Neural adaptive tracking control of a DC motor

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Abstract

This paper presents a neural network-based adaptive control strategy for speed or position tracking of a DC motor with unknown system nonlinearities. In the proposed scheme, we successfully integrate some existing techniques, such as the input–output linearization technique used to cancel the nonlinearities, and neural networks used to implement the linearizing control law. The network approximation errors are compensated by using the sliding mode control scheme. Moreover, the neural network parameters are updated according to the Lyapunov approach. It is shown that, through the proposed control scheme, the rotor speed or position of a DC motor can follow any arbitrarily selected trajectories under variable load torque. Numerical simulation results are provided to confirm the performance and effectiveness of the proposed control approach. © 1999 Published by Elsevier Science Inc. All rights reserved.

1. Introduction

Both DC and AC motors have been extensively used in control systems but each has its own characteristics. The main advantages of DC motors are easy speed or position control and wide adjustable range. Therefore, DC motors are often used in a variety of industrial applications such as robotic manip-
ulators, where a wide range of motions are required to follow a predetermined speed or position trajectory under variable load. DC motors are customarily modeled by linear systems, and then linear control strategies can be designed. However, regarding the armature reaction and windings compensation, DC motors are generally nonlinear systems. Most of the existing linear controllers generally cannot lead to good tracking and regulation responses when the controlled system has a wide range of operating conditions. Recently, there has been considerable development in nonlinear control schemes for the high performance servo systems with their industrial applications, e.g. input–output linearization and adaptive nonlinear control [1–4]. Such control methodologies are being rapidly explored for speed and position control of servo motors [5–8].

In general, the architecture designed to solve the tracking control problem for the nonlinear plants assumes that the control methods of these systems with known parameters are well developed [1,2]. In particular, when the plant nonlinearities are known, adaptive tracking control methods with the technique of exact input–output linearization are well analyzed and understood. They may be applied when the plant nonlinearities can be expressed in terms of known functions linearly scaled by unknown parameters [9,10].

The neural network area consists of a very promising direction to solve the problems relating to unknown nonlinear systems. Hence, neural networks appear as a powerful tool for learning static and highly nonlinear dynamic systems. Their massive parallelism, very fast adaptation, and inherent approximation capability, have attracted extensive researches in the field of system engineering, especially in the areas of system identification and control. In the literatures, the tracking control by using neural network architecture for complex and unknown nonlinear dynamic systems has become a topic of considerable importance [11–14].

In this paper, a neural adaptive control law incorporated with sliding mode control technique, based on approximate input–output linearization of the plant dynamics, is proposed for the tracking task of a DC motor modeled by a nonlinear dynamics with unknown system nonlinearities. By incorporating the sliding mode control, the proposed adaptive controller compensates the network approximation errors within the state region in which the networks are used. This allows one to reduce the asymptotic tracking error to a neighborhood of zero, which is determined solely by the inherent network approximation errors. The neural network parameters are updated according to the Lyapunov approach, and therefore the stability of the closed-loop system can be guaranteed by the Lyapunov method.

Throughout this paper, the notation $\| \cdot \|$ denotes the usual Euclidean norm or the corresponding induced matrix norm.
2. Preliminaries

2.1. Input–output linearization technique

Generally speaking, input–output linearization is an approach to nonlinear control design for tracking problems, by differentiating output $y$ with respect to time repeatedly until an explicit relationship between $y$ and control input $u$ appears. Then, a controller is formulated to cancel the nonlinearity and transfer the nonlinear system dynamics into a (fully or partly) linear one, so that linear control techniques can be applied. If the output function is to be differentiated $r$ times for the input $u$ to appear, the system is said to have “relative degree” $r$. It can be shown that for any controllable system of order $n$, $r \leq n$ [1].

