Equivalence of SLNR Precoder and RZF Precoder in Downlink MU-MIMO Systems

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***Abstract*—The signal-to-leakage-and-noise ratio (SLNR) precoder is widely used for MU-MIMO systems in many works, and observed with improved performance from zeroforcing (ZF) precoder. Our work proofs SLNR precoder is completely equivalent to conventional regulated ZF (RZF) precoder, which has significant gain over ZF precoder at low SNRs. Therefore, with our conclusion, the existing performance analysis about RZF precoder can be readily applicable to SLNR precoder.**

I. Introduction

Downlink Multi-user Multiple-Input-and-Multiple-Output (MU-MIMO) techniques have attracted much attention in the last decades [1–3]. It is promising to increase the system throughput in the future industrial standards, such as LTE-A and WiMax.

To support multiple user in downlink, it is important to suppress interference between users. As is known, the sum capacity of downlink MU-MIMO can be achieved by using Dirty-Paper-Coding (DPC) techniques [4], which is however with huge complexity. To reduce the complexity, various linear precoders are proposed [1, 3].

Among all, the Zeroforcing (ZF) based precoders by using channel inversions are widely used in MU-MIMO systems because of low complexity [1, 2]. These schemes impose a restriction on the system that the interference between users are cancellated. To avoid the poor performance of pure ZF precoder, regulated ZF precoder was proposed in [3].

Different from ZF based precoders, a leakage-based precoder proposed in [5]. The precoders are designed based on the concept of signal "leakage", which refers to the interference caused by the signal intended for a desired user to the remaining users. Such precoder is to maximize the signal-to-leakage-and-noise ratio (SLNR) for all users. It was observed in [5] SLNR precoder has significant improvement from ZF precoder. A comprehensive comparison between ZF, MRT and SLNR precoders can be found in [6] the comparison between SLNR and ZF that I have madeThe SLNR precoder is with decoupled nature analytical closed form and thus widely adopted in many optimization applications [7, 8].

In this work, we proof that the SLNR precoder, although inspired from different idea, is equivalent to the conventional RZF precoder. The equivalence not only explains why SLNR precoder is observed to be better than ZF precoder, but also makes the existing analysis of RZF precoder readily available to SLNR precoder.

II. System Model

A. Downlink channel

We consider a downlink MU-MIMO system where a BS with N_t antennas serves K single-antenna users. The received signal at the kth user can be expressed as

$$y_k = \mathbf{h}_k^H \mathbf{w}_k s_k + \sum_{j=1, j\neq k}^{K} \mathbf{h}_k^H \mathbf{w}_j s_j + n_k, \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{N_t\times 1}$ is the channel vector of the kth user, $\mathbf{w}_k \in \mathbb{C}^{N_t\times 1}$ is a unit-norm precoder, s_k is the data symbol with unit variance destined to user k, and n_k is the additive white Gaussian noise with zero mean and variance σ^2. With loss of generalities, we assume $\sigma^2 = 1$. $()^H$ denotes the conjugate-transpose operation.

B. Existing ZF, RZF, & SLNR precoder

In downlink MU-MIMO systems, Zeroforcing (ZF) precoder is chosen such that the interference between users is nulled, i.e.,

$$\mathbf{h}_k^H \mathbf{w}_j = 0, \tag{2}$$

when $k \neq j$.

A pseu-inverse based ZF precoder $\mathbf{w}_{\text{ZF},k}$ [1] is as

$$\mathbf{w}_{\text{ZF},k} \propto (\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{h}_k. \tag{3}$$

where $\propto$ means linear proportionality, and $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_K]$.

A regulated ZF precoder (RZF) is proposed in [3] to improve pure ZF precoder as

$$\mathbf{w}_{\text{RZF},k} \propto (\alpha\mathbf{I} + \mathbf{H}\mathbf{H}^H)^{-1}\mathbf{h}_k, \tag{4}$$

where α is the regulation parameter. It is worthy to note that optimal α varies with system configurations. However, a most frequently choice is $\alpha = \sigma^2$, as recommended in [3], which is optimal when K is large and works well even when K is small.

Different from previous precoders, the signal-to-leakage-noise ratio (SLNR) precoder adopts a leakage-base solution. The SLNR of single stream is defined as

$$\begin{aligned}\text{SLNR}_k &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{1 + \sum_{j\neq k} |\mathbf{h}_j^H \mathbf{w}_k|^2} \\ &= \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{1 + \|\mathbf{H}_{-k}^H \mathbf{w}_k\|^2}\end{aligned} \tag{5}$$

where $\mathbf{H}_{-k} = [\mathbf{h}_1, \cdots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \cdots, \mathbf{h}_K]$. The denominator in the above equation defines the sum of the noise power and the total interference power leaked from one user to the other users.

