

Tiny Graviton Matrix Theory/SYM Correspondence: Analysis of BPS States

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Abstract

In this paper we continue analysis of the Matrix theory describing the DLCQ of type IIB string theory on $AdS_5 \times S^5$ (and/or the plane-wave) background, *i.e.* the Tiny Graviton Matrix Theory (TGMT) [1]. We study and classify 1/2, 1/4 and 1/8 BPS solutions of the TGMT which are generically of the form of rotating three brane giants. These are branes whose shape are deformed three spheres and hyperboloids. In lack of a classification of such ten dimensional type IIB supergravity configurations, we focus on the dual $\mathcal{N} = 4$ four dimensional 1/2, 1/4 and one 1/8 BPS operators and show that they are in one-to-one correspondence with the states of the same set of quantum numbers in TGMT. This provides further evidence in support of the Matrix theory.

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1 Introduction

The AdS/CFT [2] has provided us with a very elegant framework for studying quantum gravity (string theory) via a dual gauge theory and vice-versa. This correspondence (duality), however, is a kind of strong-weak duality in the sense that generically only one side of the

duality is perturbatively accessible. Therefore, most of the analysis and checks of the duality has been limited to the BPS sectors, the information of which can be safely used at strongly coupled regime.

It has, however, been noted that in some specific limits/sectors both sides of the AdS/CFT duality can be perturbative and hence giving a window for a direct check of the duality. The most extensively studied such sector is the so-called BMN sector [3]. The BMN sector is obtained through the translation of taking the Penrose limit (which is an operation on the geometry/gravity side) into the dual $\mathcal{N} = 4$ SYM language (for review e.g. see [4]).

Despite the large number of works devoted to the analysis of the AdS/CFT duality, a non-perturbative description of either side is still lacking. Inspired by the example of the BFSS matrix theory [5], according which the discrete light-cone quantization (DLCQ) of M-theory is described by a $0 + 1$ SYM theory, one may hope that type IIB string theory on $AdS_5 \times S^5$, at least in the DLCQ description, admits a Matrix theory formulation. In [1] it was argued that this is indeed the case. To argue for existence of such a Matrix theory description three observations were noted in [1]: 1) The DLCQ description is very similar to a description in infinite momentum frame (IMF) [5] and in the IMF what is viewed from the AdS geometry is its Penrose limit, the corresponding (maximally supersymmetric) plane-wave geometry. 2) The DLCQ of M-theory on the maximally supersymmetric *eleven dimensional* plane-wave geometry, the geometry which is obtained as Penrose limit of either of the $AdS_{4,7} \times S^{7,4}$ geometries, is described by a $0 + 1$ supersymmetric gauge theory, the BMN Matrix theory [3]. 3) It was shown in [6] that, similarly to the BFSS case [7], the BMN matrix theory is a theory of discretized membranes in the 11 dimensional plane-wave background. Moreover, it was noted that the $1/2$ BPS vacuum solutions of the BMN matrix model are of the form of concentric giant membrane gravitons [6]. These giant membrane gravitons in the BMN matrix model notation appear as fuzzy two spheres. As pointed out in [1] these membranes can be interpreted as a collection of “tiny membrane gravitons” blown up (by the Myers effect [8]) to cover a two sphere. Tiny membrane gravitons play the role of D0-branes in the BFSS theory. Hence, the BMN (or plane-wave) matrix model, which is describing the DLCQ of M-theory on the $AdS_{4,7} \times S^{7,4}$ and/or the 11 dimensional plane-wave, is nothing but a *tiny (membrane) graviton matrix theory*.

A similar idea may also be applied to obtain the DLCQ formulation of type IIB string theory on the $AdS_5 \times S^5$ background and/or the corresponding plane-wave. In this matrix theory, however, we should employ the *tiny three-brane gravitons*. The action for the tiny three-brane graviton theory, or the TGMT in short, in analogy with the 11 dimensional case, is obtained from discretization of a three-brane action on the 10 dimensional plane-wave background [1]. In section 2, we review the statement of the TGMT conjecture, its

action and symmetry structure. As the DLCQ formulation of string theory on the $AdS_5 \times S^5$, one expects the TGMT to be in correspondence with both type IIB supergravity and the dual $\mathcal{N} = 4$, $D = 4$ SYM theory (see Fig.6 of [9] which illustrates AdS-TGMT-CFT triality).

In this paper we continue the analysis started in [1, 9] to provide further supportive evidence for the conjecture. We study the BPS configurations of the TGMT. In section 3, we review and expand results of [9] by exhausting the 1/2 BPS configurations of the TGMT and showing that they are very closely related to the same configurations in the type IIB supergravity, the LLM geometries [10], and the $\mathcal{N} = 4$ dual gauge theory [12, 11]. Here we show that the non-commutativity of the (x_1, x_2) -plane in the LLM geometries [13] comes as a natural outcome in the tiny graviton matrix theory.

In section 4, we extend our analysis to the less BPS, i.e. 1/8 and 1/4 BPS, configurations. The BPS configurations that we study here are all of the form of *transverse* three branes of various shape and topology and fall into two classes, those which are completely specified with the *shape* of the brane, i.e. geometric fluctuations of three branes. And those in which the non-geometric, internal degrees of freedom of the three brane (*i.e.* the gauge fields on the D3-brane) is turned on. We give explicit matrix representations of these deformed three sphere giants and show that they are completely labelled by the number of giants and at most four integers corresponding to their angular momenta.

In section 5, we review the BPS states/multiplets of the $\mathcal{N} = 4$ SYM and show that all the BPS states we have studied in the TGMT have correspondents in the dual gauge theory. This, via the AdS/CFT, provides further supportive evidence for the TGMT conjecture. We end this article by summary of our results and the outlook.

In the TGMT Lagrangian *four brackets* has been introduced and used. Since four brackets, unlike the usual two brackets, *i.e.* commutators, are not so familiar we find it useful to present some identities regarding computations with these brackets. These and some notation-fixings are gathered in Appendix A and Appendix B contains the explicit form of the TGMT superalgebra, $PSU(2|2) \times PSU(2|2) \times U(1)$, in terms of the matrices.

Note added: When this work was finished [14] appeared on the arxiv which has some overlap in the subject of the current work.

2 Review of the Tiny Graviton Matrix Theory

In this section we briefly review basics of the tiny graviton Matrix theory, TGMT. It is essentially a very short summary of [1]. The TGMT proposal states that the DLCQ of type IIB strings on the $AdS_5 \times S^5$ or the 10 dimensional plane-wave background in the sector with J units of light-cone momentum is described by a $U(J)$ supersymmetric gauge theory.

2.1 The TGMT action

Dynamics of the TGMT is governed by the action which is the regularized (discretized) form of a *single* D3-brane action on the plane-wave background, once the light-cone gauge is fixed and while the gauge field living on the brane is turned off. Fixing the light-cone gauge does not remove all the unphysical gauge degrees of freedom. It fixes the part of four dimensional diffeomorphisms which mixes temporal and spatial directions on the brane world-volume. The parts which rotate spatial directions among themselves remains unfixed. The spatial part of the diffeomorphisms, after the prescribed “regularization” method for discretizing the three-brane [1], constitute the $U(J)$ gauge symmetry of the TGMT. As is evident from the above construction we expect in $J \rightarrow \infty$ limit to recover the volume-preserving diffeomorphisms. This parallels the discussions that the $U(N)$ gauge symmetry of the BFSS matrix model is nothing but the discretized version of the area preserving diffeomorphism on a membrane [5, 7]. Again, in analogy with the BFSS case, the TGMT can also be understood as a theory of J tiny gravitons. Tiny gravitons, similarly to the D0-branes, are on one hand gravitons and on the other hand (tiny, spherical) D3-branes and hence show the remarkable property of gauge symmetry enhancement when become coincident. According to the TGMT conjecture everything, including the fabric of space-time, is made out of different configurations of tiny (three-brane) gravitons.

The TGMT action is then

$$\mathcal{S} = \int d\tau \mathbf{L}, \quad (2.1)$$

with the Lagrangian

$$\begin{aligned} \mathbf{L} = R_- \text{Tr} & \left[\frac{1}{2R_-^2} [(\mathcal{D}_0 X_i)^2 + (\mathcal{D}_0 X_a)^2] - \frac{1}{2} \left(\frac{\mu}{R_-} \right)^2 (X_i^2 + X_a^2) \right. \\ & - \frac{1}{2 \cdot 3! g_s^2} ([X^i, X^j, X^k, \mathcal{L}_5][X^i, X^j, X^k, \mathcal{L}_5] + [X^a, X^b, X^c, \mathcal{L}_5][X^a, X^b, X^c, \mathcal{L}_5]) \\ & - \frac{1}{2 \cdot 2g_s^2} ([X^i, X^j, X^a, \mathcal{L}_5][X^i, X^j, X^a, \mathcal{L}_5] + [X^a, X^b, X^i, \mathcal{L}_5][X^a, X^b, X^i, \mathcal{L}_5]) \\ & + \frac{\mu}{3! R_- g_s} (\epsilon^{ijkl} X^i [X^j, X^k, X^l, \mathcal{L}_5] + \epsilon^{abcd} X^a [X^b, X^c, X^d, \mathcal{L}_5]) \\ & + \left(\frac{i}{R_-} \right) \left(\theta^{\dagger\alpha\beta} \mathcal{D}_0 \theta_{\alpha\beta} + \theta^{\dagger\dot{\alpha}\dot{\beta}} \mathcal{D}_0 \theta_{\dot{\alpha}\dot{\beta}} \right) - \left(\frac{\mu}{R_-} \right) \left(\theta^{\dagger\alpha\beta} \theta_{\alpha\beta} - \theta_{\dot{\alpha}\dot{\beta}} \theta^{\dagger\dot{\alpha}\dot{\beta}} \right) \\ & - \frac{1}{2g_s} \left(\theta^{\dagger\alpha\beta} (\sigma^{ij})_{\alpha}^{\delta} [X^i, X^j, \theta_{\delta\beta}, \mathcal{L}_5] + \theta^{\dagger\alpha\beta} (\sigma^{ab})_{\alpha}^{\delta} [X^a, X^b, \theta_{\delta\beta}, \mathcal{L}_5] \right) \\ & \left. - \frac{1}{2g_s} \left(\theta^{\dagger\dot{\alpha}\dot{\beta}} (\sigma^{ij})_{\dot{\alpha}}^{\dot{\delta}} [X^i, X^j, \theta_{\dot{\delta}\dot{\beta}}, \mathcal{L}_5] + \theta^{\dagger\dot{\alpha}\dot{\beta}} (\sigma^{ab})_{\dot{\alpha}}^{\dot{\delta}} [X^a, X^b, \theta_{\dot{\delta}\dot{\beta}}, \mathcal{L}_5] \right) \right], \quad (2.2) \end{aligned}$$

where the X 's and θ 's are $J \times J$ matrices in the adjoint of $U(J)$, $i = 1, 2, 3, 4$ and $a = 5, 6, 7, 8$.

The above action besides the $U(J)$ gauge symmetry has $SO(4)_i \times SO(4)_a$ global symmetry, as well as a \mathbb{Z}_2 which exchanges i, a directions and the fermionic indices α, β which run over 1, 2 are Weyl indices of either of the $SO(4)_i$ or $SO(4)_a$ symmetries.

In our conventions R_- , which is the radius of compactification of the light-like direction X^- in the plane-wave geometry, similarly to the μ , has dimension of energy. In fact the X^- compactification radius (in string units) is R_-/μ which is a free parameter once TGMT is used as string theory on the plane-wave background. Besides μ/R_- in the action we have another dimensionless parameter, g_s . As discussed in [1] and reviewed in the introduction, TGMT may also be used as DLCQ formulation of strings on the $AdS_5 \times S^5$ geometry. In this case R_-/μ is related to the AdS radius, $(l_s^4 g_s N)^{1/4}$, as [1]

$$\left(\frac{R_-}{\mu}\right)^2 = g_s N = \frac{R_{AdS}^4}{l_s^4}. \quad (2.3)$$

In the above action \mathcal{L}_5 is a fixed (non-dynamical) $SO(4)_i \times SO(4)_a$ invariant, Hermitian $J \times J$ matrix defined through [1, 9]^{1,2}

$$\text{Tr } \mathcal{L}_5^2 = 1, \quad \text{Tr } \mathcal{L}_5 = 0. \quad (2.4)$$

Also, \mathcal{D}_0 is the covariant derivative of the 0 + 1 dimensional $U(J)$ gauge theory

$$\mathcal{D}_0 = \partial_0 + i[\mathcal{A}_0, \cdot] \quad (2.5)$$

$\mathcal{A}_0 = \mathcal{A}_0^m T_m$ is the only component of the gauge field and T_m , $m = 1, 2, \dots, J^2$ are the generators of the $U(J)$ gauge symmetry. We can use the temporal gauge and set $\mathcal{A}_0 = 0$.

To ensure this gauge condition, all of our physical states must satisfy the Gauss law constraint arising from equations of motion of \mathcal{A}_0 . These constraints, which consist of $J^2 - 1$ independent conditions are

$$\left(\Phi_{J \times J} \equiv i[X^i, P^i] + i[X^a, P^a] + \{\theta^{\dagger\alpha\beta}, \theta_{\alpha\beta}\} + \{\theta^{\dagger\dot{\alpha}\dot{\beta}}, \theta_{\dot{\alpha}\dot{\beta}}\}\right) |Phys\rangle = 0 \quad (2.6)$$

where $P^I = D_0 X^I$. Note that (2.6) is an operator equation which should be satisfied with all physical states and the commutators are matrix commutators.

It is useful to derive the light-cone Hamiltonian of the theory using Legendre transformation

$$\mathbf{H} = \text{Tr } P^I \dot{X}^I - \mathbf{L} + \text{Tr } \mathcal{A}_0 \Phi \quad (2.7)$$

¹These relations are $U(J)$ invariant and hence one can choose the basis in which \mathcal{L}_5 is diagonal. This specifies \mathcal{L}_5 up to permutation of the eigenvalues, \mathcal{S}_J .

²There has been another proposal for the plane-wave matrix string theory which does not involve the \mathcal{L}_5 [15]. This theory is governed by the action for J coincident ‘‘gravitons’’ [16]. Matrix model of [15], however, only exhibits $(SU(2) \times U(1))^2$ out of $SO(4) \times SO(4)$.

where $I = 1, 2, \dots, 8$,

$$P_I = \frac{\partial \mathbf{L}}{\partial \dot{X}^I}$$

and the last term is Lagrange multiplier \mathcal{A}_0 , times the equation of motion of \mathcal{A}_0 . The explicit form of the Hamiltonian in the temporal gauge is given in the Appendix B.