Consider the single-input/single-output system

$$\dot{x} = f_0(x) + g_0(x)u,$$
$$y = h(x),$$

where $x \in \mathbb{R}^n$ is the state vector, and $f_0, g_0 : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $h : \mathbb{R}^n \rightarrow \mathbb{R}$ are smooth enough functions. Moreover, $f_0(0) = 0, h(0) = 0$ and state vector $x$ is assumed available. Differentiating $y$ with respect to time, one obtains:

$$\dot{\dot{y}} = \frac{\partial h}{\partial x} \dot{x} = \frac{\partial h}{\partial x} f_0(x) + \frac{\partial h}{\partial x} g_0(x)u := f_1(x) + g_1(x)u. \quad (2)$$

If $g_1(x)$ in (2) is bounded away from zero, then we can choose the linearizing feedback control law $u$ in the following form to cancel the nonlinearity in (2),

$$u = \frac{r - f_1(x)}{g_1(x)}, \quad (3)$$

system (2) becomes a linear differential equation

$$\dot{\dot{y}} = r. \quad (4)$$

Suppose the control purpose is to steer the plant output $y(t)$ tracking a reference trajectory $y_d(t)$. Then the new control input can be chosen as follows:

$$r = \dot{y}_d - \alpha (y - y_d), \quad (5)$$

where $\alpha > 0$. It will result the following tracking error equation:

$$\dot{e} + \alpha e = 0, \quad (6)$$

where $e = y - y_d$ and $\alpha$ is a gain factor. It is clear that (6) represents an exponentially stable error dynamics and $e(t)$ will exponentially converge to zero. If $e(0) = \dot{e}(0) = 0$, $e(t)$ will be equal to zero for all $t \geq 0$, i.e. perfect tracking.
2.2. Sliding mode control

Variable structure control with sliding mode control is one of the main approaches for dealing with uncertain nonlinear systems. It first defines the sliding surface in the error state space and forms a switching state feedback control using the bounds on the uncertainties. The high-speed switching forces the error to slide along the sliding surface until it converges and then the tracking is attained. For ensuring the reachability and sliding along with the switching surface of the control trajectory, a feedback control law is derived to meet the so-called sliding condition given by

$$\sigma \dot{\sigma} \leq -\rho |\sigma|,$$

where $\rho > 0$ and $\sigma$ is the designed sliding surface. The existence of a sliding mode implies that upon intersection with $\sigma = 0$, and the system trajectories remain along the switching surface for subsequent time. When in the sliding mode, the system is insensitive to uncertainties.

3. Tracking control of a DC motor

3.1. DC motor model

A separately excited DC motor dynamic model given in [6] is used for designing the tracking controller. This model is customarily considered a linear system by neglecting the armature reaction effect or by assuming that the compensating windings completely remove such an effect. Thus, introducing the armature reaction leads to a nonlinear system and hence, a nonlinear approach might become appropriate. The nonlinearity may be considered as follows [15]:

$$k_m = A + Bi_a,$$

where $A$ is the no load machine constant and $B$ is a small negative number that represents the armature current, $i_a$, effect on the machine constant. Then the state space form of the model can be rewritten as:

$$\dot{x} = f_0(x) + g_0(x)u,$$

where $x = [x_1 \ x_2 \ x_3]^T = [\delta \ \omega \ i_a]^T \in \mathbb{R}^3$ is the motor state variable vector, $\delta$ the rotor position (radians), $\omega$ the rotor speed (rad/s) and $u = V_i$ is the input. The functions $f_0$ and $g_0$ are smooth vector field in $\mathbb{R}^3$ and defined as follows:

$$f_0(x) = \begin{bmatrix} x_2 \\ k_1x_2 + k_2x_3 + k_3x_3^2 + k_4T_i \\ k_5x_2 + k_6x_2x_3 + k_7x_3 \end{bmatrix}, \quad g_0(x) = \begin{bmatrix} 0 \\ 0 \\ k_8 \end{bmatrix},$$

where $k_1 = -F/J$, $k_2 = A/J$, $k_3 = B/J$, $k_4 = -1/J$, $k_5 = -A/L$, $k_6 = -B/L$, $k_7 = -R/L$, $k_8 = -1/L$, $R$ and $L$ are the armature resistance ($\Omega$) and induc-
tance (H), respectively, $T_l$ the load torque (N/m), $v_t$ the terminal voltage (volt) and $J$ is the rotor inertia (kg/m$^2$) while $F$ is the friction.

