Nonlinear Identification and Robust Tracking Control of A Camless Engine Valve Actuator Based on A Volterra Series Representation

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Abstract—This paper presents the nonlinear identification and the robust position tracking control of a camless engine valve actuator in frequency domain. If a periodic signal excites a nonlinear system, it turns out to generate output spectrum at multiple harmonic frequencies other than that of the excitation. Therefore, such nonlinear features should be taken account in tracking control system design to improve tracking performance. First, nonlinear identification with a Volterra series representation is proposed to capture nonlinearities. Then, robust tracking control of an uncertain Volterra system based on the internal model principle is addressed. It argues that an internal model unit should embed the extended generating dynamics to suppress tracking error occurring at multiple harmonics. To validate the control design method, the tracking results of two different generating dynamics are compared. From the comparison, tracking performance advances through the extended generating dynamics.

I. INTRODUCTION

Advanced technologies of micro-computers, sensors, and actuators have led to broad application of mechatronic systems into modern internal combustion engines. It is attributable to that mechatronic systems enable flexible modulation of engine operation so that the optimal conditions of emissions and fuel efficiency can be attained [1], [2]. And increasingly stringent regulations on emissions and fuel efficiency have accelerated such a technical tendency.

In the paper, a camless engine valve system, particularly, an electro-hydraulic valve actuator, which is one of the most significant elements for advanced air management is concerned. A conventional engine is configured with camshafts, which is mechanically linked to a crankshaft by a timing belt. Therefore, the engine valve operation is synchronized with the rotation of the crankshaft. Fixed valve timings and the valve lift are planned with a specific condition (for example, full load). As a contrast, a camless engine uses a mechatronic actuator to operate poppet valves actively [3]. Its high flexibility allows optimal valve flow control during intake and exhaust strokes over a wide range of engine operating conditions. However, fundamental challenges of modeling and control system design should be resolved for practical use of the mechatronic engine valve system.

Physics-based models and analysis of electro-hydraulic systems are extensively studied in [4]. Regarding the frequency range of interest, the reduced model is developed for computational efficiency [5]. The least squares method is utilized to estimate some of physical parameters [6]. However, estimation of all physical parameters is not a simple task due to complexity and sensitivity. Hence, a standard linear model identification through a frequency response test is preferably used [7]. However, since it is unable to retain nonlinearities of a system, tracking performance can be degraded. To achieve greater tracking performance, nonlinear dynamics should be regarded in control system design.

Many studies of nonlinear identification have been conducted [8]. It is shown that a wide class of nonlinear systems can be approximated by a functional representation, namely Volterra series, with acceptable accuracy [9]. With the Volterra series representation, nonlinear identification method in frequency domain is well documented in [10]. To be specific, identification methods with the Wiener structure which is made up of a preceding linear dynamical system and a following polynomial function are presented in [11], [12]. In [10], [12], the Volterra system are identified by a somewhat similar way, with the single harmonic excitation of different amplitudes and frequencies. But, their experimental costs are very expensive. The method in [11] allows general nonlinear functions in the Wiener structure, but it needs calculation in time- and frequency domain both.

The control system for a camless engine valve actuator demands robust stability and high tracking performance, concurrently. Substantial works of model-based control system design have been performed. The adaptive control law and H∞ control are used for robust tracking control [13], [14], [15]. The internal model principle-based tracking control of a camless engine valve system is presented in [7].

The internal model principle-based control has been widely used in servomechanism. It consists of two sub-blocks: a stabilizer and an internal model unit [16]. The internal model unit includes reference dynamics explicitly. And the stabilizer unit is designed such that the unforced closed-loop system is robustly stable. A repetitive control is the specific design method based on the internal model principle designed to track repetitive reference [17]. The principle was extended to a nonlinear system [18].

The rest of the paper is organized as follows: In Sec. II, the frequency domain nonlinear identification method is proposed for an electro-hydraulic valve actuator with the Volterra series representation and the Wiener model structure. Then, the robust position tracking control of an uncertain Volterra system claimed in [19] is applied in Sec. III. Finally, the control method is validated through the simulations and the experiments in Sec. IV.
II. MODEL IDENTIFICATION

In this section, the efficient frequency domain nonlinear identification technique with the Wiener model structure is developed using a Volterra series representation. The critical benefit of the method is that it requires the single amplitude of harmonic excitation only, which is experimentally cheap.

