Consumption and Wealth Inequality with External Habit Formation. *

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Abstract

In the last two decades the U.S. economy has experienced an upsurge in its levels of income and wealth inequality which has not been accompanied by a corresponding rise in consumption inequality. This paper attends to answer the following question: Can a external habit formation help to understand those differences in trends in the evolution of consumption and wealth inequality? We study the evolution of wealth and consumption distribution along the transition path of a one sector growth model. Households differ in their initial holdings of wealth and display a catching up with the Joneses behavior when taking their consumption decisions. Theoretically it is shown that the evolution of consumption inequality is driven by the evolution of wealth inequality but with less variation. Furthermore, an aggregate shock affects the distribution of income, wealth and consumption along the transition since it hits more severe to poor individuals' savings rates than that of rich ones. On the quantitative part of this work, the model is calibrated to match some key statistics of the U.S. economy. We observe that the level of wealth and consumption inequality and their evolution resembles that of the U.S. economy. TO BE FINISHED

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1 Introduction

The sharp increase in income and wealth inequality for the U.S. economy in the last 30 years is a well-documented fact. Budria et all (2001) and Wolff (1994, 1998) between others have found that the dispersion of U.S. household income and wealth have a strong upward trend. Table 1 presents some US wealth distribution data[1] from several authors,

Table 1. Wealth Distribution in the US

	Wealth	Percentage wealth in top				Top 1/Btm 40
Year	Gini	1%	5%	20%	40%	Ratio
1979	-	20.05	-	-	-	-
1983	0.72	28	49	75	89	-
1992	0.78	29.55	53.5	79.49	92.92	875
1998	0.803	34.70	57.8	81.7	93.9	1,335

Source: data for 1979 from Lindert (2000), for 1983 from Wolff (1994) and rest from Budria et all (2001).

On the other hand, the raising income and wealth inequality has not been accompanied by a corresponding rise in consumption inequality. Using data from the Consumer Expenditure Survey and the Current Population Survey, Krueger and Perri (2002) document that there exist a surprising lack of increase in consumption inequality, finding that is robust to different definitions of consumption and different measures of inequality. Table 2 summarized evolution of consumption inequality (using three different definitions of consumption).

Table 2. Consumption Inequality in the US

	%Δ Std. Dev. Cons			% Share Btm.	90/10 ratio	Gini
Period	ND+	ND	TE	Quint. ND+	ND+	ND+
72-73	0	0	0	9.2	3.09	0.2460
80-81	NA	-4.6	0.4	9.57	3.15	-
85-86	1.7	-2.6	7.9	9.45	3.13	0.268
90-91	0.6	-4.5	5.6	9.67	3.14	0.264
97-98	1.5	-4.0	7.4	9.24	3.35	0.272

Source: Krueger & Perri (2002)

[1] Wealth is define as the difference in value between total assets and total liabilities and debt.

ND+ includes nondurable consumption expenditures (ND) plus expenditure in household equipment plus imputed services from houses and cars, and TE is total consumption expenditures[2]. Both measures of inequality (Gini coefficient and percentage increase in standard deviation of consumption - using period 72-73 to compare) confirm that consumption inequality has been quite stable, with inequality in nondurable consumption expenditures actually decreasing and inequality in total consumption expenditures modestly increasing. The share of ND+ consumption expenditures of the poorest 20% of the population has also remained stable (see also Slesnick 2001). The 90/10 ratio is the ratio between the ND+ consumption of the household at the 90% percentile and the ND+ consumption of the household at the 10% percentile of the distribution. Again, this ratio reveals a similar pattern, displaying a small increase in consumption inequality. Looking to Gini Coefficients, we also confirm the modest increase in consumption inequality (1.49 percen in period 1985-1998).

Taking those facts into account, the paper attends to answer the following question: Can a external habit formation help to understand those differences in trends in the evolution of consumption and wealth inequality?

We consider a neoclassical one sector growth model in which households differ in the initial holdings of capital or wealth. When taking their decisions on consumption and savings, households care about their consumption related to an existing stock of reference. The reference stock is based in past average consumption in society. In the literature, this feature is known as external habit formation "a la" catching up with the Joneses[3].

Catching up with the Joneses feature can generate wealth induced differences in savings rates. As an external habit formation in consumption, it can also help to explain the excess of smoothness that the demand of consumption exhibits as in fact consumption does not respond immediately to news (Cambel & Deaton 1989, Fuhrer 2000). This two properties can be key to understand the evolution of wealth and consumption inequality. Recently, Maurer and Meier (2003), using US panel data from the PSID, conclude that there are correlated effects that accounts for the existence of external habit formation

[2]The definition of non durable consumption expenditures (ND) is the one used by Attanasio and Davis (1996). See Krueger and Perri (2002) for a more precise description of the three definitions of consumption.

[3]This feature is also known as "outward looking comparison utility" (as in Carroll et all, 1997), "interdependent preferences" or the "relative income" hypothesis.

In the model, the existence of habit formation introduces a interdependence between intertemporal elasticity of substitution (IES) and household wealth. Therefore, in the transition path towards the steady state IES differs between individuals (with different levels of wealth) and in time. These intra and intertemporal differences generates disparities in savings rates between households, which implies changes in wealth and consumption distribution along the economy transition path. Furthermore, expressing relative consumption between individuals as a function of relative wealth between individuals, it is shown that the evolution of consumption inequality follows that of wealth but with less variation than the evolution of inequality in wealth.

Another feature that the model display is that the evolution of saving rates is affected by the economic growth. Hence, a slowdown in Total Factor Productivity (TFP) can affect the distribution of wealth and consumption along the transition. This second "source" of generating inequality appears also because of the existence of the habit: the decline of income reduces poor households' (in terms of wealth) saving rates in a more severe way than that of the rich households. As a result, inequality in wealth increases but distribution of consumption is less affected.

On the quantitative side of this work, the model is calibrated to match some key statistics of the US. economy. The behavior that the model exhibits generates an evolution of wealth and consumption inequality similar to the one observe in the data. TO BE WRITTEN.

This paper shares part of the aim with Krueger & Perri (2002). They provide the empirical findings on the evolution of the distribution of consumption (some commented in this introduction) and argue that the different trends of income and consumption inequality are consistent with the hypothesis that an increase in income volatility has been an important cause of the increase in income inequality, but at the same time has lead to an endogenous development of credit markets, allowing households to better smooth their consumption against idiosyncratic income fluctuations. They develop a consumption model in which the sharing of income risk is limited by imperfect enforcement of credit contracts and in which the development of financial markets depends on the volatility of the individual income process. The present paper emphasizes on the different trends that the evolution of wealth and consumption inequality display. Opposite to Krueger and Perri (2002), we focus on the social environment and the household's comparison utility based on an external habit as the key hypothesis and this idea is developed in a growth model.

Regarding the habit feature, there is a bast literature that analyzes the implications of its existence on different contest. Carroll, Overland and Weil (1997) examines the dynamic of an AK growth model where the representative individual have comparison utility (i.e. both internal and external habit) focusing on the respond of saving and growth to a negative shock to capital. In the asset pricing field the habit formation feature has been used to separates the risk aversion from the intertemporal elasticity of substitution, solving in this way the equity premium puzzle (see Abel (1990), Constantinides (1990), Boldrin & Fisher (1997) or Campbell & Cochrane (1999)). In this paper habit formation is brought to the cool again for the implications that this feature has on the evolution of wealth and consumption inequality. Diaz, Mas Pigou and Rios-Rull (2002) studies the role of habit formation in shaping the amount of precautionary savings and the wealth distribution in a model with idiosyncratic uncertainty, focusing in the steady state of the economy.