The above model is now used to design an input–output linearization-based algorithm for rotor position and speed tracking.

3.2. Position tracking

Choose motor position to be the system output, then the state space equation of the DC motor model can be rewritten as

$$
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= k_1 x_2 + k_2 x_3 + k_3 x_3^2 + k_4 T_l, \\
\dot{x}_3 &= k_5 x_2 + k_6 x_2 x_3 + k_7 x_3 + k_8 u_\delta, \\
y_\delta &= x_1.
\end{align*}
$$

(11)

Using the input–output linearization technique, the direct relationship between the output and the input is

$$
\begin{align*}
y^{(3)}_\delta &= \ddot{x}_2 = k_1 \dot{x}_2 + k_2 \dot{x}_3 + 2 k_3 x_3 \dot{x}_3, \\
&=: f_1(x) + g_1(x)u_\delta.
\end{align*}
$$

(12)

Remark 1. For all $x = \{x \in \mathbb{R}^3 | x_3 \neq A/2B\} \in Q$ if $g_1(x)$ is bounded away from singularity, then $g^{-1}_1(x)$ exists and has bounded norm over a compact set $q \subset \mathbb{R}^3$. Specifically,

$$
|g_1(x)| \geq k > 0 \quad \forall x \in Q.
$$

(13)

If $f_1(x)$ and $g_1(x)$ are known, choose input $u_\delta$ in (12) as (3),

$$
u_\delta = g_1^{-1}(x)[-f_1(x) + r_\delta],
$$

(14)

it will serve to linearize and decouple the input–output map of system (9) such that

$$
y^{(3)}_\delta = r_\delta,
$$

(15)

where $r_\delta$ is an exogenous input.

Define a sliding surface

$$
\sigma_\delta = \ddot{e}_\delta + \alpha_1 \dot{e}_\delta + \alpha_2 e_\delta,
$$

(16)

where $\alpha_i \geq 0$ are chosen such that the following polynomials of $s$

$$
\tilde{H}(s) = \tilde{s} + \alpha_1 \dot{\tilde{s}} + \alpha_2 \tilde{s} + \alpha_3 s + \alpha_4
$$

(17)

is Hurwitz, and $e_\delta = y_\delta - y^{\delta}_d$ represents the tracking error. Let the control input $r_\delta$ be given by
where $\xi_\delta(x, t)$ is defined as
\[ \xi_\delta(x, t) = y^{(j)}_{\delta d} - x_t \dot{\sigma}_\delta - x_2 \ddot{\sigma}_\delta, \]
\(\rho\) is the switching gain, and
\[ \text{sgn}(\sigma_\delta) = \begin{cases} 
1, & \sigma_\delta > 0, \\
0, & \sigma_\delta = 0, \\
-1, & \sigma_\delta < 0, 
\end{cases} \]
where $y_{\delta d}$, $y^{(j)}_{\delta d}$, $j = 1, 2, 3$ are the desired output trajectories and their derivatives, respectively. Note that $\xi_\delta(x, t)$ is assumed to be a realizable signal. With the construction of the control laws (14) and (18), it can easily verify that the sliding condition (7) holds. If $\sigma_\delta \equiv 0$ in (16), all $\varepsilon_i$ will tend to zero as the time goes to infinity, then the remaining task is to steer the trajectories of the closed loop system toward the sliding surface $\sigma_\delta \equiv 0$ and remaining in the surface thereafter. Therefore, the dynamics of $\sigma_\delta$ can be described as
\[ \dot{\sigma}_\delta = f_1(x) + g_1(x)u_\delta - \xi_\delta(x, t), \]
which is crucial for the design of the neural parameter updating law.

### 3.3. Speed tracking

Choose the system output $y_\omega = \omega = x_2$. Following a similar procedure, we can get
\[ \dot{y}_\omega = \dot{x}_2 = f_1(x) + g_1(x)u_\omega. \]

The linearizing feedback control
\[ u_\omega = g^{-1}_1(x)[-f_1(x) + r_\omega] \]
results in a second-order system in input–output form. Therefore, one state is made unobservable by the state feedback, and the system may become internally unstable even when the output tracking error is reduced to zero.