2.2 The symmetry structure

The plane-wave is a maximally supersymmetric background, *i.e.* it has 32 fermionic isometries, which can be arranged into two sets of 16 in kinematical supercharges and dynamical supercharges, and 30 bosonic isometries. More details can be found in [4].

On the other hand, TGMT, has a large number of local and global symmetries. Let us work with the Hamiltonian in $\mathcal{A}_0 = 0$ gauge given in (B.4). This Hamiltonian still enjoys the *time independent* part of the $U(J)$ gauge symmetry, which appears as a global symmetry. Moreover, it has $PSU(2|2) \times PSU(2|2) \times U(1)_{\mathbf{H}} \times U(1)_{p^+}$, with the generators $Q_{\alpha\dot{\beta}}$, $Q_{\dot{\alpha}\beta}$, J^{ij} , J^{ab} , \mathbf{H} , p^+ . That is,

$$[P^+, Q_{\alpha\dot{\beta}}] = 0, \quad [P^+, Q_{\dot{\alpha}\beta}] = 0, \quad [\mathbf{H}, Q_{\alpha\dot{\beta}}] = 0, \quad [\mathbf{H}, Q_{\dot{\alpha}\beta}] = 0. \quad (2.8)$$

$$\{Q_{\alpha\dot{\beta}}, Q^{\dagger\rho\lambda}\} = \delta_{\alpha}^{\rho}\delta_{\dot{\beta}}^{\lambda}\mathbf{H} + \frac{\mu}{2}(i\sigma^{ij})_{\alpha}^{\rho}\delta_{\dot{\beta}}^{\lambda}\mathbf{J}_{ij} + \frac{\mu}{2}\delta_{\alpha}^{\rho}(i\sigma^{ab})_{\dot{\beta}}^{\lambda}\mathbf{J}_{ab} - \mu(i\sigma^{ij})_{\alpha}^{\rho}(i\sigma^{ab})_{\dot{\beta}}^{\lambda}\mathcal{R}_{ijab} \quad (2.9a)$$

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\rho\lambda}\} = \delta_{\dot{\alpha}}^{\rho}\delta_{\beta}^{\lambda}\mathbf{H} + \frac{\mu}{2}(i\sigma^{ij})_{\dot{\alpha}}^{\rho}\delta_{\beta}^{\lambda}\mathbf{J}_{ij} + \frac{\mu}{2}\delta_{\dot{\alpha}}^{\rho}(i\sigma^{ab})_{\beta}^{\lambda}\mathbf{J}_{ab} - \mu(i\sigma^{ij})_{\dot{\alpha}}^{\rho}(i\sigma^{ab})_{\beta}^{\lambda}\mathcal{R}_{ijab} \quad (2.9b)$$

$$\{Q_{\alpha\dot{\beta}}, Q^{\dagger\rho\lambda}\} = \mu(\sigma^i)_{\alpha}^{\rho}(\sigma^a)_{\dot{\beta}}^{\lambda}(C_{ia} + i\hat{C}_{ia}) \quad (2.9c)$$

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\rho\lambda}\} = \mu(\sigma^i)_{\dot{\alpha}}^{\rho}(\sigma^a)_{\beta}^{\lambda}(C_{ia} - i\hat{C}_{ia}) \quad (2.9d)$$

$$\{Q_{\dot{\alpha}\beta}, Q_{\rho\lambda}\} = 0, \quad \{Q_{\alpha\dot{\beta}}, Q_{\rho\lambda}\} = 0, \quad \{Q_{\dot{\alpha}\beta}, Q_{\rho\lambda}\} = 0. \quad (2.9e)$$

The other bosonic generators, R_{ijab} , C_{ia} , \hat{C}_{ia} are constituting some of the possible extensions of the minimal $PSU(2|2) \times PSU(2|2) \times U(1)$ which are inherent to the TGMT formulation [17]. In our conventions the complex conjugate \dagger just raises and lowers the fermionic indices. To find the explicit form of (some of) the extensions in terms of matrices one should perform a careful computation of various anticommutators of the above dynamical supercharges, this has been carried out in [17]. (A similar calculation can be carried out for kinematical and mixed supercharges.) For the computation we need to use the following basic *operator* (to be compared with matrix) commutation relations:

$$\begin{aligned} [X_{pq}^I, P_{rs}^J] &= i\delta^{IJ}\delta_{ps}\delta_{qr} \\ \{(\theta^{\dagger\alpha\beta})_{pq}, (\theta_{\rho\gamma})_{rs}\} &= \delta_{\rho}^{\alpha}\delta_{\gamma}^{\beta}\delta_{ps}\delta_{qr} \\ \{(\theta^{\dagger\dot{\alpha}\dot{\beta}})_{pq}, (\theta_{\dot{\rho}\dot{\gamma}})_{rs}\} &= \delta_{\dot{\rho}}^{\dot{\alpha}}\delta_{\dot{\gamma}}^{\dot{\beta}}\delta_{ps}\delta_{qr}, \quad p, r, s, t = 1, 2, \dots, J \end{aligned} \quad (2.10)$$

The two $SO(4)_i$ and $SO(4)_a$ rotations act on i and a vector indices of the bosonic X^i, P_i and X^a, P_a fields and on the spinor (Weyl) indices of fermionic $\theta_{\alpha\beta}$ as

$$\begin{aligned} X_{rs}^i &\rightarrow \tilde{X}_{rs}^i = R_{ij}^i X_{rs}^j \\ (\theta_{\alpha\beta})_{rs} &\rightarrow (\tilde{\theta}_{\alpha\beta})_{rs} = R_{\alpha\gamma}(\theta_{\gamma\beta})_{rs} \\ \mathcal{L}_5 &\rightarrow \mathcal{L}_5, \end{aligned} \tag{2.11}$$

where $R_{ij} = e^{i\omega_{ij}\gamma^{ij}}$, $R_{\alpha\gamma} = e^{i\omega_{ij}\sigma^{ij}}$ are respectively 4×4 and 2×2 $SO(4)$ rotation matrices and r, s are $J \times J$ indices. There is another \mathbb{Z}_2 symmetry which changes the orientation of the X^i and X^a *simultaneously* (i.e. $\epsilon_{ijkl}, \epsilon_{abcd} \rightarrow -\epsilon_{ijkl}, -\epsilon_{abcd}$) together with sending $\mathcal{L}_5 \rightarrow -\mathcal{L}_5$.

Under the $U(J)$ rotations all the dynamical fields as well as the \mathcal{L}_5 are in the adjoint representation:

$$X_I \rightarrow \hat{X}^I = UX^I U^{-1}, \quad \mathcal{L}_5 \rightarrow U\mathcal{L}_5 U^{-1} \tag{2.12}$$

where $U \in U(J)$. There is a $U(1)$ subgroup of $U(J)$, $U(1)_\alpha$ which is generated by \mathcal{L}_5 :

$$U_\alpha = e^{i\alpha\mathcal{L}_5}. \tag{2.13}$$

This subgroup has the interesting property that keeps the \mathcal{L}_5 invariant. We will discuss the $U(1)_\alpha$ further in the following sections.

2.3 Classical BPS equations: Matrix regularized form

Given the superalgebra we are now ready to study BPS configurations of the TGMT, which is the main theme of this paper. By definition, a BPS state is a field configuration which is invariant under some specific supersymmetry transformations. For the configurations in which the spinors θ 's are turned off the non-zero SUSY variations are only $\delta_\epsilon\theta$'s and hence the BPS equations read as

$$\delta_\epsilon\theta = \{\epsilon^\dagger Q + \epsilon Q^\dagger, \theta\} = 0 \tag{2.14}$$

for classical configurations, and $\delta_\epsilon\theta|BPS\rangle = 0$ for quantum BPS states, for some spinor ϵ . The number of independent ϵ 's which satisfy either of the above equations determines how much BPS our configuration is. Explicitly, if there are n ϵ 's the configuration is $n/32$ BPS.

TGMT is a DLCQ formulation of strings and as such the kinematical supercharges $q_{\alpha\beta}, q^{\dagger\alpha\beta}, q_{\dot{\alpha}\dot{\beta}}, q^{\dagger\dot{\alpha}\dot{\beta}}$ which anticommute to light-cone momentum p^+ are not preserved by any physical state. Hence the BPS configurations of TGMT are 1/2 or less BPS. To check what

portion of the dynamical supercharges, Q 's, is preserved we write out (2.14) explicitly

$$\begin{aligned}
\delta_\epsilon(\theta_{\alpha\beta})_{pq} &= \left\{ \epsilon^{\dagger\gamma\delta} Q_{\gamma\delta} + \epsilon^{\dagger\dot{\gamma}\dot{\delta}} Q_{\dot{\gamma}\dot{\delta}} + \epsilon_{\gamma\dot{\delta}} Q^{\dagger\gamma\dot{\delta}} + \epsilon_{\dot{\gamma}\delta} Q^{\dagger\dot{\gamma}\delta}, (\theta_{\alpha\beta})_{pq} \right\} \\
&= \left((P^d + i\frac{\mu}{R_-} X^d) + \frac{i}{3!g_s} \epsilon^{abcd} [X^a, X^b, X^c, \mathcal{L}_5] \right)_{pq} (\sigma^d)_{\beta}^{\dot{\delta}} \epsilon_{\alpha\dot{\delta}} \\
&\quad + \frac{1}{2g_s} [X^a, X^i, X^j, \mathcal{L}_5]_{pq} (\sigma^a)_{\beta}^{\dot{\delta}} (i\sigma^{ij})_{\alpha}^{\gamma} \epsilon_{\gamma\dot{\delta}} \\
&\quad + \left((P^l + i\frac{\mu}{R_-} X^l) + \frac{i}{3!g_s} \epsilon^{ijkl} [X^i, X^j, X^k, \mathcal{L}_5] \right)_{pq} (\sigma^l)_{\alpha}^{\dot{\gamma}} \epsilon_{\gamma\beta} \\
&\quad + \frac{1}{2g_s} [X^i, X^a, X^b, \mathcal{L}_5]_{pq} (\sigma^i)_{\alpha}^{\dot{\gamma}} (i\sigma^{ab})_{\beta}^{\delta} \epsilon_{\gamma\delta}
\end{aligned} \tag{2.15}$$

and

$$\begin{aligned}
\delta_\epsilon(\theta_{\dot{\alpha}\dot{\beta}})_{pq} &= \left\{ \epsilon^{\dagger\gamma\dot{\delta}} Q_{\gamma\dot{\delta}} + \epsilon^{\dagger\dot{\gamma}\delta} Q_{\dot{\gamma}\delta} + \epsilon_{\gamma\dot{\delta}} Q^{\dagger\gamma\dot{\delta}} + \epsilon_{\dot{\gamma}\delta} Q^{\dagger\dot{\gamma}\delta}, (\theta_{\dot{\alpha}\dot{\beta}})_{pq} \right\} \\
&= \left((P^d + i\frac{\mu}{R_-} X^d) + \frac{i}{3!g_s} \epsilon^{abcd} [X^a, X^b, X^c, \mathcal{L}_5] \right)_{pq} (\sigma^d)_{\dot{\beta}}^{\delta} \epsilon_{\alpha\dot{\delta}} \\
&\quad + \frac{1}{2g_s} [X^a, X^i, X^j, \mathcal{L}_5]_{pq} (\sigma^a)_{\dot{\beta}}^{\delta} (i\sigma^{ij})_{\alpha}^{\dot{\gamma}} \epsilon_{\gamma\dot{\delta}} \\
&\quad + \left((P^l + i\frac{\mu}{R_-} X^l) + \frac{i}{3!g_s} \epsilon^{ijkl} [X^i, X^j, X^k, \mathcal{L}_5] \right)_{pq} (\sigma^l)_{\dot{\alpha}}^{\gamma} \epsilon_{\gamma\dot{\beta}} \\
&\quad + \frac{1}{2g_s} [X^i, X^a, X^b, \mathcal{L}_5]_{pq} (\sigma^i)_{\dot{\alpha}}^{\gamma} (i\sigma^{ab})_{\dot{\beta}}^{\delta} \epsilon_{\gamma\dot{\delta}} .
\end{aligned} \tag{2.16}$$

The BPS equation, that is the demand that (2.15), (2.16) vanish, will specify a set of ϵ 's for a given configuration.

To analyze and solve BPS equations, here we use the method introduced and used in [18, 19]. The idea is that one can always define a Hermitian projector \mathcal{P} which is a 16×16 matrix, and has $16 - n$ zero eigenvalues for an $n/32$ BPS configuration, that is $\mathcal{P}^2 = \mathcal{P}$, $\text{Tr } \mathcal{P} = n$. Choosing ϵ to be of the form

$$\epsilon = \mathcal{P}\epsilon_0,$$

we demand that the BPS equation should then hold for any arbitrary spinor ϵ_0 . That is,

$$\delta_\epsilon \theta = \dots \mathcal{P}\epsilon_0 = 0 \tag{2.17}$$

where \dots stands for the expression in terms of X 's and P 's given above. Since ϵ_0 is an arbitrary spinor, drops out and hence the BPS equation becomes an equation only among matrix degrees of freedom

$$\mathcal{F}_{\mathcal{P}}[X, \theta, \mathcal{A}_0; \mathcal{L}_5] = 0. \tag{2.18}$$

We should still make sure that the configuration which solves (2.18) is physical in the sense that it is compatible with the Gauss law

$$[X^i, \dot{X}^i + i[\mathcal{A}_0, X^i]] + [X^a, \dot{X}^a + i[\mathcal{A}_0, X^a]] = 0. \quad (2.19)$$

As it was shown in [18, 19] the BPS equation (2.18) can be considerably simplified by choosing an appropriate form for \mathcal{A}_0 using the $U(J)$ gauge symmetry. It turns out (it will become clear in section 4) that

$$i[\mathcal{A}_0, \Psi] = -\frac{1}{g_s}[\Psi, X^1, X^2, \mathcal{L}_5] - \frac{1}{g_s}[\Psi, X^3, X^4, \mathcal{L}_5], \quad (2.20)$$

where Ψ is a generic function of matrix degrees of freedom, is a suitable choice for our purpose. In this gauge our BPS equations reduces to a simple first order equations in time which could be solved by a simple exponential. It, however, remains to ensure the Gauss law to obtain the BPS configurations. This Gauss law is the main equation to be solved and for the configurations in which X^a 's are turned off, in the above gauge it takes the form:

$$[X^i, \dot{X}^i] - \frac{1}{g_s} ([X^i, [X^i, X^1, X^2, \mathcal{L}_5]] + [X^i, [X^i, X^3, X^4, \mathcal{L}_5]]) = 0. \quad (2.21)$$

Before discussing various solutions of the BPS equations, it is worth noting that (2.20) does not completely fix the $U(J)$ gauge symmetry. As $[\mathcal{L}_5, X, Y, \mathcal{L}_5] = 0$, it fixes \mathcal{A}_0 only up to $U(1)_\alpha$ rotations (*cf.* (2.13)).