Since the external habit formation can be interpreted as a way of endogenizing a minimum consumption requirement, this paper is also close to Alvarez-Peláez and Díaz (2003), where they use that requirement as the key feature to investigates quantitatively how initial wealth holding differences across households are propagated through time in a one sector growth model economy.

The rest of the paper is organized as follows: Section 2 describes the economic environment. Section 3 shows some theoretical results and establish the connection between level of per capita income and wealth inequality. In Section 4 we present the calibrated version of the model and the results of the simulations. Finally, Section 5 concludes.

2 The model

The model is a discrete time infinite horizon economy, populated by a measure one of individuals that live forever. Each period, individuals obtain utility from consuming a commodity that is produced using physical capital and labor. The representative firm uses a Cobb-Douglas technology to produce the consumption good,

$$Y_t = A_t K_t^{\theta} N_t^{1-\theta}, \quad \theta \in (0,1), \tag{2.1}$$

where K_t denotes aggregate capital, and N_t aggregate labor. A is the exogenous technological progress factor that grows at a rate η. Capital depreciates at a constant rate, $\delta \in (0,1)$.

There exits perfect capital markets, i.e., individuals are able to borrow and lend without any restriction at the market interest rate.

Output is used for consumption, C_t, and investment, I_t

$$C_t + I_t = Y_t$$

The aggregate stock of capital, K, changes according to

$$I_t = K_{t+1} + \delta K_t, \qquad \delta \geq 0,\ K_0 > 0 \text{ given.} \tag{2.2}$$

Each period individuals are endowed with one unit of labor. They do not value leisure. Individuals differ only in their initial holdings of capital or wealth, k_0^j, where j is just an index to order types of individuals according to their level of initial wealth. There are J types of individuals. We assume all types have the same measure.

At each point in time, individual j's utility depends on the consumption of the homogeneous good, c_t^j, compared with a pondered stock of reference or habit in consumption, α_t. Individual j's lifetime utility is given by

$$U(c_t^j, \alpha_t) = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^j - \gamma \alpha_t)^{1-\sigma}}{1-\sigma}, \quad \sigma > 1, \gamma \in [0, 1). \tag{2.3}$$

where the reference stock of habit displays an evolution over time specified as

$$\alpha_{t+1} = \rho c_t + (1-\rho)\alpha_t. \tag{2.4}$$

The expression above considers that the habit is external and based on the previous period average level of consumption in the economy, c_t, that is, the catching up with the Joneses feature. The parameter ρ determines the persistence of the stock of habit: the larger is ρ, the more important is average consumption in the recent past and the smaller is the importance of the old values of the stock of habit.

The stock of reference is pondered by parameter γ which indexes the importance of the comparison of consumption and the stock of reference in the instantaneous utility function. $\gamma = 0$ implies that the habit in consumption is not considered, and only the absolute level of consumption is important. When γ is considered in the interval $[0, 1)$, the higher γ, the greater the importance of consumption compared with the stock of reference.

The existence of an habit introduces a interdependence between intertemporal elasticity of substitution (IES) and the level of individual consumption. It also makes the intertemporal elasticity of substitution (IES) to change between individuals and in time. By partially differentiating the logarithm of the intertemporal marginal rate of substitution between periods t and $t+1$ with respect to the logarithm of (c_{t+1}^j / c_t^j) and taking its inverse we get the expression for the IES to be

$$IES_t^j = \frac{1}{\sigma} \frac{c_{t+1}^j - \gamma \alpha_{t+1}}{c_{t+1}^j}.$$

Notice that parameter γ is part of the expression for IES_t^j; the higher γ the lower the IES is and the smother the consumption path for individual j.

In this economy, a feasible path $(C, K, Y, \alpha)_{t=0}^{\infty}$ is called a steady state if C, K, Y and α are strictly positive and grow at a constant (though not necessarily equal or positive) rates. Let y, k, and c be per capita output, capital and consumption respectively. Let the rate of growth of a per capita variable x be denoted $g_x \equiv \frac{x_{t+1}}{x_t} - 1$.

Lemma 1 *In steady state $g_y = g_k = g_c = (1+\eta)^{\frac{1}{1-\theta}} - 1$. Further, c/α is constant, so $g_c = g_\alpha$.*

Proof. Consider a steady state. By definition, $y > 0$. From (2.1) $g_y = (1+\eta)(1+g_k)^\theta - 1$, which gives $g_y = g_k = (1+\eta)^{\frac{1}{1-\theta}} - 1$. By (2.2) $(1+g_k) = \frac{y_t}{k_t} - \frac{c_t}{k_t} - \delta$ so $\frac{c_t}{k_t}$ is constant and, hence, $g_c = g_k$. Further, since g_α is constant, from (2.4) we have that c/α is constant. Therefore $g_c = g_\alpha$. ■

2.1 The firm's problem

The firm faces a series of static one-period profit maximization problems

$$\max_{K_t, N_t} A_t K_t^\theta N_t^{1-\theta} - w_t N_t - r_t K_t$$

Solving this problem we get that the wage rate, w_t, and the real rental price of capital, r_t, are equal to the marginal productivity of the production factors in equilibrium.

2.2 The individual j's problem

In this section we specify the modified model economy we are going to study throughout this paper. This economy exhibits a balanced growth path, along which the rate of growth of the per capita variables is g. We detrain all the variables to eliminate the long run growth. Once done this, the individual i's problem can be written as

$$\begin{array}{ll} \max & \sum_{s=t}^{\infty} \phi^{s-t} \frac{(c_s^j - \gamma\alpha_s)^{1-\sigma}}{1-\sigma} \\ \text{s.t.} & c_s^j + (1+g)k_{s+1}^j \leq w_s + (1 + r_s - \delta)k_s^j \\ & (1+g)\alpha_{s+1} = \rho c_s + (1-\rho)\alpha_s \\ & \alpha_t, k_t^j > 0 \text{ given.} \end{array} \tag{2.5}$$

where $\phi = \beta(1+g)^{1-\sigma}$ due to the technological progress. Notice that variables in (2.5) are not the same that those considered before in expression (2.3). If we for example renamed consumption in expression (2.3) as $\tilde{c}_t^j$ then the relation is $\tilde{c}_t^j = c_t^j \cdot (1+g)^t$.

Solving the individual problem above using the lifetime budget constraint

$$\sum_{s=t}^{\infty} p_s c_s^j \leq \sum_{s=t}^{\infty} p_s w_s + (1 + r_t - \delta)\, k_t^j,$$

we get the following expression for the demand function

$$c_t^j = \gamma\alpha_t + \frac{1}{M_t}\left[\sum_{s=t}^{\infty} \frac{p_s}{p_t}(w_s - \gamma\alpha_s) + (1 + r_t - \delta)\, k_t^j\right] \tag{2.6}$$

where

$$M_t = \sum_{s=t}^{\infty} \phi^{\frac{s-t}{\sigma}} \left(\frac{p_s}{p_t}\right)^{\frac{\sigma-1}{\sigma}},$$

and p_t is the price of consumption good in period t in terms of consumption good in period 0.