The SLNR precoder to maximize the SLNR in (5) is as [5]

$$\mathbf{w}_{\text{SLNR},k} = \text{max. eigenvector}\left((\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H\right) \quad (6)$$

In the case of multiple receiver antennas at the user, extensions of the SLNR precoder from single stream to multi stream can refer to [8]. Our work focus on single stream case and multi stream case can be investigated in further work. This approach is also supported by the prevalent LTE-standards, where each stream is treated as an individual users irrespective of multiple receiver antennas.

III. Equivalence of SLNR and RZF precoder

We will show that the SLNR precoder is equivalent to RZF precoder. To do this, we first introduce the following lemma.

Lemma 1: The two vectors satisfy $(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k \propto (\mathbf{I}+\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{h}_k$.

Proof: From matrix inverse identity [9]

$$(\mathbf{A}+\mathbf{x}\mathbf{x}^H)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{x}\mathbf{x}^H\mathbf{A}^{-1}}{1+\mathbf{x}^H\mathbf{A}^{-1}\mathbf{x}}, \quad (7)$$

we have

$$\begin{aligned}(\mathbf{A}+\mathbf{x}\mathbf{x}^H)^{-1}\mathbf{x} &= \mathbf{A}^{-1}\mathbf{x} - \frac{\mathbf{A}^{-1}\mathbf{x}\mathbf{x}^H\mathbf{A}^{-1}}{1+\mathbf{x}^H\mathbf{A}^{-1}\mathbf{x}}\mathbf{x} \\ &= \mathbf{A}^{-1}\mathbf{x} - \frac{\mathbf{x}^H\mathbf{A}^{-1}\mathbf{x}}{1+\mathbf{x}^H\mathbf{A}^{-1}\mathbf{x}}\mathbf{A}^{-1}\mathbf{x} \\ &= \frac{1}{1+\mathbf{x}^H\mathbf{A}^{-1}\mathbf{x}}\mathbf{A}^{-1}\mathbf{x}, \quad (8)\end{aligned}$$

which means

$$(\mathbf{A}+\mathbf{x}\mathbf{x}^H)^{-1}\mathbf{x} \propto \mathbf{A}^{-1}\mathbf{x}. \quad (9)$$

By setting $\mathbf{A} = \mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H$ and $\mathbf{x} = \mathbf{h}_k$, we immediately obtain Lemma 1.■

Therom 1: The SLNR precoder obtained in (6) is equivalent to the RZF precoder obtained in (4).

Proof: Define $\tilde{\mathbf{w}}_k = \frac{1}{\sqrt{\gamma}}(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k$, where $\gamma = \|(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\|^2$. We observe that

$$\begin{aligned}&(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H\tilde{\mathbf{w}}_k \\ &= (\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H\frac{1}{\sqrt{\gamma}}(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k \\ &= \mathbf{h}_k^H\frac{1}{\sqrt{\gamma}}(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k \\ &\triangleq \lambda\tilde{\mathbf{w}}_k \quad (10)\end{aligned}$$

where $\lambda = \mathbf{h}_k^H(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k > 0$. It indicates that $\tilde{\mathbf{w}}_k$ is an eigenvector of $(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H$.

Moreover, due to the matrix product rank inequality $\text{rank}(\mathbf{AB}) \leq \min\{\text{rank}(\mathbf{A}), \text{rank}(\mathbf{B})\}$, we find that $\text{rank}\left((\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H\right) \leq \text{rank}\left(\mathbf{h}_k\mathbf{h}_k^H\right) = 1$. Because $(\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H$ is a nonzero matrix, we have

$$\text{rank}\left((\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H\right) = 1. \quad (11)$$

Since there is only one eigenvector, we know that $\tilde{\mathbf{w}}_k = \text{max. eigenvector}\left((\mathbf{I}+\mathbf{H}_{-k}\mathbf{H}_{-k}^H)^{-1}\mathbf{h}_k\mathbf{h}_k^H\right)$.

From Lemma 1, $\mathbf{w}_{\text{SLNR},k} = \tilde{\mathbf{w}}_k \propto \mathbf{w}_{\text{RZF},k}$. Due to unit norm constraint, we finally conclude that $\mathbf{w}_{\text{SLNR},k} = \mathbf{w}_{\text{RZF},k}$.■

To this end, we show that SLNR precoder, though inspired from different precoding strategy, is equivalent to conventional RZF precoder.

IV. Discussions

The reason that SLNR precoder outperform ZF precoder can be consequently illustrated. It is well known that RZF precoder can significantly increase downlink MU-MIMO systems from pure ZF precoder at low SNRs. In fact, this is exactly analogous to the difference between ZF equalization and minimum mean-square error (MMSE) equalization: while zero-forcing results in complete cancelation of user interference, an MMSE equalizer allows a measured amount of interference such that the output SNR is maximized.

By using the equivalence of SLNR and RZF precoder, existing works about RZF precoder can be readily applied to SLNR precoder. For example, to get performance analysis of SLNR precoder, details can be found in [3], where RZF precoder is analyzed.

It is shown in [5] the SLNR precoder does not require that transmit antenna number should be no less than the number of streams supported in the downlink. However, it can be expected that even SLNR precoder working in this configuration will result in severe interference between users, which is obivous for RZF precoders.

