A New Optimization Framework To Solve The Optimal Feeder Reconfiguration And Capacitor Placement Problems

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Abstract: This paper introduces a new stochastic optimization framework based bat algorithm (BA) to solve the optimal distribution feeder reconfiguration (DFR) as well as the shunt capacitor placement and sizing in the distribution systems. The objective functions to be investigated are minimization of the active power losses and minimization of the total network costs. In order to consider the uncertainties of the active and reactive loads in the problem, point estimate method (PEM) with 2m scheme is employed as the stochastic tool. The feasibility and good performance of the proposed method are examined on the IEEE 69-bus test system.

Index Terms: Optimal Distribution Feeder Reconfiguration (DFR), shunt capacitor placement, Point Estimate Method (PEM), Bat Algorithm (BA).

1 INTRODUCTION

Distribution system is a significant part of the electric power network that delivers power to the consumers. Therefore, the situation of this part will affect the power quality of the electrical services greatly. In this way, distribution feeder reconfiguration (DFR) and shunt capacitor placement (SCP) are two of the most significant strategies that can improve the total situation of the distribution networks. By definition, DFR is defined as the process of changing the topology of the radial networks using some normally open switches (tie switch) and normally closed switches (sectionalizing switches). One significant limitation in the DFR strategy is preserving the radiality of the network before and after the reconfiguration. While DFR is a valuable strategy in the power system, the intrinsic complex and discrete of this technique is a barrier for its usage. The literature introduces many methods for solving this problem. Some of the most significant methods can be named as heuristic techniques [1], neural network [2], expert systems [3], optimum flow pattern [4], brute-force approach [5], hybrid simulated annealing algorithm and Tabu search [6], graph theory [7] and ant colony optimization algorithm [8].

Other works exits that have assessed the effect of DFR on the other targets such as voltage profile enhancement [9], load balance increase [10] and reliability improvement [11]. The other valuable reinforcement strategy is SCP. SCP can help the system by injecting reactive power locally to the buses and thus releasing the capacity of the feeders. This issue can result in many benefits in the long time for the system from both power losses and costs. Generally, the methods of solving the SCP can be categorized in four groups [12]: analytical methods [13-14], numerical programming [15], heuristic search [16] and Artificial Intelligence (AI) methods [17-19]. Nevertheless, the recent tendency has been toward the AI methods which among them evolutionary algorithms have the most popularity. The special characteristics of the evolutionary algorithms are no need to derivatives and ability of solving both continuous and discrete optimization problems.

2 Problem formulation

In this section, the objective functions and the limitations are described.

2.1 Control Vector

The control vector $X$ includes 1) optimal location of shunt capacitors, 2) optimal size of shunt capacitors, 3) optimal status of tie switches and 4) optimal status of sectionalizing switches as below:

$$X = \begin{bmatrix} i\text{Tie}, i\text{Sw}, i\text{T} \end{bmatrix}$$

(1)

$$i\text{Tie} = \begin{bmatrix} T_{i1}, T_{i2}, \ldots, T_{i\text{nsw}} \end{bmatrix}$$

(2)

$$i\text{Sw} = \begin{bmatrix} S_{i1}, S_{i2}, \ldots, S_{i\text{nsw}} \end{bmatrix}$$

(3)

$$i\text{T} = \begin{bmatrix} T_1, T_2, \ldots, T_{\text{nsw}} \end{bmatrix}$$

(4)

$$i\text{Cap} = \begin{bmatrix} Q_{i1}, Q_{i2}, \ldots, Q_{i\text{nsw}} \end{bmatrix}$$

(5)

where $T_i$ shows that if the $i^{th}$ bus is a good candidate for reactive power compensation or not ($T_i = 1$ shows a good candidate), $T_{ie}$ is the status of $i^{th}$ tie switch ($T_{ie} = 1$ shows this tie switch should be closed), $S_{iw}$ is the status of $i^{th}$
sectionalizing switch \((S_{W_i} = 1)\) shows this sectionalizing switch should be opened), \(N_{sw}\) is the number of sectionalizing switches in the grid, \(Q^c_i\) is the capacity of shunt capacitor that should be installed on \(l^th\) bus, \(N_{sw}\) is the number of tie switches in the grid, and \(N_{bus}\) is the number of buses in the grid.