This expression for the demand function (expression 2.6) tell us that the amount consumed above the pondered stock of habit each period is the fraction $\frac{1}{M_t}$ of life-time wealth net of future needs of consumption.

This demand function is an affine function of the individual asset holdings. The linearity of the Engels' curves ensure that the aggregate stock of capital next period does not depend on the distribution of wealth. Therefore, in this economy growth affects the evolution of wealth inequality, but inequality does not affect growth, as in Alvarez-Peláez and Díaz (2003). Thus, the dynamics of the aggregate variables is identical to the dynamics of the representative agent version of this economy. This result is due to the specific function used and the assumption that individuals do not obtain utility from leisure.

3 Transition path and the distribution of wealth and consumption

To study the evolution of the distribution of wealth, and therefore, how wealth inequality changes, let us obtain the law of motion of individual j's wealth. Substituting the demand function into the individual budget constraint (2.5) we get

$$\begin{aligned} k_{t+1}^j &= B_t + D_t k_t^j \qquad \text{where} \qquad (3.1)\\ B_t &= \frac{1}{1+g}\left[w_t - \gamma\alpha_t + \frac{1}{M_t}\sum_{s=t}^{\infty}\frac{p_s}{p_t}(w_s-\gamma\alpha_s)\right]\\ D_t &= \frac{1+r_t-\delta}{1+g}(1-\frac{1}{M_t}). \end{aligned}$$

Since $\frac{1}{M_t}$ is the consumption fraction over life-time wealth, factor D_t is the fraction of current wealth after interest rate has been paid that is saved at period t. Regarding B_t, this factor is the amount of current labor income above the pondered stock of habit, $\gamma\alpha_t$, that is saved at period t.

Using these expressions, we can write individual j's saving rate as

$$s_t^j = \frac{k_{t+1}^j - (1-\delta)k_t^j}{y_t^j} = \left[\frac{B_t}{k_t^j} + D_t - (1-\delta)\right]\frac{k_t^j}{w_t + r_t k_t^j}$$

Observe that $\frac{B_t}{k_t^j}$ is the element that allows changes in the savings rate for different levels of wealth. Even more, the differences in savings rates are implied by the sign of factor B_t. Recall that $\frac{1}{M_t}\sum_{s=t}^{\infty}\frac{p_s}{p_t}(w_s-\gamma\alpha_s)$ is the fraction of present value of labor earnings above the pondered stock of habit that finances consumption over the habit, $c_t^j - \gamma\alpha_t$, at period t. Hence, if $B_t < 0$, households cannot finance current consumption only with current labor earnings and they need to either use capital earnings, to deplete their stock of capital or to borrow.

To study the implications of factor B_t on the evolution of wealth inequality, let us examine the relation of the savings rate with the level of wealth:

$$\frac{\partial s_t^i}{\partial k_t^i} = \frac{1}{y_t^i}\left[(D_t - 1 + \delta)w_t - B_t r_t\right]$$

Notice that if B_t is negative or $|B_t r_t| < (D_t - 1 + \delta)w_t$ the savings rates are higher the higher the level of wealth since $\frac{\partial s_t^j}{\partial k_t^j} > 0$.[4] If $|B_t r_t| > (D_t - 1 + \delta)w_t B_t$ or $B_t > 0$ the opposite happens, since

[4] Along the transition path that starts with a level of capital smaller than the steady state level of capital,

poorer individuals in society are the one who save a larger fraction of their income, compared to the saving rate of the wealthier individuals ($\frac{\partial s_t^j}{\partial k_t^j} < 0$). Consequently, factor B_t is going to govern the evolution of savings rate, and therefore, wealth and consumption inequality.

Savings rates differences between individuals (with different levels of wealth) and in time is due to the existence of habit formation. As we commented before, the habit makes the intertemporal elasticity of substitution (IES) to change between individuals and in time.

Notice that the dependence of consumption for individual j on factor B_t is opposite to the one the savings rates have, which it seems natural. If we express consumption for individual j as function of factors B_t and D_t we get

$$c_t^j = w_t - B_t + (1 + r_t - \delta - D_t)k_t^j,$$

which allow us to get consumption relative to the individual level of wealth as

$$\frac{c_t^j}{k_t^j} = \frac{w_t - B_t}{k_t^j} + (1 + r_t - \delta - D_t).$$

As we can see, the evolution of this ratio also depends on the sign of factor B_t.

To see the law of motion of relative wealth of individual j respect to the average, the following ratio is defined

$$KX_t^j \equiv \frac{k_t^j}{k_t} = \frac{B_t}{k_t} + D_t\frac{k_{t-1}^j}{k_t}$$

Notice that the evolution of the ratio KX_t^j respect to the average is given by

$$KX_{t+1}^j - 1 = \frac{D_t k_t}{B_t + D_t k_t}(KX_t^j - 1) \tag{3.2}$$

The expression shows that the share KX_{t+1}^j gets closer to (further away from) the average when the factor $\frac{D_t k_t}{B_t + D_t k_t}$ is smaller (greater) then one, which confirms the importance of factor B_t in the study of wealth inequality.

In the same spirit, the ratio of individual $j's$ consumption to aggregate consumption is defined as

$$CX_t^j \equiv \frac{c_t^j}{c_t} = \frac{w_t - B_t + (1 + r_t - \delta - D_t)k_t^j}{w_t - B_t + (1 + r_t - \delta - D_t)k_t}$$

$M_t > 1$ which implies $0 < \frac{1}{M_t} < 1$. This, together the fact that $1 + r_t - \delta > 0$, implies that $D_t - 1 + \delta = \frac{1+r_t-\delta}{1+g}(1 - \frac{1}{M_t}) - 1 + \delta$ is positive.

The evolution of the ratio CX_t^j respect to the average can be decomposed into four components, as the following expression shows

$$\frac{CX_{t+1}^j - 1}{CX_t^j - 1} = \frac{1 + r_{t+1} - \delta - D_{t+1}}{1 + r_t - \delta - D_t} \frac{k_{t+1}}{k_t} \frac{c_t}{c_{t+1}} \frac{KX_{t+1}^j - 1}{KX_t^j - 1} \tag{3.3}$$

The last expression allow us to see the evolution of inequality of consumption as a function of the evolution of inequality of wealth. Notice that the evolution of inequality in consumption will follow the evolution of wealth inequality softened by the evolution of factor D_t and the growth rate of both aggregate capital and consumption in the transition. Since M_t is always positive and decreasing in time towards its steady state value 1, factor D_t is also decreasing in its transition towards the steady state[5]. This implies that the first fraction in the above expression is smaller than one. In other words, the evolution of consumption inequality follows that of wealth but with less variation than the evolution of inequality in wealth. Notice as well that inequality in consumption will be constant in the steady state since wealth inequality and factor D_t will be constant also and the aggregate variables capital and consumption will grow at the same rate.

The latest analysis allow us to study only of the evolution of wealth inequality since the evolution in consumption can be derived from that and expression (3.3).

3.1 Distribution of wealth and consumption inequality

Let us study the distribution of wealth and consumption inequality by looking at the evolution of the ratio KX_t^j respect to the average, that is given by

$$KX_{t+1}^j - 1 = \frac{D_t k_t}{B_t + D_t k_t}(KX_t^j - 1) \tag{3.4}$$

As we commented before, the expression above shows that the share KX_{t+1}^j gets closer to (further away from) the average when the factor $\frac{D_t k_t}{B_t + D_t k_t}$ is smaller (greater) then one.