3 Half BPS Solutions

This part is mainly a review of [9], we repeat it here for completeness. We'll also add some more physical discussions regarding the connection of these solutions and the LLM geometries [10]. Half BPS configurations are those which preserve all the dynamical supercharges and hence both equations (2.15) and (2.16) must vanish for these configurations. In terms of the projector method, we should choose $\mathcal{P} = \mathbf{1}$ as the projection matrix. The BPS equations imply that X 's should be time independent and

$$[X^i, X^j, X^k, \mathcal{L}_5] = -\frac{\mu g_s}{R_-} \epsilon^{ijkl} X^l \quad (3.1a)$$

$$[X^a, X^b, X^c, \mathcal{L}_5] = -\frac{\mu g_s}{R_-} \epsilon^{abcd} X^d \quad (3.1b)$$

$$[X^a, X^b, X^i, \mathcal{L}_5] = [X^a, X^i, X^j, \mathcal{L}_5] = 0. \quad (3.1c)$$

For half BPS configurations $P_i = P_a = 0$ and hence the Gauss law is trivially satisfied.

The simplest solution is of course $X^i = X^a = 0$. The first non-trivial set of solutions is obtained by setting X^a (or X^i 's) equal to zero. In this case, (3.1b,c) is evident. To solve (3.1a), let us introduce

$$X^i = \left(\frac{\mu g_s}{R_-}\right)^{1/2} \mathcal{J}^i, \quad \mathcal{L}_5 \equiv \mathcal{J}_5, \quad (3.2)$$

where \mathcal{J}^i 's are in $J \times J$ representations of $Spin(4)$ and \mathcal{J}_5 is the chirality matrix, satisfying [9] (see Appendix A for the explicit form of \mathcal{J}^i matrices)

$$[\mathcal{J}^i, \mathcal{J}^j, \mathcal{J}^k, \mathcal{J}_5] = -\epsilon^{ijkl} \mathcal{J}^l. \quad (3.3)$$

The above specifies \mathcal{J} 's, they can however be in reducible or irreducible representations of $Spin(4)$. If in the irreducible representation, \mathcal{J} 's fulfill

$$\sum_{i=1}^4 \delta_{ij} \mathcal{J}^i \mathcal{J}^j = J \mathbf{1}_{J \times J}. \quad (3.4)$$

In the case of reducible representations, $\sum_{i=1}^4 \delta_{ij} \mathcal{J}^i \mathcal{J}^j$ becomes block diagonal and in each block its value is equal to the size of the block [9].

In the irreducible representation $X^i = \left(\frac{\mu g_s}{R_-}\right)^{1/2} \mathcal{J}^i$ defines a *single fuzzy three sphere* of radius^{3,4}

$$R^2 = l^2 J, \quad l^2 \equiv \frac{\mu g_s}{R_-} l_s^2. \quad (3.5)$$

In the reducible cases we have concentric multi fuzzy three sphere configurations whose radii squared sum to R^2 . These fuzzy spheres are in fact “quantized” spherical three brane giants.⁵ The multi giant solutions is completely specified once we give the set of $\{J_k\}$'s whose sum is J :

$$\sum_{k=1} J_k = J. \quad (3.6)$$

³It is remarkable that in terms of the $AdS_5 \times S^5$ parameters, the “fuzziness” or the *tiny graviton scale* l in Planck units is related to $1/N$ as [1] (*cf.* (2.3))

$$l^4 = \frac{1}{N} l_p^4.$$

⁴We should point out that, the appearance of \mathcal{L}_5 in the definition of the fuzzy three sphere, as explained in [1, 9], is related to the fact that in the $SO(\text{even})$ groups (in particular $SO(4)$) we have the chirality operator. More intuitively, a fuzzy three sphere may be obtained from a fuzzy four sphere once we restrict ourselves to a narrow strip around the equator; due to the fuzziness this cutting cannot be done exactly at the equator [9]. At the level of TGMT and quantum type IIB string theory, however, \mathcal{L}_5 is the reminiscent of the 11th circle once we compactify M-theory on a shrinking two torus to obtain type IIB string theory [9]. In this viewpoint the spherical three branes are M-theory five branes wrapping the T^2 .

⁵In terms of the matrix model presented in [15] the vacuum solutions differ from our fuzzy three spheres in the sense that they are of the form of three spheres which are realized as fuzzy two spheres with an *Abelian* $U(1)$ fiber over it.

The above equation, the problem of partition of a given integer into non-negative integers, is solved by Young tableaux of J boxes, with J_k number of boxes in each column (or row). Our solution has a \mathbb{Z}_2 symmetry which exchanges the columns and rows on the Young tableau, corresponding to giants and dual giants exchange symmetry [13, 20].

One can now find the most general solutions with both X^i and X^a non-zero, using \mathcal{J} 's. It is enough to take both of X^i and X^a to be proportional to \mathcal{J} 's but choose \mathcal{J} 's to be in *reducible* representations in such a way that non-zero parts of X^i 's and X^a 's do not overlap, *i.e.* $[X^i, X^a] = 0$. In this case we have a collection of both giants and dual giants of various radii. The giants are centered at $X^a = 0$ while the dual giants at $X^i = 0$.

It should be noted that equations (3.2) and (3.3) define the half BPS configurations up to $U(J)$ and $SO(4)$ rotations, e.g. obviously if X^i is a fuzzy sphere solution $\tilde{X}^i = UX^iU^{-1}$, $U \in U(J)$ and $\tilde{X}^i = R_{ij}X^j$, $R \in SO(4)$ also describe the same physical configuration.

In [9] a one-to-one connection between half BPS configurations of the TGMT and the LLM geometries which are deformations about the plane-wave geometry were demonstrated. These deformations in the LLM language are given by a ladder type black and white configuration in the (x_1, x_2) -plane [10] and x_1 is to be identified with the X^- direction. In the TGMT X^- is compactified on a circle of radius R_-/μ in string units. The x_2 direction, however, is half of the difference of the radii squared of the two three spheres in the LLM geometry (*cf.* equation (2.20) of [10]). Since we should be able to wrap our fuzzy spheres on these three spheres, or equivalently for the LLM geometries viewed and probed by the “quantized” spherical three brane probes, equation (3.5) implies that the spectrum of x_2 should be quantized in units of l^2 . (Note that in the LLM conventions x_1 and x_2 both have dimension length squared.) That is

$$x_2 = l^2 \times k, \quad k \in \mathbb{Z}. \quad (3.7)$$

In other words, $\Delta x_2 = l^2$. Since $\Delta x_1 = \frac{R_-}{\mu} l_s^2$,

$$\text{smallest volume on } (x_1, x_2)\text{-plane} = \Delta x_1 \Delta x_2 = l_s^4 g_s = l_p^4. \quad (3.8)$$

This is remarkable because that is exactly the result coming from the semiclassical analysis of quantization of the (five-form) flux in the LLM geometries [10]. This result can, however, also come from $[x_1, x_2] = il_p^4$ relation, if one assumes (x_1, x_2) -plane to be a noncommutative Moyal plane. In the case of our interest we are in fact dealing with a noncommutative cylinder (e.g. see [21]) and the noncommutative structure is a direct outcome of the TGMT formulation.

In the TGMT setup with finite J there is an upper limit on the radii of the three spheres set by J . That is, to cover the whole LLM geometry one should take infinite J limit. This limit could be taken either keeping R_-/μ fixed or sending it to infinity, keeping g_s fixed.

The other conclusion inferred from the quantization of the three sphere radii in the LLM geometries is that the spectrum of the y coordinate in the LLM coordinate system is also quantized, *i.e.* $\Delta y \sim l^2$. This latter result is not an outcome of the LLM geometry setup, or a direct consequence of the corresponding dual $\mathcal{N} = 4$ SYM analysis. (In the finite J , y also has a finite extent and it has been cut off at $R^2/2 = l^2 J/2$.) Moreover, if we follow the same line of arguments we find that the function z in the LLM geometries [10] should also be quantized, and at arbitrary y , z should only take fractional values, ranging from $-1/2$ to $+1/2$ [22].

4 Less BPS Solutions

Having briefly studied 1/2 BPS configurations, now we investigate less BPS configurations including those of 1/4 and 1/8 BPS solutions. Following the strategy explained in section 2.3, we should find the appropriate projector \mathcal{P} for solving (2.15) and (2.16) as well as the Gauss law constraint. In this section we will focus on the projectors which in general keep 1/16 or 1/8 of supersymmetry. Since our fermions carry two sets of fermionic indices we should in principle introduce two projectors for each of the two $SO(4)$ subspaces. In section 4.1 we study configurations coming from deformations of the 1/2 BPS fuzzy sphere solutions of the previous section. Here we have 1/8 and 1/4 BPS states. Within our construction the 1/4 BPS states are obtained as special cases of 1/8 BPS states and 1/2 BPS configurations as special case of 1/4 BPS. In section 4.2 we study 1/8 BPS configurations which are not related to 1/2 BPS fuzzy sphere solutions for any value of the parameters defining the solutions. In section 4.3 we double check our BPS analysis by working out the energy and other quantum numbers of the configurations and directly verify the BPS condition using the right hand side of the superalgebra.

4.1 Multi spin 1/2 BPS spherical branes

Here we identify a class of configurations which keep 2 or 4 number of supercharges. All the solutions of this class physically correspond to rotating 1/2 BPS spherical three branes of the previous section. In section 4.1.1 we work out and analyze 1/8 BPS configurations and in section 4.1.2 we study 1/4 BPS configurations.

4.1.1 1/8 BPS configurations

To obtain 1/8 BPS configurations coming from deformations of fuzzy three spheres, let us start with the case in which we have a single fuzzy three sphere in the X^i directions, setting

$X^a = P^a = 0$. The appropriate projection is

$$\bar{\mathcal{P}} = \frac{1}{2}(1 + i\bar{\sigma}^{12}) \quad \text{OR} \quad \mathcal{P} = \frac{1}{2}(1 + i\sigma^{12}). \quad (4.1)$$

Note that these projectors only act on the first index of our fermions, *i.e.* the Weyl indices of $SO(4)_i$. $\text{Tr } \bar{\mathcal{P}}$ or $\text{Tr } \mathcal{P}$ over the space of Weyl indices of $SO(4)_i \times SO(4)_a$ equals to 4 and hence we can describe 1/8 BPS states with either of these projectors. Plugging $\bar{\mathcal{P}}$ into the fermion SUSY variations we obtain

$$\delta_\epsilon(\theta_{\alpha\beta})_{rs} = \left(P^l + i\frac{\mu}{R_-}X^l + \frac{i}{3!g_s}\epsilon^{ijkl}[X^i, X^j, X^k, \mathcal{L}_5] \right)_{rs} (\sigma^l)_\alpha^\gamma \frac{1}{2}(\delta_\gamma^\eta + i\sigma_\gamma^{12\eta})\epsilon_{\eta\beta}^0 \quad (4.2)$$

while $\delta_\epsilon(\theta_{\dot{\alpha}\dot{\beta}})$ can never become zero. If we choose \mathcal{P} , $\delta_\epsilon(\theta_{\alpha\beta})$ is non-vanishing and

$$\delta_\epsilon(\theta_{\dot{\alpha}\dot{\beta}})_{rs} = \left(P^l + i\frac{\mu}{R_-}X^l + \frac{i}{3!g_s}\epsilon^{ijkl}[X^i, X^j, X^k, \mathcal{L}_5] \right)_{rs} (\sigma^l)_{\dot{\alpha}}^\gamma \frac{1}{2}(\delta_\gamma^\eta + i\sigma_\gamma^{12\eta})\epsilon_{\eta\dot{\beta}}^0 \quad (4.3)$$

Expanding and setting coefficient of different σ 's and $\bar{\sigma}$'s to zero independently, we have

$$(P^1 + i\frac{\mu}{R_-}X^1) - i(P^2 + i\frac{\mu}{R_-}X^2) - \frac{i}{g_s}[X^2, X^3, X^4, \mathcal{L}_5] + \frac{1}{g_s}[X^1, X^3, X^4, \mathcal{L}_5] = 0, \quad (4.4a)$$

$$(P^3 + i\frac{\mu}{R_-}X^3) - i(P^4 + i\frac{\mu}{R_-}X^4) - \frac{i}{g_s}[X^1, X^2, X^4, \mathcal{L}_5] + \frac{1}{g_s}[X^1, X^2, X^3, \mathcal{L}_5] = 0, \quad (4.4b)$$

for the projector $\bar{\mathcal{P}}$ (resulting form (4.2)) and

$$(P^1 + i\frac{\mu}{R_-}X^1) - i(P^2 + i\frac{\mu}{R_-}X^2) - \frac{i}{g_s}[X^2, X^3, X^4, \mathcal{L}_5] + \frac{1}{g_s}[X^1, X^3, X^4, \mathcal{L}_5] = 0, \quad (4.5a)$$

$$(P^3 + i\frac{\mu}{R_-}X^3) + i(P^4 + i\frac{\mu}{R_-}X^4) - \frac{i}{g_s}[X^1, X^2, X^4, \mathcal{L}_5] - \frac{1}{g_s}[X^1, X^2, X^3, \mathcal{L}_5] = 0, \quad (4.5b)$$

resulting from (4.3) for the other possible projector \mathcal{P} .