Define the mapping
\[ z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = T(x) = \begin{bmatrix} x_2 \\ k_1 x_2 + k_2 x_3 + k_3 x_3^2 + k_4 T_1 \\ x_1 \end{bmatrix}, \]
where $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a diffeomorphism, and system (11) can be transformed into
\[ \begin{cases} 
\dot{z}_1 = z_2, \\
\dot{z}_2 = f_1(z) + g_1(z)u, \\
\dot{z}_3 = z_1, 
\end{cases} \]
where \( f_1(z) \) and \( g_1(z) \) represent the \( f_1(x) \) and \( g_1(x) \) in the \( z \)-coordinate, respectively. To obtain tracking of the desired output trajectory with internal stability, a usual minimum-phase assumption about the system, i.e., the zero-dynamics [2] must be asymptotically stable, will be needed.

**Remark 2.** When the speed tracking has been obtained, in other words \( z_1 \) converges to a desired speed \( z_{1d} \), the dynamics of Eq. (25b) will be unbounded except that \( z_{1d} \) is a sinusoidal signal. However, the motor is in the rotational motion when the speed tracking control is obtained, and then the position increases with time but it does not destroy the stability of the overall system.

Let the control \( r_\omega \) be given by
\[
    r_\omega(x, t) \equiv \xi_\omega(x, t) - \rho \text{sgn}(\sigma_\omega),
\]
where \( \xi_\omega \) is defined as
\[
    \xi_\omega(x, t) = \dot{y}_{\text{med}} - x_1 \dot{e}_\omega
\]
and the sliding surface is
\[
    \sigma_\omega = \dot{e}_\omega + x_1 e_\omega,
\]
where \( e_\omega = y_\omega - y_{\text{med}} \) represents tracking error. It is clear that \( x_1 \)'s can be selected to minimize the error in both position and speed tracking, and the time is taken to reduce the error to its minimum.

**Remark 3.** Due to use of an input–output linearization technique, the proposed method needs some limitations about the system nonlinearities: (i) the output \( y \) can be repeatedly differentiable, and the input \( u \) can appear in the differentiated term; (ii) the system of interest must be minimum phase; (iii) \( g_1(x) \) must be nonsingular for all states \( x \), and all states must be measurable; (iv) the structure of the nonlinear system can be represented in the form of (1), and the nonlinear functions \( f_0(x) \) and \( g_0(x) \) must be smooth functions; and (v) the output \( y \) must not contain high frequency noise.

4. Neural adaptive controller design

In Section 3, the control algorithm (14) for system (9) is described under the assumption that the functions \( f_1(x) \) and \( g_1(x) \) are known exactly. If \( f_1(x) \) and \( g_1(x) \) are unknown, we can use two multilayered neural networks \( \hat{f}_1(x, w) \) and \( \hat{g}_1(x, v) \), as shown in Fig. 1, to model \( f_1(x) \) and \( g_1(x) \), respectively. An assumption is made in the following.
Assumption. There exist coefficients $w$ and $v$ such that $\hat{f}_1$ and $\hat{g}_1$ approximate the continuous functions $f_1$ and $g_1$, with accuracy $\varepsilon$ over a compact subset $Q \subset \mathbb{R}^3$, that is, there exist $w$ and $v$ such that

$$\max_k \|\hat{f}_1(x,w) - f_1(x)\| \leq \varepsilon \quad \text{and} \quad \max_k \|\hat{g}_1(x,v) - g_1(x)\| \leq \varepsilon \quad \forall x \in Q. \quad (29)$$