We now introduce the notion of inequality. We give the definition of Lorenz-dominance in terms of our notation.

Definition 1 *Let all the agents be ordered according to their initial level of wealth and let J be the number of types of individuals according to their level of wealth. $\frac{1}{J}KX_t^j$ is the share of*

[5] The factor $p_s/p_t\phi^{s-t}$ is always less than one along a transition path that starts with a level of capital smaller that the steady state level of capital.

wealth held by group j. Then, the distribution of wealth at period $t+1$ is more egalitarian that the distribution at period t if and only if it is satisfied that for $1 \leq L \leq J$

$$\sum_{j=1}^{L} \frac{1}{J} KX_{t+1}^{j} \geq \sum_{j=1}^{L} \frac{1}{J} KX_{t}^{j} \tag{3.5}$$

The following Proposition relates the level of inequality, measured using the concept of Lorenz-dominace, with the aggregate dynamics of our model.

Proposition 1 *The distribution of capital at period $t+1$ is more egalitarian than the distribution of wealth at period t if and only if factor B_t is non negative.*

Proof. *See expression (3.4) and Definition 1 above.* ■

Using that ratio of individual j́s wealth to aggregate capital, we can write the Gini Coefficient as

$$KG_t = \frac{\sum_{j=1}^{J} (\sum_{l=1}^{j} \frac{1}{J}(1 - KX_t^l)}{\sum_{j=1}^{J} \sum_{l=1}^{j} \frac{1}{J}}$$

Substituting by the value of KX^j at any period t we can get

$$KG_t = \prod_{r=0}^{t} \frac{D_r k_r}{B_r + D_r k_r} KG_0$$

Therefore, the Gini Coefficient at any period t depends on the initial distribution and the evolution of the aggregate variables.

Our claim considers that if B_t is negative, then savings rate will increase with the level of wealth. In that case, wealth inequality will increase. We can also see how savings rates react to a reduction in income when B_t is negative, since then $\frac{\partial s^j}{\partial y^j} > 0$, i.e., savings rates decreases when income decrease, and the effect is stronger the lower the level of wealth is, since $\frac{\partial s^j}{\partial y^j \partial k^j} < 0$. Therefore, if $B_t < 0$ the model predicts that after an aggregate negative shock, the fall in the savings rate is more severe among households in the low wealth ranges than among wealthier households.

Bosworth, Burtless and Sabelhaus (1991) found that the decline in the savings rate during the 1980's was actually smaller among bonds and stockholders than it was among households with no marketable financial assets. Wolff (1998) reports that more than 43 percent of the wealth of the richest 20 percent of the households takes the form of investment assets. In contrast, almost two thirds of the wealth of the bottom 80 percent of the households was invested in their own home. This evidence leads us to think that the decline in the savings rate of wealthy households was less severe than that of the poor households, as our model will predict if $B_t < 0$.

4 Calibration issues

In this section we discuss the calibration of the model economy. We focus our study on the period 1983-1998. We are going to assume that the U.S. economy is on its balanced growth path throughout the period 1998-2002. To calibrate the representative agent version of our model economy we use data from the *National Income and Product Account* (NIPA) and *Fixed Reproducible Wealth* published by the Bureau of Economic Analysis.

The first step in this calibration procedure is to construct measures of output, Y, investment, I, and aggregate capital, K, according to the model and its objectives. Since the focus of the paper is on the distribution of privately owned wealth and consumption, we have decided not to include neither government owned capital (as part of the aggregate capital) nor government expenses as part of consumption.[6] Therefore, we define output as measured GDP minus government expenditures plus the estimated value of the flow of services of consumer durable goods (scd). The stock of capital is the sum of private fixed assets, the stock of inventories and the stock of consumer durables. And aggregate investment is the sum of investment in private assets plus the change in inventories, net exports (since our economy is a close economy) and durables expenditures.

We proceed as in Cooley and Prescott (1995) to find the capital share of output. Income from capital is related to the stock of capital by

$$Y_{Kp} = (i_{Kp} + \delta_{Kp})K_p$$

[6] Since our model does not consider government sector, the corresponding government expenses should appear as part of consumption, which it would disturb our study on distribution.

from where we find the return to private capital, considering that income of private fixed capital, Y_{Kp}, is the sum of the rents of residential (r_{rKp}) and non residential (r_{nrKp}) private stocks of capital. Diaz and Luengo-Prado consider the same expression and calculate that the average estimated value of the return to capital for the period 1954-1999 is 8.42%. We use this estimated return to calculate services from the stock of consumer durables. Therefore,

$$share\ of\ capital = \frac{r_{rKp} + r_{nrKp} + scd}{Y}$$

The estimated value for the period 1987-2001 is 42.6%.

The average capital-output ratio for the period 1987-1997 is 4.08 and the investment-capital ratio is 0.0662. Furthermore, we set the initial level of Total Factor Productivity A_0 equal to 1.

In order to choose η, the Total Factor Productivity growth rate, we consider the desire length of the transitional dynamics, since we want the model to display 15 years of transition. The choice of the growth rate η has to be consistent with the accumulated growth in per capita income observed in the period covered in this paper. According to NIPA, real GDP is 1.4 times higher in 1998 than in 1983. We assume as in King and Rebelo (1993) that Total Factor Productivity growth explains one half of total growth of the period. Therefore, we set the annual growth rate of TFP so that

$$(1+\eta)^{15} = 1.4^{0.5}$$

which implies $\eta = 0.0113$ and a balance growth rate g equal to 1.97 percent.

To find the initial level of capital, we consider that as growth in capital is affected by TFP growth, it must be true that

$$(1+g)^{15\theta} \frac{K_{1998}^{\theta}}{K_{1983}^{\theta}} = 1.4^{0.5} \tag{4.1}$$

Therefore, the value of the initial stock of capital is 94.2% of the level of capital in the steady state.

Regarding the initial level of the stock of habit, we go back to 1950 and set $\alpha_{1950} = 0$. Using the law of motion of the stock of habit given by expression (2.4) and the historic private consumption series back to 1950 we calculate the corresponding real value of the stock of habit for our initial year 1983. We consider different initial values of the stock of habit depending on the value of parameter ρ used to calculate the series. The following table resume those values together with the corresponding initial values of the ratios stock of habit and consumption (α_{1983}/c_{1983}) and the stock of habit and output (α_{1983}/y_{1983})

Table 3. Stock of habit and ρ

	α_0/c_0	α_0/y_0
$\rho = 0.1$	0.76	0.59
$\rho = 0.2$	0.88	0.68
$\rho = 0.3$	0.91	0.71
$\rho = 0.5$	0.94	0.73

Regarding preferences parameters, we have left σ, and γ. Since σ is the main parameter governing the length of the transition (parameter γ also affects, but in much less proportion), we calibrate σ to match that length. Parameter γ is a very difficult parameter to calibrate since is part of a very concrete specification of the utility function. Therefore, we leave this parameter "free" and we analyze have the results change with modification of the value of this parameter.

Respect to the initial wealth distribution, we choose the distribution across deciles corresponding to 1983, as report in Table 1.