Looking for 1/8 BPS configurations we should only consider one set of the above equations. For 1/4 BPS states, however, (4.4) and (4.5) should be solved simultaneously, this will be done in further detail in section 4.1.2. From now on we only focus on (4.4a), (4.4b). These complicated looking equations are simplified drastically once we fix the (2.20) gauge:

$$(\dot{X}^2 + i\dot{X}^1) + \frac{i\mu}{R_-}(X^2 + iX^1) = 0, \quad (4.6a)$$

$$(\dot{X}^4 + i\dot{X}^3) + \frac{i\mu}{R_-}(X^4 + iX^3) = 0, \quad (4.6b)$$

whose solution are

$$(X^2 + iX^1) = (X_0^2 + iX_0^1) \exp\left(-\frac{i\mu}{R_-}t\right), \quad (4.7a)$$

$$(X^4 + iX^3) = (X_0^4 + iX_0^3) \exp\left(-\frac{i\mu}{R_-}t\right). \quad (4.7b)$$

In this gauge, the Gauss law takes a non-trivial form

$$\begin{aligned} \frac{2\mu}{R_-} [X^1, X^2] + \frac{1}{g_s} \left([X^1, [X^1, X^3, X^4, \mathcal{L}_5]] + [X^2, [X^2, X^3, X^4, \mathcal{L}_5]] \right) + \\ \frac{2\mu}{R_-} [X^3, X^4] + \frac{1}{g_s} \left([X^3, [X^1, X^2, X^3, \mathcal{L}_5]] + [X^4, [X^1, X^2, X^4, \mathcal{L}_5]] \right) = 0. \end{aligned} \quad (4.8)$$

It is readily seen that the time dependence in the Gauss law drops out and (4.8) reduces to an equation among X_0^i 's. To simplify the notation we will use X^i instead of X_0^i 's. The main task is now to solve (4.8).

To solve (4.8) we note that all the geometric fluctuations of a three sphere can be expanded in terms of $SO(4)$ spherical harmonics, and hence one may try the ansatz

$$X^i \sim T^i_{i_1 i_2 \dots i_n} \mathcal{J}^{i_1} \mathcal{J}^{i_2} \dots \mathcal{J}^{i_n} \quad (4.9)$$

where T is a tensor of $SO(4)_i$ of rank $n + 1$.

Conventions:

Hereafter, unless explicitly specified, we take \mathcal{J} 's to be in $J \times J$ irreducible representation of $SO(4)$. In our notation \sim means that X^i 's are measured in units of $l = \left(\frac{\mu g_s}{R_-}\right)^{1/2} l_s$. That is, $X^i \sim \mathcal{J}^i$ means $X^i = l \mathcal{J}^i$.

In general one may search for solutions with arbitrary rank n . In the present work we restrict ourselves to $n = 1$. The general case will be considered elsewhere [23]. For the $n = 1$ case, we have

$$X^i \sim T^i_j \mathcal{J}^j. \quad (4.10)$$

Plugging (4.10) into (4.8) and using (3.3), the Gauss law equation reduces to simple algebraic equation for the tensor T . Recalling that we also have the \mathcal{L}_5 in our disposal, (4.10) is not the most general X^i 's at $n = 1$ level, it is rather

$$X^i \sim M^i_j \mathcal{J}^j + N^i_j i \mathcal{J}_5 \mathcal{J}^j \quad (4.11)$$

where M and N are 4×4 $SO(4)$ covariant matrices. Of course not all M and N matrices lead to physically distinct configurations. For example, (\tilde{M}, \tilde{N})

$$\begin{aligned} \tilde{M}^i_j &= \cos 2\alpha M^i_j + \sin 2\alpha N^i_j \\ \tilde{N}^i_j &= \cos 2\alpha N^i_j - \sin 2\alpha M^i_j \end{aligned} \quad (4.12)$$

and (M, N) , which are related by a U_α transformation, are physically equivalent. (Recall that fixing the gauge by (2.20) the U_α rotations remains unfixed.) To obtain (4.12) we have used the fact that $\{\mathcal{J}^i, \mathcal{J}_5\} = 0$ (*cf.* (A.8)).

To classify all possible (M, N) configurations it is useful to think them as exponentials of elements of $so(4)_i$ algebra. The 4×4 matrices are then related to symmetric traceless, anti-symmetric, or singlet irreducible representations of $so(4)$. The symmetric M which can be

written as $K - \frac{1}{4}(\text{Tr } K)\mathbf{1}$, for some symmetric matrix K , corresponds to symmetric traceless representation. $M \propto \mathbf{1}$ is of course the singlet representation and if M is in the form of $SO(4)$ rotation matrix it can be written as the exponential of an anti-symmetric $so(4)$ tensor. Let us, for the time being, set $N = 0$. It is evident that the anti-symmetric representation for M does not correspond to a physical deformation of the fuzzy three sphere, it corresponds to zero mode fluctuations [1]. The singlet representation, which is related to the breathing mode of a three sphere brane does not solve the BPS equation (similar statements is also true for membranes of the BMN matrix model [6, 24]). Therefore, for the $N = 0$ the only remaining option is choosing M to be of the form

$$M^i_j = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$$

where we have used six independent $SO(4)$ rotations to bring the M into the diagonal form.

It is straightforward to check that the Gauss law (4.8) is solved with

$$X_1 \sim a\mathcal{J}_1, \quad X_2 \sim b\mathcal{J}_2, \quad X_3 \sim c\mathcal{J}_3, \quad X_4 \sim d\mathcal{J}_4 \quad (4.13)$$

provided that

$$(a^2 + b^2)\frac{cd}{ab} = 2, \quad (c^2 + d^2)\frac{ab}{cd} = 2,$$

or equivalently

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2} + \frac{1}{d^2}, \quad \frac{a}{b} + \frac{b}{a} = \frac{2}{cd}. \quad (4.14)$$

The solutions to (4.14) can be parameterized by the angles θ, ϕ :

$$a^{-1} = \kappa \sin \theta, \quad b^{-1} = \kappa \cos \theta, \quad c^{-1} = \kappa \sin \phi, \quad d^{-1} = \kappa \cos \phi \quad (4.15)$$

where

$$\kappa = \left(\frac{1}{2} \sin 2\theta \sin 2\phi \right)^{-1/2}. \quad (4.16)$$

For $\theta = \phi = \pi/4$, $a = b = c = d = 1$, and hence the above solution reduces to the half BPS spherical solution. This is of course expected, because this family of 1/8 BPS configurations should contain 1/2 BPS ones as special cases. If we expand the solution about $\theta = \phi = \pi/4$ as $\theta = \pi/4 + \delta$, $\phi = \pi/4 + \epsilon$, $\det M = abcd = 1 + \mathcal{O}(\epsilon^2, \delta^2)$. This confirms the expectation that the above BPS modes are coming from the exponentials of symmetric traceless $so(4)$ representations.

As we have already mentioned, the above solution is describing geometric fluctuations (deformations) of a spherical three brane. The shape of the deformed three brane, in the

frame which is rotating in both $X^1 + iX^2$ and $X^3 + iX^4$ directions with frequency μ/R_- (cf. (4.7)), can be easily worked out:

$$\frac{1}{a^2}X_1^2 + \frac{1}{b^2}X_2^2 + \frac{1}{c^2}X_3^2 + \frac{1}{d^2}X_4^2 = R^2 \mathbf{1} = l^2 J \mathbf{1}_{J \times J}$$

or equivalently

$$\sin^2 \theta X_1^2 + \cos^2 \theta X_2^2 + \sin^2 \phi X_3^2 + \cos^2 \phi X_4^2 = R^2 / \kappa^2 \quad (4.17)$$

In the large matrices limit the above equation is describing a three brane of an ellipsoidal form. Although generic cross sections of the above ellipsoid is like an ellipse, it has two independent circular sections, e.g. along 2-plane

$$X_3 = AX_1, \quad X_4 = 0$$

where $A^2 = \frac{\cos 2\theta}{\sin^2 \phi}$ and 2-plane

$$X_2 = 0, \quad X_1 = BX_3$$

where $B^2 = \frac{\cos 2\phi}{\sin^2 \theta}$. In both cases the section is circular with radii squares $R_1^2 = R^2 \tan \theta \sin 2\phi$ and $R_2^2 = R^2 \tan \phi \sin 2\theta$, respectively. This configuration, hence, generically preserves $U(1) \times U(1)$ isometry out of the $SO(4)$. There are special cases with larger isometry subgroup which will be discussed in section 4.1.2.

The total energy and the angular momentum for this configuration can be evaluated once we plug the ansatz (4.13) into (B.4), (B.2)

$$\mathbf{H} = \frac{\mu^2 l^2}{4R_-} \left((a - bcd)^2 + (b - acd)^2 + (c - abd)^2 + (d - abc)^2 \right) \text{Tr} (\mathcal{J}^2) \quad (4.18)$$

$$\mathbf{J}_{12} = \frac{\mu l^2}{4R_-} (a^2 + b^2 - 2abcd) \text{Tr} (\mathcal{J}^2) \quad (4.19)$$

$$\mathbf{J}_{34} = \frac{\mu l^2}{4R_-} (c^2 + d^2 - 2abcd) \text{Tr} (\mathcal{J}^2) \quad (4.20)$$

and other angular momentum components vanish. (Note that we have used the fact that $\text{Tr} (\mathcal{J}_1^2) = \text{Tr} (\mathcal{J}_2^2) = \text{Tr} (\mathcal{J}_3^2) = \text{Tr} (\mathcal{J}_4^2) = \frac{1}{4} \text{Tr} (\mathcal{J}^2) = J^2/4$). Using (4.14), energy of the 1/8 BPS configurations becomes

$$\mathbf{H} = \frac{\mu}{2g_{eff}^2} \left(\frac{\sin 2\phi}{\sin 2\theta} + \frac{\sin 2\theta}{\sin 2\phi} - 2 \sin 2\theta \sin 2\phi \right) \quad (4.21)$$

where as discussed in [1, 25, 26]

$$g_{eff}^2 = \left(\frac{R_-}{\mu J} \right)^2 \frac{1}{g_s} = \frac{1}{(\mu p^+)^2 g_s} \quad (4.22)$$

is the effective coupling of the field theory on a giant graviton of radius R , $R^2/l_s^2 = \mu p^+ g_s$ (3.5). This result is expected if the above deformations are parameterizing the field excitations of an effective (Yang-Mills) gauge theory of coupling g_{eff} which resides on a giant graviton. Similar results was obtained for deformations of the giant in a background NSNS B -field, or when the magnetic field on the brane is turned on [26]. The angular momenta is obtained to be

$$\begin{aligned} \mathbf{J}_{12} &= \frac{1}{2g_{eff}^2} \left(\frac{\sin 2\phi}{\sin 2\theta} - \sin 2\theta \sin 2\phi \right) \\ \mathbf{J}_{34} &= \frac{1}{2g_{eff}^2} \left(\frac{\sin 2\theta}{\sin 2\phi} - \sin 2\theta \sin 2\phi \right). \end{aligned} \tag{4.23}$$

As we see and expected under the exchange of θ and ϕ \mathbf{J}_{12} goes over to \mathbf{J}_{34} while the expression for energy is invariant.

Here we have only considered the classical matrix theory, if we performed computations with the quantized matrices we should obtain quantized values for angular momenta. Imposing semi-classical quantization of the angular momenta by hand we see that the deformation parameters, θ, ϕ cannot take continuous values and become quantized as well.

Deformations of a three brane (giant graviton) besides the geometric modes that we have considered also contains internal ‘‘photon’’ modes which one would expect to be in the same super-multiplet as the geometric fluctuations [25]. These photon modes are also among the 1/8 (and also possibly 1/4) BPS states. As we showed the $N = 0$ deformations only include the geometric modes. To see the photon modes one should then consider $N \neq 0$ cases. To simplify the argument let us only consider the infinitesimal deformations of the giant from the spherical case. Since we only want to focus on the photonic modes, or the \mathcal{L}_5 piece, we may take $M = \mathbf{1}$, *i.e.* $X^i = \mathcal{J}^i + iN^i{}_j \mathcal{L}_5 \mathcal{J}^j$. One can show that among the three possibilities for N , only those which are in the anti-symmetric representation of $so(4)$ lead to independent (non-geometric) modes. The computations may be performed using the identities presented in Appendix A. Since they are tedious and not illuminating we will not present them here. However, here is a simple intuitive argument for this statement. The geometric fluctuations (which are linear in \mathcal{J}_i) will all show up in the equation defining the shape, $g_{ij} X^i X^j \propto \mathbf{1}_{J \times J}$ where g_{ij} is a symmetric 4×4 matrix. It is now easy to check that for $X^i \sim \mathcal{J}^i + iN^i{}_j \mathcal{L}_5 \mathcal{J}^j$ where N is the infinitesimal deformation parameter, the anti-symmetric part of N , F_{ij} , is not completely captured in the shape equation. For anti-symmetric F , $g_{ij} X^i X^j$ has a term of the form $iF_{ij} \mathcal{L}_5 [\mathcal{J}^i, \mathcal{J}^j]$. This F_{ij} may directly be related to the gauge field strength living on the brane.⁶ Using $SO(4)$ rotations any anti-symmetric $so(4)$ tensor F_{ij} can always be

⁶Recall that to obtain the TGMT action we discretized the brane action in which the gauge field was turned off. In the discretization procedure, however, we introduced the \mathcal{L}_5 . It is now apparent that intro-

brought to the skew symmetric form

$$F_{ij} = \begin{pmatrix} 0 & f_1 & 0 & 0 \\ -f_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_2 \\ 0 & 0 & -f_2 & 0 \end{pmatrix} \quad (4.24)$$

where f_1 and f_2 are the two parameters corresponding to the two polarizations of photons.

In sum, the infinitesimal physical BPS deformations of three sphere giants which are linear in \mathcal{J}^i 's can be parameterized as $X^i = \mathcal{J}^i + S^i_j \mathcal{J}^j + iF^i_j \mathcal{L}_5 \mathcal{J}^j$ where S is a symmetric traceless and F an anti-symmetric $so(4)$ tensor. This result is compatible with those of continuum limit [25].

In this section we considered the single giant configurations. Multi giant configurations, where $\sum_i^4 (\mathcal{J}^i)^2$ is block diagonal can also be treated in a very similar manner and hence we do not repeat the details of computations here. For example for a double giant configuration, one has two possibilities: to put the deformations on either of the giants and keep the other one intact, or generically deform both of the giants in such a way that each of them individually satisfy the BPS equation (4.14). In the language of the notations we used here, that is deforming each giant with a set of θ, ϕ parameters.

4.1.2 1/4 BPS configurations

As mentioned earlier the BPS configurations resulting from the projectors \mathcal{P} or $\bar{\mathcal{P}}$ (4.1), *i.e.* solutions to (4.4) or (4.5), preserve 4 supersymmetries, the first preserving two of $Q_{\alpha\dot{\beta}}$ type supercharges while killing all $Q_{\dot{\alpha}\beta}$ supercharges and the other keep two of $Q_{\dot{\alpha}\beta}$, killing $Q_{\alpha\dot{\beta}}$ type supercharges. It is, however, possible to impose both of the projectors simultaneously and obtain a configuration which preserves 8 supercharges.