2.2 Objective Function

The first objective function is the total resistive losses of the network that should be minimized. Since the problem is investigated in the stochastic framework, the symbol \(E\) or \(\mathbb{E}\) is used to show expected value here. The amount of active power losses can be computed as follows:

\[
f_1(X) = E(P_{loss}(X)) = \sum_{i=1}^{N_{bus}} R_i \times |E(I_i)|^2
\]

(6)

where \(f_i\) is the amount of current in the \(i^{th}\) branch, \(R_i\) is the resistance of the \(i^{th}\) branch and \(N_{br}\) is the number of branches.

The second target is the total network costs that includes both the cost of power losses and the cost of capacitor placement as follows:

\[
f_2(X) = E(\cos (X)) = K_p \times E(P_{loss}) + \sum_{i=1}^{N_{bus}} K^c_i \times Q^c_i
\]

(7)

where \(K_p\) is the equivalent annual cost per unit of power loss and \(K^c_i\) is annual capacitor installation cost.

2.3 Constraints and Limitations

There are some limitations that should be preserved as follows: - Distribution power flow equations

The power flow equations are supposed as equality constraints of the problem:

\[
E(P_{ij}) = \sum_{i=1}^{N_{bus}} [E(V_i)] [E(V_j)] [Y_{ij} \cos (\theta_{ij} - E(\delta_j)) + E(\delta_j)]
\]

(8)

\[
E(Q_{ij}) = \sum_{i=1}^{N_{bus}} [E(V_i)] [E(V_j)] [Y_{ij} \sin (\theta_{ij} - E(\delta_j)) + E(\delta_j)]
\]

where \(N_{bus}\) is the number of buses, \(\delta_i\) is the voltage angle of \(i^{th}\) bus, \(Y_{ij}\) is the admittance of the feeder between the buses \(i\) and \(j\), \(V_i\) is the voltage of \(i^{th}\) bus, \(\theta_{ij}\) is the angle of the admittance of the feeder between the buses \(i\) and \(j\).

- Capacitor sizes

Shunt capacitors are made in discrete sizes and generally an integer multiple of the smallest capacitor size as follows:

\[
Q^c_{min} = L \times Q^c_0
\]

(9)

where \(Q^c_0\) is the size of the smallest capacitor in kVar and \(L\) is an integer. The available capacitor sizes and their cost per kVar are given in Table 1.

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q^c) (kVar)</td>
<td>150</td>
<td>300</td>
<td>450</td>
<td>600</td>
<td>750</td>
<td>900</td>
<td>105</td>
<td>120</td>
<td>135</td>
</tr>
<tr>
<td>$/ kVar</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

- Bus voltage limitation

From the operation view, the voltage of the buses should be preserved in the limited ranges as follows:

\[
V_{min} \leq E(V_i) \leq V_{max}
\]

(10)

where \(V_{min}\) / \(V_{max}\) are the minimum / maximum voltage level of the buses.

- Feeder maximum power flow limitation

Maximum transfer power of the feeders is preserved according to their thermal capacity as below:

\[
|E(P_{ij}^{max})| \leq P_{ij}^{max}
\]

(11)

where \(P_{ij}^{max}\) is the amount of power flow in the line between the buses \(i\) and \(j\) and \(P_{ij}^{max}\) is the amount of maximum power flow in the line between the buses \(i\) and \(j\).

- Radial structure of the network limitation

The radial structure of the network should be preserved before and after the DFR. The number of main loops of the network is calculated as:

\[
N_{rl} = N_{br} - N_{bus} + 1
\]

(12)

3. Modeling Uncertainty

In order to model the uncertainties of the problem, this paper proposes PEM as a suitable tool. PEM uses the idea of replacing each random variable \(z_i\) with a PDF function \(f(z_i)\). Then, by the use of 2m scheme (wherein \(m\) shows the number of uncertain variables), each PDF is replaced by two concentration points \(z_{i1}\) and \(z_{i2}\). Fig. 1 shows the schematic diagram of the 2m PEM.
According to Fig. 1, the uncertainty of the input variables is transferred to the output variables through the PEM. The mean values of $z_{1}$ and $z_{2}$ are calculated as follows:

\[ \xi_{i,k} = \mu_{i} + \xi_{i,k} \cdot \sigma_{i} \quad k = 1,2 \quad (13) \]

where $\sigma_{i}$ is the standard deviation of the $f_{i}$. Each time that the load flow is run, one of these two concentration points will replace the mean value of the relevant uncertain parameter as follows:

\[ S = F(\mu_{1}, \mu_{2}, ..., \mu_{i}, ..., \mu_{m}) \quad k = 1,2 \quad (14) \]

where $\mu_{i}$ is the mean value of the PDF of the random variable. $S$ is the output of the load flow equations and $z_{i,k}$ is the $k^{th}$ concentration point of $i^{th}$ uncertain variable $z_{i}$ that is calculated using (13). In the (13), the value of $\xi_{i,k}$ is calculated as:

\[ \xi_{i,k} = \frac{\lambda_{i,k} - m}{\sqrt{1 - m^2}} \quad k = 1,2 \quad (15) \]