5 Quantitative implications of the model

Let us analyze the quantitative predictions of the model about the level of wealth and consumption inequality along the transition path. The linearity of the Engel curves will allow us to study this economy with heterogeneous individuals in two steps: we will analyze the evolution of prices and aggregate variables in the representative agent version of this model and we will turn afterwards to the full model to study the evolution of the wealth and consumption distribution.

5.1 Dynamic of the aggregate variables

As an initial point, we look for an adequate value of parameter σ. We summarize the values of the parameters in the following tables:

Table 4. Technology parameters

A	θ	δ	g
1.0000	0.4260	0.0465	0.0197

Table 5. Preferences parameters

α_0/c_0	β	γ	ρ
0.880	0.967	0.200	0.050

We generate the dynamics of the aggregate variables for three different values of parameter σ: $\sigma = 1.2$, 1.8, 2.2. Figures 1a, 1b and 1c show the growth rate of output, aggregate consumption and stock of capital both as a percentage of its respective steady state values and the evolution of the ratio capital- stock of habit for the three different values of σ respectively. In all of the figures we can see that the evolution of the aggregate variables is very similar, besides the length of the transition. For $\sigma = 1.2$ the stock of capital reaches the 99 percent of its steady state value after 21 periods, while for $\sigma = 1.8$ and $\sigma = 2.2$ it takes 32 and 35 periods respectively. The evolution of the rate of growth and aggregate consumption do not have different evolution for the different values of σ and in the case of the ratio stock of capital to stock of habit the shape of the transition is the same although the steady state value of this ratio is not the same in the three cases. We should remember that this ratio is the one governing the transition of the model economy.

A similar conclusion we can obtain from observing Figures 2a, 2b and 2c: The evolution of gross aggregate savings rate and real interest rate do not differ for the three values of parameter σ considered. The interest rate is a bit too high compare to the one of the US economy, but we have to consider that the interest rate of our model economy has no counterpart in the US economy since we are assuming that aggregate capital includes consumer durables.

Regarding the evolution of factor B, for all the different values of σ that factor has the same behavior: It takes negative values during the whole transition and it converges to zero as its steady state value. This behavior suggest that our election on the parameter σ will have no further implications for the evolution of wealth and consumption inequality since inequality will increase in the transition for the three values of σ.

Since the dynamics generated by the value of parameter $\sigma = 1.2$ is the one that replicate better the length of the transition, we will consider this as the benchmark.

5.2 Sensitivity analysis for parameters ρ and γ

Let us analyze how the evolution of the aggregate variables change as a respond to changes in the persistence of the stock of habit, ρ, or the importance of the comparison of consumption and the stock of habit in the instantaneous utility function, γ.

Parameter ρ. We consider two additional possible values for the persistence of the stock of habit. Table 6 shows the results for some aggregate variables for $\rho = 0.5$ (the value considered in the previous section) 0.2 (low persistence) and 0.7 (high persistence):

Table 6. Agg. variables for different values of ρ

ρ	α_0/c_0	α_{ss}/c_{ss}	$0.99 * Ks$	s_0	B_0
0.2	0.88	0.9982	22 periods	0.2871	-0.0899
0.5	0.94	0.9986	21 periods	0.2868	-0.0921
0.7	0.95	0.9987	21 periods	0.2868	-0.0920

The transition of the aggregate variables does not change substantially. The persistence of the stock of habit has very little influence on the ratio stock of habit-consumption and that little influence is reflected in small changes in the length of the transition. Mainly, it affects to the initial condition $(\alpha_{0/}c_0)$ and that is the reason for the small changes of factor B.

Parameter γ. Due to the high initial calibrated values for the ratio stock of habit-consumption, the comparison parameter can not have very high values in the interval $[0, 1)$. Therefore, in Table 7 we present the values of the aggregate variables for two additional values: $\gamma = 0.3$ and $\gamma = 0.4$

Table 7. Agg. variables for different γ

γ	α_{ss}/c_{ss}	$0.99 * Ks$	s_0	B_0
0.2	0.999	21 periods	0.2868	-0.0921
0.3	0.999	21 periods	0.2860	-0.0763
0.4	0.999	22 periods	0.2853	-0.0603

In the same line that the changes in ρ, variations in γ do not have significant influence in the transition of the aggregate variables. It has a minor effect on the length of the transition but it does not alter much the initial savings rate or the steady state value of the ratio stock of habit to aggregate consumption. In contrast, factor B is affected by changes in γ: as expected, an increase in the "comparison" parameter reduces the initial value of factor B. This suggest that we should consider different values of γ when study the evolution of wealth and consumption inequality where the evolution of factor B is determinant.

5.3 The evolution of inequality

In this subsection we analyze the size and trend of the variation in inequality generated in the transition path. We confront the result to the wealth and consumption distribution data available and presented in Table 1 and 2. To do this, we consider the same stratification that the data present(top 1, 5, 20 and 40 percent richest) and we fix as initial distribution the data for the distribution of wealth in 1983. The evolution of consumption, investment and capital across individuals are obtained using expressions (2.6), (3.1) and the constrain for the individual problem (2.5).

As we suggest in the previous section, we first present the distribution generated using the benchmark model (with the parameter values in Section 5.1). Figure 3 shows the evolution of the percentage of wealth hold by the top 1%, 5%, 20% and 40% of the population only for the first 15 periods of the transition and Table 8 sumarizes the results in wealth distribution.

Table 8. Wealth Distribution (model)

	Percentage wealth in top			
Year	1%	5%	20%	40%
1983	28	49	75	89
1992	29.79	51.92	78.65	92.25
1998	31.30	54.38	81.73	94.99

We can see that all of the shares in wealth has increased in the evolution: the amount of wealth hold by the 1% richest individuals increase 6.39 percent in the first 9 years, while the increase in the US data is 5.54 percent. From 1992 to 1998 the model considers that the wealth hold by this group raise a 5.05 percent, while the data registers a higher increase (17.44%).

Therefore the model under stimate the cumulative increase of the wealth hold by the top 1% during the period considered (11.74%), since the total increase of the wealth hold by this group is 23.93%. The same happens with the other percentage: top 5%, 20% and 40% raise a 5.96 percent, 4.87 percent and 3.65 percent respectively between 1983 and 1992, while the data shows higher increases: 9.18 percent for the top 5%, 5.99 percent for the top 20% and 4.40 percent for the 40% in the same period. Respect to the increases in the whole period, the data shows that in period 1983-1998 the share of the top 5% raises a 17.96 percent and those of the top 20% and 40% a 8.93 percent and 5.51 percent respectively, while the model generates only increases of 10.98 percent, 8.93 percent and 6.73 percent for the share of the top 5%, top 20% and top 40%.

About consumption inequality, the model generates a very mild increase in the Gini coefficient of a 0.15 percent, increase not comparable with the data, that suggests an increase of 1.49 percent between 1985 and 1998.