To obtain such 1/4 BPS configurations one then needs to find solutions to all four (4.4a), (4.4b), (4.5a) and (4.5b) equations. It is readily seen that such solution must be static in 34 directions, that is $\mathbf{J}_{34} = 0$. Moreover, as solutions to (4.4a), (4.4b), one may use (4.23) and impose the $\mathbf{J}_{34} = 0$ condition. This is possible if $\phi = \frac{\pi}{4}$ or $\frac{3\pi}{4}$. Therefore, quarter BPS solutions constitute a one parameter family of the rotating spherical branes with

$$\mathbf{H} \equiv \mu \mathbf{J}_{12} = \frac{\mu}{2g_{eff}^2} \left(\frac{1}{\sin 2\theta} - \sin 2\theta \right), \quad (4.25)$$

which is positive definite and is zero for $\theta = \pi/4$ where we recover 1/2 BPS configurations. One can also work out the shape of the 1/4 BPS rotating branes:

$$2 \sin^2 \theta X_1^2 + 2 \cos^2 \theta X_2^2 + X_3^2 + X_4^2 = R^2 \sin 2\theta \quad (4.26)$$

duction of the \mathcal{L}_5 is crucial for reviving the gauge field which lives on the three brane giants. Needless to emphasize that presence of this photon modes and gauge fields is necessary to have a supersymmetric theory.

First we note that it has a two sphere cross section. To see this set $X_1 = r \sin \alpha$ and $X_2 = r \cos \alpha$. For $\alpha = \pi/4$ we recover a 2-sphere of radius squared $R^2 \sin 2\theta$ in $r34$ -space. This exhibits the $SU(2)$ isometry of the solution. There is a circular cross section e.g. at $X_4 = 0$, $X_3 = AX_1$ ($A^2 = 2 \cos \theta$), with radius squared $R^2 \tan 2\theta$. Therefore, altogether for 1/4 BPS configuration we have $SU(2) \times U(1)$ isometries out of whole $SO(4)$.

For the multi giant case, the configuration which is made of several concentric 1/4 BPS states of the same kind (*i.e.* e.g both have vanishing \mathbf{J}_{34}) still remains 1/4 BPS. However, if we have concentric 1/4 BPS states of different kind, e.g. one of them has a vanishing \mathbf{J}_{34} and the other vanishing \mathbf{J}_{12} , the system altogether is a 1/8 BPS configuration.

We discussed that there is another class of 1/8 BPS states which are not of the form of geometric fluctuations of a spherical brane, rather related to turning on a gauge field on the brane. These states are specified by an anti-symmetric rank two tensor of $SO(4)$ F_{ij} , that is a $\mathbf{6}$ of $SO(4)$. In terms of the $SU(2) \times SU(2)$ it is $(\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3})$. If we take the self-dual (or anti-self dual) part of F_{ij} , in the notation of (4.24) that is $f_1 = f_2$ (or $f_1 = -f_2$), we will have 1/4 BPS state. As is obvious, for these cases one of the $SU(2)$'s, in which the F_{ij} is a singlet, remains intact and hence the symmetry is $SU(2) \times U(1)$.

4.2 Other 1/8 BPS configurations

So far we have analyzed configurations which could be obtained as (continuous) deformations of spherical 1/2 BPS three brane giants. In this section, we consider cases which are not of this kind. We discuss two examples. One of them, the hyperboloid three brane, only exists in the infinite J limit while the other are generically of the form of deformed (rotating) spherical branes which are extended in 12 and 56 directions.

4.2.1 Hyperboloid case

Another class of solutions can be extracted out of matrices \mathcal{K}_i which satisfy the following algebraic structure

$$\begin{aligned} [\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{L}_5] &= -\mathcal{K}_4, & [\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_4, \mathcal{L}_5] &= -\mathcal{K}_3 \\ [\mathcal{K}_1, \mathcal{K}_3, \mathcal{K}_4, \mathcal{L}_5] &= -\mathcal{K}_2, & [\mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4, \mathcal{L}_5] &= -\mathcal{K}_1. \end{aligned} \quad (4.27)$$

Here \mathcal{K}_i 's are in the representations of $SO(2, 2)$, rather than $SO(4)$ ($[\mathcal{K}_i, \mathcal{K}_j]$ form generators of $so(2, 2)$). It is easy to check that the Gauss law can be solved by

$$X_1 \sim a\mathcal{K}_1, \quad X_2 \sim b\mathcal{K}_2, \quad X_3 \sim c\mathcal{K}_3, \quad X_4 \sim d\mathcal{K}_4 \quad (4.28)$$

provided that

$$(a^2 - b^2) \frac{cd}{ab} = 2, \quad (c^2 - d^2) \frac{ab}{cd} = 2 \quad (4.29)$$

or equivalently

$$\frac{1}{b^2} - \frac{1}{a^2} = \frac{1}{d^2} - \frac{1}{c^2}, \quad \frac{a}{b} - \frac{b}{a} = \frac{2}{cd} \quad (4.30)$$

The solution can be identified by θ, ϕ as

$$a^{-1} = \kappa \sinh \theta, \quad b^{-1} = \kappa \cosh \theta, \quad c^{-1} = \kappa \sinh \phi, \quad d^{-1} = \kappa \cosh \phi \quad (4.31)$$

where

$$\kappa = \left(\frac{1}{2} \sinh 2\theta \sinh 2\phi \right)^{-1/2} \quad (4.32)$$

In this case, the Casimir operator is $\mathcal{K}^2 = -\mathcal{K}_1^2 + \mathcal{K}_2^2 - \mathcal{K}_3^2 + \mathcal{K}_4^2$ and therefore, the shape can be described by

$$-\sinh^2 \theta X_1^2 + \cosh^2 \theta X_2^2 - \sinh^2 \phi X_3^2 + \cosh^2 \phi X_4^2 = R^2 / \kappa^2 \quad (4.33)$$

The above describes a hyperboloid which is extended off to infinity in all directions. It has two independent circular cross sections while generically we also have hyperbolic sections as well. This configuration generically preserves $U(1) \times U(1)$ symmetry out of the $SO(4)$. As we can see there is no real θ and ϕ for which the above goes to a spherical brane. Similar configurations (hyperbolic membrane solutions) in the BMN matrix model, i.e. the tiny membrane graviton matrix theory, has been constructed in [18].

It is straightforward to see that the above solutions *formally* could be obtained from those of section 4.1 by Wick rotation on θ and ϕ . Doing so, noting that \mathcal{J} 's are hermitian the corresponding \mathcal{K} 's cannot be hermitian. The hermiticity problem of X^i is resolved once we note that $so(2, 2)$, unlike $so(4)$, is non-compact and has no finite dimensional unitary representations.⁷ Therefore, solutions of this class only exist for infinite J . This is compatible with the intuition that an infinite size brane cannot be built out of finite number of building blocks, the tiny gravitons. One should however, note that the infinite J limit should be taken with a special care to keep physical light-cone moment finite, that is, $J, R_- \rightarrow \infty, \mu p^+ = \frac{\mu J}{R_-} = \text{fixed}$. In the large matrices limit, one can understand this hyperbolic solutions are infinite deformation limit of the spherical branes of the previous section.

Total energy and the angular momentum for this configuration can be evaluated

$$\mathbf{H} = \frac{\mu^2 l^2}{4R_-} \left(-(a + bcd)^2 + (b - acd)^2 - (c + abd)^2 + (d - abc)^2 \right) \text{Tr} (\mathcal{K}^2) \quad (4.34)$$

$$\mathbf{J}_{12} = \frac{\mu l^2}{4R_-} (b^2 - a^2 - 2abcd) \text{Tr} (\mathcal{K}^2) \quad (4.35)$$

⁷Using the Wick rotation trick it can be checked that hermiticity of the four bracket equations (4.27) forces us to Wick rotate θ and ϕ together. Hence, it is not possible to obtain a solution based on the generators of $so(3, 1)$.

$$\mathbf{J}_{34} = \frac{\mu l^2}{4R_-} (d^2 - c^2 - 2abcd) \text{Tr}(\mathcal{K}^2) \quad (4.36)$$

and other angular momentum component vanish. Note that in the above equations, we have used the fact that $\text{Tr}(\mathcal{K}_2^2) = \text{Tr}(\mathcal{K}_4^2) = -\text{Tr}(\mathcal{K}_1^2) = -\text{Tr}(\mathcal{K}_3^2) = \frac{1}{4}\text{Tr}(\mathcal{K}^2)$. Using (4.31), energy and angular momenta read

$$\mathbf{H} = \frac{\mu}{2g_{eff}^2} \left(\frac{\sinh 2\phi}{\sinh 2\theta} + \frac{\sinh 2\theta}{\sinh 2\phi} + 2 \sinh 2\theta \sinh 2\phi \right) \quad (4.37)$$

$$\begin{aligned} \mathbf{J}_{12} &= \frac{1}{2g_{eff}^2} \left(\frac{\sinh 2\phi}{\sinh 2\theta} + \sinh 2\theta \sinh 2\phi \right) \\ \mathbf{J}_{34} &= \frac{1}{2g_{eff}^2} \left(\frac{\sinh 2\theta}{\sinh 2\phi} + \sinh 2\theta \sinh 2\phi \right) \end{aligned} \quad (4.38)$$

where, despite of having infinite size matrices, g_{eff} is still given by (4.22), that is $g_{eff}^{-2} = (\mu p^+)^2 g_s$. From the energy expression it is readily seen that the Hamiltonian is positive definite and its value is finite.

It is worth noting that, unlike the case of previous section, here we do not have special cases in which we recover 1/4 BPS solutions and where the $U(1) \times U(1)$ isometry enhances to $SU(2) \times U(1)$. That is because \mathbf{J}_{34} or \mathbf{J}_{12} never vanish while the other one remains finite.

4.2.2 Deformed spherical branes in X^1, X^2, X^5, X^6 subspace

Let us now consider the case which involves both X^i and X^a directions in a non-trivial way. Take $X^3 = X^4 = X^7 = X^8 = 0$ while the other X 's are turned on. For this case (2.15) takes the form

$$\begin{aligned} \delta_\epsilon(\theta_{\alpha\beta}) &= \left((\sigma^1)_{\dot{\alpha}}^{\dot{\rho}} (P^1 + \frac{i\mu}{R_-} X^1) \delta_{\dot{\beta}}^{\dot{\rho}} + (\sigma^2)_{\dot{\alpha}}^{\dot{\rho}} (P^2 + \frac{i\mu}{R_-} X^2) \delta_{\dot{\beta}}^{\dot{\rho}} \right. \\ &\quad \left. + \frac{1}{2g_s} (\sigma^1)_{\dot{\alpha}}^{\dot{\rho}} (i\sigma^{56})_{\dot{\beta}}^{\dot{\rho}} [X^1, X^5, X^6, \mathcal{L}_5] + \frac{1}{2g_s} (\sigma^2)_{\dot{\alpha}}^{\dot{\rho}} (i\sigma^{56})_{\dot{\beta}}^{\dot{\rho}} [X^2, X^5, X^6, \mathcal{L}_5] \right) \epsilon_{\dot{\rho}\dot{\beta}}^0 \\ &\quad + \left((\sigma^5)_{\dot{\beta}}^{\dot{\rho}} (P^5 + \frac{i\mu}{R_-} X^5) \delta_{\dot{\alpha}}^{\dot{\rho}} + (\sigma^6)_{\dot{\beta}}^{\dot{\rho}} (P^6 + \frac{i\mu}{R_-} X^6) \delta_{\dot{\alpha}}^{\dot{\rho}} \right. \\ &\quad \left. + \frac{1}{2g_s} (\sigma^5)_{\dot{\beta}}^{\dot{\rho}} (i\sigma^{12})_{\dot{\alpha}}^{\dot{\rho}} [X^5, X^1, X^2, \mathcal{L}_5] + \frac{1}{2g_s} (\sigma^6)_{\dot{\beta}}^{\dot{\rho}} (i\sigma^{12})_{\dot{\alpha}}^{\dot{\rho}} [X^6, X^1, X^2, \mathcal{L}_5] \right) \epsilon_{\dot{\rho}\dot{\alpha}}^0 \end{aligned} \quad (4.39)$$

Similarly one may work out $\delta_\epsilon(\theta_{\dot{\alpha}\dot{\beta}})$ using (2.16).

We choose our projector to be

$$\mathcal{P} = \mathcal{P}_1 \mathcal{P}_2, \quad \mathcal{P}_1 = \frac{1}{2}(1 + i\sigma^{12}) \quad \mathcal{P}_2 = \frac{1}{2}(1 + i\sigma^{56}) \quad (4.40)$$

Inserting projection and Expanding and setting coefficient of different σ 's equal to zero independently, the BPS equations are obtained to be

$$(P^1 + i\frac{\mu}{R_-}X^1) - i(P^2 + i\frac{\mu}{R_-}X^2) - \frac{i}{g_s}[X^2, X^5, X^6, \mathcal{L}_5] + \frac{1}{g_s}[X^1, X^5, X^6, \mathcal{L}_5] = 0 \quad (4.41a)$$

$$(P^5 + i\frac{\mu}{R_-}X^5) - i(P^6 + i\frac{\mu}{R_-}X^6) - \frac{i}{g_s}[X^6, X^1, X^2, \mathcal{L}_5] + \frac{1}{g_s}[X^5, X^1, X^2, \mathcal{L}_5] = 0 \quad (4.41b)$$

These are similar to (4.4a), (4.4b) if one exchanges $X^5 \rightarrow X^3$, $X^6 \rightarrow X^4$ and hence could be solved with the same trick (note that this exchange should also be done in the gauge fixing condition (2.20)). That is, one may insert $X^1 \sim a\mathcal{J}_1$, $X^2 \sim b\mathcal{J}_2$, $X^5 \sim c\mathcal{J}_3$, $X^6 \sim d\mathcal{J}_4$ into (4.41) where a, b, c and d are of the form (4.15).

