PULSE-EXCITED RC NONAUTONOMOUS CHAOTIC OSCILLATOR STRUCTURES

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Received January 9, 2004; Revised June 1, 2004

Two novel circuit-independent RC nonautonomous chaotic oscillator structures are presented. Both structures rely on a periodic pulse-train as the driving force and on a comparator as the only source of nonlinearity.

Keywords: Nonautonomous chaotic oscillators, pulse-excitation.

1. Introduction
A technique for generating nonautonomous chaotic oscillators by using an exciting periodic pulse-train voltage-source was recently introduced in [Elwakil, 2002] and [Elwakil & Özoguz, 2003]. This technique also involves a comparator (or digital inverter) as the only nonlinear element and was used to generate chaos in [Elwakil, 2002] from Chua’s circuit third-order passive structure and in [Elwakil & Özoguz, 2003] from the active second-order series LC resonator.

In this Letter, we prove the generality and validity of this technique when applied to two circuit-independent second-order RC oscillator structures. The technique was tested on several RC oscillator network and a selected example is shown. Pulse-driven nonautonomous chaotic oscillators are important for neural network modeling and the generation of pseudo-random sequences in clocked systems [Nakano & Saito, 2001; Aihara, 2002].

2. Background
An important goal that has recently been emphasized in the area of chaotic oscillator design is to move the design process from the circuit-specific level to a more general circuit-independent level in which the functionality of the building blocks, rather than their construction, is significant. In this regards, it was first shown in [Elwakil & Kennedy, 2000] and later in [Elwakil & Kennedy, 2001] that the core engine necessary to generate chaos in any chaotic oscillator, which is a sinusoidal oscillator, can acquire a second-order RC circuit-independent structure. Briefly, we recall here two of these structures.

Consider the general sinusoidal oscillator, shown partially in Fig. 1(a), and composed of an arbitrary active first-order RC network (containing a capacitor $C_2$) driving a separate series $R_1C_1$ network by a voltage $V_S$. This voltage can be expressed in general form as a function of the two state variables $(X, Y)$ as:

$$V_S = \pm K_1X \mp K_2Y$$

where $K_1$ and $K_2$ are constants any of which might be zero. With $C_1 = C_2$, this sinusoidal oscillator is ideally described as [Elwakil & Kennedy, 2000]:

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} = \begin{pmatrix} \pm K_1 - 1 \\ \mp K_2 \end{pmatrix} \frac{n \pm \mp (K_1 - 1)^2}{\pm K_2} \begin{pmatrix} X \\ Y \end{pmatrix} \quad (1)$$
where $X = V_{C1}/V_{cc}$, $Y = V_{C2}/V_{cc}$ ($V_{cc}$ is the DC supply voltage of the active network). Here, time is normalized with respect to the time constant $\tau = R_1 C_1$ and hence the oscillation frequency is $\omega_0 = \sqrt{n}$; $n$ is an arbitrary frequency scaling constant.

Next, consider the sinusoidal oscillator, shown partially in Fig. 1(b), where the active network supplies a current $I$ to the parallel $R_1 C_1$ branch. This current can be expressed as: $I = \pm g_1 V_{C1} \mp g_2 V_{C2}$ where $g_1$ and $g_2$ are constant transconductances. This sinusoidal oscillator can then be described as in [Elwakil & Kennedy, 2000]:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \pm K_1 - K_2 & \mp 1 \\ \pm [nK_2^2 + (\pm K_1 - K_2)] & K_2 \mp K_1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

Here, $K_1 = g_1/g_2$, $K_2 = 1/(R_1 g_2)$ and time is normalized with respect to $\tau = C_1/g_2$.

3. Proposed Pulse-Excited Chaotic Oscillators

To generate a nonautonomous chaotic oscillator, the proposed technique is systematic and implies [see Fig. 1(a)] adding to the circuit-independent sinusoidal oscillator (i) feedback comparator, whose output voltage $V_F$ switches according to the input voltage $V_S$, and (ii) a periodic pulse-train excitation $V_P$. $V_F$ and $V_P$ are given respectively by:

$$V_F = V_{cc} \text{sgn}(V_S) \quad \& \quad V_P = V_{cc} \text{sgn}(\sin(\omega_P t)) \quad (3)$$

where $\omega_P$ is the excitation frequency. The effect of the driving force $V_P$ and the comparator output $V_F$ are added together after being converted into weighted currents through resistors $R_P$ and $R_F$. 

![Fig. 1. Proposed pulse-excited RC nonautonomous chaotic oscillator structures: (a) with a series $R_1 C_1$ network; (b) with a parallel $R_1 C_1$ network; (c) circuit design example.](image-url)
respectively. The combined current is then used to excite the node across $C_1$. The resulting nonautonomous oscillator of Fig. 1(a) is described as:

\[
\begin{align*}
\dot{X} &= -(K_P + K_P - K_1 + 1)X - K_2Y \\
\dot{Y} &= \frac{n + (1 - K_1)^2}{K_2}X + (1 - K_1)Y
\end{align*}
\]  

(4a)

(4b)

where $f(X, Y)$ and $p(\tau)$ are given respectively as:

\[
\begin{align*}
f(X, Y) &= \text{sgn}(K_1X - K_2Y) \\
&= \begin{cases} 
1 & K_1X - K_2Y \geq 0 \\
-1 & K_1X - K_2Y < 0 
\end{cases} \\
p(\tau) &= \text{sgn}(\sin(\phi\tau)) \\
&= \begin{cases} 
1 & \sin(\phi\tau) \geq 0 \\
-1 & \sin(\phi\tau) < 0 
\end{cases}
\end{align*}
\]  

(5)

(6)

Here, we have chosen $V_S = 0.8V_{SS}$ ($\dot{X} = \dot{X}_+$ & $\dot{Y} = \dot{Y}_+$ in (1)) and defined $K_P = R_1/R_F$, $K_P = R_1/R_F$ in addition to $\phi = \omega rR_0C_1$.