- Percentage increase in the standard deviation in consumption

5.4 Changes in inequality due to changes in γ

Here we analyze how inequality in wealth and consumption change when we modify the parameter value γ. TO BE WRITTEN

6 Aggregate shocks and inequality

Here we analyze how aggregate shocks affects the distribution of income, wealth and consumption along the transition. TO BE WRITTEN

7 Conclusions

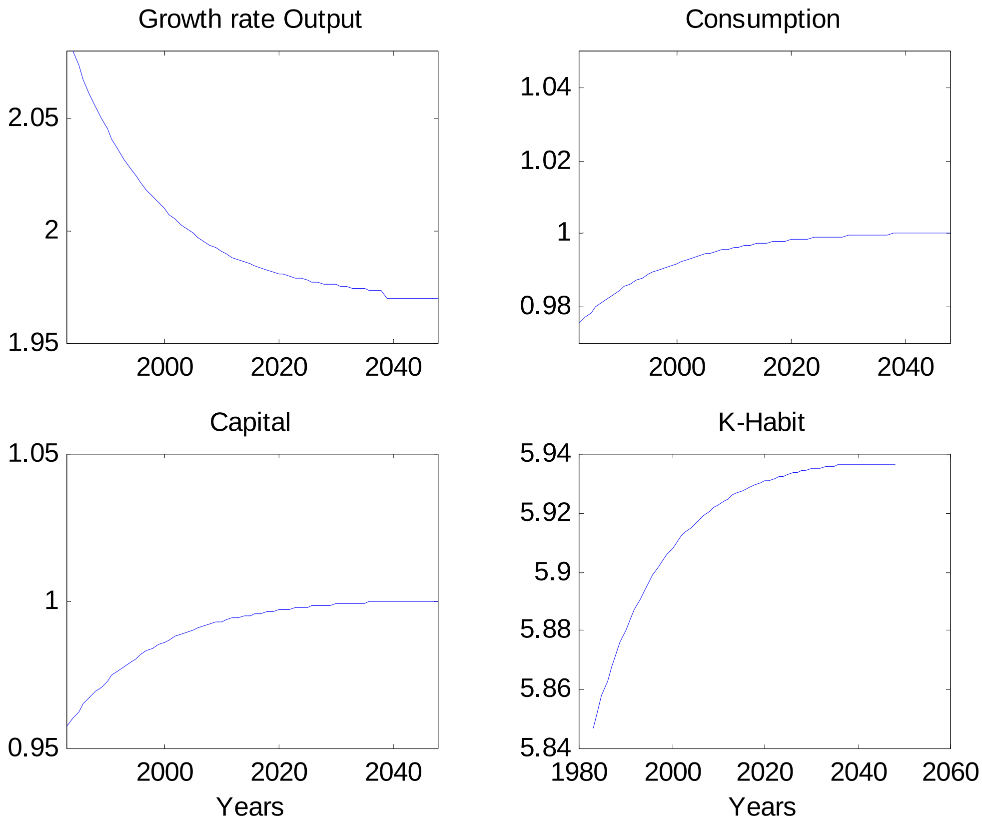

Figure 1a. Evolution of Output, Consumption and Capital for $\sigma = 1.2$

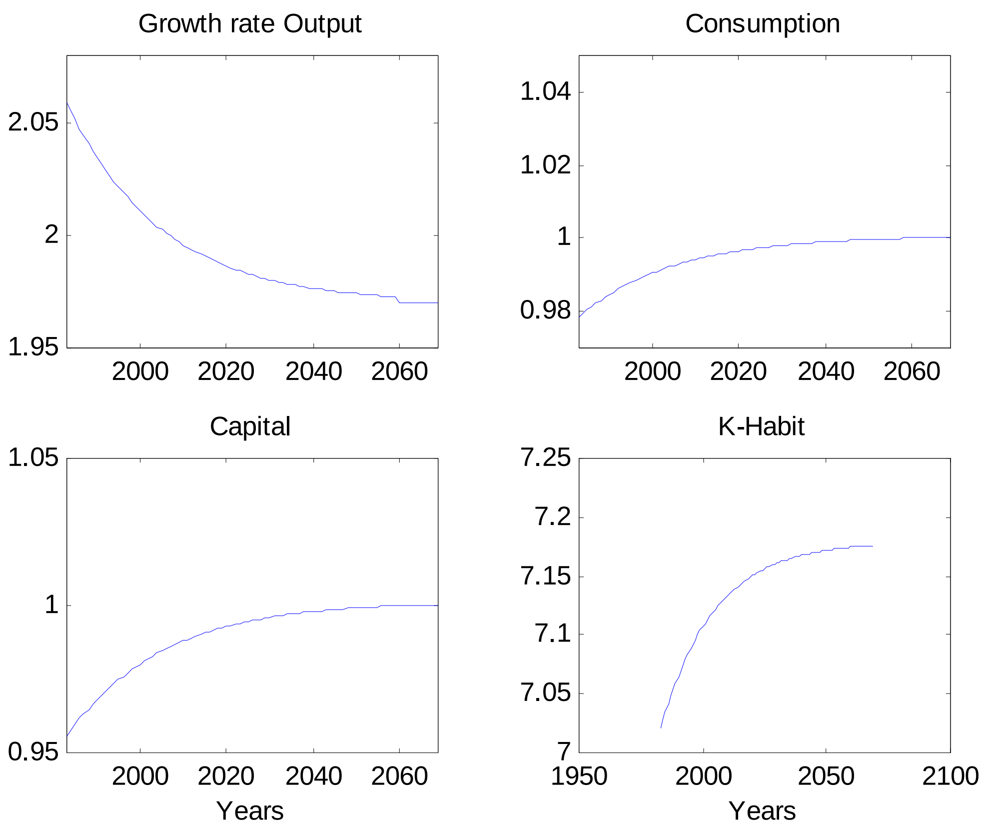