There is, however, a subtlety here. It is easy to verify that $\text{Tr } \mathcal{P} = 2$, rather than 4, which is needed for obtaining 1/8 BPS configuration and it may seem that the above equations are describing 1/16 BPS solutions. This issue is resolved once we note that there is another projector, namely

$$\bar{\mathcal{P}} = \bar{\mathcal{P}}_1\bar{\mathcal{P}}_2, \quad \bar{\mathcal{P}}_1 = \frac{1}{2}(1 + i\bar{\sigma}^{12}) \quad \bar{\mathcal{P}}_2 = \frac{1}{2}(1 + i\bar{\sigma}^{56}) \quad (4.42)$$

which also leads to the same BPS equations as (4.41). Therefore, solutions of (4.41) will preserve both of the supersymmetries resulting from \mathcal{P} and $\bar{\mathcal{P}}$ projectors and hence they are describing 1/8 BPS states.

Although very similar in equations, the above deformed three sphere configurations are physically totally distinct from those of section 4.1.1. For this class of solutions the (central) extension \mathcal{R}_{ijab} (B.7) does not vanish. Parameterizing the solutions of (4.41) by θ and ϕ , as is done in (4.15), one can now calculate the quantum numbers associated with the above solutions:

$$\mathbf{H} = \frac{\mu}{2g_{eff}^2} \left(\frac{\sin 2\phi}{\sin 2\theta} + \frac{\sin 2\theta}{\sin 2\phi} \right) \quad (4.43a)$$

$$\mathbf{J}_{12} = \frac{1}{2g_{eff}^2} \left(\frac{\sin 2\phi}{\sin 2\theta} - \sin 2\theta \sin 2\phi \right) \quad (4.43b)$$

$$\mathbf{J}_{56} = \frac{1}{2g_{eff}^2} \left(\frac{\sin 2\theta}{\sin 2\phi} - \sin 2\theta \sin 2\phi \right) \quad (4.43c)$$

$$\mathcal{R}_{1256} = \frac{1}{g_s} \text{Tr} ([X^1, X^2, X^5, X^6]\mathcal{L}_5) = +\frac{1}{4g_{eff}^2} \sin 2\theta \sin 2\phi \quad (4.43d)$$

It is note worthy that among these solutions we have 1/4 BPS configurations in special case of $\mathbf{J}_{12} = \mathbf{J}_{56} = 0$ which happens for $\theta = \phi = \pi/4$, \mathcal{R}_{1256} has its maximal value. For this case we have spherical three branes. Out of the $SO(4)_i \times SO(4)_a$ symmetry this configuration preserves $SO(4)_{diag}$ and two extra $U(1)$'s. The radius of this sphere if \mathcal{J} 's are

in the irreducible representations of $SO(4)$ is equal to R (3.5). As discussed in [17] non-zero \mathcal{R}_{1256} corresponds to the (self-dual) four form dipole moment of the spherical three brane.

4.3 Analysis from the superalgebra

In the previous sections we focused on the definition of the BPS states (configurations) which is resulting from the vanishing of supersymmetry variations of the fermions. One can, however, equivalently use the right-hand-side (RHS) of the superalgebra to identify the BPS states. That is, the configurations which have specific relations between their energy and other quantum charges for which the RHS of the supercharge anticommutators vanish are BPS. The number of supersymmetries preserved is then equal to the number of the zero eigenvalues the RHS of the superalgebra has. We analyze the cases of those only in X^i subspace and in X^i, X^a subspaces separately in sections 4.3.1 and 4.3.2.

4.3.1 Deformed 1/2 BPS cases

This case consists of the configurations which have vanishing X^a, P^a , and hence for these configurations $\mathbf{J}_{ab} = 0$ and $\mathcal{R}_{ijab} = 0$. The RHS of the both of the $PSU(2|2)$ superalgebra factors are

$$\{Q_{\alpha\dot{\beta}}, Q^{\dagger\rho\dot{\gamma}}\} = \delta_{\dot{\gamma}}^{\dot{\beta}} \left(\delta_{\alpha}^{\rho} \mathbf{H} + \mu(i\sigma^{12})_{\alpha}^{\rho} \mathbf{J}_{12} + \mu(i\sigma^{34})_{\alpha}^{\rho} \mathbf{J}_{34} \right) \quad (4.44a)$$

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\dot{\rho}\gamma}\} = \delta_{\beta}^{\dot{\rho}} \left(\delta_{\dot{\alpha}}^{\gamma} \mathbf{H} + \mu(i\bar{\sigma}^{12})_{\dot{\alpha}}^{\dot{\rho}} \mathbf{J}_{12} + \mu(i\bar{\sigma}^{34})_{\dot{\alpha}}^{\dot{\rho}} \mathbf{J}_{34} \right) \quad (4.44b)$$

It is convenient to choose the basis such that

$$i\sigma^{12} = i\sigma^{34} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad i\bar{\sigma}^{12} = -i\bar{\sigma}^{34} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (4.45)$$

The demand of having BPS configurations leads to

$$\begin{pmatrix} \mathbf{H} & 0 \\ 0 & \mathbf{H} \end{pmatrix} + \mu \begin{pmatrix} +\mathbf{J}_{12} + \mathbf{J}_{34} & 0 \\ 0 & -\mathbf{J}_{12} - \mathbf{J}_{34} \end{pmatrix} = 0 \quad (4.46)$$

and

$$\begin{pmatrix} \mathbf{H} & 0 \\ 0 & \mathbf{H} \end{pmatrix} + \mu \begin{pmatrix} +\mathbf{J}_{12} - \mathbf{J}_{34} & 0 \\ 0 & -\mathbf{J}_{12} + \mathbf{J}_{34} \end{pmatrix} = 0 \quad (4.47)$$

Dealing with complex supercharges, one should remember that the number of preserved real supersymmetries is twice the number of the zero eigenvalues.

For the configuration with energy and angular momenta of (4.21) and (4.23), it is evident that $\mathbf{H} = \mu(\mathbf{J}_{12} + \mathbf{J}_{34})$ and hence the configuration is 1/8 BPS, as discussed in the previous section. As we see for this case both of the preserved supercharges are coming from a single $PSU(2|2)$.

The 1/4 BPS configurations in this class, as is readily seen from (4.46) and (4.47), can only be obtained if either of \mathbf{J}_{12} or \mathbf{J}_{34} vanishes while the other one is equal to the energy (up to a factor of μ). This is exactly the case for the configurations of section 4.1.2. In this case two of the supercharges are coming from one $PSU(2|2)$ and two of them from the other $PSU(2|2)$. In a similar manner, half BPS configurations must have $\mathbf{J}_{12} = \mathbf{J}_{34} = 0$.

The hyperbolic case which discussed in 4.2.1 can also be analyzed in the same way. For this case only non-vanishing generators are $\mathbf{H}, \mathbf{J}_{12}$ and \mathbf{J}_{34} . For this configuration as seen from (4.37), (4.38) satisfies $\mathbf{H} = \mu(\mathbf{J}_{12} + \mathbf{J}_{34})$ and hence is 1/8 BPS.

4.3.2 Configurations with non-zero \mathcal{R}_{ijab}

For the configurations lying in 1256 direction the algebra reads

$$\{Q_{\alpha\dot{\beta}}, Q^{\dagger\rho\lambda}\} = \delta_{\alpha}^{\rho}\delta_{\dot{\beta}}^{\lambda}\mathbf{H} + \mu(i\sigma^{12})_{\alpha}^{\rho}\delta_{\dot{\beta}}^{\lambda}\mathbf{J}_{12} + \mu\delta_{\alpha}^{\rho}(i\bar{\sigma}^{56})_{\dot{\beta}}^{\lambda}\mathbf{J}_{56} - 4\mu(i\sigma^{12})_{\alpha}^{\rho}(i\bar{\sigma}^{56})_{\dot{\beta}}^{\lambda}\mathcal{R}_{1256} \quad (4.48a)$$

$$\{Q_{\dot{\alpha}\beta}, Q^{\dagger\dot{\rho}\lambda}\} = \delta_{\dot{\alpha}}^{\dot{\rho}}\delta_{\beta}^{\lambda}\mathbf{H} + \mu(i\bar{\sigma}^{12})_{\dot{\alpha}}^{\dot{\rho}}\delta_{\beta}^{\lambda}\mathbf{J}_{12} + \mu\delta_{\dot{\alpha}}^{\dot{\rho}}(i\sigma^{56})_{\beta}^{\lambda}\mathbf{J}_{56} - 4\mu(i\bar{\sigma}^{12})_{\dot{\alpha}}^{\dot{\rho}}(i\sigma^{56})_{\beta}^{\lambda}\mathcal{R}_{1256} \quad (4.48b)$$

In a convenient basis for σ 's both of the above equations take the same form as

$$\mathbf{H} + \mu(s_1\mathbf{J}_{12} + s_2\mathbf{J}_{56} - 4s_1s_2\mathcal{R}_{1256}) = 0 \quad (4.49)$$

where s_1, s_2 are taking ± 1 values. Therefore the configuration of section 4.2.2 which has $\mathbf{H} = \mu(\mathbf{J}_{12} + \mathbf{J}_{56} + 4\mathcal{R}_{1256})$, satisfies (4.49) for $s_1 = s_2 = -1$ and is a 1/8 BPS configuration. It is notable that each couple of the four preserved supercharges are coming from one the $PSU(2|2)$ superalgebras. For the special case of $\mathbf{J}_{12} = \mathbf{J}_{56} = 0$ the above equation finds a new set of solutions for $s_1 = s_2 = +1$ and hence the preserved SUSY is doubled leading to a 1/4 BPS solution.

5 BPS States in the Dual SYM Theory

The TGMT is conjectured to be the DLCQ of type IIB string theory on the $AdS_5 \times S^5$ or the plane-wave backgrounds. As such, one expects the BPS states of the TGMT analyzed in the previous sections to have counterparts in the dual $\mathcal{N} = 4$ $U(N)$ SYM theory. In this section we construct *local* operators in the $\mathcal{N} = 4$ SYM which correspond to these BPS states.

The $\mathcal{N} = 4$, $D = 4$, $U(N)$ SYM theory is a superconformal field theory and has a large supergroup, $PSU(2, 2|4)$. All the gauge invariant operators of the gauge theory fall into various multiplets of unitary representations of this superconformal group which may be labelled by the quantum numbers of the bosonic subgroup: $SU(2, 2) \times SU(4) \simeq SO(4, 2) \times$

$SO(6)$ with

$$\begin{aligned} SU(2, 2) &\approx SO(4, 2) \supset SO(4) \times SO(2) \approx \overbrace{SU(2) \times SU(2)}^{(s_+, s_-)} \times \overbrace{U(1)}^{\Delta} \\ \underbrace{SU(4)}_{[r_1, r_2, r_3]} &\approx SO(6) \supset SO(4) \times SO(2) \approx \overbrace{SU(2) \times SU(2)}^{(t, u)} \times \overbrace{U(1)}^J \end{aligned} \quad (5.1)$$

In order to explicitly write the operators we need to recall the field content of the $U(N)$ SYM theory which consists of a spin one gauge field A_μ , four spin 1/2 Weyl fermion fields ψ_α^I and six spin zero scalar fields ϕ^i , all in the $N \times N$ representation of $U(N)$. Under $SU(4)$ R -symmetry, A_μ is a singlet, ψ is a **4**, and ϕ is a rank two anti-symmetric **6**. These fields naturally fall into an (ultra) short representation of $psu(2, 2|4)$.

Gauge invariant operators are obtained by summing over all $U(N)$ indices of any combination of the covariant derivative $D_\mu = \partial_\mu + i[A_\mu, \cdot]$, ψ_α^I and the three complex scalars X, Y, Z defined as

$$X = \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2), \quad Y = \frac{1}{\sqrt{2}}(\phi^3 + i\phi^4), \quad Z = \frac{1}{\sqrt{2}}(\phi^5 + i\phi^6). \quad (5.2)$$

Among these operators there are the BPS operators whose scaling dimension is protected and completely specified by their $SO(4) \subset SO(4, 2)$ representations and R -charges. Almost always people use Tr over a product of $N \times N$ matrices to form gauge invariant operators. For our purposes, where we are dealing with the brane-type states, the giant gravitons, the subdeterminant [27] or Schur polynomial bases are more appropriate [11]. Let us focus on the subdeterminant basis, generalization of the discussions to the other basis is straightforward.

5.1 Systematics of BPS operators

Independently of the Tr , subdeterminant or Schur polynomial bases, one can study and analyze the BPS operators just by their $SO(4, 2) \times SO(6)$ quantum numbers, as these are the groups appearing in the $psu(2, 2|4)$ superalgebra. The classification of these BPS operators has been extensively studied in the literature [28, 29, 30] but generically with some emphasis on the trace basis. As most of the arguments are basis independent, we will be very brief on that.

All physical operators of the $\mathcal{N} = 4$ SYM fall into unitary (or more generally unitarizable) multiplets of $psu(2, 2|4)$ superalgebra. There is, however, a one-to-one correspondence between superconformal chiral-primary operators and unitary superconformal multiplets; the superconformal chiral-primaries appear as the highest weight state of the superconformal BPS multiplet. Hence, the multiplets can be named after their highest weight state. *We should, however, stress that here we only analyze individual states/configurations and not*

the multiplets. The chiral-primary operators are only made out of scalar fields ϕ^i and hence completely specified with their $SU(4)$ R -symmetry representation.

Systematic analysis show that there are four distinct classes [29, 30]:

- $\Delta = r_1 + r_2 + r_3$
- $\Delta = \frac{3}{2}r_1 + r_2 + \frac{1}{2}r_3 \geq 2 + \frac{1}{2}r_1 + r_2 + \frac{3}{2}$
- $\Delta = \frac{1}{2}r_1 + r_2 + \frac{3}{2}r_3 \geq 2 + \frac{3}{2}r_1 + r_2 + \frac{1}{2}$
- $\Delta \geq \text{Max}[2 + \frac{1}{2}r_1 + r_2 + \frac{3}{2}r_3; 2 + \frac{3}{2}r_1 + r_2 + \frac{1}{2}]$

where r_i are the Dynkin labels of $SU(4)$ irreducible representation (*cf.* (5.1)). The first 3 cases correspond to discrete series of representation for which at least one of the supercharges commutes with the primary operator. These states are BPS states. A given BPS state of specific charges in general can be either in the highest weight representation of a BPS multiplet, or a descendent of another BPS state. The fourth case corresponds to continuous series of representations, for which no supercharges commute with the primary operator. They are referred to as non-BPS operators.