Numerical integration of the above system using an adaptive-step Runge–Kutta algorithm was performed after taking $K_1 = 0.85$, $K_2 = K_P = 1$, $K_P = 2$ and $n = 0.1$. The observed chaotic attractor projection in the $X - Y$ plane is shown in Fig. 2. It is clear that the system has four equilibrium points

\[
(x_0, y_0) = \frac{\pm K_P \pm K_P}{n + (K_1 - 1)(K_P + K_P)} \\
\cdot \left[ (K_1 - 1), \frac{n + (K_1 - 1)^2}{K_2} \right]
\]  

(7)

The value of $K_2$ does not affect the chaotic dynamics but only results in a mirror image of the attractor, as shown in the upper right corner of Fig. 2 for $K_2 = -1$ [Elwakil & Kennedy, 2001]. As explained in [Elwakil & Özoguz, 2003], the excitation $p(\tau)$ can be replaced with a nonlinearity $f(Z) = \text{sgn}(Z)$ where $Z(\tau) = \sin(\phi\tau)$ is the solution to the second-order differential equation $\ddot{Z} + \phi^2Z = 0$. Hence, a view of the chaotic attractor in the $X - Y - Z$ subspace can be constructed and is shown in Fig. 3.

Next we consider the structure of Fig. 1(b). With the proposed modification technique applied and choosing $I = I_+$ ($\dot{X} = \dot{X}_+$ & $\dot{Y} = \dot{Y}_+$ in (2)), the resulting nonautonomous oscillator is described as:

\[
\begin{align*}
\dot{X} &= (K_1 - K_2 - K_P - K_P)X - Y \\
\dot{Y} &= nK_2^2 + (K_1 - K_2)X + (K_2 - K_1)Y
\end{align*}
\]  

(8a)

(8b)

Fig. 2. $X - Y$ projection of the chaotic attractor observed from the structure of Fig. 1(a).
with $f(X) = \text{sgn}(X)$ and $p(\tau) = \text{sgn}(\sin(\phi \tau))$; 
$\phi = \omega t C_1 / g_2$.

A chaotic attractor similar to that in Fig. 2 is observed from the above system when numerically integrated with the same values of $K_1$, $K_2$, $K_F$, $K_p$, $\phi$ and with $n$ increased to 0.25. The four equilibrium points are located at

$$
(x_0, y_0) = \frac{\pm K_F \pm K_F}{(K_1 - K_2)(K_F + K_F) + nK_2^2}
\left[ (K_1 - K_2), nK_2^2 + (K_1 - K_2)^2 \right]
$$

Fig. 3. Three-dimensional view of the chaotic attractor in the $X - Y - Z$ subspace.

Fig. 4. Chaotic $V_{C1} - V_{C2}$ trajectory (X axis: 0.5 V/div, Y axis: 1 V/div) and sample chaotic output and periodic input pulse-trains.
4. Design Example

A circuit design example is shown in Fig. 1(c). The AD844 current feedback op amp, employed as a noninverting amplifier, along with $R_1, R_2, C_1$ and $C_2$ represent a sinusoidal oscillator which can be modeled by (1). Applying the pulse-excitation technique to the node across $C_1$ and taking $C_1 = C_2 = C$, the resulting circuit is described by:

$$\dot{X} = -(K_F + K_P + 1)X + K_P Y$$
$$+ K_P f(X) + K_P p(\tau) \quad (10a)$$
$$\dot{Y} = -X + (K_2 - K_1) Y \quad (10b)$$

where $K_1 = R_1/R_2$, $K_2 = R_5/(R_4 + R_5)$ and $f(X) = \text{sgn}(X)$. $K_F, K_P, \phi$ and $p(\tau)$ are as defined before. Compared to Fig. 1(a), note that the comparator is directly controlled by $V_{C1}$ as a simplified case.

The circuit was experimentally tested with $C = 10 \text{nF}$, $R_1 = 2R_2 = R_1 = R_5 = 2R_5 = 10 \text{k}\Omega$, $R_5 = 38 \text{k}\Omega$ and using a TL082 op amp for the comparator. Figure 4 represents the observed $V_{C1} - V_{C2}$ chaotic trajectory. A sample of the comparator output waveform along with the periodic pulse-train excitation ($\phi = 0.5$) are also shown.

5. Conclusion

We have proposed two classes of inductorless nonautonomous chaotic oscillators excited by a periodic pulse-train with self-feedback provided through a comparator. These oscillators are suitable for the generation of pseudo-random sequences in digitally clocked systems.

Acknowledgments

This work has been supported by the Turkish Academy of Sciences, in the framework of the Young Scientist Award Program (ISO/TUBA-GEPIB/2002-1-16) and by ITU Research Activity Secretariat.

References