A. Output Spectrum of the Wiener Model

As presented in [9], a nonlinear system can be approximated by the preceding linear dynamical part and the following static nonlinear part. The Wiener model which is shown in Fig. 1 is one of such approximations.

\[ f(x(t)) = \sum_{i=1}^{N} c_i x(t)^i \]  

Based on the Volterra operator, the \( n \)th order of the output spectrum of the Wiener model is determined as [12]:

\[
\begin{align*}
Y_n(j\omega) &= \frac{1}{(2\pi)^{n-1}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{i=1}^{n} G(j\omega_i) U(j\omega) \\
&\times d\omega_1 \cdots d\omega_{n-1} \\
&= \frac{1}{(2\pi)^{n-1}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} H_n(j\omega_1, \cdots, j\omega_n) \\
&\times \prod_{i=1}^{n} U(j\omega_i) d\omega_1 \cdots d\omega_{n-1}
\end{align*}
\]

where \( \omega = \sum_{i=1}^{n} \omega_i \). Then, the final output spectrum is determined by the summation of the \( N \) terms as below:

\[
Y(j\omega) = \sum_{n=1}^{N} Y_n(j\omega)
\]

\( U(j\omega) \) and \( Y(j\omega) \) indicate the input- and the output spectrum obtained by discrete Fourier transform (DFT) of \( u(t) \) and \( y(t) \), respectively. \( G(j\omega) \) is the frequency response function (FRF) of the linear dynamical part. In the second equation of Eq. (2), \( H_n(j\omega_1, \cdots, j\omega_n) \) is the generalized frequency response function (GFRF) determined as:

\[
H_n(j\omega_1, \cdots, j\omega_n) = c_n \prod_{i=1}^{n} G(j\omega_i)
\]

The GFRF is an extension of the FRF (in other words, transfer function) of a linear system to a nonlinear system.

When a single frequency input of \( \omega \) is applied, the output spectrum at harmonic frequencies can be calculated using Eq. (2) as shown in Eqs. (5)-(9). The input frequency \( \omega \) is the element of the finite set \( \Omega = [\omega_1, \cdots, \omega_q] \). \( q \) is the number of input frequencies.

\[
\begin{align*}
Y(0) &= \frac{2c_2}{2^1} X(j\omega) X(-j\omega) + \frac{6c_4}{2^2} X(j\omega)^2 X(-j\omega)^2 \\
Y(j\omega) &= \frac{c_1}{2^0} X(j\omega) + \frac{3c_3}{2^2} X(j\omega)^2 X(-j\omega) \\
Y(2j\omega) &= \frac{c_2}{2^1} X(j\omega)^2 + \frac{4c_4}{2^4} X(j\omega)^3 X(-j\omega) \\
Y(3j\omega) &= \frac{c_3}{2^3} X(j\omega)^3 \\
Y(4j\omega) &= \frac{c_4}{2^4} X(j\omega)^4
\end{align*}
\]

where \( X(j\omega) = G(j\omega)U(j\omega) \) is the internal state spectrum. Here, the plant is assumed as a fourth order Volterra system regarding spectral observation. Eqs. (5)-(9) explain why multiple harmonic components including zero frequency are created even with a single harmonic input. This phenomenon is called as intermodulation which is one of the main limiting factors of tracking performance of a nonlinear system.

B. Linear Part Estimate

The fundamental element of Eq. (6) is rewritten as Eq. (10). From this, the phase angle relationship of Eq. (11) can be obtained.

\[
\begin{align*}
Y(j\omega) &= \left( \frac{c_1}{2^0} + \frac{3c_3}{2^2} X(j\omega) X(-j\omega) \right) X(j\omega) \\
\angle Y(j\omega) &= \angle X(j\omega)
\end{align*}
\]

Using this fact, the linear dynamical part represented by a fractional FRF as Eq. (12) can be identified. It is immediate that the phase angle of \( G(j\omega) \) can be known from the phase angle difference between the input and the output as Eq. (13).

\[
\begin{align*}
G(j\omega) &= \left( \frac{j\omega)^m + b_{m-1}(j\omega)^{m-1} + \cdots + b_0}{(j\omega)^n + a_{n-1}(j\omega)^{n-1} + \cdots + a_0} \right) \\
\angle G(j\omega) &= \angle Y(j\omega) - \angle U(j\omega)
\end{align*}
\]