1/2 BPS operators

Half-BPS operators sit in $[0, k, 0]$, $k \geq 2$ (Dynkin label) representation of R -symmetry [29]. These chiral primaries are annihilated by half of the super-Poincare charges, Q 's, and appear as the highest weight state of a short (1/2 BPS) multiplet with spin ranging from 0 to 2. The chiral-primary states of R -charge k are totally symmetric traceless rank k tensors of $SU(4)$. In terms of the $\mathcal{N} = 4$ fields, the simplest such *local* operator is \mathcal{O}_k :

$$\mathcal{O}_k = \frac{1}{\sqrt{k!(N-k)!}} \mathcal{E}_{i_1 i_2 \dots i_k}^{j_1 j_2 \dots j_k} : Z_{i_1}^{j_1} Z_{i_2}^{j_2} \dots Z_{i_k}^{j_k} : \quad (5.3)$$

where

$$\mathcal{E}_{i_1 i_2 \dots i_k}^{j_1 j_2 \dots j_k} \equiv \epsilon_{i_1 i_2 \dots i_k i_{k+1} \dots i_N} \epsilon^{j_1 j_2 \dots j_k i_{k+1} \dots i_N}. \quad (5.4)$$

(5.3) corresponds to a giant graviton of radius $R^2 \sim k$. Note that the indices on Z_j^i are running from 1 to N and are $U(N)$ indices.

The most general half-BPS operators of R -charge J is a multi-subdeterminant operator of the form

$$: \mathcal{O}_{k_1}(x) \mathcal{O}_{k_2}(x) \dots \mathcal{O}_{k_n}(x) : , \quad J = \sum_{i=1}^n k_i . \quad (5.5)$$

The above is describing a multi-giant configuration consisting of n concentric giants, whose radii squared sum to J . As is evident all of these operators have $\Delta - J = 0$.

In order to compare to the TGMT it is more convenient to label them by $SO(4) \times U(1)_\Delta \times SU(2) \times SU(2) \times U(1)_J$ subgroup of $SO(4, 2) \times SO(6)$ and identify the light-cone Hamiltonian as [1, 4]

$$\mathbf{H} = \Delta - J.$$

The chiral primary operators which are singlets of $SO(4) \times SO(4)_R$ correspond to the zero energy half BPS solutions of the TGMT [9], see also section 3. In fact there is an exact one-to-one correspondence between the TGMT 1/2 BPS configurations and the $\mathcal{N} = 4$ local chiral-primary operators.

5.2 Less BPS operators

As discussed 1/2 BPS operators are those which are only made out of one kind of the complex scalars, e.g. Z . The 1/4 (and 1/8) BPS chiral operators are those which besides Z also involve Y (and X, Y). Here is a more systematic analysis.

5.2.1 1/4 BPS operators

Quarter-BPS operators are the next simplest, killed by four super-Poincare charges. 1/4 BPS chiral operators sit in Dynkin label $[l, J - l, l]$, $J \geq l \geq 2$ representation of the R -symmetry. These states can appear either as descendent of a chiral primary or as the highest weight of a 1/4 BPS multiplet in which spin of the states ranges from 0 to 3. The latter is only possible if we have multi (at least two) trace/subdeterminant operator [30].

For example operators of the form

$$\hat{\mathcal{O}}_{J,l} = \mathcal{E}_{i_1 i_2 \dots i_{J+l}}^{j_1 j_2 \dots j_{J+l}} : X_{i_1}^{j_1} X_{i_2}^{j_2} \dots X_{i_l}^{j_l} Z_{i_{l+1}}^{j_{l+1}} \dots Z_{i_{J+l}}^{j_{J+l}} : \quad (5.6)$$

correspond to a deformed single giant, with $\mathbf{J}_{34} = 0$ and $\mathbf{J}_{12} \propto l$. These operators have $\mathbf{H} = \Delta - J = l$. For $l = 0$ these operators reduce to the chiral primaries of (5.3). Operators of this kind has been considered in [31]. One can consider multi-subdeterminant operators. These are of the form

$$: \hat{\mathcal{O}}_{J_1, l_1}(x) \hat{\mathcal{O}}_{J_2, l_2}(x) \dots \hat{\mathcal{O}}_{J_n, l_n}(x) : , \quad \sum_{i=1}^n J_i = J, \quad \sum_{i=1}^n l_i = l,$$

which is of the form of n concentric deformed spherical giants. Obviously some of the l_i 's can be zero.

There is another class of 1/4 BPS states which is obtained by insertion of the covariant derivative, instead of X into the sequence of Z 's in the 1/2 BPS operator (5.3). In order to obtain a 1/4 BPS states, however, we should insert a self-dual gauge field strength (*cf.* discussions in the end of section 4.1.2).

5.2.2 1/8 BPS operators

The last family of chiral operators are 1/8 BPS operators sit in $[l-m, J-l, l+m]$, $l \geq m \geq 2$ representation of R -symmetry and are annihilated by two supercharges. These operators are constructed from J number of Z 's, l number of X 's and m of Y 's, e.g.

$$\tilde{\mathcal{O}}_{J,l,m} = \mathcal{E}_{i_1 i_2 \dots i_{J+l+m}}^{j_1 j_2 \dots j_{J+l+m}} : X_{i_1}^{j_1} X_{i_2}^{j_2} \dots X_{i_l}^{j_l} Y_{i_{l+1}}^{j_{l+1}} \dots Y_{i_{l+m}}^{j_{l+m}} Z_{i_{l+m+1}}^{j_{l+m+1}} \dots Z_{i_{J+l+m}}^{j_{J+l+m}} : \quad (5.7)$$

or one may have insertions of X 's or Y 's (and not both) together with insertion of a self-dual $F_{\mu\nu}$. In the TGMT, although possible, we did not specify configurations corresponding to the latter case. $\tilde{\mathcal{O}}_{J,l,m}$ operators correspond to states with numbers $\mathbf{J}_{12} \propto l$ and $\mathbf{J}_{34} \propto m$ and $\mathbf{H} = \Delta - J = l + m$, in the matrix theory side. These operators are invariant under $SO(4) \times U(1) \times U(1)$. Similarly to the previous cases we can construct multi-subdeterminants, corresponding to multi giant states.

In section 4.2.1 we analyzed hyperbolic configurations. The gauge theory operators corresponding to this configuration can be obtained from operators of the form (5.7) in the appropriate $J, N \rightarrow \infty$ limit, keeping μp^+ fixed, that is the BMN limit [3, 4]. One should also scale l and m such that $l/N, m/N$ remain finite. Intuitively, one would expect that for large angular momenta the deformation of the rotating three brane from the spherical shape is so large that it deforms to a hyperboloid shape brane.

6 Discussion and Outlook

In this paper we continued analysis of the Tiny Graviton Matrix Theory (TGMT) by classifying 1/4 and 1/8 BPS states of the Matrix theory. These are generically of the form of deformed three sphere giant gravitons or rotating spherical branes. We then compared these configurations with the states in the dual $\mathcal{N} = 4$ $U(N)$ SYM theory and explicitly constructed the *local* operator corresponding to each configuration. In this way we provided further evidence in support of the TGMT conjecture.

In the $\mathcal{N} = 4$ SYM theory, however, one can have BPS *non-local* operators. For example, there are 1/2 BPS Wilson lines [32]. The supergravity solutions dual to such operators have also been constructed [33]. It is interesting to find the description of these non-local operators in the TGMT language. Presumably such a configuration, which should have a manifest $SO(5) \times SO(3)$ isometry [33], could be obtained from turning on the X^i 's and one of the X^a 's, say X^5 , as $X^i = \mathcal{J}^i$ and $X^5 = \mathcal{L}_5$. In this way we have the desired $SO(5) \times SO(3)$ symmetry. The detailed analysis of this 1/2 BPS configuration is postponed to an upcoming work [34].

We did not present an explicit gauge theory description for the configurations of section 4.2.2 which involves \mathcal{R}_{ijab} , and cannot be obtained along the ideas discussed here in section 5, *i.e.* by insertion of other fields into sequence of chiral primaries. As they involve turning on a central extension in the extended $psu(2, 2|4)$ superalgebra [17] one would expect that these operators should also correspond to non-local gauge theory operators. Identifying these operators, and operators corresponding to the other possible (central) extensions of the $psu(2, 2|4)$ is another interesting direction to pursue.

Here we only studied transverse three branes, mainly those which are topologically spherical ones. As another interesting direction for further analysis, one can study BPS configurations corresponding to longitudinal three branes or longitudinal D-strings, or D5-branes on the plane-wave background analyzed in [35] or branes which are of the form of fuzzy tubes or brane on surfaces of higher genus topologies, such as the ones constructed in [36].

Finally all the configurations we discussed at the level of the TGMT are “classical” ones, in the sense that the entries of our $J \times J$ matrices are c -numbers, rather than operators. In order to see quantization of \mathbf{J}_{12} , \mathbf{J}_{34} and \mathcal{R}_{ijab} , which is required for matching to the $\mathcal{N} = 4$ SYM analysis, one needs to perform quantization. This requires a the systematic analysis of the Tiny Graviton Matrix Perturbation Theory. This work, which parallels a similar analysis on the BMN matrix model [6], was started in [1] and is in need of a thorough study [23].

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A Conventions and Useful Identities

The four brackets are not as familiar objects as the usual commutators, and in fact it is much more involved to work with them. In this appendix we gather some useful identities for handling and carrying out four bracket manipulations.

Let us first start with the definition of the four bracket:

$$\begin{aligned}
 [A, B, C, D] \equiv & \frac{1}{24} \left([A, B][C, D] + [C, D][A, B] \right. \\
 & - [A, C][B, D] - [B, D][A, C] \\
 & \left. + [A, D][B, C] + [B, C][A, D] \right).
 \end{aligned}
 \tag{A.1}$$

From the above definition one can work out the generalized ‘‘Jacobi’’ identity:

$$\begin{aligned}
[A, B, C, [D, E]] &= [[A, B, C, D], E] + [D, [A, B, C, E]] \\
&\quad - [[A, B], C, D, E] - [[C, A], B, D, E] - [[B, C], A, D, E].
\end{aligned}
\tag{A.2}$$

As a useful special case when $E = C$, we obtain

$$[[A, B, C, D], D] = [[A, D], B, C, D] + [A, [B, D], C, D] + [A, B, [C, D], D]. \tag{A.3}$$

Trace of a four bracket is zero,

$$\text{Tr} ([A, B, C, D]) = 0 \tag{A.4}$$

and under the trace we have the ‘‘by-part integration’’ property, that is

$$\text{Tr} ([A, B, C, D]E) = -\text{Tr} ([A, B, C, E]D). \tag{A.5}$$

One can find, through representation theory of $SO(4)$ (for details see [9]), four \mathcal{J}^i matrices such that

$$[\mathcal{J}^i, \mathcal{J}^j, \mathcal{J}^k, \mathcal{L}_5] = -\epsilon^{ijkl} \mathcal{J}^l \tag{A.6}$$

\mathcal{J}^i and \mathcal{L}_5 have the following explicit matrix form:

$$\mathcal{J}^i = \left(\begin{array}{c|c} 0 & \Sigma^i \\ \hline \bar{\Sigma}^i & 0 \end{array} \right), \quad \mathcal{L}_5 = \left(\begin{array}{c|c} \mathbf{1} & 0 \\ \hline 0 & -\mathbf{1} \end{array} \right), \tag{A.7}$$

where $\bar{\Sigma}^i = (\Sigma^i)^\dagger$ and the blocks are $J/2 \times J/2$ matrices. These \mathcal{J}^i 's and Σ^i 's are generalizations of the usual Dirac gamma matrices and the σ^i matrices, respectively. \mathcal{L}_5 is a direct generalization of the chirality matrix γ^5 . It is evident from the above form that \mathcal{L}_5 *anti-commutes* with \mathcal{J}^i 's, *i.e.*

$$\{\mathcal{J}^i, \mathcal{L}_5\} = 0 \tag{A.8}$$

Therefore,

$$[\mathcal{J}^i, \mathcal{J}^j \mathcal{L}_5] = \{\mathcal{J}^i, \mathcal{J}^j\} \mathcal{L}_5, \quad [\mathcal{J}^i, \mathcal{L}_5] = -2\mathcal{L}_5 \mathcal{J}^i,$$

moreover, \mathcal{L}_5 commutes (anti-commutes) with any product of even (odd) number of \mathcal{J}^i 's.

Using \mathcal{J}^i one can obtain a $J \times J$ representation of the $so(4)$ algebra whose generators are $\mathcal{J}^{ij} \equiv [\mathcal{J}^i, \mathcal{J}^j]$. In the explicit matrix form

$$\mathcal{J}^{ij} = \left(\begin{array}{c|c} \Sigma^{ij} & 0 \\ \hline 0 & \bar{\Sigma}^{ij} \end{array} \right) \tag{A.9}$$

where $\Sigma^{ij} = \Sigma^i \bar{\Sigma}^j - \Sigma^j \bar{\Sigma}^i$ and $\Sigma^{ij} = \Sigma^i \bar{\Sigma}^j - \Sigma^j \bar{\Sigma}^i$. It is obvious that $[\mathcal{J}^{ij}, \mathcal{L}_5] = 0$. The Σ^i 's are chosen in such a way that

$$[\mathcal{J}^{ij}, \mathcal{J}^k] = 4i(\delta^{jk} \mathcal{J}^i - \delta^{ik} \mathcal{J}^j) \quad (\text{A.10})$$

Using the above identities, and with some (perhaps tedious) algebra one can show that

$$[\mathcal{J}^i, \mathcal{J}^j, i\mathcal{L}_5 \mathcal{J}^k, \mathcal{L}_5] = -\epsilon^{ijkl} i\mathcal{L}_5 \mathcal{J}^l + \frac{i}{3!} (\mathcal{J}^j \mathcal{J}^k \mathcal{J}^i - \mathcal{J}^i \mathcal{J}^k \mathcal{J}^j) \quad (\text{A.11a})$$

$$[\mathcal{J}^i, i\mathcal{L}_5 \mathcal{J}^j, i\mathcal{L}_5 \mathcal{J}^k, \mathcal{L}_5] = -\epsilon^{ijkl} \mathcal{J}^l + \frac{1}{3!} \mathcal{L}_5 (\mathcal{J}^k \mathcal{J}^i \mathcal{J}^j - \mathcal{J}^j \mathcal{J}^i \mathcal{J}^k) \quad (\text{A.11b})$$

$$[i\mathcal{L}_5 \mathcal{J}^i, i\mathcal{L}_5 \mathcal{J}^j, i\mathcal{L}_5 \mathcal{J}^k, \mathcal{L}_5] = -\epsilon^{ijkl} i\mathcal{L}_5 \mathcal{J}^l. \quad (\text{A.11c})$$