Figure 1b. Evolution of Output, Consumption and Capital for $\sigma = 1.8$

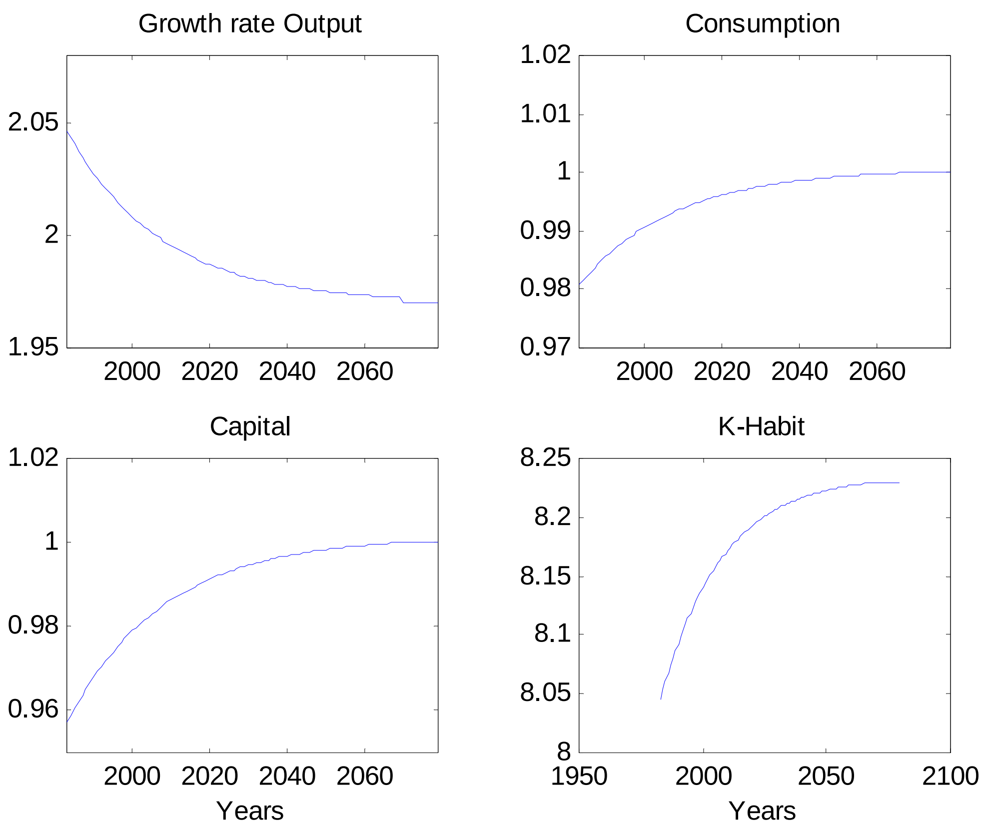

Figure 1c. Evolution of Output, Consumption and Capital for $\sigma = 2.2$

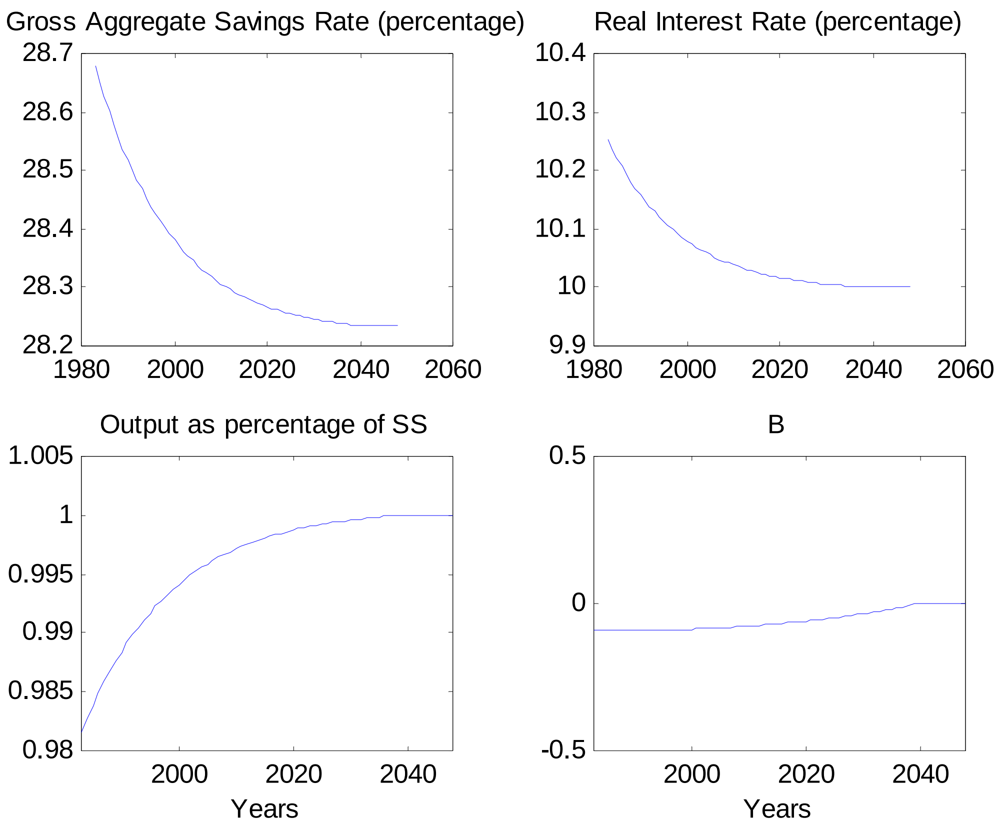

Figure 2a. Evolution of gross saving rates,.real interest rate, output as a percentage of its value in steady state and B for $\sigma = 1.2$.

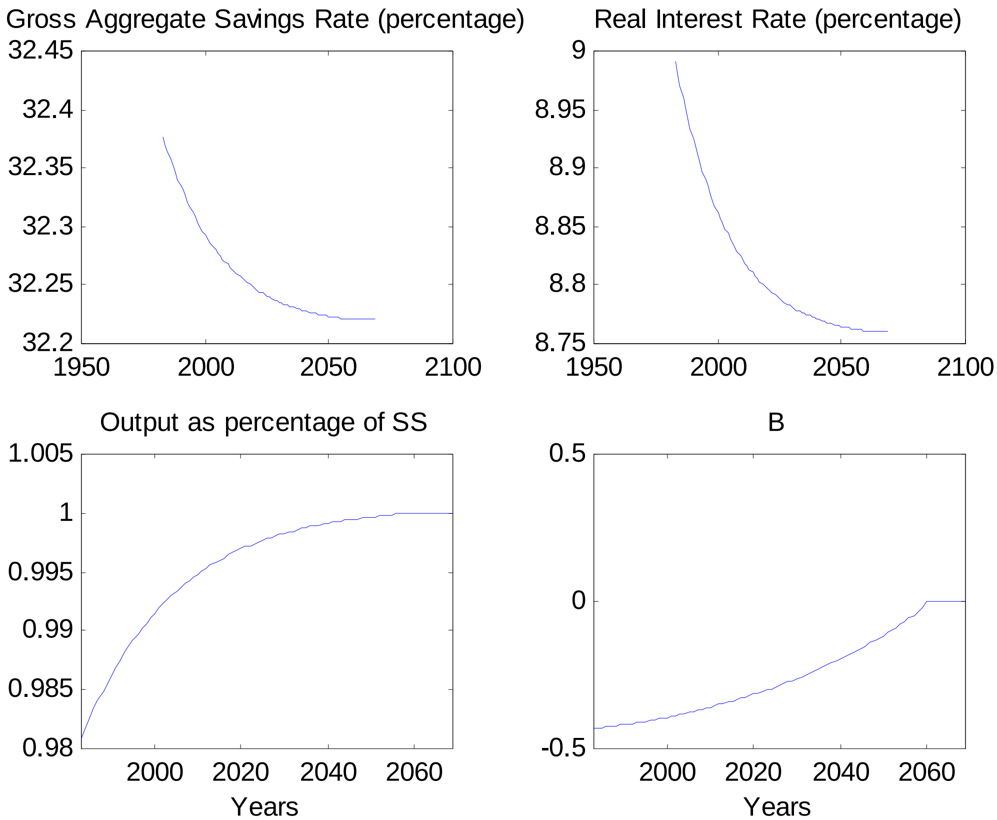

Figure 2b. Evolution of gross saving rate, real interest rate, output as a percentage of its steady state value and B for $\sigma = 1.8$.

