employ the following strategy. Using some measure we determine the topological content of the

near-zero modes and use the zero modes as a comparison since we know that these are topological in origin. To be more specific we define the chiral density $\omega_5(x) = \psi^\dagger(x)\gamma_5\psi(x)$ and the topological charge density $q(x)$ of the background gauge fields obtained after cooling and calculate the dimensionless overlap [5]

$$C_d^5 = \int d^4x |\omega_5(x)|^d |q(x)|^{1-d} \times \text{sign}(\omega_5(x))\text{sign}(q(x)). \quad (1)$$

The advantage of using such a definition is that it is scale invariant for any d as long as only one scale is involved in both ω_5 and q .

For SU(3) we find that the overlap is comparable for the zero modes and the near-zero modes and that it decreases as the eigenvalues of the near-zero modes increase. This remains qualitatively true as $V \rightarrow \infty$ and $a \rightarrow 0$. We therefore conclude that topology indeed drives chiral symmetry breaking in SU(3) and that the influence of topology is weakening for higher lying modes.

As N grows we find that the overlaps become smaller for both the zero modes and the near-zero modes, and that this happens approximately at the same rate. As a consequence the ratio between overlaps from the zero modes and near-zero modes is roughly constant for all N (see fig. 2). Again, this remains qualitatively true as $V \rightarrow \infty$ and $a \rightarrow 0$. All these findings suggest that topology drives chiral symmetry breaking for all N , despite the fact that the local chirality becomes weaker as N grows.

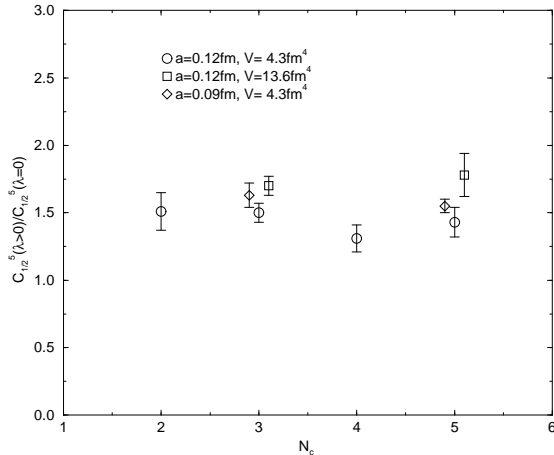


Figure 2. Ratio between the scale invariant overlaps of the zero and near-zero modes for $d = 1/2$.

4. INSTANTONS...?

In order to investigate whether instantons can usefully describe topology at large N , we repeated the calculations on cooled configurations as well as on artificial semi-classical instanton configurations. Again we calculated the lowest few eigenmodes, determined the scale invariant overlaps, the local chiral and topological structures and the instanton size distributions from the chiral densities of the eigenmodes. As N grows we find that small instantons (dislocations) are suppressed and that the typical instanton size grows. Furthermore the eigenmodes become rapidly less chiral while the gauge fields themselves are less (anti-)selfdual. We therefore conclude that instantons seem to become less useful for describing topology at large N , despite the fact that topology remains important.

An interesting point to note, however, is the following [6,7]. Several instantons which overlap strongly in coordinate space may interact only weakly when they occupy mutually commuting $SU(2)$ subgroups of $SU(N)$. So the density of instantons grows $\propto N$ and the local chiral density of the eigenmodes as well as the (anti-)selfduality of the gauge fields appear to become weaker, although the instanton liquid remains dilute in the sense that the interactions are weak. Calculations using different $SU(2)$ subgroup projections seem

to indicate that this possibility is indeed partly realised, however, its effect appears to be very weak (at least for the gauge groups we have considered).

Without commenting on its relevance for $SU(N)$ Yang-Mills we note as a curious side remark, that in $\mathcal{N} = 4$ SUSY Yang-Mills at $N = \infty$ the so-called 'master field', i.e. the field configuration dominating the path integral, is an instanton cluster in which all instantons share a common location and size [8].

5. CONCLUSIONS

We investigated the role of topology in the breaking of chiral symmetry in $SU(N)$ gauge theories. Calculations for $N = 2, 3, 4, 5$ allow us to make statements about all N . We calculated the low lying eigenmodes of the Dirac operator which drive chiral symmetry breaking and determined their topological content. We performed calculations at two lattice spacings and two volumes so as to have some control over the corresponding corrections. We obtain convincing evidence that topology does indeed drive chiral symmetry breaking for $SU(3)$ and that this remains so for all N . We find that dislocations are suppressed and that the local chirality and the (anti-)selfduality of the vacuum becomes weaker as N grows. Whether the topology can be usefully described in terms of instantons is thus not at all clear.

REFERENCES

1. N. Cundy, M. Teper, U. Wenger, Phys. Rev. D66 (2002), 094505.
2. E. Witten, Nucl. Phys. B149 (1979) 285.
3. B. Lucini, M. Teper, U. Wenger, hep-lat/0307017; Phys. Lett. B545 (2002) 197.
4. J. Kiskis, R. Narayanan, H. Neuberger, Phys. Rev. D66 (2002) 025019.
5. N. Cundy, M. Teper, U. Wenger, hep-lat/0209104.
6. T. Schäfer, hep-ph/0204026.
7. G. Carter, E. Shuryak, Phys. Lett. B524 (2002) 297.
8. N. Dorey et al., Nucl. Phys. B552 (1999) 88.