The weighting factors $\omega_{i1}$, $\omega_{i2}$ are calculated as follows:

\[ \omega_{i1} = \frac{1}{m} \left( \frac{\xi_{i1}}{\xi_{i2}} \right) \quad (17) \]

Finally, the expected value of the output variables (here targets) are calculated as follows:

\[ \sigma = \sqrt{\text{var}(S)} = \sqrt{E(S^{2})} - E(S) \quad (18) \]

The variance var shows the variance math operator and $S_{i}$ is $i^{th}$ moment of $i^{th}$ objective function $S_{i}$.

4. Bat Algorithm (BA)

BA is a population based evolutionary optimization algorithm that imitates the behavior of bat animals to take their prey. The superiority of BA is demonstrated over some others such as honey bee mating optimization [25-28], particle swarm optimization (PSO) [29-30], teacher learning algorithm [31], shuffled frog leaping algorithm [32], cuckoo search algorithm [33] and firefly algorithm [34]. BA is constructed based on four main concepts: 1) bats utilize echolocation process to find distance and identify the difference between prey and food; 2) Each bat in the position $X_{i}$ flies randomly with the velocity of $V_{i}$ producing a pulse with the frequency and loudness of $f_{i}$ and $A_{i}$ respectively; 3) The loudness of $A_{i}$ differs in many ways such as reducing from a large value to a low value; and 4) The frequency $f_{i}$ and rate $r_{i}$ of each pulse is regulated automatically. After the generation of the initial random bat population, the objective function is calculated for all bats and the best bat $G_{best}$ is stored.

\[ V_{i}^{\text{new}} = V_{i}^{\text{old}} + f_{i}(G_{best} - X_{i}); i = 1, ..., N_{\text{bat}} \quad (19) \]

\[ X_{i}^{\text{new}} = X_{i}^{\text{old}} + V_{i}^{\text{new}}; i = 1, ..., N_{\text{bat}} \]

where $N_{\text{bat}}$ is the number of bats in the population; $\varphi_{i}$ is a random value in the range [0,1] and $f_{i}^{\max}$ and $f_{i}^{\min}$ are the maximum/minimum frequency values of the $i^{th}$ bat. The above formulation is an improvisation stage for shifting the bat population toward the best bat. The second movement is done using a randomly generated value $\beta$. If $\beta$ is bigger than $r_{i}$ a new solution around $X_{i}$ is generated as follows:

\[ X_{i}^{\text{new}} = X_{i}^{\text{old}} + \varphi_{i} A_{i}^{\text{old}}; i = 1, ..., N_{\text{bat}} \quad (20) \]

where $\epsilon$ is a random value in the range of [-1,1] and $A_{i}^{\text{old}}$ is the mean value of the loudness of all bats. If the random value $\beta$ is less than $r_{i}$ then a new position $X_{i}^{\text{new}}$ is generated randomly. The new position $X_{i}^{\text{new}}$ will be acceptable if it satisfy the below conditions:

\[ \beta < A_{i} \quad \& \quad f(X_{i}) < f(G_{best}) \quad (21) \]

Also, loudness and rate values will be updated as follows:

\[ A_{i}^{\text{new}} = \alpha A_{i}^{\text{old}} \quad (22) \]

\[ r_{i}^{\text{iter}+1} = r_{i}^{0} [1 - \exp(-\gamma \times \text{iter})] \]

where $\alpha$ and $\gamma$ are constant values and $\text{iter}$ is the number of the iteration during the optimization process.

5. Simulation Results

In this section, IEEE 69-bus test system is used as the case study [35]. This test system has 69 sectionalizing switches and 5 tie switches (dotted lines) with the voltage of 12.66 kV. The compelle data of the network can be found in [35]. The amounts of active and reactive loads are 3,802.19 kW and 2,694.59 kVar, respectively. The value of $K_{p}$ is assumed as 168 ($$/kW$$-$$yr$$) as determined by the literature.
At first, the results of applying just DFR on the network are shown. Table 2 shows the effect of DFR for reducing the active power losses function. The last column shows the open switches. According to this table, the optimal DFR has reduced the initial active losses from 225 kW to 99.62 kW after reconfiguration.