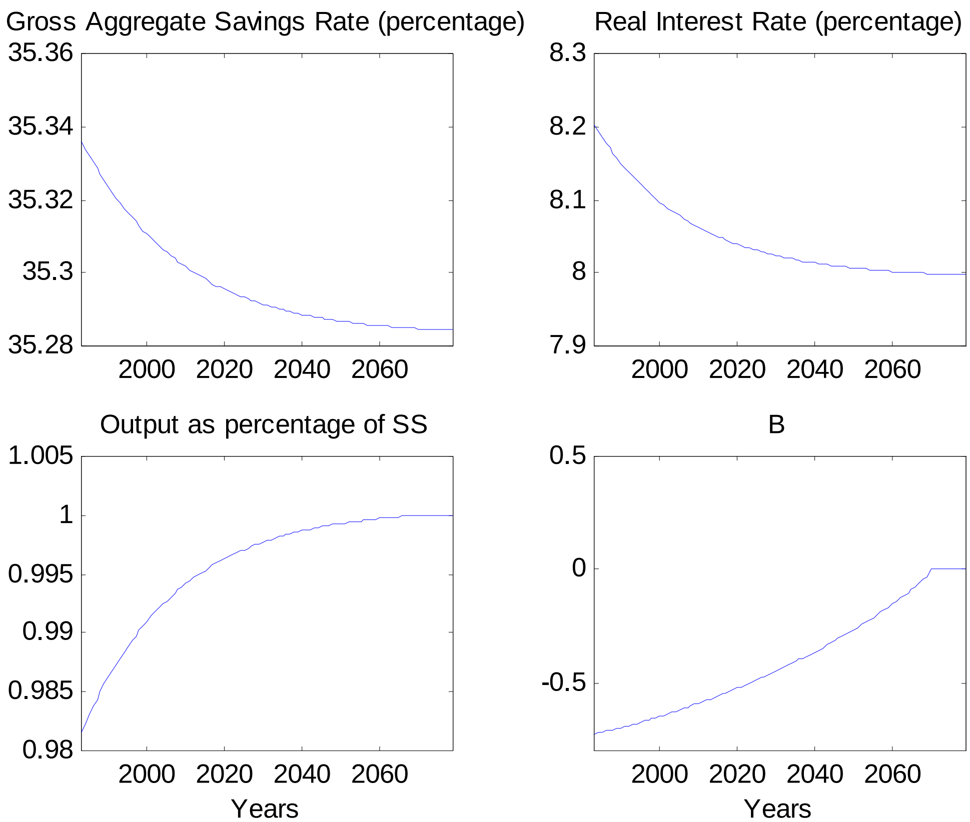

Figure 2c. Evolution of gross saving rate, real interest rate, output as a percentage of its steady state value and B for $\sigma = 2.2$

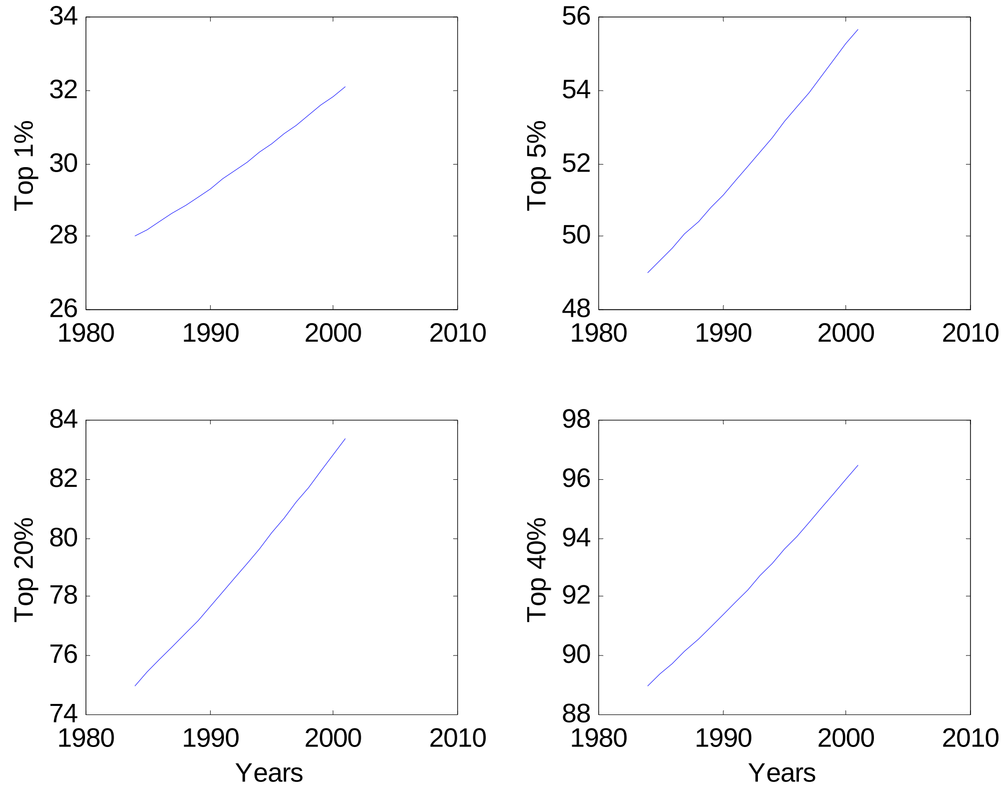

Figure 3. Evolution of the percentage of wealth hold by the top 1%, 5%, 20% and 40%.

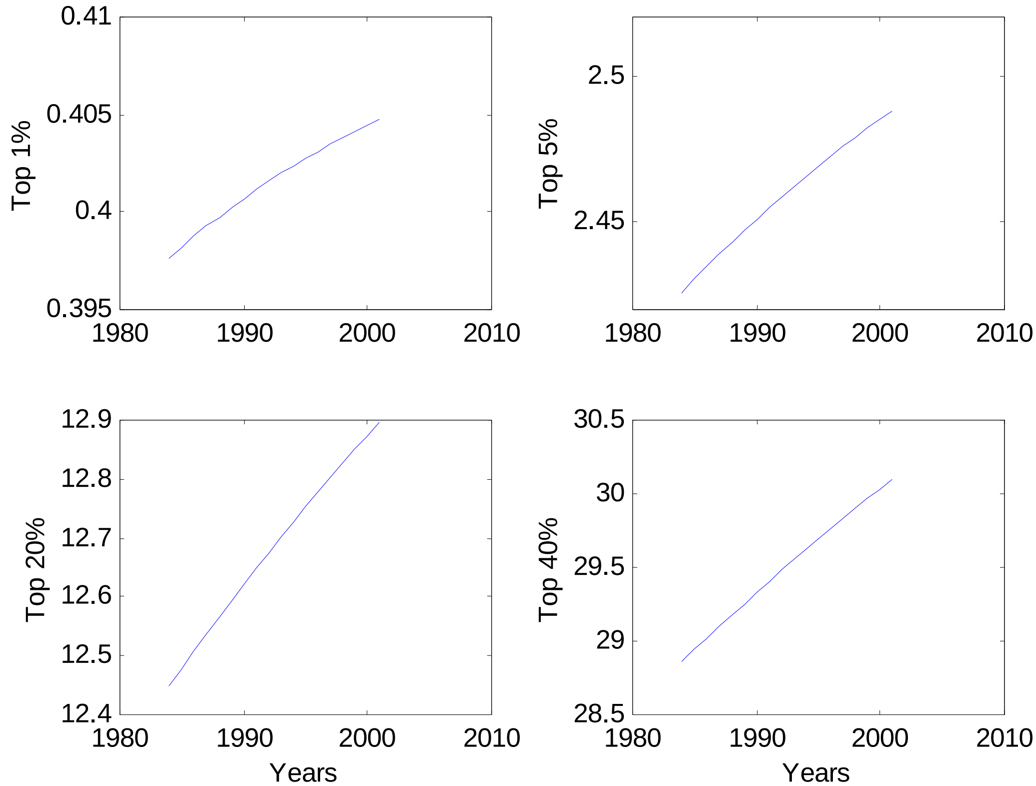

Figure 4. Evolution of the savings rate of the top 1%, 5%, 20% and 40% of population.

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