It is noted that both the numerator and the denominator in Eq. (12) are monic. It is because that the gains of the static nonlinear part and the linear dynamical part are not unique in the serial structure. Any pair of (\( \alpha g(t) \), \( f(x(t)/\alpha) \)) would produce identical output measurement with non-zero \( \alpha \). Therefore, the gain of the linear dynamical part is intentionally constrained by monic polynomials. Using the nonlinear least squares method, the unique parameters in Eq. (12) are estimated. Eventually, if the polynomial function in the nonlinear block is estimated, the gain of the linear dynamical part will be rescaled.
C. Nonlinear Part Estimate

If the linear dynamical part is estimated once, the parameters of the nonlinear part can be estimated. Eqs. (5)-(9) can be rewritten in a matrix form as:

\[ Y = \Pi X \]  

(14)

where \( Y \) is the column vector of the measured output spectrum at zero frequency and the multiple harmonic frequencies when each input frequency \((\omega \in \Omega)\) is applied, \( Y = [Y(0) Y(j\omega) Y(2j\omega) Y(3j\omega) Y(4j\omega)]^T \). \( \Pi \) is the matrix with the appropriate dimension \((5 \times 4)\) which is obtained from the estimated FRF of the linear dynamical block and the measured input spectrum for each input frequency. \( X \) is the column vector whose elements are the polynomial coefficients, \( X = [c_1 \ c_2 \ c_3 \ c_4]^T \). Then, the polynomial coefficients of the nonlinear block are estimated using the least squares method as Eq. (15).

\[ X = \text{Re} \left( \left( \sum_{\omega \in \Omega} \Pi^T \Pi \right)^{-1} \left( \sum_{\omega \in \Omega} \Pi^T Y \right) \right) \]  

(15)

Nonlinear parameters are normalized by \( c_1 \) as \( \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4 \) such that \( \hat{c}_1 = 1 \). And the FRF of the linear dynamical part is amplified with \( c_1 \).

D. Application to the Electro-hydraulic Valve Actuator

The proposed identification method is applied to the electro-hydraulic valve actuation system shown in Fig. 2.

Fig. 2. A picture of the electro-hydraulic valve actuator

Its nonlinearities are associated with the orifice flow and the chamber pressure dynamics. All other sub-elements (the spool and the actuator) are like a mass-damper-spring system [4]. Such a mild nonlinear system can be represented by Volterra series of a finite order with acceptable accuracy. The system is made up of the voice coil motor, the spool, the hydraulic fluid supply, the actuator, and poppet valve assembly. The hydraulic fluid is supplied by the reservoir after highly pressurized by the pump. The stem of the poppet valve is tapered, so that its position is measured by the non-contact sensor (Microstrain NC-DVRT 1.0). The voice coil motor (BEI Kimco Magnetics: LA13-12-000A) with the sensitivity of \( K_f = 9.79 \) N/A is used to modulate the spool position. A high-bandwidth power amplifier (Advanced Motion Controls: 12A8) is tuned to generate the current of the voice coil which is proportional (0.1 A/V) to the control signal. For the frequency response test, the system is stabilized by the scheduled proportional control in the inner-loop first as shown in Fig. 3.

Fig. 3. A schematics of the experimental setup

Fig. 4 depicts the identified second order linear dynamical block model amplified by \( c_1 \). The red curves and the blue asterisks indicate the magnitude (upper) and the phase angle (lower) of the estimated linear dynamical part and the measured output spectrum at the input frequency, respectively. It is noted that the magnitude mismatch between them is found, even in the low frequency range. But it is obviously because the former includes the first term only, however, the latter does the first- and third terms as given in Eq. (6).

Fig. 4. Linear Block Estimate - curve: estimated, asterisk: measured

Fig. 5 shows the estimate of the normalized parameters of the polynomial equation in the nonlinear block. The red circles are the parameter estimates which are independent of the input frequency from Eq. (15). On the other hand, the blue asterisks are frequency-dependent estimates obtained from Eq. (15), but without summation of input frequencies and restriction to a real number, that is, \( X(\omega) = (\Pi^T \Pi)^{-1} (\Pi^T Y) \).