Table 2
Optimization of the Active Power Losses in the 69-Bus Test System using DFR strategy (Deterministic Framework)

<table>
<thead>
<tr>
<th>Method</th>
<th>Loss [kW]</th>
<th>Loss Cost [$]</th>
<th>Voltage (pu) (×10⁻²)</th>
<th>Open switches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Condition</td>
<td>225.0</td>
<td>37,800.0</td>
<td>909.21</td>
<td></td>
</tr>
<tr>
<td>Liu and Chen</td>
<td>102.6</td>
<td>17,236.8</td>
<td>932.11</td>
<td>s11-66,s13-20,s14-15,s50-51,s44-45,s11-66,s13-20,s14-15,s50-50,54,39-48,s27-66,s49-54,51,39-48</td>
</tr>
<tr>
<td>Bi, Liu [37]</td>
<td>102.1</td>
<td>17,152.8</td>
<td>932.11</td>
<td>s50-50,54,39-48</td>
</tr>
<tr>
<td>Shirmohammadi</td>
<td>106.63</td>
<td>17,914.2</td>
<td>932.11</td>
<td>s21-22,s47-48,s11-66,s13-20,s14-15,s50-50,51,47-48,s11-66,s13-20,s14-15,s50-50,51,47-48</td>
</tr>
<tr>
<td>Li et al. [38]</td>
<td>99.62</td>
<td>16,736.2</td>
<td>942.82</td>
<td>s50-50,51,47-48</td>
</tr>
<tr>
<td>NAGAI [40]</td>
<td>99.62</td>
<td>16,736.2</td>
<td>942.82</td>
<td>s50-50,51,47-48</td>
</tr>
<tr>
<td>Proposed BA</td>
<td>99.62</td>
<td>16,736.2</td>
<td>942.82</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 shows the results of optimizing the cost function using the SCP problem. Table 3 shows the optimal location and size of shunt capacities to be 300 kVar and 1200 kVar on buses 22 and 60, respectively. By comparison of the results, the superiority of the proposed method over the other well-known methods in the table can be deduced. The initial cost of the system is 37,800 $ which is reduced to 24,764.57 $ after the SCP. BA could reach the cost with minimum value in this table.

Table 3
Optimization of the Cost objective in the 69-Bus Test System using SCP strategy (Deterministic Framework)

<table>
<thead>
<tr>
<th>Items</th>
<th>Initial Status</th>
<th>Using optimal DFR</th>
<th>Using optimal SCP</th>
<th>Using DFR and SCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Losses(kW)</td>
<td>225.0</td>
<td>99.62</td>
<td>145.5689</td>
<td>88.4131</td>
</tr>
<tr>
<td>Loss reduction (%)</td>
<td>----</td>
<td>55.72</td>
<td>35.30</td>
<td>60.70</td>
</tr>
<tr>
<td>Annual cost ($/yr)</td>
<td>37,800</td>
<td>16,736.2</td>
<td>24,764.57</td>
<td>15,780.11</td>
</tr>
<tr>
<td>Net saving($/yr)</td>
<td>----</td>
<td>21,063.8</td>
<td>13,035.43</td>
<td>22,019.89</td>
</tr>
<tr>
<td>Saving (%)</td>
<td>----</td>
<td>55.72</td>
<td>34.48</td>
<td>58.25</td>
</tr>
</tbody>
</table>
Finally, the simulation results of the stochastic analysis are provided considering the uncertainty of active and reactive loads. The simulation results are shown in Table 6. For better comparison, the results of the deterministic framework are also shown comparatively. According to Table 6, considering uncertainty has resulted in incremental value in all objective functions. This is the cost that we pay to have real outputs with more dependability. Fig. 6 shows the voltage profile of the system for the three cases of optimizing the network using only DFR, only SCP and both DFR and SCP.

<table>
<thead>
<tr>
<th>Items</th>
<th>Active power losses</th>
<th>Maximum Voltage Deviation</th>
<th>Total system costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic Framework</td>
<td>88.4131</td>
<td>0.04386635</td>
<td>15,780.11</td>
</tr>
<tr>
<td>Stochastic Framework</td>
<td>90.4762</td>
<td>0.05133782</td>
<td>15,995.32</td>
</tr>
</tbody>
</table>

7. References


6. Conclusion

The focus of this paper was on the optimal operation and management of the DFR and SCP in the radial distribution system to solve them in the same framework. The simulation results on the IEEE 69-bus test system showed that the proposed BA can solve the DFR and SCP optimally. In addition, it was seen that these strategies should be solved in the same framework to reach the maximum efficiency in the targets. From the stochastic framework, it was deduced that considering uncertainty will result in incremental values in the objectives.