Let $w_t$ and $v_t$ denote the estimates of $w$ and $v$ at time $t$, respectively. Suppose that $\hat{g}_1(x,v_t)$ is nonsingular in the adaptation process, then the control law $u(t)$ can be defined as follows:

$$u = \hat{g}_1^{-1}(x,v_t)[-\hat{f}_1(x,w_t) + r]. \quad (30)$$

Fig. 2 shows the closed-loop neural-based adaptive control scheme, with the control $u_\delta(T)$ defined in (30), then (21) becomes
\[
\dot{\delta} = f_1(x) + g_1(x)u_\delta - \xi_\delta(x,t)
\]
\[
= f_1(x) + [g_1(x) + \hat{g}_1(x,v_t)\dot{g}_1(x,v_t)]u_\delta - \xi_\delta(x,t)
\]
\[
= f_1(x) + [g_1(x) - \hat{g}_1(x,v_t)\dot{g}_1^{-1}(x,v_t)(-\dot{f}_1(x,v_t) + r_\delta)]
\]
\[
- \xi_\delta(x,t)
\]
\[
= [f_1(x) - \dot{f}_1(x,v_t)] + [g_1(x) - \hat{g}_1(x,v_t)]u_\delta + r_\delta - \xi_\delta(x,t)
\]
\[
= [f_1(x) - \dot{f}_1(x,v_t)] + [g_1(x) - \hat{g}_1(x,v_t)]u_\delta + \xi_\delta(x,t) - \rho \text{sgn}(\sigma_\delta)
\]
\[
= [\dot{f}_1(x,w) - \dot{f}_1(x,v_t)] + [(\hat{g}_1(x,v) - \dot{g}_1(x,v_t)]u_\delta
\]
\[
+ [(\dot{f}_1(x) - \dot{f}_1(x,w)] + (\hat{g}_1(x) - \dot{g}_1(x,v))u_\delta - \rho \text{sgn}(\sigma_\delta).
\] (31)

Denote the parameter vector of the networks associated with the \(i\)th output at time \(t\) as
\[
\Theta_t = [w_t \quad v_t]^T,
\] (32)
and the parameter error as
\[
\tilde{\Theta}(t) = \Theta_t - \Theta,
\] (33)
where \(\Theta = [w \quad v]^T\). Then (31) can be represented as
\[
\dot{\sigma}_\delta = \rho \text{sgn}(\sigma_\delta) + [-\tilde{\Theta}^TJ + \eta(t)],
\] (34)
where
\[
J = \left[\left(\frac{\partial \hat{f}_1(x,w)}{\partial w}\right)_{w} \left(\frac{\partial \hat{g}_1(x,v)}{\partial v}\right)_{v}\right]^T
\]
and
\[
\eta(t) = O\left(\|\tilde{\Theta}\|^2\right) + O(\varepsilon).
\]
The weights of the networks are updated according to the following law:

Updating law:
\[
\dot{\Theta}_t = J\sigma_\delta.
\] (35)

**Main Theorem.** Consider system (9), satisfying Remark 1 and Assumption, with the prescribed desired position trajectories \(y_{d,j}\), \(j = 1, 2, 3\), which are all bounded. Then under the control (30), the output \(y_\delta\) and their derivatives up to order 3, of the closed-loop system asymptotically track the desired position trajectories \(y_{d,j}\).
and the corresponding output derivatives while maintaining the boundedness of all signals inside the system, and the switching gain can be chosen to satisfy $\rho(t) > \|\eta(t)\|$.

**Proof.** Define the Lyapunov function candidate as

$$V(\sigma_\delta, \Theta) = \frac{1}{2}\sigma_\delta^2 + \frac{1}{2}\tilde{\Theta}_\delta^T \tilde{\Theta},$$

then we have

$$\dot{V} = \sigma_\delta \dot{\sigma}_\delta + \tilde{\Theta}_\delta^T \dot{\tilde{\Theta}}$$

$$= -\rho\|\sigma_\delta\|_1 + \sigma_\delta [-\tilde{\Theta}_\delta^T J + \eta] + \tilde{\Theta}_\delta^T \dot{\tilde{\Theta}}$$

$$= -\rho\|\sigma_\delta\|_1 + \sigma_\delta \eta$$

$$\leq -\rho\|\sigma_\delta\|_1 + \|\eta\|\|\sigma_\delta\|$$

$$= -(\rho - \|\eta\|)\|\sigma_\delta\|$$

$$\leq 0,$$

where $\|\cdot\|_1$ denotes the 1-norm. Obviously, $V$ is a positive definite and decreasing function, and (37) implies that $\sigma_\delta$, $\Theta$ are uniformly stable at the equilibrium point