V. Simulation Results

In this section, we assume that the channel is subjected to i.i.d. flat Rayleigh fading. Moreover, we assume homogeneous users in the downlink where users are with equal SNRs and the transmit power is equally allocated to each users.

In the first simulation, we investigate the sum rate of MU-MIMO systems versus different SNRs with different precoders: ZF, SLNR and RZF. The rate is calculated using the Shannon's formula by assume perfect link adaptation. Two configurations are considered. One is $N_t = K = 4$, and the other is $N_t = K = 2$. As shown in the figure, in spite of different duplexing gain, we find that in both two cases the sum rate of MU-MIMO system using SLNR precoder is the same with that of using RZF precoder. SLNR/RZF precoder outperforms that of using ZF at low SNRs and converges to ZF precoder at high SNRs. The simulation results support our analysis.

In the second simulation, we investigate the bit error rate (BER) of MU-MIMO systems using 4 QAM with different precoders: ZF, SLNR and RZF. Two configurations are considered. We set $K = 4$ and in the first configuration $N_t = 4$, and in the second is $N_t = 6$. As shown in the figure, the diversity order of BER in the second case is higher than that in the first one. In both cases, we find that the BER of MU-MIMO system using SLNR precoder is the same with that of using RZF precoder, outperforming ZF precoder. Similar results can be observed from other configurations where our analysis is still valid.

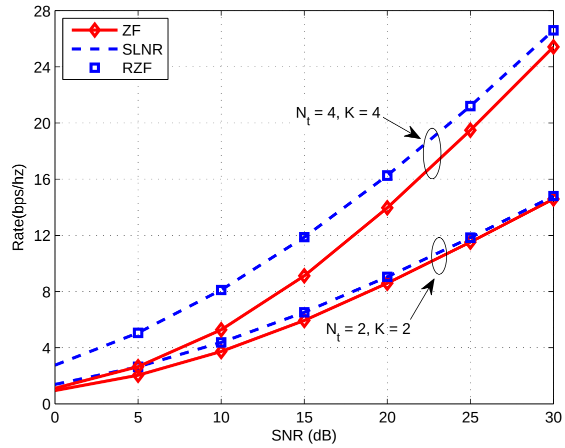

Fig. 1. Sum rate of MU-MIMO systems

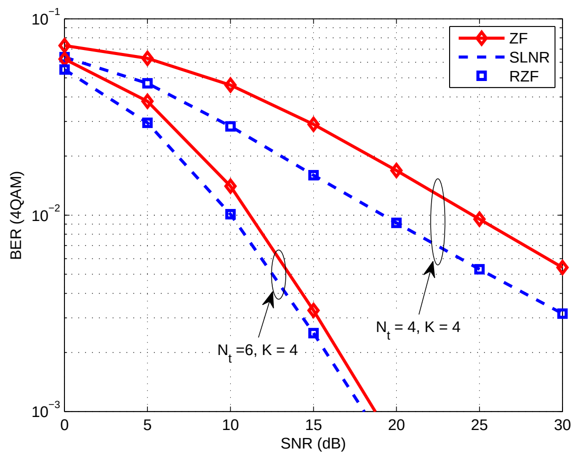

Fig. 2. BER of MU-MIMO systems with 4 QAM

Conclusions

In this work, we have proofed that the SLNR precoder is equivalent to conventional regulated ZF precoder, which illustrates why SLNR precoder outperforms ZF precoders in many applications. Our work makes the previous analysis of RZF precoders available for SLNR precoders.

References

[1] T. Haustein, C. von Helmolt, E. Jorswieck, V. Jungnickel, and V. Pohl, "Performance of MIMO systems with channel inversion," in *in Proc. of IEEE Vehi. Tech. Conf. (Spring)*, 2002.

[2] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inform. Theory*, vol. 52, no. 11, pp. 5045–5060, 2006.

[3] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication-part i: channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, 2005.

[4] M. Costa, "Writing on dirty paper (corresp.)," *IEEE Trans. Inform. Theory*, vol. 29, no. 3, pp. 439–441, 1983.

[5] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, 2007.

[6] E. Björnson, R. Zakhour, D. Gesbert, and B. Ottersten, "Cooperative multicell precoding: Rate region characterization and distributed strategies with instantaneous and statistical csi," *IEEE Trans. Signal Processing*, vol. 58, no. 8, pp. 4298–4310, 2010.

[7] V. Sharma and S. Lambotharan, "Multiuser downlink MIMO beamforming using an iterative optimization approach," in *in Proc. of IEEE Vehi. Tech. Conf. (Fall)*, 2006.

[8] P. Cheng, M. Tao, and W. Zhang, "A new SLNR-based linear precoding for downlink multi-user multi-stream MIMO systems," *IEEE Commun. Lett.*, vol. 14, no. 11, pp. 1008–1010, 2010.

[9] H. L. V. Trees, *Optimum Array Processing, Part IV of Detection, Estimation and Modulation Theory*. Wiley, 2002.