The purpose of the frequency-dependent estimate is to identify the multiplicative uncertainty of the first Volterra
term which results from the use of the unique model structure (i.e. Wiener structure). This uncertainty model will be utilized in robust control design in the later section. Based on Eq. (4), the first Volterra term can be written as the following:

$$H_1(s) = \left(1 + \frac{c_k^i - c_1}{c_1}\right) c_1 G(s)$$

(16)

where $c_k^i$ is the input frequency-dependent estimate when $k$th input frequency among $\Omega$ is applied. It is noted that $c_k^i$ is a complex number, whereas $c_1$ is a real number. Then, the multiplicative uncertainty of the first Volterra term is defined as Eq. (17). The uncertainty and its upper bound are shown in Fig. 6.

$$W_u = \frac{c_k^i - c_1}{c_1}$$

(17)

Fig. 6. Uncertainty of the 1st Volterra term - circle: measured, curve: upper-bound

III. CONTROL DESIGN

Robust asymptotic reference tracking control of an uncertain Volterra system was introduced in [19]. It claimed that even a linear control can achieve robust asymptotic tracking of an uncertain Volterra system, but necessarily with the extended generating dynamics. Tracking performance of two well-known control methods in terms of the extended generating dynamics are examined in this section.

A. Robust Asymptotic Tracking Control

Any periodic reference concerned can be approximated by a sum of harmonic functions with good accuracy as below:

$$r(t) = \frac{1}{2} \sum_{i=-p,i\neq 0}^{p} A_i \exp(i\omega_it)$$

(18)

where $A_i$ and $A_{-i}$ are a complex number and its conjugate, respectively. And $\omega_i = -\omega_i$. As presented in Sec. II, a nonlinear system output includes many harmonic elements at combination of reference frequencies; $\omega = \pm \omega_1 \pm \cdots \pm \omega_n > 0$ for $n = 1,2, \cdots, N$, and $1 \leq i_1, \cdots, i_n \leq p$. $N$ is the order of Volterra series.

The Volterra series representation from $u$ to $y$ is given in Eq. (2). The one from $r$ to $e$ is defined below:

$$E_n(j\omega) = \frac{1}{2^{n-1}} \sum_{\omega_1+\cdots+\omega_n=\omega} F_n(j\omega_1, \cdots, j\omega_n) \times A_{i_1} \cdots A_{i_n}$$

(19)

where $E_n(j\omega)$ is the $n$th order error spectrum, and $F_n$ is the $n$th GFRF from $r$ to $e$. From Eq. (19), the following sufficient and necessary condition is derived to achieve zero steady state tracking error:

$$F_n(j\omega_1, \cdots, j\omega_n) = 0, \forall n$$

(20)

The $n$th GFRF from $e$ to $y$, $P_n$ is shown below:

$$P_n(j\omega_1, \cdots, j\omega_n) = \prod_{i=1}^{n} C(j\omega_i) H_n(j\omega_1, \cdots, j\omega_n)$$

(21)

where $C$ is the FRF of the linear control. The $n$th GFRF from $u$ to $y$ is described by the fractional form as:

$$H_n(j\omega_1, \cdots, j\omega_n) = \frac{N_{h,n}(j\omega_1, \cdots, j\omega_n)}{D_{h,n}(j\omega_1, \cdots, j\omega_n)}$$

(22)

By applying the growing exponent method to the negative feedback loop, the $n$th GFRF from $r$ to $e$ becomes [10]:

$$F_1(j\omega) = \frac{1}{1 + F_1(j\omega)}$$

(23)

$$\vdots$$

$$F_n(j\omega_1, \cdots, j\omega_n) = -F_1(j\omega_1 + \cdots + j\omega_n) \times Q_n(j\omega_1, \cdots, j\omega_n)$$

(24)

The details of $Q_n$ are explained in [10], [19]. Then, the linear control of Eq. (25) is solvable for robust asymptotic tracking of an uncertain Volterra system. It is designed such that Eqs. (26) and (27) are satisfied.

$$C(j\omega) = \frac{N_c(j\omega)}{D_c(j\omega) \phi(j\omega)}$$

(25)

$$F_1(j\omega) : \text{stable}$$

(26)

$$\phi(j\omega) = 0, \forall \omega = \omega_1 + \cdots + \omega_n$$

(27)

The linear control is able to stabilize $F_1$, since $F_1$ is linear (see Eq. (23)). $\phi(j\omega)$ is the extended generating dynamics in terms of the internal model principle. It should be zero at any possible harmonic frequencies depending on the reference frequencies and the order of the Volterra series. Then, the sufficient and necessary condition given in Eq. (20) is satisfied, since $F_1(j\omega_1, \cdots, F_1(j(\omega_1 + \cdots + \omega_n))$ become zero. It is the extension of internal model principle from a linear system to a Volterra system. To examine tracking performance with respect to the generating dynamics, two well-known control methods are compared.
B. Harmonic Internal Model Unit