$$\sigma_\delta = 0, \quad w_t = w, \quad v_t = v,$

of the adaptive system [9]. To give an estimate $\Omega$, the region of attraction of the equilibrium, we first note that $\Omega$ must be a subset of our estimate $\Psi$ of the feasibility region. Let $\Omega(c)$ be an invariant set of the dynamics defined by $V < c$ and let $c^*$ be the largest constant $c$ such that $\Omega(c) \subset \Psi$, then the estimate $\Omega$ of the attraction is given by

$$\Omega = \Omega(c) = \{V < c^*\}, \quad c^* = \arg \sup_{\Omega(c) \subset \Psi} \{c\}.$$  

Since $e^{j_1} = [sH^{-1}(s)]_j$, $j = 0, \ldots, 2$, the stability properties of stable transfer functions imply that $y_\delta(j), j = 1, 2, 3$, are uniformly bounded [16]. By a diffeomorphism transformation, the state $x$ is also uniformly bounded. Note that in the above derivation, we have used the fact that $\|\cdot\|_1 \geq \|\cdot\|$. Next, by Barb-alat’s lemma [17], if for all initial conditions $(x, v_t, w_t)_{t=0} \in \Omega$, then $\sigma_\delta \in L_\infty \cap L_2$ and $\sigma_\delta \in L_\infty$, which in turn implies $\sigma_\delta(t) \to 0$ as $t \to \infty$. This completes our proof. □

For speed tracking, by following a similar procedure, we can get

$$\dot{\sigma}_\omega = \rho \text{sgn}(\sigma_\omega) + [-\tilde{\Theta}_\delta^T J + \eta(t)].$$

And updating law

$$\tilde{\Theta}_t = J \sigma_\omega.$$
Corollary. Consider system (9), satisfying Remark 2 and assumption, with the prescribed desired position trajectories $y_{\text{d}j}^{(i)}$, $j = 1, 2, 3$, which are all bounded. Then under the control (30) of the closed-loop system, the output $y_o$ and their derivatives up to order 2, of the closed-loop system asymptotically track the desired speed trajectories $y_{\text{d}j}$. The corresponding output derivatives while maintaining the boundedness of all signals inside the system except the position signal, and the switching gain can be chosen to satisfy $\rho(t) > \|\eta(t)\|$.

5. Numerical simulations

In this section, we present some simulations to evaluate the effectiveness of the proposed tracking controller for position and speed tracking of a separately excited DC motor model. Let us consider a 110 V, 1800 rpm, 2.5 Hp motor with machine parameters: $R = 1$ $\Omega$, $L = 0.046$ H, $F = 0.008$, $J = 0.093$ kgm$^2$, $A = 0.57$, $B = -0.01$, and load torque $T_l = a_1\omega^2$, where $a_1 = 0.00028$. A three-input two-output three-layered neural network with 18 sigmoid function-based nodes in the hidden layer is constructed to implement the control law. Two tracking strategies have been studied.

Case 1: Position tracking. Consider a desired position trajectory as follows:

$$\delta_d = 100 \cos t + \cos 2t.$$  \hspace{1cm} (40)

Fig. 3 shows the position tracking trajectories, at 1 s, the motor actual trajectory met the desired trajectory.

Case 2: Speed tracking. In this case, the desired speed track is chosen independently and does not necessarily guarantee rotor position tracking. A desired speed track was chosen as follows:

$$\omega_d = 100 + 12 \cos t + 5 \cos(2t + 20).$$  \hspace{1cm} (41)

Fig. 4 shows that after 2 s of the initial speed error, motor speed trajectory follows the desired trajectory very well.

6. Conclusion

A neural adaptive tracking control strategy of DC motors with unknown nonlinearities was developed. With the control scheme, the speed and position of the rotor shaft are forced to follow any arbitrarily selected trajectory under variable load torque. The simplicity and generality of the proposed controller makes it suitable for many servo system applications employing any type of motor. Simulation results are provided to confirm the effectiveness of the proposed controller, thus making it suitable for high performance DC motor tracking applications.
Fig. 3. The position tracking trajectories.

Fig. 4. The speed tracking trajectories.
References