For studying the 1/4 BPS configurations it is useful to introduce $\mathcal{Z}_0, \mathcal{W}_0$ matrices

$$\begin{aligned} \mathcal{Z}_0 &= \frac{1}{\sqrt{2}} (\mathcal{J}^1 + i\mathcal{J}^2) \\ \mathcal{W}_0 &= \frac{1}{\sqrt{2}} (\mathcal{J}^3 + i\mathcal{J}^4) \end{aligned} \quad (\text{A.12})$$

In terms of these matrices the $SU(2) \times U(1)$ part of $SO(4)$ is manifest. (In the sense that the $U(1)$ is rotating both $\mathcal{Z}_0, \mathcal{W}_0$ with the same phase, while under the $SU(2)$, $(\mathcal{Z}_0, \mathcal{W}_0)$ rotate as a doublet.) The $\mathcal{J}^{12}, \mathcal{J}^{34}$ generators then become

$$\mathcal{J}^{12} = i[\mathcal{Z}_0, \bar{\mathcal{Z}}_0], \quad \mathcal{J}^{34} = i[\mathcal{W}_0, \bar{\mathcal{W}}_0].$$

and hence

$$[[\mathcal{Z}_0, \bar{\mathcal{Z}}_0], \mathcal{Z}_0] = 4\mathcal{Z}_0, \quad [[\mathcal{Z}_0, \bar{\mathcal{Z}}_0], \mathcal{W}_0] = 0 \quad (\text{A.13})$$

In terms of $\mathcal{Z}_0, \mathcal{W}_0$,

$$[\mathcal{Z}_0, \mathcal{W}_0, \bar{\mathcal{W}}_0, \mathcal{L}_5] = \mathcal{Z}_0 \quad (\text{A.14a})$$

$$[\bar{\mathcal{Z}}_0, \bar{\mathcal{W}}_0, \mathcal{W}_0, \mathcal{L}_5] = \bar{\mathcal{Z}}_0 \quad (\text{A.14b})$$

$$[\mathcal{W}_0, \mathcal{Z}_0, \bar{\mathcal{Z}}_0, \mathcal{L}_5] = \mathcal{W}_0 \quad (\text{A.14c})$$

$$[\bar{\mathcal{W}}_0, \bar{\mathcal{Z}}_0, \mathcal{Z}_0, \mathcal{L}_5] = \bar{\mathcal{W}}_0 \quad (\text{A.14d})$$

$$[i\mathcal{L}_5 \mathcal{Z}_0, i\mathcal{L}_5 \mathcal{W}_0, i\mathcal{L}_5 \bar{\mathcal{W}}_0, \mathcal{L}_5] = i\mathcal{L}_5 \mathcal{Z}_0 \quad (\text{A.15})$$

and

$$[\mathcal{Z}_0, \mathcal{W}_0, i\mathcal{L}_5 \bar{\mathcal{W}}_0, \mathcal{L}_5] = [\mathcal{W}_0, i\mathcal{L}_5 \bar{\mathcal{W}}_0, \mathcal{Z}_0, \mathcal{L}_5] = i\mathcal{L}_5 \mathcal{Z}_0 - \frac{i}{6} [\mathcal{Z}_0, \{\mathcal{W}_0, \bar{\mathcal{W}}_0\}] \quad (\text{A.16a})$$

$$[\mathcal{Z}_0, i\mathcal{L}_5 \mathcal{W}_0, \bar{\mathcal{W}}_0, \mathcal{L}_5] = [i\mathcal{L}_5 \mathcal{W}_0, \bar{\mathcal{W}}_0, \mathcal{Z}_0, \mathcal{L}_5] = i\mathcal{L}_5 \mathcal{Z}_0 + \frac{i}{6} [\mathcal{Z}_0, \{\mathcal{W}_0, \bar{\mathcal{W}}_0\}] \quad (\text{A.16b})$$

$$[i\mathcal{L}_5 \mathcal{Z}_0, \mathcal{W}_0, \bar{\mathcal{W}}_0, \mathcal{L}_5] = [\mathcal{W}_0, \bar{\mathcal{W}}_0, i\mathcal{L}_5 \mathcal{Z}_0, \mathcal{L}_5] = i\mathcal{L}_5 \mathcal{Z}_0 - \frac{i}{3} (\mathcal{W}_0 \mathcal{Z}_0 \bar{\mathcal{W}}_0 - \bar{\mathcal{W}}_0 \mathcal{Z}_0 \mathcal{W}_0) \quad (\text{A.16c})$$

$$[i\mathcal{L}_5\mathcal{Z}_0, i\mathcal{L}_5\mathcal{W}_0, \bar{\mathcal{W}}_0, \mathcal{L}_5] = [i\mathcal{L}_5\mathcal{W}_0, \bar{\mathcal{W}}_0, i\mathcal{L}_5\mathcal{Z}_0, \mathcal{L}_5] = \mathcal{Z}_0 - \frac{1}{6}\mathcal{L}_5[\mathcal{Z}_0, \{\mathcal{W}_0, \bar{\mathcal{W}}_0\}] \quad (\text{A.17a})$$

$$[i\mathcal{L}_5\mathcal{Z}_0, \mathcal{W}_0, i\mathcal{L}_5\bar{\mathcal{W}}_0, \mathcal{L}_5] = [\mathcal{W}_0, i\mathcal{L}_5\bar{\mathcal{W}}_0, i\mathcal{L}_5\mathcal{Z}_0, \mathcal{L}_5] = \mathcal{Z}_0 + \frac{1}{6}\mathcal{L}_5[\mathcal{Z}_0, \{\mathcal{W}_0, \bar{\mathcal{W}}_0\}] \quad (\text{A.17b})$$

$$[\mathcal{Z}_0, i\mathcal{L}_5\mathcal{W}_0, i\mathcal{L}_5\bar{\mathcal{W}}_0, \mathcal{L}_5] = [i\mathcal{L}_5\mathcal{W}_0, i\mathcal{L}_5\bar{\mathcal{W}}_0, \mathcal{Z}_0, \mathcal{L}_5] = \mathcal{Z}_0 - \frac{1}{3}\mathcal{L}_5(\mathcal{W}_0\mathcal{Z}_0\bar{\mathcal{W}}_0 - \bar{\mathcal{W}}_0\mathcal{Z}_0\mathcal{W}_0) \quad (\text{A.17c})$$

B SUSY Generators in Terms of Matrices

Here we present the generators of the TGMT superalgebra $PSU(2|2) \times PSU(2|2) \times U(1)$ in the Matrix realization.

$$P^+ = -P_- = \frac{1}{R_-} \text{Tr } \mathbf{1} \quad , \quad P^- = -P_+ = -\mathbf{H} \quad (\text{B.1})$$

$$\mathbf{J}_{ij} = \text{Tr} (X^i\Pi^j - X^j\Pi^i - 2\theta^{\dagger\alpha\beta}(i\sigma^{ij})_{\alpha}^{\rho}\theta_{\rho\beta} + 2\theta^{\dagger\dot{\alpha}\dot{\beta}}(i\sigma^{ij})_{\dot{\alpha}}^{\dot{\rho}}\theta_{\dot{\rho}\dot{\beta}}) \quad (\text{B.2})$$

$$\mathbf{J}_{ab} = \text{Tr} (X^a\Pi^b - X^b\Pi^a - 2\theta^{\dagger\alpha\beta}(i\sigma^{ab})_{\beta}^{\rho}\theta_{\alpha\rho} + 2\theta^{\dagger\dot{\alpha}\dot{\beta}}(i\sigma^{ab})_{\dot{\beta}}^{\dot{\rho}}\theta_{\dot{\alpha}\dot{\rho}}) \quad (\text{B.3})$$

$$\begin{aligned} \mathbf{H} = R_- \text{Tr} & \left[\frac{1}{2}(P_i^2 + P_a^2) + \frac{1}{2} \left(\frac{\mu}{R_-} \right)^2 (X_i^2 + X_a^2) \right. \\ & + \frac{1}{2 \cdot 3!g_s^2} ([X^i, X^j, X^k, \mathcal{L}_5][X^i, X^j, X^k, \mathcal{L}_5] + [X^a, X^b, X^c, \mathcal{L}_5][X^a, X^b, X^c, \mathcal{L}_5]) \\ & + \frac{1}{2 \cdot 2g_s^2} ([X^i, X^j, X^a, \mathcal{L}_5][X^i, X^j, X^a, \mathcal{L}_5] + [X^a, X^b, X^i, \mathcal{L}_5][X^a, X^b, X^i, \mathcal{L}_5]) \\ & - \frac{\mu}{3!R_-g_s} (\epsilon^{ijkl} X^i[X^j, X^k, X^l, \mathcal{L}_5] + \epsilon^{abcd} X^a[X^b, X^c, X^d, \mathcal{L}_5]) \\ & + \left(\frac{\mu}{R_-} \right) (\theta^{\dagger\alpha\beta}\theta_{\alpha\beta} - \theta_{\dot{\alpha}\dot{\beta}}\theta^{\dagger\dot{\alpha}\dot{\beta}}) \\ & + \frac{1}{2g_s} (\theta^{\dagger\alpha\beta}(\sigma^{ij})_{\alpha}^{\delta}[X^i, X^j, \theta_{\delta\beta}, \mathcal{L}_5] + \theta^{\dagger\alpha\beta}(\sigma^{ab})_{\alpha}^{\delta}[X^a, X^b, \theta_{\delta\beta}, \mathcal{L}_5]) \\ & \left. + \frac{1}{2g_s} (\theta^{\dagger\dot{\alpha}\dot{\beta}}(\sigma^{ij})_{\dot{\alpha}}^{\dot{\delta}}[X^i, X^j, \theta_{\dot{\delta}\dot{\beta}}, \mathcal{L}_5] + \theta^{\dagger\dot{\alpha}\dot{\beta}}(\sigma^{ab})_{\dot{\alpha}}^{\dot{\delta}}[X^a, X^b, \theta_{\dot{\delta}\dot{\beta}}, \mathcal{L}_5]) \right] \quad (\text{B.4}) \end{aligned}$$

$$\begin{aligned}
Q_{\dot{\alpha}\dot{\beta}} = & \sqrt{\frac{R_-}{2}} \text{Tr} \left[(\Pi^i - \frac{i\mu}{R_-} X^i) (\sigma^i)_{\dot{\alpha}}^{\rho} \theta_{\rho\dot{\beta}} + (\Pi^a - \frac{i\mu}{R_-} X^a) (\sigma^a)_{\dot{\beta}}^{\dot{\rho}} \theta_{\dot{\alpha}\dot{\rho}} \right. \\
& - \frac{i}{3!g_s} (\epsilon^{ijkl} [X^i, X^j, X^k, \mathcal{L}_5] (\sigma^l)_{\dot{\alpha}}^{\rho} \theta_{\rho\dot{\beta}} + \epsilon^{abcd} [X^a, X^b, X^c, \mathcal{L}_5] (\sigma^d)_{\dot{\beta}}^{\dot{\rho}} \theta_{\dot{\alpha}\dot{\rho}}) \\
& \left. + \frac{1}{2g_s} ([X^i, X^a, X^b, \mathcal{L}_5] (\sigma^i)_{\dot{\alpha}}^{\rho} (i\sigma^{ab})_{\dot{\beta}}^{\gamma} \theta_{\rho\gamma} + [X^a, X^i, X^j, \mathcal{L}_5] (\sigma^a)_{\dot{\beta}}^{\dot{\gamma}} (i\sigma^{ij})_{\dot{\alpha}}^{\dot{\rho}} \theta_{\dot{\rho}\dot{\gamma}}) \right]
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
Q_{\alpha\beta} = & \sqrt{\frac{R_-}{2}} \text{Tr} \left[(\Pi^i - \frac{i\mu}{R_-} X^i) (\sigma^i)_{\alpha}^{\dot{\rho}} \theta_{\dot{\rho}\beta} + (\Pi^a - \frac{i\mu}{R_-} X^a) (\sigma^a)_{\beta}^{\rho} \theta_{\alpha\rho} \right. \\
& - \frac{i}{3!g_s} (\epsilon^{ijkl} [X^i, X^j, X^k, \mathcal{L}_5] (\sigma^l)_{\alpha}^{\dot{\rho}} \theta_{\dot{\rho}\beta} + \epsilon^{abcd} [X^a, X^b, X^c, \mathcal{L}_5] (\sigma^d)_{\beta}^{\rho} \theta_{\alpha\rho}) \\
& \left. + \frac{1}{2g_s} ([X^i, X^a, X^b, \mathcal{L}_5] (\sigma^i)_{\alpha}^{\dot{\rho}} (i\sigma^{ab})_{\beta}^{\dot{\gamma}} \theta_{\dot{\rho}\dot{\gamma}} + [X^a, X^i, X^j, \mathcal{L}_5] (\sigma^a)_{\beta}^{\dot{\gamma}} (i\sigma^{ij})_{\alpha}^{\rho} \theta_{\rho\dot{\gamma}}) \right]
\end{aligned} \tag{B.6}$$

$$\mathcal{R}^{ijab} = \frac{1}{g_s} \text{Tr} ([X^i, X^j, X^a, X^b] \mathcal{L}_5) \tag{B.7}$$

$$\begin{aligned}
C^{ia} = & \frac{R_-}{\mu} \text{Tr} \left[P^i P^a - \left(\frac{1}{2g_s} \right)^2 \epsilon^{abcd} \epsilon^{ijkl} [X^j, X^b, X^c, \mathcal{L}_5] [X^d, X^k, X^l, \mathcal{L}_5] \right. \\
& \left. + \left(\frac{\mu}{R_-} X^i + \frac{1}{3!g_s} \epsilon^{ijkl} [X^j, X^k, X^l, \mathcal{L}_5] \right) \left(\frac{\mu}{R_-} X^a + \frac{1}{3!g_s} \epsilon^{abcd} [X^b, X^c, X^d, \mathcal{L}_5] \right) \right]
\end{aligned} \tag{B.8}$$

$$\hat{C}^{ia} = \frac{R_-}{\mu} \frac{1}{2g_s} \text{Tr} \left(\epsilon^{ijkl} P^j [X^a, X^k, X^l, \mathcal{L}_5] + \epsilon^{abcd} P^c [X^i, X^c, X^d, \mathcal{L}_5] \right) \tag{B.9}$$

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