First, the internal model principle-based control design [16] is concerned for tracking control of a Volterra system. The control shown in Eq. (28) is designed, based on the first Volterra term given in Eq. (29). In Eq. (30), $\Gamma(s)$ is the reference dynamics itself. $R(s)$ is the reference.

$$
C(s) = \frac{N_C(s)}{D_C(s)\Gamma(s)}
$$

(28)

$$
H_1(s) = \frac{N_{H_1}(s)}{D_{H_1}(s)}
$$

(29)

$$
\Gamma(s)R(s) = 0
$$

(30)

$D_C(s)$ and $N_C(s)$ should be determined to stabilize the following closed loop system regardless of the uncertainty in the first Volterra term shown in Fig. 6.

$$
E(s) = \frac{D_C(s)D_{H_1}(s)\Gamma(s)R(s)}{\Gamma(s)D_C(s)D_{H_1}(s) + N_C(s)N_{G_1}(s)}
$$

(31)

C. Prototype Robust Repetitive Control

Second, the discrete robust repetitive control [17] is concerned. The control is composed of a constant control gain $K_r$, a stable filter $M(z^{-1})$, a delay operator $z^{-N}$, and a low pass filter $Q(z^{-1})$ as Eq. (32). Eq. (33) is a discrete form of Eq. (29). Reference dynamics is given in Eq. (34). $Q(z^{-1})$ is designed regarding the uncertainty in the first Volterra term similarly to retain robust stability. For details of control synthesis, see [17].

$$
C(z^{-1}) = K_r M(z^{-1}) \frac{Q(z^{-1})z^\delta z^{-N}}{1 - Q(z^{-1})z^{-N}}
$$

(32)

$$
G(z^{-1}) = \frac{N_{H_1}(z^{-1})}{D_{H_1}(z^{-1})}
$$

(33)

$$
(1 - z^{-N})r(k) = 0
$$

(34)

Both control methods satisfy the internal model principle. The difference between them places on the different descriptions of the reference signal, Eqs. (30) and (34). Their different inherent features are shown in Fig. 7. The single harmonic signal of 10 Hz plus a positive constant offset is regarded as a reference. The first control possesses two peak gains at 0 and 10 Hz. However, the second has peak gains at multiple harmonics. It means that the extended generating dynamics is inherently present in the repetitive control. But, it is noted that if $Q(z^{-1})$ is 1, it is strictly the extended generating dynamics. If not, the condition of Eq. (27) is collapsed. However, still the repetitive control plays a role of a robust filter though perfect asymptotic tracking is sacrificed due to robust stability [19].

IV. SIMULATIONS & EXPERIMENTS

Simulations are conducted with two control methods presented in Sec. III, and their tracking performances are compared. The identified model established in Sec. II is utilized as a plant. The control purpose is to track the single sinusoidal reference of 10 Hz plus the positive constant offset. Figs. 8 and 9 show simulation results in time and frequency domains, respectively. From the figures, the second control shows dramatic improvement of tracking performance. The error spectrum is nearly zero at all harmonics.

Both controls are implemented into the actual electro-hydraulic valve actuation system. Figs. 10 and 11 show experimental results. Similarly, the second shows better tracking performance. These support why the repetitive control shows better tracking performance if the actual plant possesses nonlinearities.

V. CONCLUSIONS

The nonlinear identification and robust tracking control of a camless engine valve actuator are presented. First, with the Volterra series representation, the proposed frequency domain nonlinear identification technique is applied to the electro-hydraulic engine valve actuator. Second, the robust tracking control of an uncertain Volterra system based on the internal model principle is presented with emphasis on the extended generating dynamics. For validation, two different methods in terms of generating dynamics are compared. One possesses the reference dynamics only, but the other has the extended one. Simulations and experiments showed that even a linear control, but necessarily with the extended generating dynamics attains robust asymptotic tracking of an uncertain Volterra system.

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Fig. 8. Time domain comparison - left: harmonic IMU, right: repetitive control

Fig. 9. Frequency domain comparison - left: harmonic IMU, right: repetitive control

Fig. 10. Time domain comparison - left: harmonic IMU, right: repetitive control

Fig. 11. Frequency domain comparison - left: harmonic IMU, right: repetitive control


